Partial chemical equilibrium

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Motivation - why partial chemical equilibrium?

- Statistical production can be used to describe hadron abundances and also their spectra
- (Simple) statistical model of interacting hadrons: interactions via inclusion of (free) resonance states [R. Dashen, S.K. Ma, H.J. Bernstein, Phys. Rev. 187 (1969) 345]

Chemical freeze-out

- Hadron abundances set by three (four) parameters: V, T_{ch}, μ_B, (γ_s)
- $T \sim 140 - 160$ MeV ($\sqrt{s_{NN}}$ dependent, above 7.7 GeV)

Kinetic freeze-out

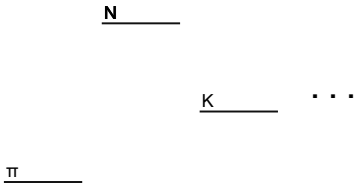
- Sets the p_T spectra
- need transverse expansion
- slope due to T_k and $\langle v_t \rangle$
- $T_k \sim 80 - 120$ MeV (also higher)

How to build a scenario with chemical and kinetic freeze-out?

- need to freeze the effective numbers of stable hadrons—projected numbers after decays of all resonances $N_h^{eff} = \sum_r p_{r\to h} \langle N_r \rangle$
- Assumption: at chemical freeze-out inelastic collisions stop and elastic continue

The chemical potentials

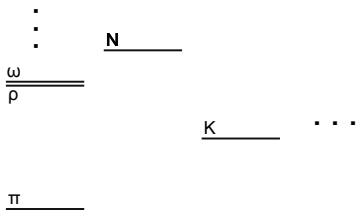
- ground state species do not change one into other $\Rightarrow$ chemical potential for each



[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B 378 (1992) 95]

The chemical potentials

- ground state species do not change one into other $\Rightarrow$ chemical potential for each
- towers of resonances above every stable hadron species
- resonances always in equilibrium with ground state $\Rightarrow$ it does not cost extra energy to produce or decay resonance into stable species
- resonance chemical potentials from those of stable hadrons, e.g. $\mu_\rho = 2\mu_\pi$, $\mu_\omega = 3\mu_\pi$



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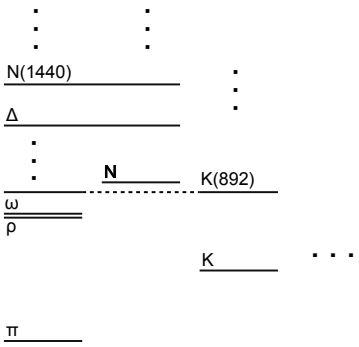

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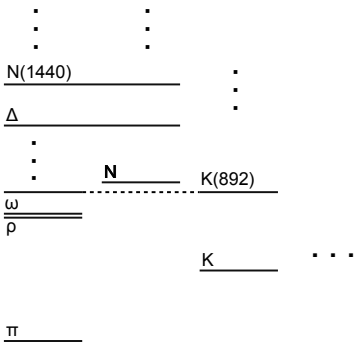


- Resonances with more decay channels, chain decays:

$$\mu_R = \sum_h p_{R\to h}\mu_h$$

[H. Bebie, P. Gerber, J.L. Goity, H. Leutwyler, Nucl. Phys. B 378 (1992) 95]

Evolution of chemical potentials

Keep the (effective stable) particle numbers constant, as a function of temperature!

$$\langle N_h^{eff} \rangle = \sum_r p_{r\to h} V(T) n_r(T, \{\mu(T)\}), \qquad \frac{\mathrm{d}\langle N_h^{eff}\rangle}{\mathrm{d}T} = 0$$

$$-\frac{\frac{\mathrm{d}V}{\mathrm{d}T}}{V} \sum_r p_{r\to h} n_r(T) = \sum_r p_{r\to h} \frac{\mathrm{d}n_r(T)}{\mathrm{d}T}$$

Obtain the derivative of volume from entropy conservation: $0 = \mathrm{d}S/\mathrm{d}T = \mathrm{d}(sV)/\mathrm{d}T$

$$-\frac{\frac{\mathrm{d}V}{\mathrm{d}T}}{V} = \frac{\frac{\mathrm{d}s}{\mathrm{d}T}}{s}$$

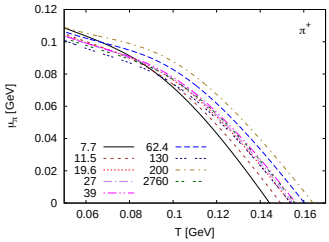
Equations for the evolution of chemical potentials

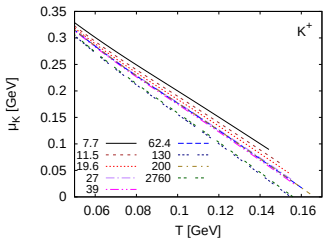
$$\frac{\sum_r p_{r\to h} \frac{\mathrm{d}n_r(T,\{\mu(T)\})}{\mathrm{d}T}}{\mathrm{d}s/\mathrm{d}T} = \frac{1}{s} \sum_r p_{r\to h} n_r(T, \{\mu(T)\})$$

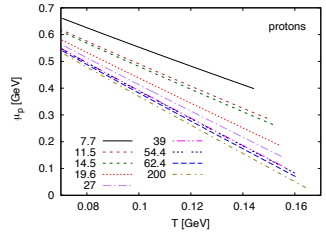
Evolution of chemical potentials: results

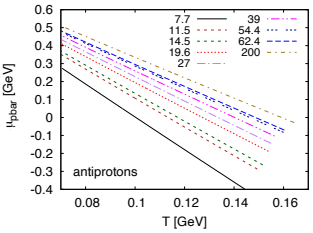
Start the evolution of chemical potentials at the chemical freeze-out

[STAR collab., Phys. Rev. C 96 (2017) 044904 and ALICE collab., Nucl. Phys. A 904-905 (2013) 531c]









Fluctuations of net-baryon number

Cumulants of the net-baryon number distribution from derivatives of $\log Z$

$$\frac{\partial \ln Z}{\partial \frac{\mu_B}{T}} = \langle B \rangle = \mu_1 = \kappa_1 = VT^3\chi_1$$

$$\frac{\partial^2 \ln Z}{\partial \left(\frac{\mu_B}{T}\right)^2} = \langle B^2 \rangle - \langle B \rangle^2 = \mu_2 = \kappa_2 = \sigma^2 = VT^3\chi_2$$

$$\frac{\partial^3 \ln Z}{\partial \left(\frac{\mu_B}{T}\right)^3} = \langle B^3 \rangle - 3\langle B^2 \rangle \langle B \rangle + 2\langle B \rangle^3 = \mu_3 = \kappa_3 = VT^3\chi_3$$

$$\frac{\partial^4 \ln Z}{\partial \left(\frac{\mu_B}{T}\right)^4} = \langle B^4 \rangle - 4\langle B^3 \rangle \langle B \rangle - 3\langle B^2 \rangle^2 + 12\langle B^2 \rangle \langle B \rangle^2 - 6\langle B \rangle^4 = \mu_4 - 3\mu_2^2 = \kappa_4 = VT^3\chi_4$$

$$\frac{\partial^5 \ln Z}{\partial \left(\frac{\mu_B}{T}\right)^5} = \kappa_5 = VT^3\chi_5\,, \qquad \frac{\partial^6 \ln Z}{\partial \left(\frac{\mu_B}{T}\right)^6} = \kappa_6 = VT^3\chi_6$$

central moments μ_i, cumulants κ_i, susceptibilities χ_i

Combinations of cumulants

variance, skewness, kurtosis, hyperskewness, hyperkurtosis

$$\sigma^2 = \kappa_2\,, \quad S = \frac{\kappa_3}{\kappa_2^{3/2}}\,, \quad \kappa = \frac{\kappa_4}{\kappa_2^2}, \quad S^H = \frac{\kappa_5}{\kappa_2^{5/2}}\,, \quad \kappa^H = \frac{\kappa_6}{\kappa_2^3},$$

These cumulants, moments, and their combinations still depend on volume

$\Rightarrow$ construct volume-independent combinations

$$\frac{\chi_2}{\chi_1} = \frac{\kappa_2}{\kappa_1} = \frac{\sigma^2}{M} \qquad \frac{\chi_3}{\chi_2} = \frac{\kappa_3}{\kappa_2} = S\sigma \qquad \frac{\chi_4}{\chi_2} = \frac{\kappa_4}{\kappa_2} = \kappa\sigma^2$$

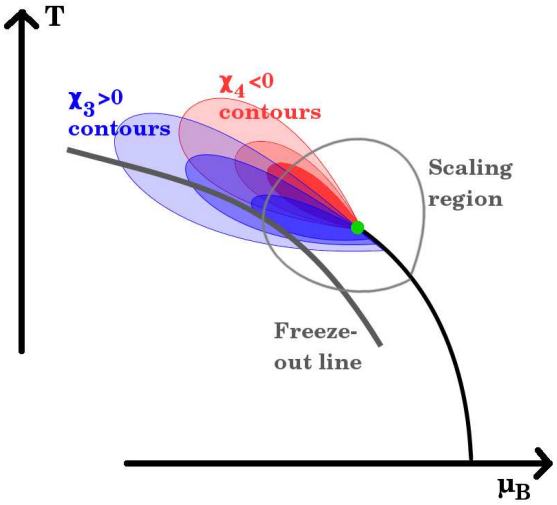
$$\frac{\chi_5}{\chi_1} = \frac{\kappa_5}{\kappa_1} = \frac{S^H\sigma^5}{M} \qquad \frac{\chi_5}{\chi_2} = \frac{\kappa_5}{\kappa_2} = S^H\sigma^3 \qquad \frac{\chi_6}{\chi_2} = \frac{\kappa_6}{\kappa_1} = \kappa^H\sigma^4$$

Connection to the phase diagram

Enhanced fluctuations close to the critical point

Illustration:
Susceptibilities from the Ising model
(same universality class)

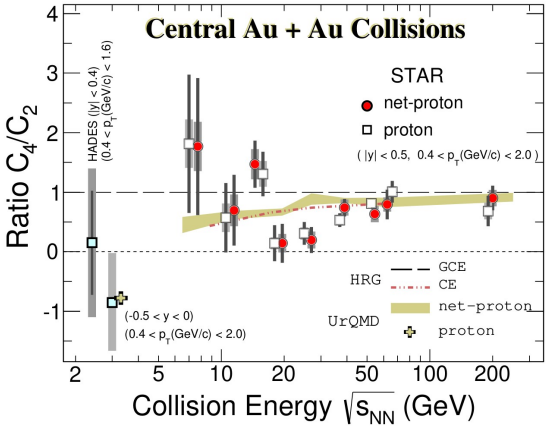
[J.W. Chen et al.: Phys. Rev. D 95 (2017) 014038]



Data: enhanced net-proton number fluctuations at $\sqrt{s_{NN}} = 7.7$ GeV

- Not all baryons are measurable
- net-proton number as proxy for the net baryon number
- enhanced κ_4/κ_2 at $\sqrt{s_{NN}} = 7.7$ GeV
- not reproduced by theoretical calculations

What would be the prediction of statistical model with PCE?



[STAR collaboration: 2112:00240]

Net-proton number fluctuations from PCE

- Not calculable as derivatives of the partition function!
  - derivatives of $\log Z$ only contain fluctuations due to exchange with the heat bath
  - decays of resonances are random and may randomize proton number (even at fixed B)
- cumulants of proton and antiproton number via derivatives of the generating function

$$\left\langle (\Delta N)^l \right\rangle_c = \left. \frac{\mathrm{d}^l K(i\xi)}{\mathrm{d}(i\xi)^l} \right|_{\xi=0}$$

$$K(i\xi) = \ln \sum_{N=0}^{\infty} e^{i\xi N} P(N) = \sum_R \ln \left\{ \sum_{N_R=0}^{\infty} P_R(N_R) \left(e^{i\xi} p_R + (1 - p_R) \right)^{N_R} \right\}$$

- $P_R(N_R)$: number probability of resonance R, furnished by statistical model
- Net-proton number cumulants obtained via

$$\left\langle (\Delta N_{p-\bar{p}})^l \right\rangle_c = \left\langle (\Delta N_p)^l \right\rangle_c + (-1)^l \left\langle (\Delta N_{\bar{p}})^l \right\rangle_c$$

Net-proton number fluctuations from PCE, part 2

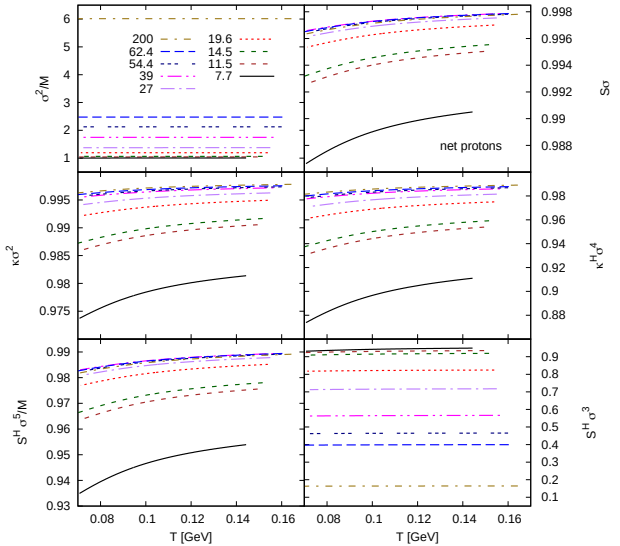
- Cumulants of the resonance number distributions

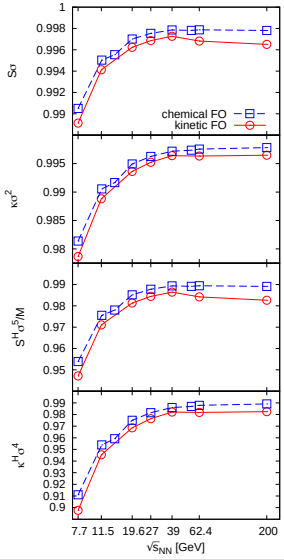
$$\langle N_R \rangle_c = \frac{g_R V}{2\pi^2} m_R^2 T \sum_{j=1}^{\infty} \frac{(\mp 1)^{j-1}}{j} e^{j\mu_R/T} K_2\left(\frac{j m_R}{T}\right),$$

$$\left\langle (\Delta N_R)^l \right\rangle_c = \frac{g_R V}{2\pi^2} m_R^2 T \sum_{j=1}^{\infty} (\mp 1)^{j-1} j^{l-2} e^{j\mu_R/T} K_2\left(\frac{j m_R}{T}\right).$$

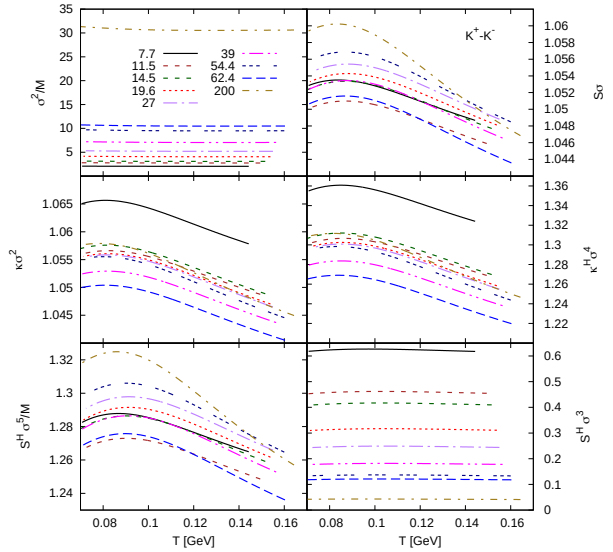
- first terms in the sums correspond to Boltzmann approximation (not BE or FD)
- In Boltzmann approximation, cumulants of all orders are the same!

Results for net-proton cumulants in PCE





Results for $K^+ - K^-$ cumulants in PCE



Production of resonances

Chemical equilibrium

- chemical potentials due to conserved quantum numbers (B, S, Q)
- resonance abundance suppressed due to their higher mass

Corrections to chemical equilibrium

- resonances decay, but their daughter particles are not reconstructed
- by interactions, resonances are restored (to what extent?)

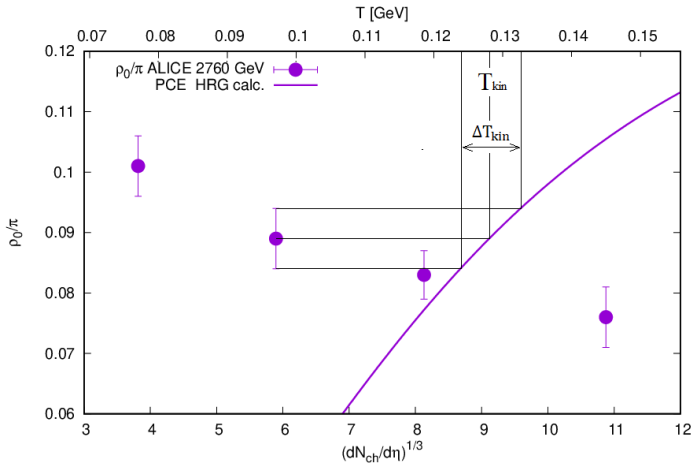
Observed abundance of resonances bears information about the microscopic dynamics within the hadronic fireball.

Partial chemical equilibrium

- At every temperature, resonances are in equilibrium with their own daughter hadrons
- specific predictions on their abundances and the ratios N_R^{eff}/N_h^{eff}; they depend on T
- effective number of resonances includes contributions from decays of heavier resonances $N_R^{eff} = \sum_r p_{r\to R} N_r$

Can Partial Chemical Equilibrium with some T reproduce data on resonance production?

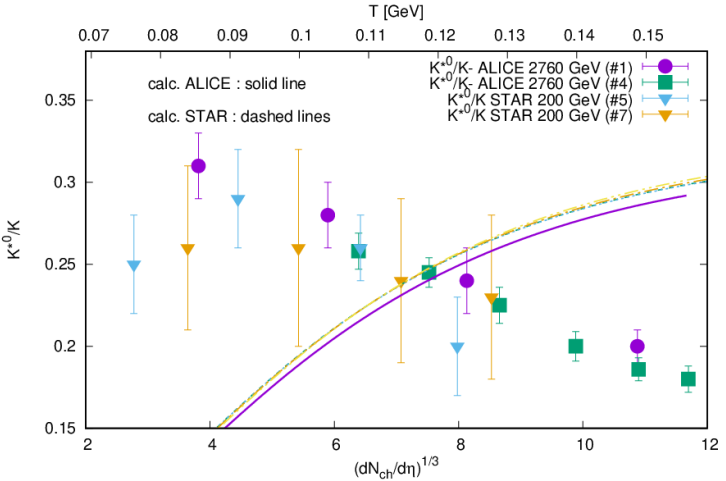
Ratio ρ_0/π (and how to read the results)



[ALICE collab., Phys. Rev. C 99 (2019) 064901] [Figure: Sándor Lökös]

Ratio K^*/K

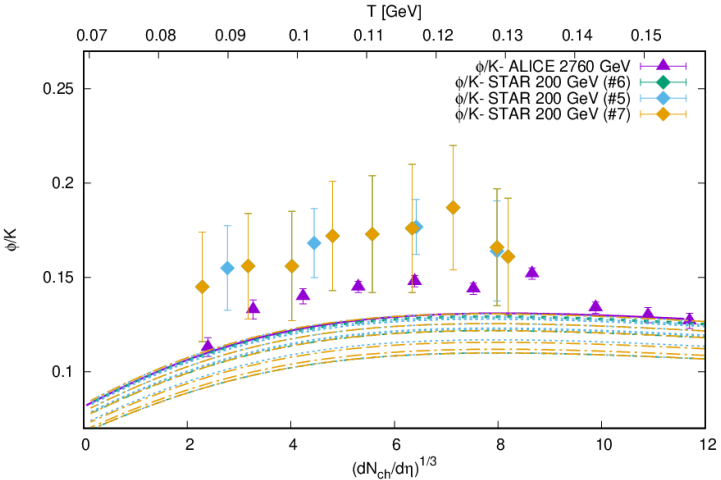
- indicated lower freeze-out temperature in central collisions
- T_{kin} never below 95 MeV



[ALICE collab., Phys. Rev. C 91 (2015) 024609]
[ALICE collab., Phys. Rev. C 95 (2017) 064606]
[STAR collab., Phys. Rev. C 84 (2011) 034909]
[STAR collab., Phys. Rev. Lett. 97 (2006) 032301]
[Figure: Sándor Lökös]

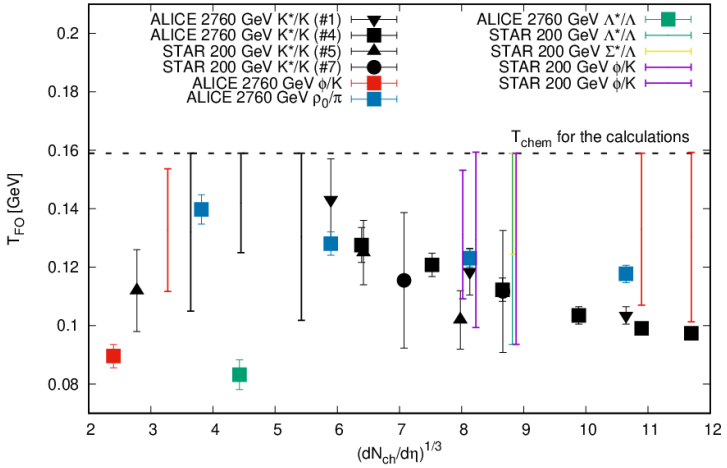
Ratio ϕ/K

- data in mid central collisions above the PCE calculations



[ALICE collab., Phys. Rev. C 91 (2015) 024609]
[STAR collab., Phys. Rev. C 70 (2009) 064903]
[STAR collab., Phys. Rev. C 84 (2011) 034909]
[STAR collab., Phys. Rev. Lett. 97 (2006) 032301]
[Figure: Sándor Lökös]

Summary of extracted temperatures



[Figure: Sándor Lökös]

Conclusions

- Partial Chemical Equilibrium
  - keeps effective numbers of stable species konstant independent of temperature
  - no reactions that would change one stable species into another (off equilibrium)
  - resonances in equilibrium with stable species
- Results from Partial Chemical Equilibrium on net-proton number fluctuations
  [B. Tomášik, P. Hillmann, M. Bleicher, Phys.Rev.C 104 (2021) 044907]
  - volume-independent ratios of cumulants of net-proton number are almost temperature independent $\Rightarrow$ they reflect values at chemical freeze-out
  - experimental data on cumulants at low energies are not reproduced
- Results for resonance production
  - ρ^0/π and K^*/K qualitatively: lower FO temperature in central collisions, quantitatively disagreement with kinetic FO fits
  - ϕ/K does not follow this trend, some centralties have too many ϕs
- Possible improvements to PCE
  - take into account entropy production
  - include hadron interactions via phase shifts