Jet transport coefficient $\hat{q}$ in lattice QCD

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Outline

- ❑ Phenomenology based extraction of transport coefficient $\hat{q}$ in heavy-ion collisions

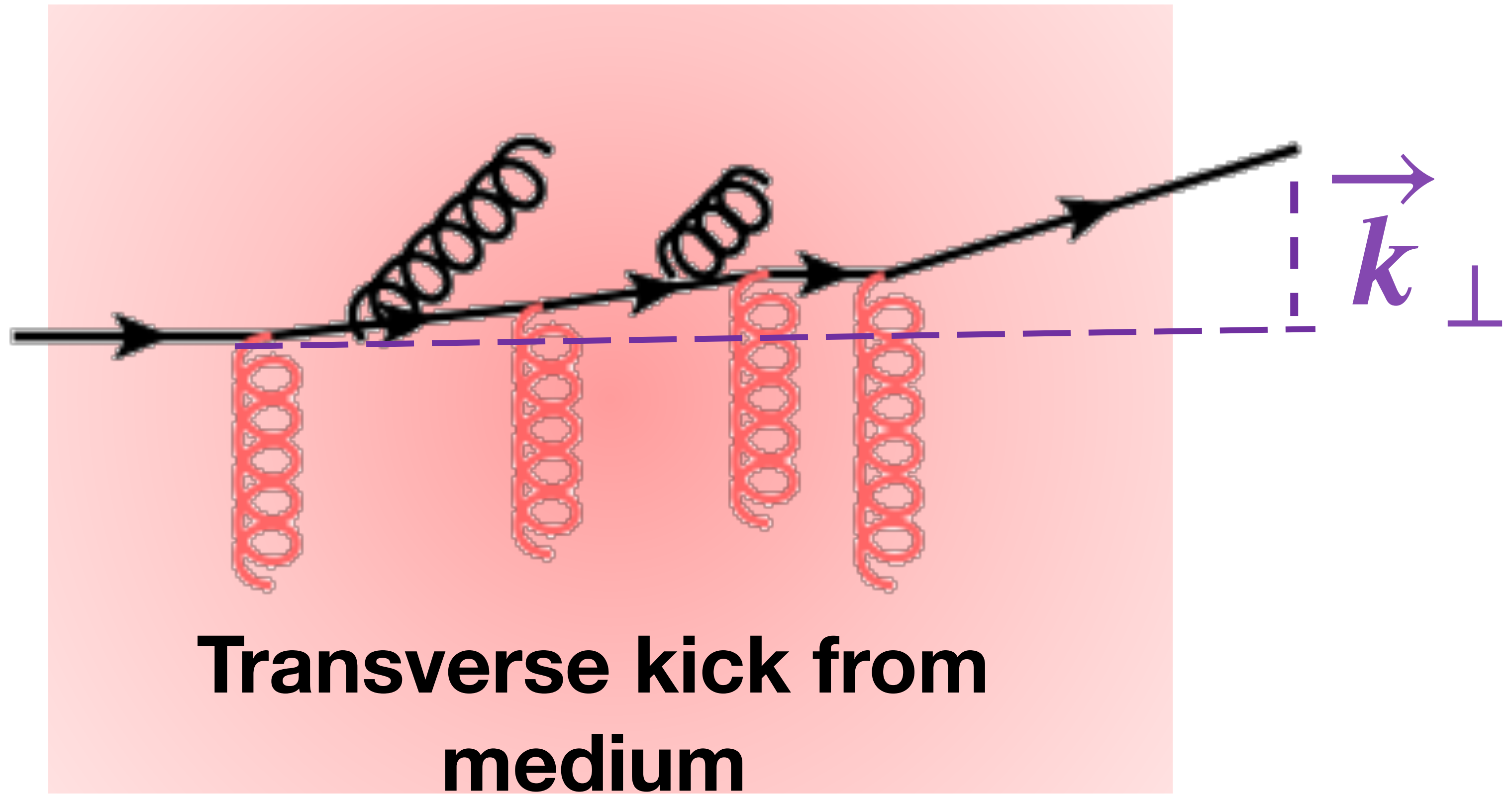
- ❑ Formulating $\hat{q}$ for hot QGP using Lattice gauge theory

1) Express $\hat{q}$ as a series of local operators using dispersion relation
2) Computing operators on quenched SU(3) lattices
3) Computing operators on unquenched SU(3) lattices

- ❑ Estimates of $\hat{q}$ for pure gluon plasma and 2+1 flavor QGP

First systematic extraction of $\hat{q}$ based on phenomenology

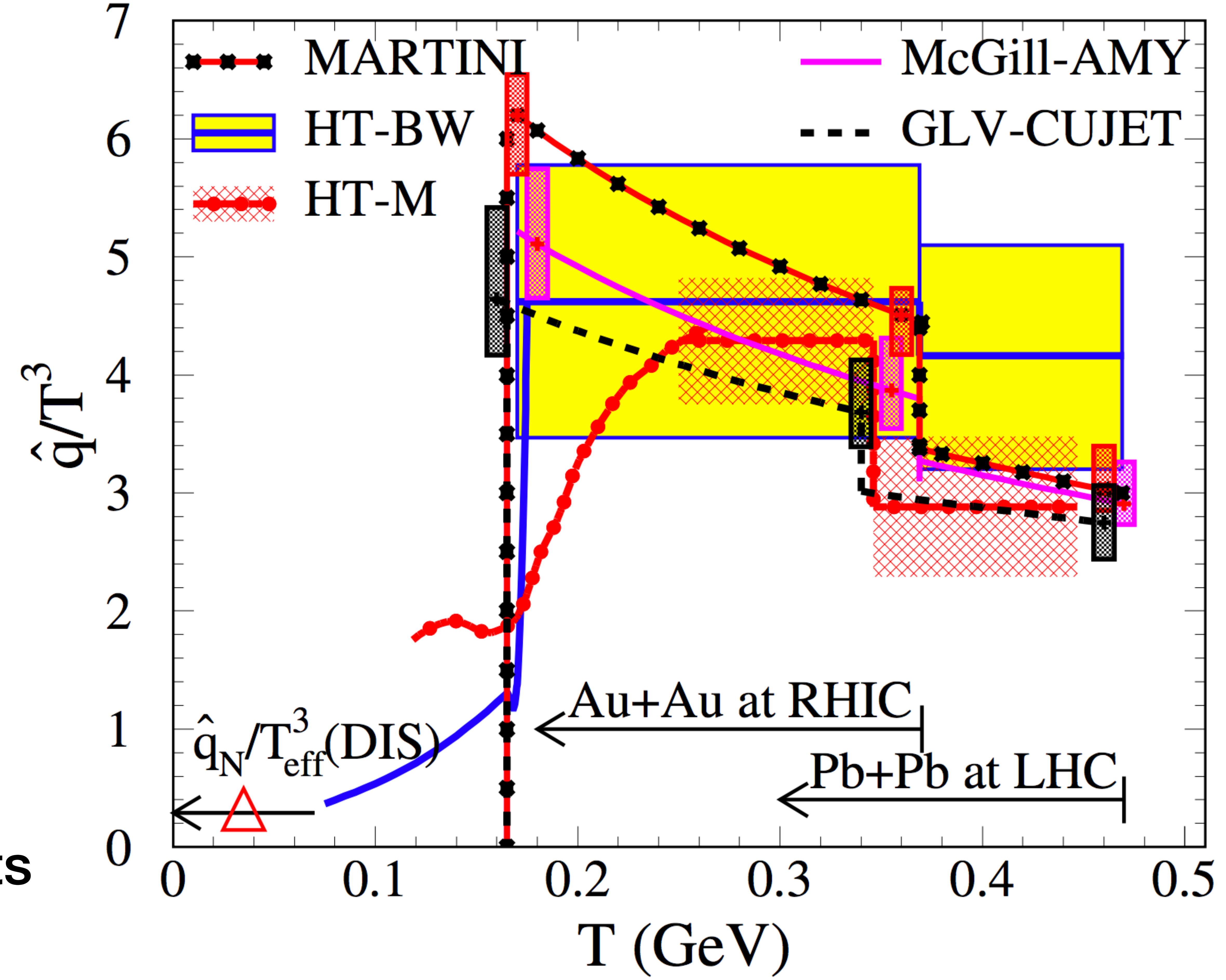
Leading parton going through medium



$$\hat{q}(\vec{r},t) = \frac{<\vec{k}_\perp^2>}{L}$$

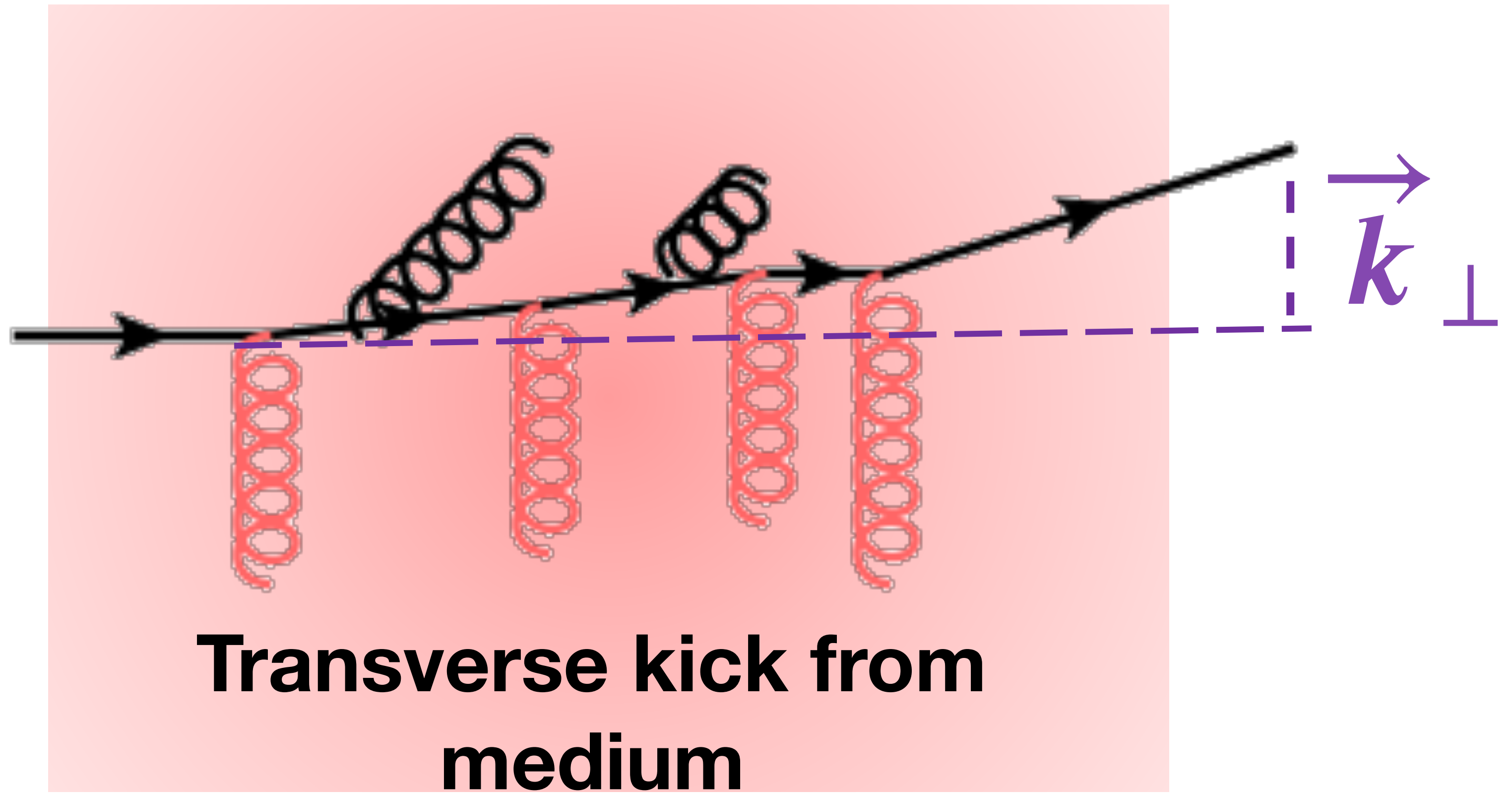
- $\dfrac{\hat{q}}{T^3} \sim$ 3-6 , Decreases as T increases
- Based on fit to hadron-R_{AA} measurements
- Based on single energy-loss scheme

JET Collaboration(Burke et al. 2014)



First Bayesian extraction of $\hat{q}$ based on phenomenology

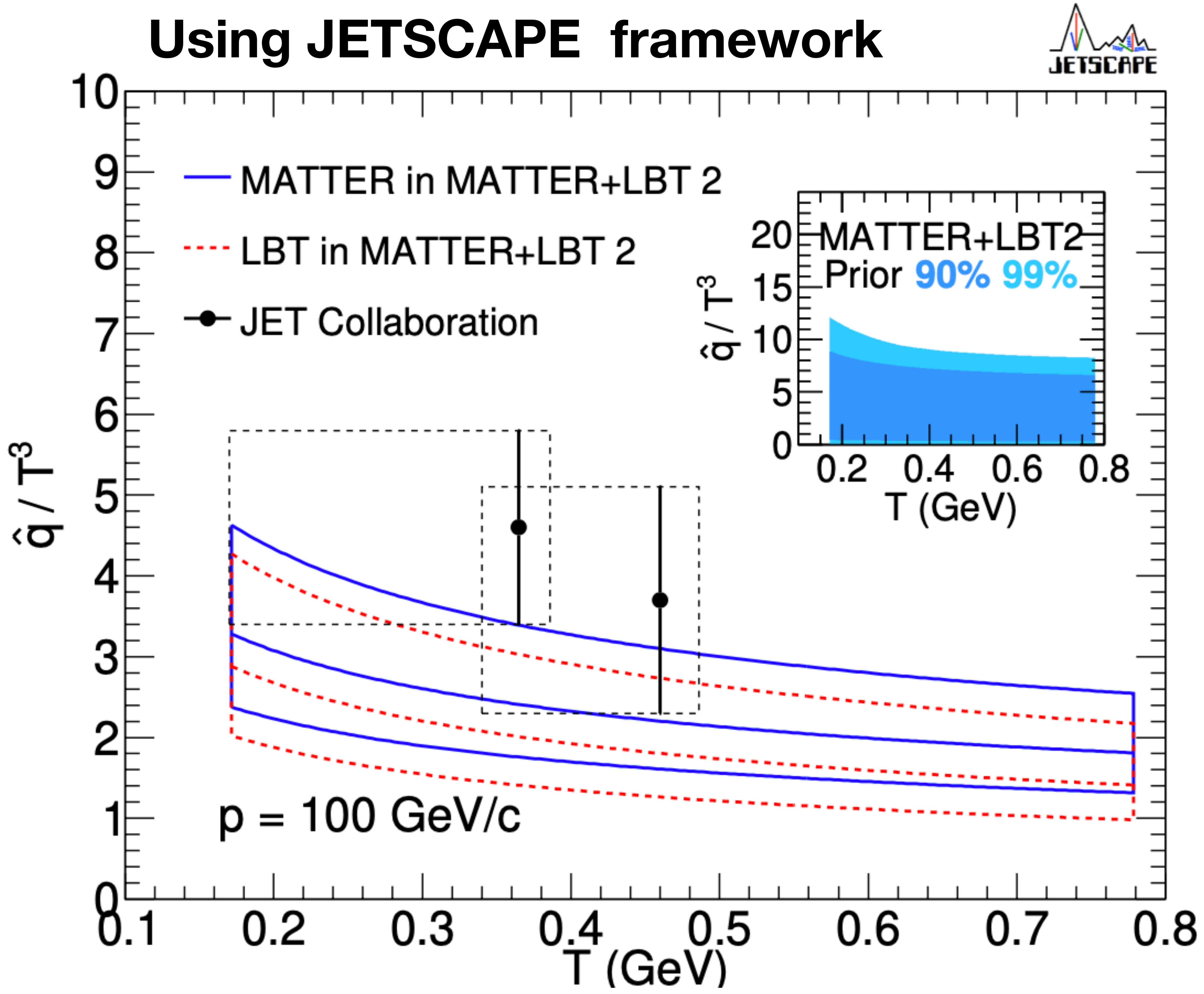
Leading parton going through medium



$$\hat{q}(\vec{r},t) = \frac{<\vec{k}_\perp^2>}{L}$$

- ❑ $\frac{\hat{q}}{T^3} \sim$ 1-4 , Decreases as T increases
- ❑ Based on fit to hadron-R_{AA} measurements
- ❑ Based on Multi-stage energy-loss scheme

Using JETSCAPE framework



JETSCAPE Coll. (S. Cao et al. PRC 104, 024905, 2021)

Lattice formulation of $\hat{q}$

A. Majumder, PRC 87, 034905 (2013)

- **Leading order (LO) process:** A high energy quark propagating (along -ve z-dir) through plasma

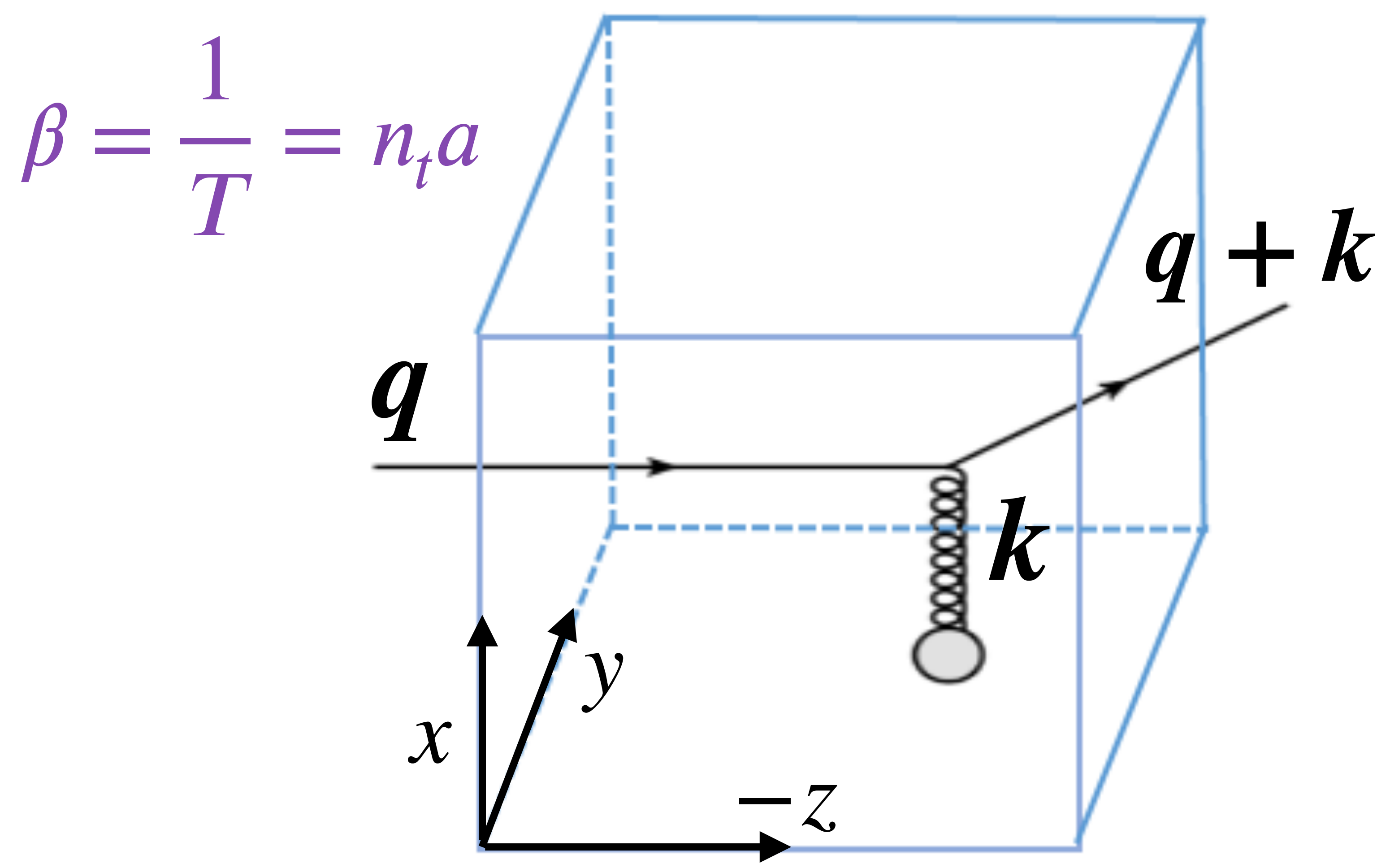
Quark momentum: $q = (\mu^2/2q^-, q^-, 0) \sim (\lambda^2, 1, 0)q^-$;

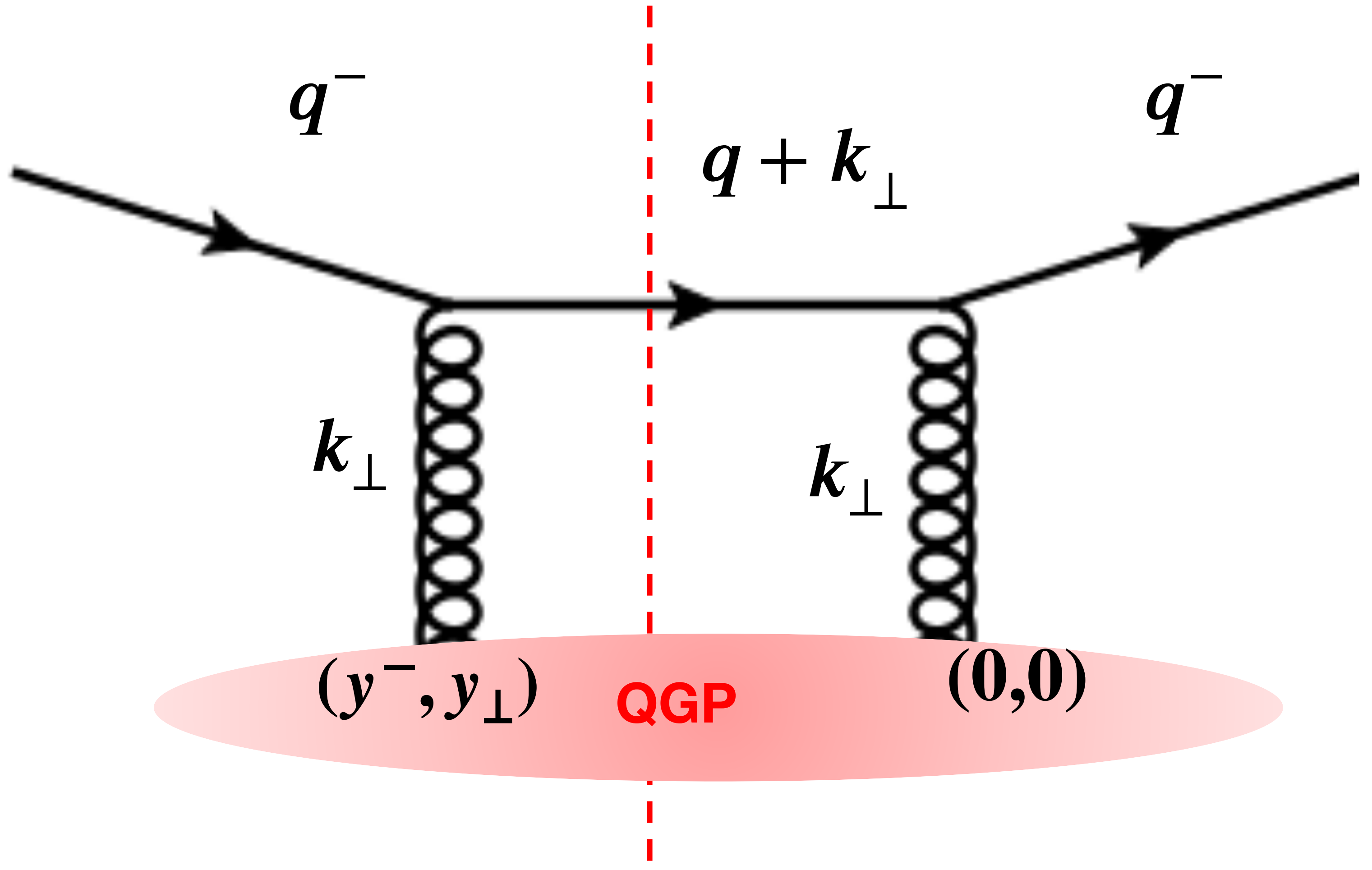
where $\lambda << 1$; $\quad q^- = (q^0 - q^3)/\sqrt{2} \equiv$ hard scale

Transverse gluon: $k = (k^+, k^-, k_\perp) \sim (\lambda^2, \lambda^2, \lambda)q^-$

$$\hat{q} = \sum_k k_\perp^2 \frac{Disc\left[\mathscr{W}(k)\right]}{L^0};$$

$\mathscr{W}(k)$: scattering amplitude





$$\hat{q} = \frac{\alpha_s}{N_c} \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} d^2 k_\perp e^{-i\frac{k_\perp^2}{2q^-}y^- + i\vec{k}_\perp \vec{y}_\perp}$$
$$\times \langle M | \mathrm{Tr}[F^{+\perp_\mu}(y^-, y_\perp) F^{+}_{\perp_\mu}(0)] | M \rangle$$

(Lattice QCD)

Non-perturbative part

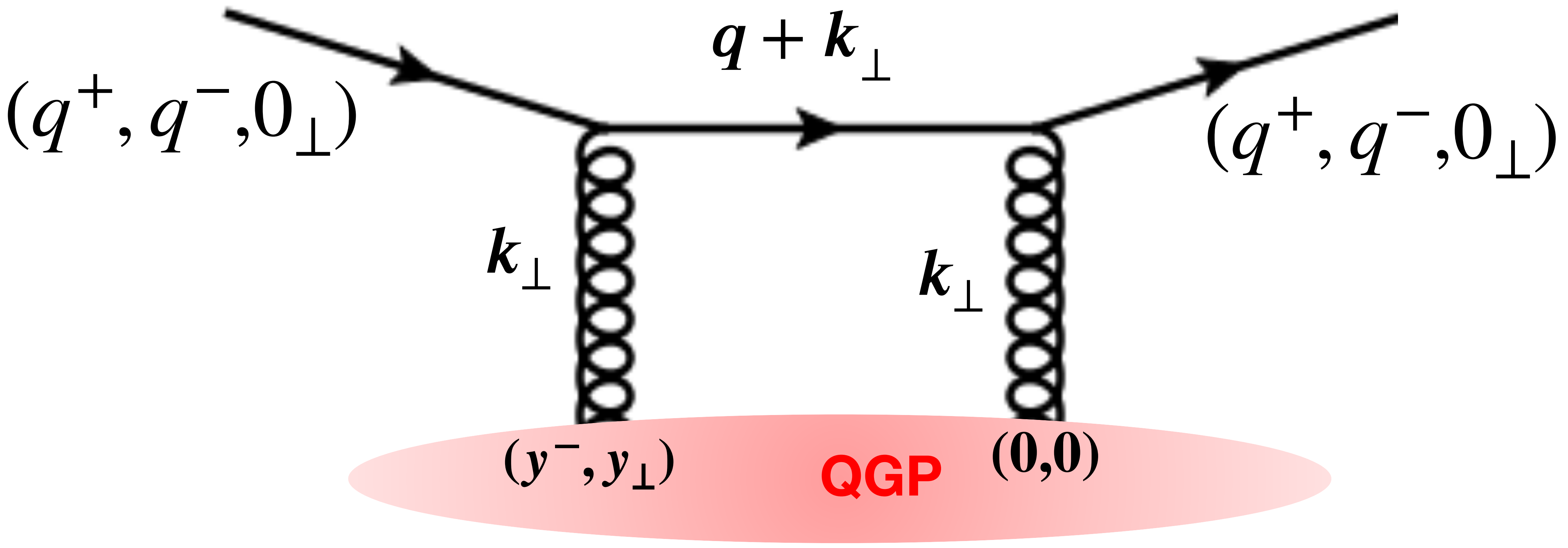
Constructing a more general expression as $\hat{Q}$

❖**Generalized object $\hat{Q}$: with q^- fixed and q^+ is variable**

$$\hat{Q} = \sum_k k_\perp^2 \frac{\mathscr{W}(k)}{L^0};$$

$$\hat{q} = \sum_k k_\perp^2 \frac{Disc\left[\mathscr{W}(k)\right]}{L^0};$$

$$\hat{Q}(q^+) = \frac{\alpha_s}{N_c} \int \frac{d^4y d^4k}{(2\pi)^4} e^{iky} q^- \frac{\langle M | \mathrm{Tr}[F^{+\perp_\mu}(0) F^+_{\perp_\mu}(y^-, y_\perp)] | M \rangle}{(q+k)^2 + i\epsilon}$$



- **When $q^+ \sim 0 << q^-$:**

$$\mathrm{Disc}\left[\hat{Q}(q^+)\right]\Big|_{q^+\sim 0} = \hat{q}$$

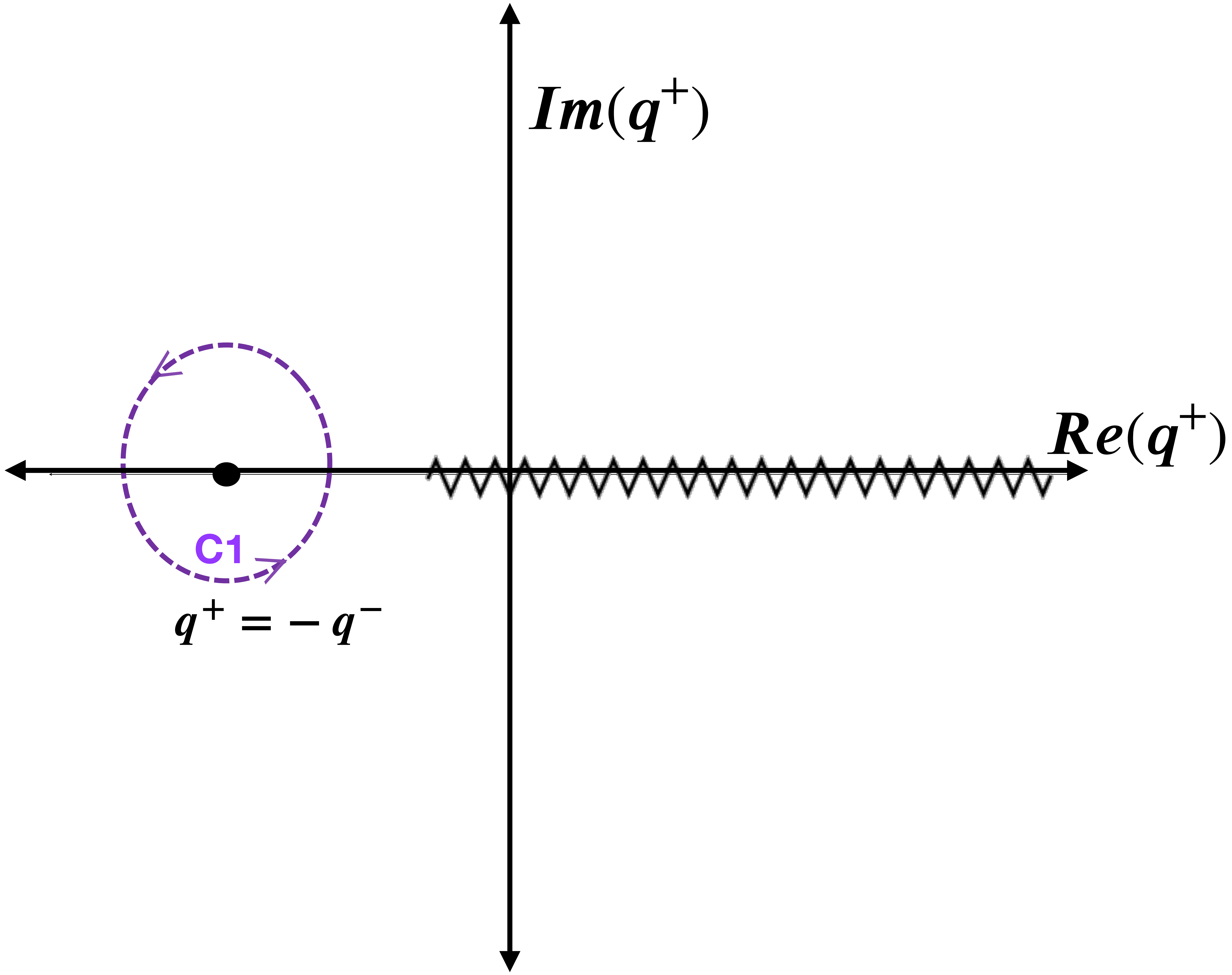
- **When $q^+ = -q^-$:**

$$\frac{1}{(q+k)^2} \simeq \frac{1}{-2q^-q^- + 2q^-(k^+ - k^-)} = -\frac{1}{2(q^-)^2}\left[1 - \left(\frac{k^+ - k^-}{q^-}\right)\right]^{-1} = -\frac{1}{2(q^-)^2}\left[\sum_{n=0}^{\infty}\left(\frac{\sqrt{2}k_z}{q^-}\right)^n\right]$$

Perform d^4y and d^4k integration

$$\hat{Q}(q^+ = -q^-) = \frac{\alpha_s}{2N_c q^-} \langle M | \mathrm{Tr}[F^{+\perp_\mu}(0) \sum_{n=0}^{\infty}\left(\frac{i\sqrt{2}D_z}{q^-}\right)^n F^+_{\perp_\mu}(0)] | M \rangle$$

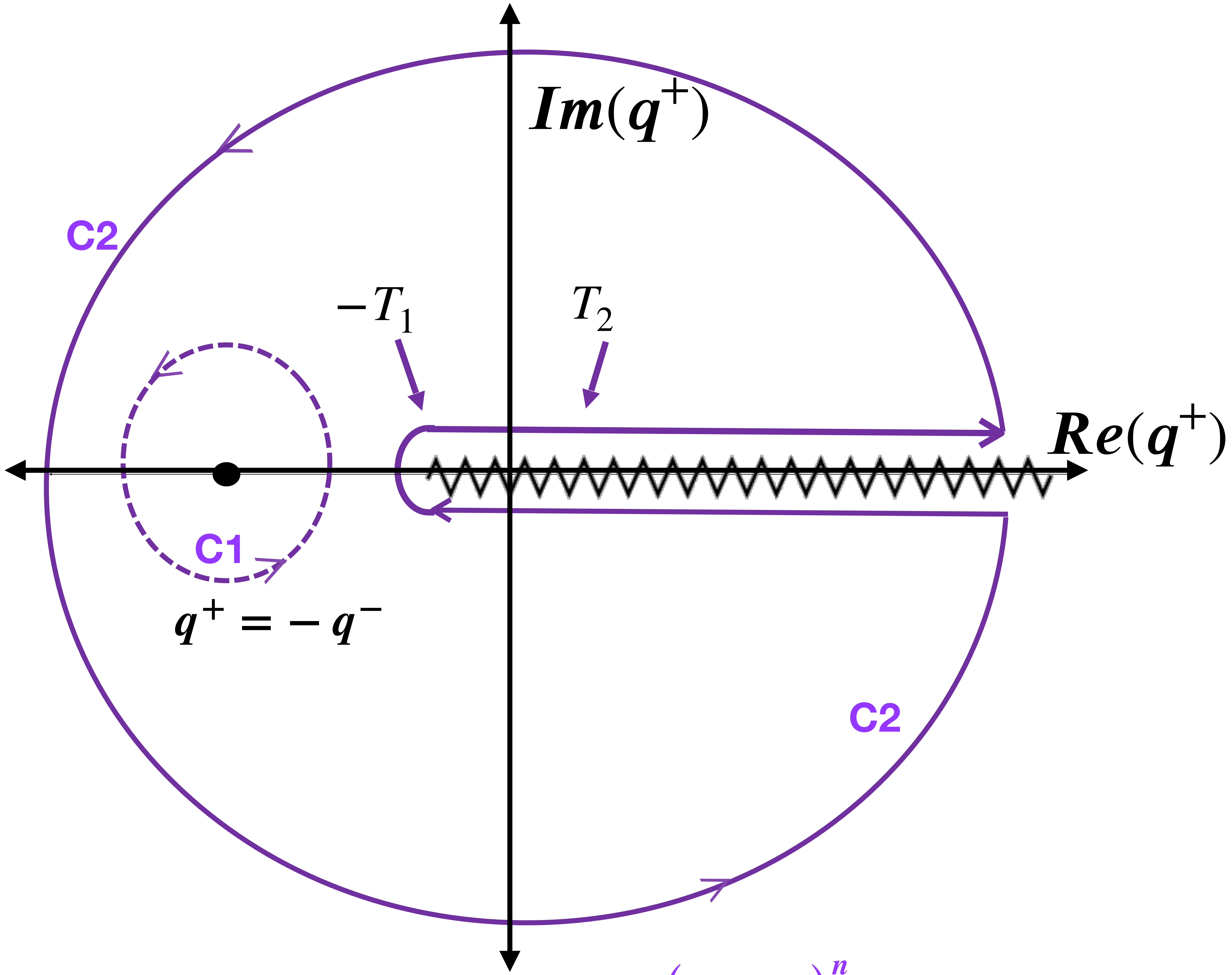
Extract $\hat{q}$ using analytic continuation of $\hat{Q}(q^+)$



Contour C1:

$$I = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + q^-)} = \hat{Q}(q^+ = -q^-)$$

Extract $\hat{q}$ using analytic continuation of $\hat{Q}(q^+)$



Contour C1:

$$I = \oint \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{(q^+ + q^-)} = \hat{Q}(q^+ = -q^-)$$

Contour C2: On extending it to infinity

$$I = \int_{-T_1}^{T_2} dq^+ \frac{\hat{q}(q^+)}{(q^+ + q^-)} + \int_0^\infty \frac{dq^+}{2\pi i} \frac{Disc[\hat{Q}(q^+)]}{(q^+ + q^-)}$$

Pure thermal part **Pure Vacuum part**

$$\hat{q} = \frac{\alpha_s}{2N_c(T_1 + T_2)} \langle M | \mathrm{Tr}[F^{+\perp_\mu}(0) \sum_{n=0}^{\infty} \left(\frac{i\sqrt{2}D_z}{q^-} \right)^n F^+_{\perp_\mu}(0)] | M \rangle_{(\mathrm{Thermal-Vacuum})}$$

Width of thermal discontinuity $\approx 2T$ (HTL analysis)

$\hat{q}$ as a series of local operators

X. Ji, PRL 110, 262002 (2013)
Parton PDF: Operator product expansion with D_z derivatives

- **Physical form of $\hat{q}$ at LO:**

$$\hat{q} = \frac{\alpha_s}{2N_c(T_1+T_2)} \langle M | \mathrm{Tr}[F^{+\perp_\mu}(0) \sum_{n=0}^{\infty} \left(\frac{i\sqrt{2}D_z}{q^-} \right)^n F^{+}_{\perp_\mu}(0)] | M \rangle_{(\mathrm{Thermal-Vacuum})}$$

Rotating to Euclidean space: $x^0 \rightarrow -ix^4;\ A^0 \rightarrow iA^4$

$$\Longrightarrow F^{0i} \rightarrow iF^{4i}$$

- **Non-zero operators:**

LO operators at n=0 : $\mathrm{Tr}\left[F^{3i}F^{3i} - F^{4i}F^{4i}\right]$

LO operators with D_z^2 derivative at n=2 : $\mathrm{Tr}\left[F^{3i}D_z^2F^{3i} - F^{4i}D_z^2F^{4i}\right]$

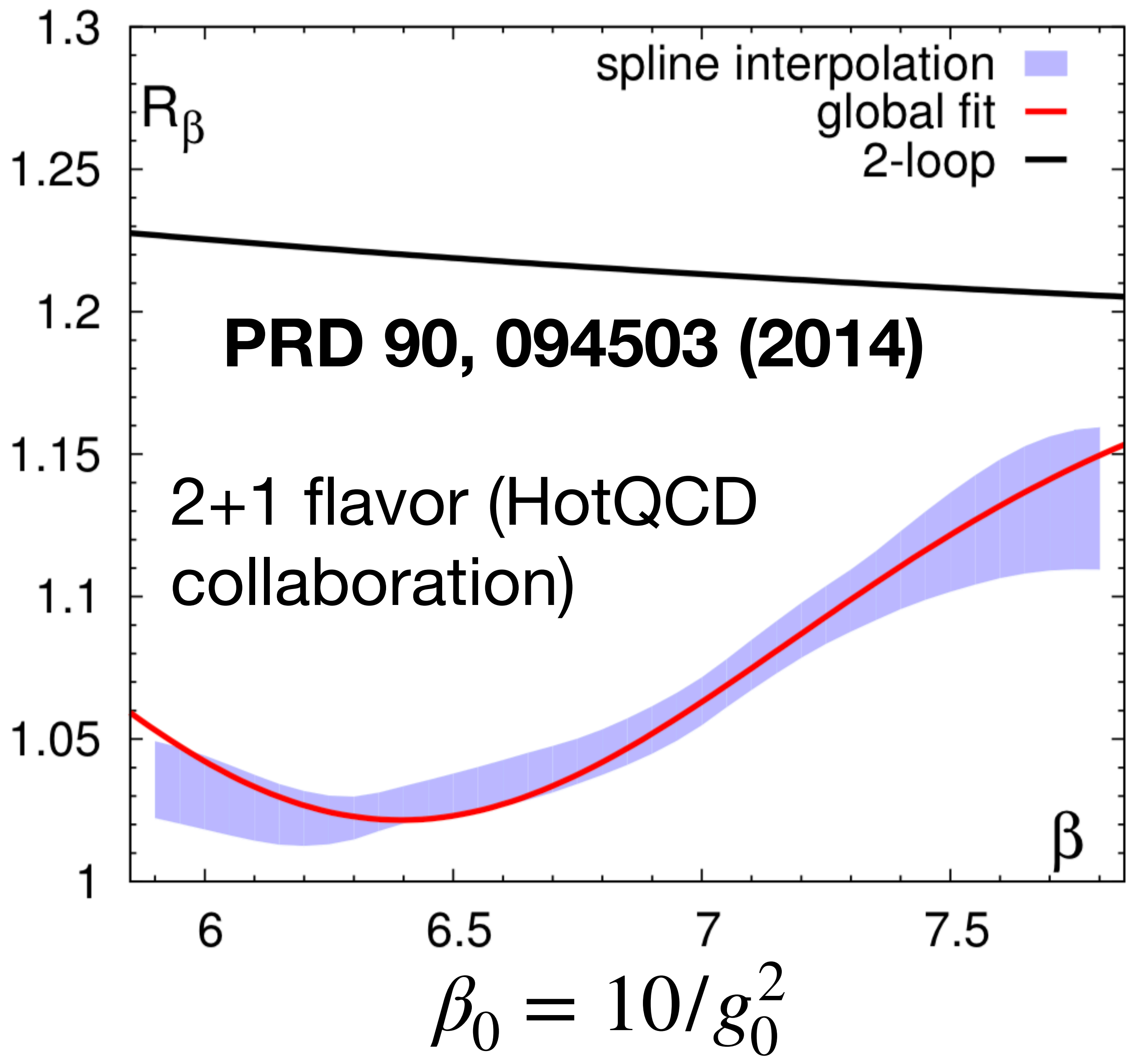
LO operators with D_z^4 derivative at n=4 : $\mathrm{Tr}\left[F^{3i}D_z^4F^{3i} - F^{4i}D_z^4F^{4i}\right]$

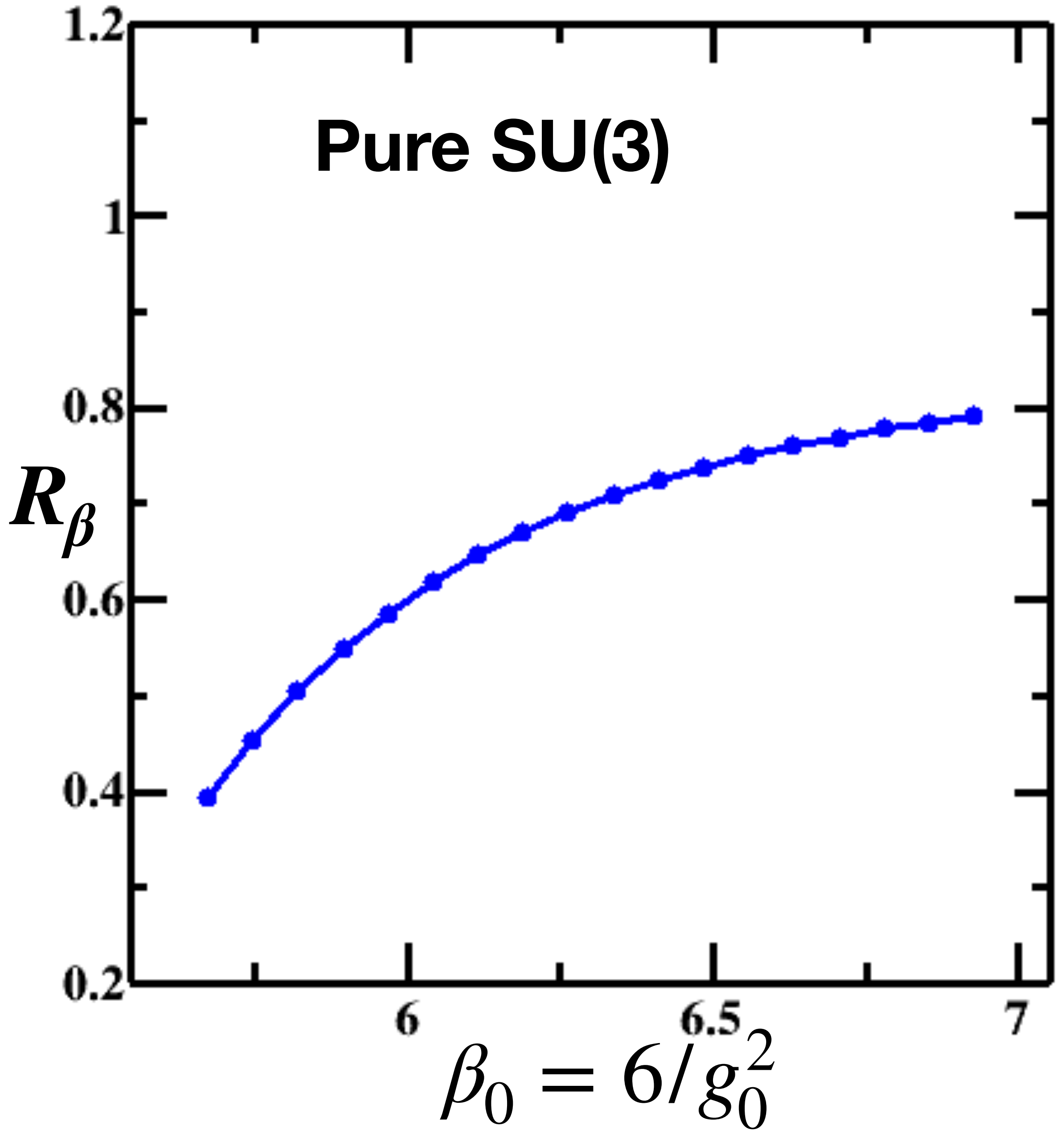
where, D_z is covariant derivative along leading parton direction (z-dir)

Basic elements in lattice QCD

(HotQCD) PRD 90, 094503 (2014)
(TUMD) PRD 98, 054511 (2018)

- **Pure SU(3) gauge action:** Wilson's gauge action $\mathcal{O}(a^2) + \mathcal{O}(g_0^2 a^2)$,
- **Full QCD action:** Highly-improved Staggered quark (HISQ) action + Tree-level Symanzik improved gauge action $\mathcal{O}(a^4) + \mathcal{O}(g_0^2 a^2)$,
  - Strange quark mass: fixed to physical value
  - Light quark mass: fixed to $m_{u,d} = m_s/20 \longrightarrow m_\pi \approx 160\text{MeV}$ (in continuum),
  - Configurations are generated using public version of MILC code
- **Lattice beta-function:** $R_\beta = -a\dfrac{d\beta_0}{da}$





- **For pure SU(3) gauge:**

$$a_L = \frac{f}{\Lambda_L}\left(\frac{11}{16\pi^2 g_0^2}\right)^{\frac{-51}{121}} exp\left(-\frac{8\pi^2}{11 g_0^2}\right)$$

$T_c = 265\text{MeV}, \Lambda_L = 5.5\text{MeV}$
Tune $f(g_0^2)$ such that T_c/Λ_L is independent of g_0^2

- **Full QCD case:**
Lattice spacing is determined using heavy quark and anti-quark potential

Renormalization of leading-twist operator

- **Symmetry group in lattice formulation:** $SO(4) \xrightarrow{\text{broken}} SW_4$ **(hypercubic group)**
- **Similarity between Leading twist operator** $Tr[FF] \equiv \mathrm{Tr}\left[F^{3i}F^{3i} - F^{4i}F^{4i}\right]$ **and energy-momentum tensor**

  Triplet component of energy momentum tensor $T^{G,(3)}_{\mu\nu} = \left[F^a_{\mu\alpha}F^a_{\mu\alpha} - F^a_{\nu\alpha}F^a_{\nu\alpha}\right]; \quad Tr[FF] = \frac{1}{2}T^{G,(3)}_{34}$

  Entropy density, $Ts = \epsilon + p = < T_{kk} - T_{44} > ; \quad Ts = T^{G(3)}_{\mu\nu}$

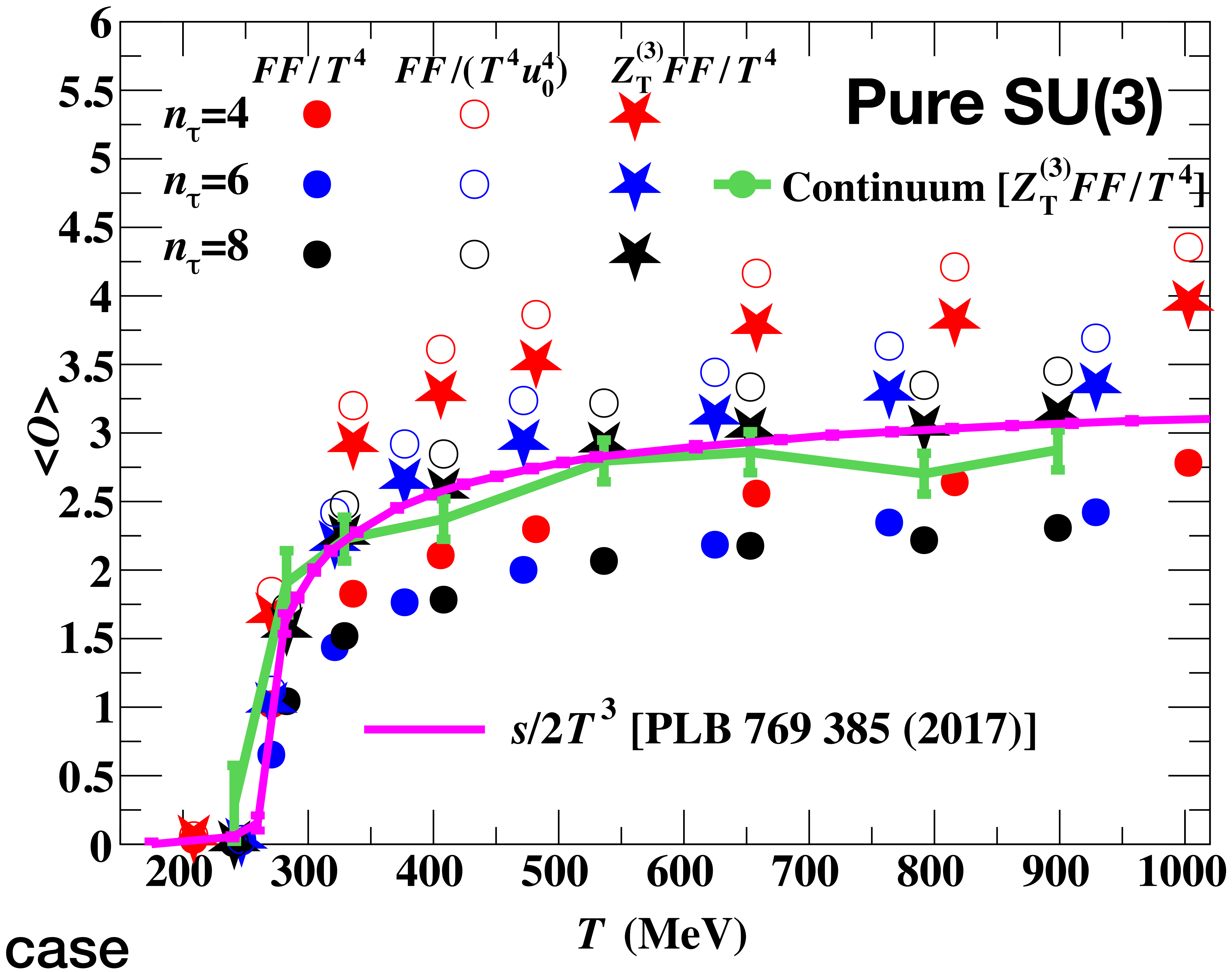
- **Renormalization of triplet component of EMT in pure SU(3) gauge** $T^{G,(3),Ren}_{\mu\nu} = Z^{(3)}T^{G,(3),Bare}_{\mu\nu}$
  - **Non-perturbative finite momentum Ward Identities (WI)** $Tr[FF]^{Ren} = Z^{(3)}Tr[FF]^{Bare}$
  - $Z^{(3)}$ **determined in** $\overline{MS}$ **for Wilson's plaquette action from** *Guisti, Pepe, PRD 91 (2015); PLB 769 (2017)*

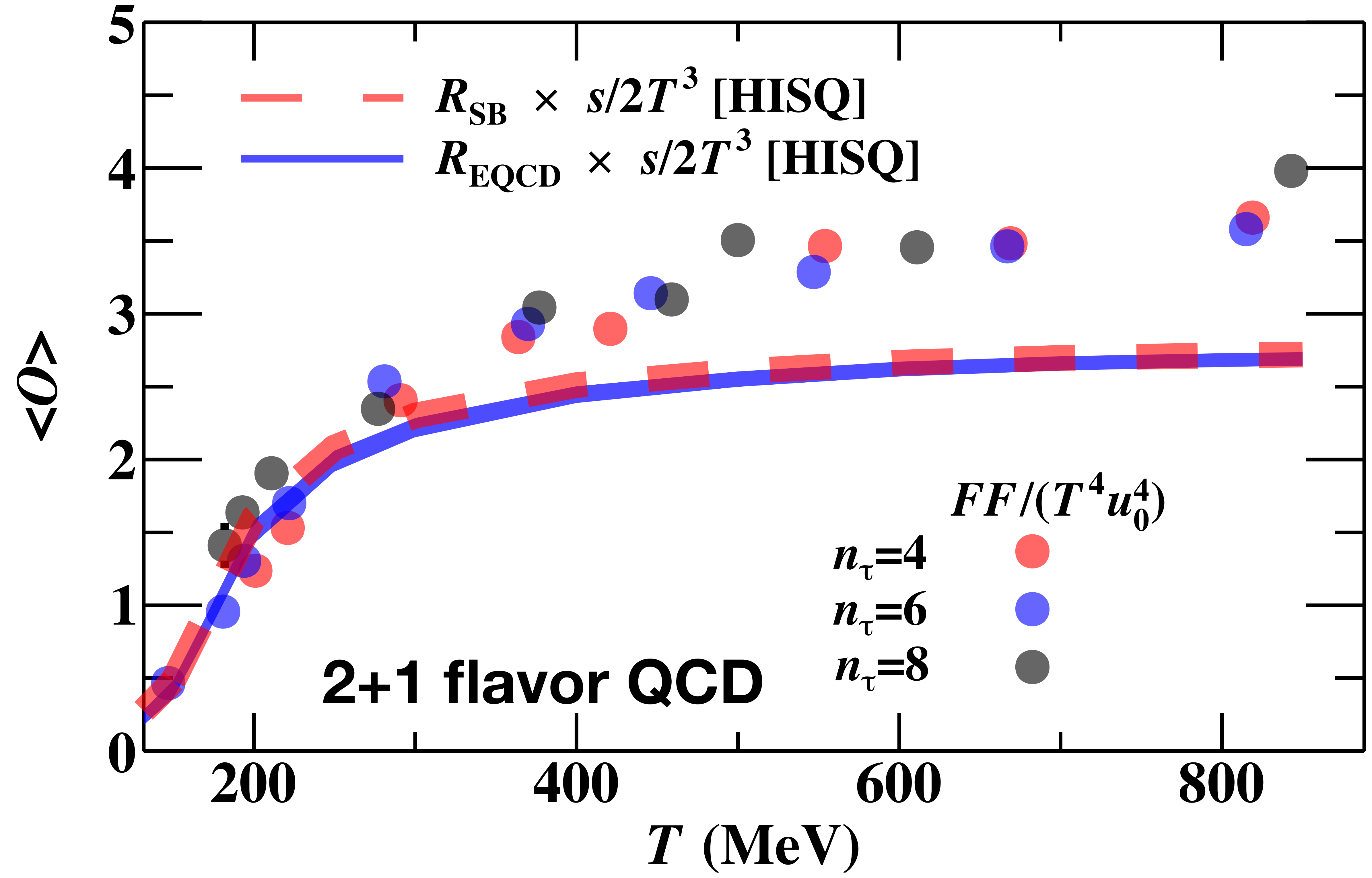
- **Renormalization of triplet component of EMT in QCD requires complete mixing matrix**

$$\begin{bmatrix} T^{(3)R}_G \\ T^{(3)R}_Q \end{bmatrix} = Z \begin{bmatrix} T^{(3)B}_G \\ T^{(3)B}_Q \end{bmatrix}, \qquad Z = \begin{bmatrix} Z^{(3)}_{GG} & Z^{(3)}_{GQ} \\ Z^{(3)}_{QG} & Z^{(3)}_{QQ} \end{bmatrix}$$

All four renormalization factors are unknown for HISQ+tree-level Symanzik improved gauge action

Leading-twist operator in quenched and unquenched SU(3) lattices





- Pure SU(3) case
  - Tad-pole (u_0) improved results and renormalized results differ by $\leq 10\,\%$
  - Continuum extrapolation of FF correlator is consistent with $T_f = 1/2$ scaled entropy density s/T^3
- Full QCD case
  - Exact renormalization factors are unknown. We use estimates from tad-pole improved results, gluon fraction to entropy density as $R(T/T_c) = \dfrac{s^{n_f=0}/T^3}{s^{n_f=3}/T^3}$, with $T_c^{n_f=0} = 270$ MeV, $T_c^{n_f=3} = 155$ MeV, Systematic uncertainty $\leq 30\,\%$

$\hat{q}$ for pure gluon plasma and 2+1 flavor QCD

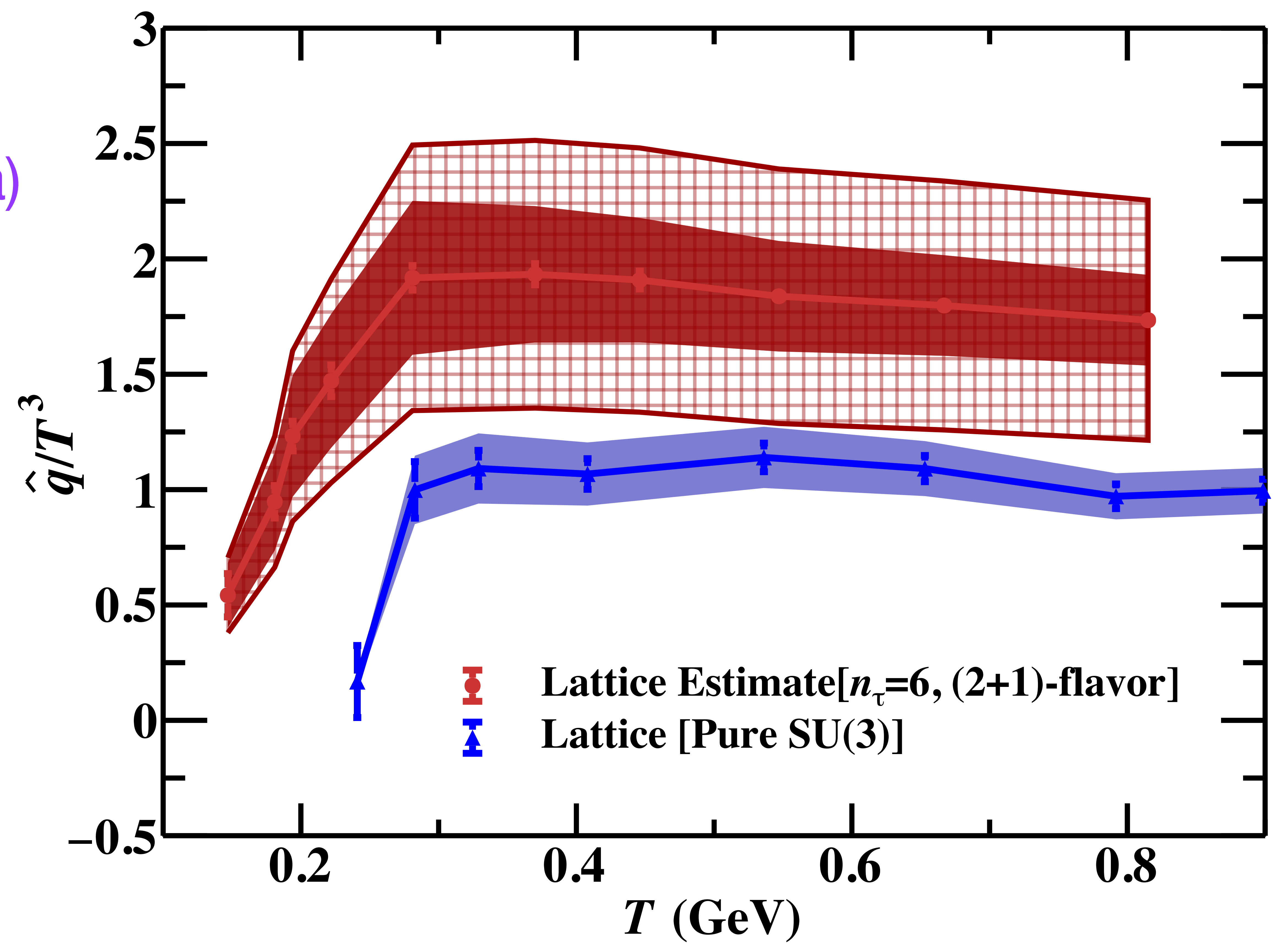
- **At high temperature:** $\hat{q} \propto T^3$
  - $\hat{q}/T^3 \sim$ 1.5-2.0 (red solid band, 2+1 flavor QCD)
  - $\hat{q}/T^3 \sim$ 1.25-2.5 (red grid, 2+1 flavor QCD plasma)
  - $\hat{q}/T^3 \sim$ 0.75-1.25 (pure gluon plasma)
- **At low temperature:**

  $\hat{q}/T^3$ becomes smaller and No signature of log-dependence as in HTL form

  $(\text{HTL formula}) \equiv \hat{q} \propto \alpha_s^2 T^3 \ln(E/T)$
- **The qualitative behavior is similar to entropy density**
- **Uncertainty**

  Grid line: 30% error from renormalization (corrections from quark and gluon renormalization)

  Solid red and blue band: uncertainty in scale $2\pi T<\mu<4\pi T$ for strong coupling constant

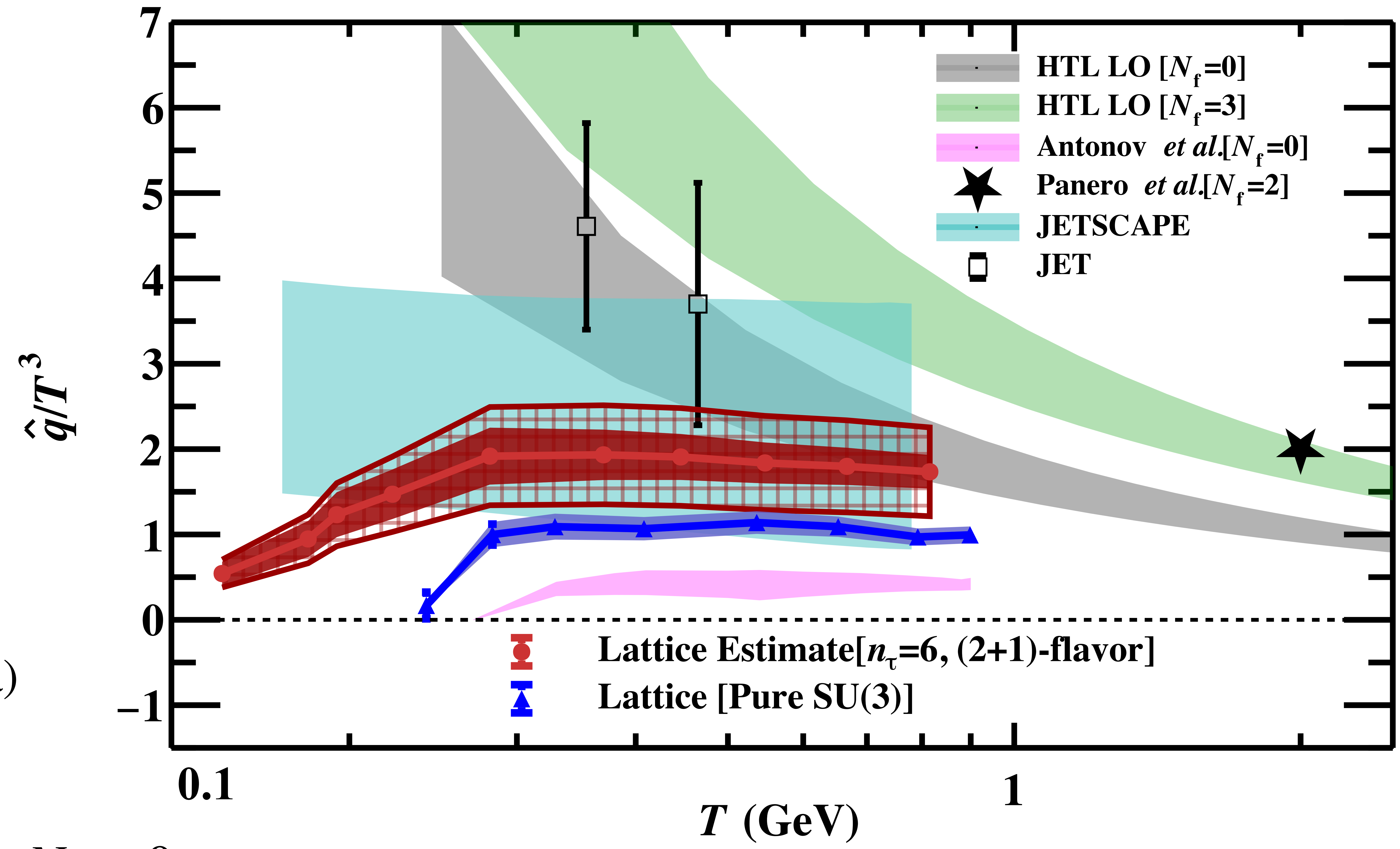


Comparison with extractions from phenomenology and other framework

- Lattice extractions are consistent to JET and JETSCAPE collaboration extraction

- Lattice extracted $\hat{q}$ does not show a log-like behavior

$$\hat{q} \propto \alpha_s^2 T^3 \ln\left(\frac{E}{T}\right) \text{ (HTL formula)}$$



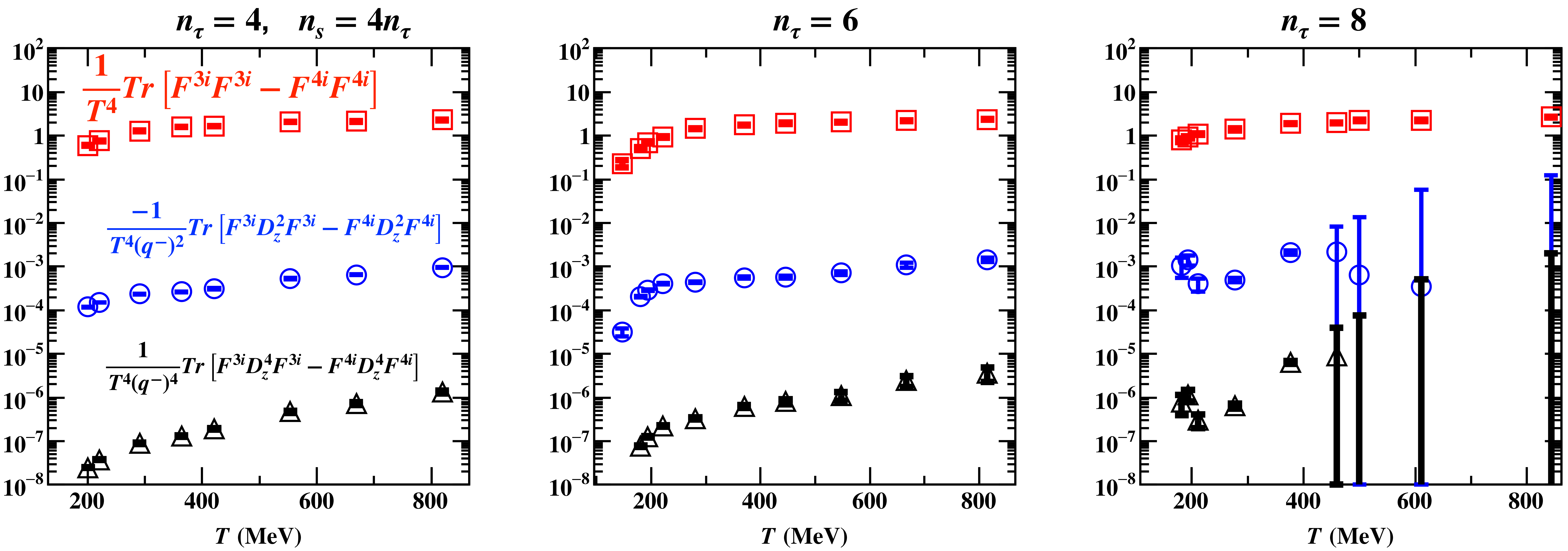
- **Stochastic vacuum model with $N_f = 0$ has a similar shape.** (Antonov, Pirner, EPJC55(2008))
- **Electrostatic QCD results with $N_f = 2$ at high temperature $T = 2$ GeV.** (Panero, et al. PRL 112(2014))
- **Hard-thermal loop results with $q^- = 100$ GeV for $N_f = 0$ and $N_f = 3$**

Summary and Future work

- **First calculation of $\hat{q}$ on 4D lattice for pure SU(3) and 2+1 flavor QCD plasma**
  - $\hat{q}/T^3 \sim$ 0.75-1.25 (pure gluon plasma) and 1.25-2.5 (QCD plasma)
  - Renormalized results in continuum for pure SU(3) case
  - Consistent with JET and JETSCAPE collaboration results within their uncertainty band
  - Analytic continuation to deep Euclidean space and expressed as local operators
  - $\hat{q}$ does not show a log-dependence as in the HTL formula

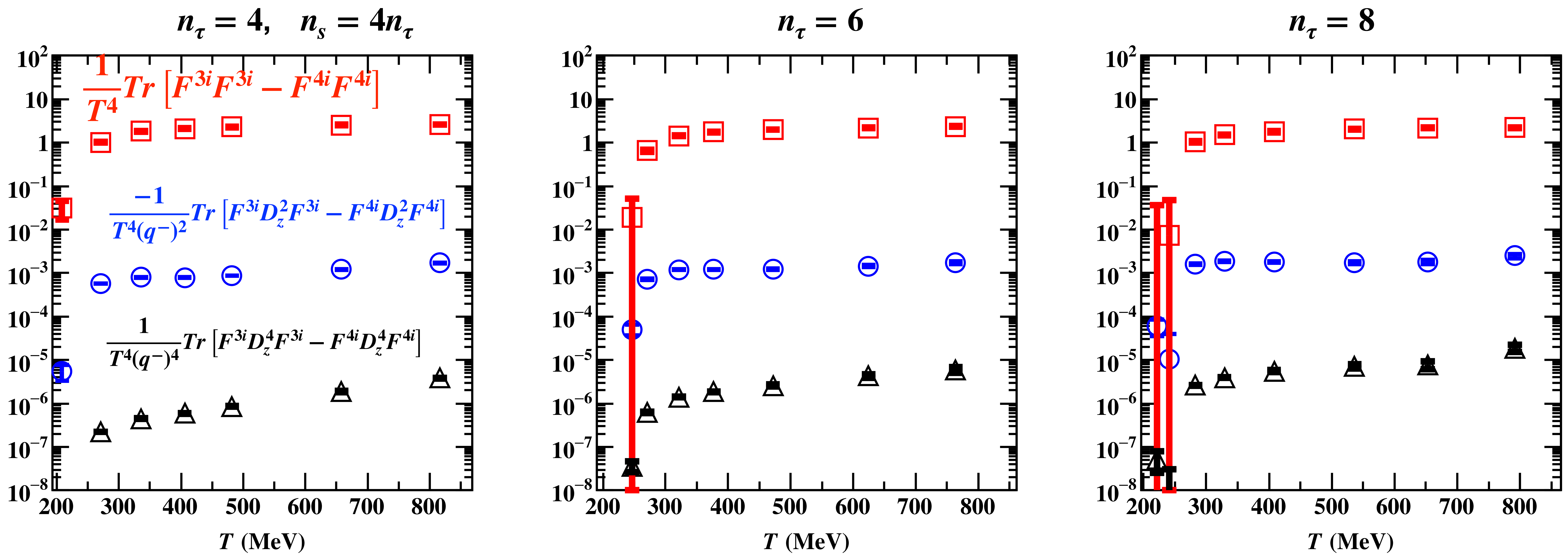
- **Future work**
  - Compute exact renormalization factors in the case of full QCD.
  - Understand the mixing fo higher-twist operators with lower-dimensional operators and proper renormalization
  - Extend calculation to finer lattices $n_\tau = 10, 12$ and perform continuum extrapolation
  - Apply this method to compute jet transport coefficient $\hat{e}_2$

FF correlator in unquenched SU(3) lattices



- LO FF correlator is dominant among all operators
- FF correlator with D_z^2 derivative are suppressed by a factor of 10^3 compared LO operator
- FF correlates with D_z^4 derivative are suppressed by a factor of 10^6 compared LO operator

FF correlator in quenched SU(3) lattices



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