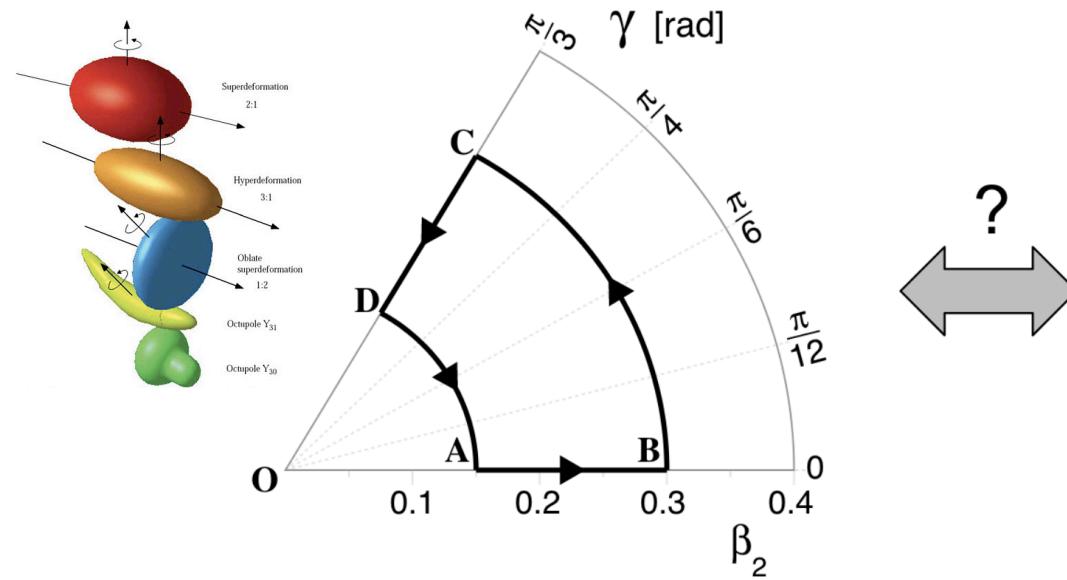




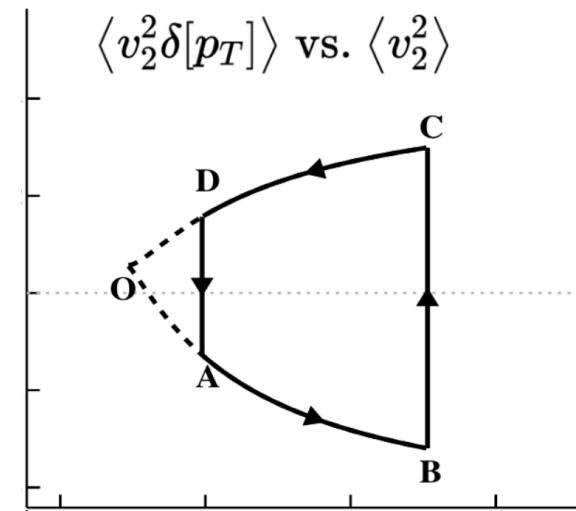
Imaging nuclear structure in heavy-ion collisions

Jiangyong Jia

Nuclear structure

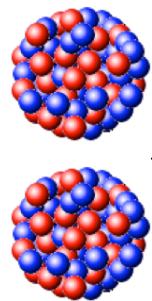


High-energy collisions



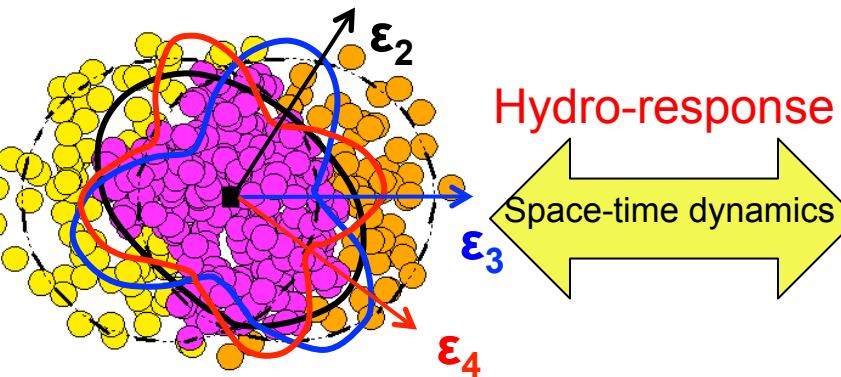
Hydrodynamic response to initial state

Nuclear Structure


 ρ_0

Imaging?

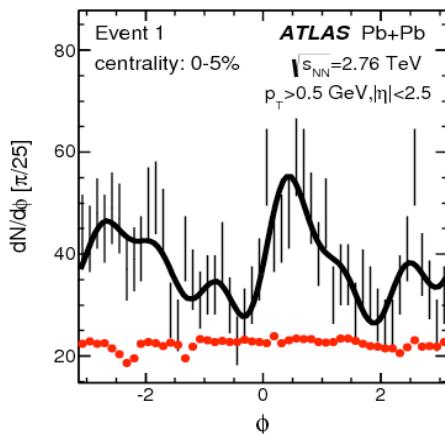
Initial State



Hydro-response

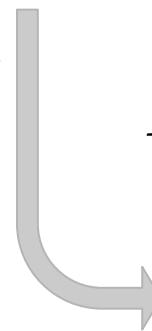
Space-time dynamics

Final Particle flow



$$\frac{1 + e^{(r - R_0(1 + \sum_n \beta_n Y_n^0(\theta, \phi)) / a_0)}}{1 + e^{(r - R_0(1 + \sum_n \beta_n Y_n^0(\theta, \phi)) / a_0)}}$$

- $\beta_2 \rightarrow$ Quadrupole deformation
- $\beta_3 \rightarrow$ Octupole deformation
- $a_0 \rightarrow$ Surface diffuseness
- $R_0 \rightarrow$ Nuclear size



Initial Size

$$R_\perp \propto \langle r_\perp^2 \rangle, \quad \mathcal{E}_n \propto \langle r_\perp^n e^{in\phi} \rangle$$

$$R_0 \quad a_0$$

Initial Shape

$$\beta_n$$

Radial Flow

$$\frac{d^2 N}{d\phi dp_T} = N(p_T) \left(\sum_n V_n e^{-in\phi} \right)$$

Harmonic Flow

arXiv:1206.1905

High energy: approx. linear response in each event:

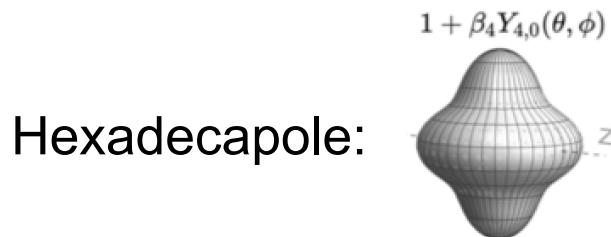
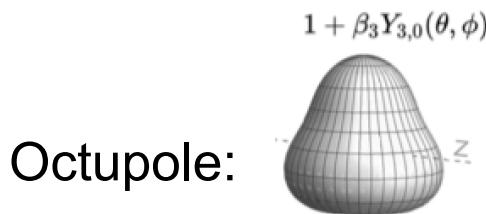
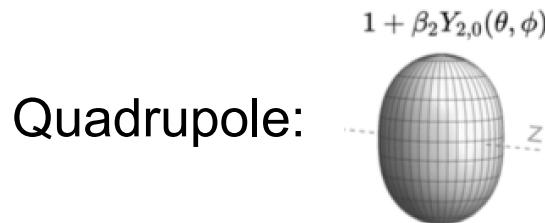
$$\frac{\delta[p_T]}{[p_T]} \propto -\frac{\delta R_\perp}{R_\perp} \quad V_n \propto \mathcal{E}_n$$

Shape of nuclei

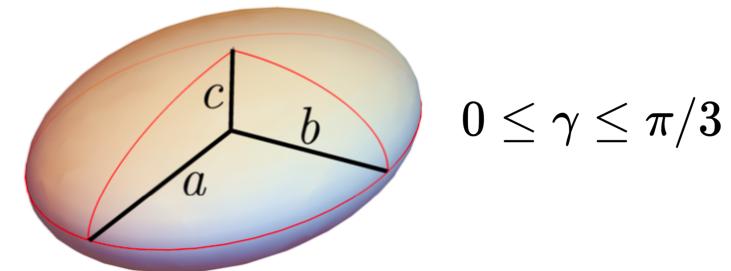
Most ground state stable nuclei are deformed

$$\rho(r, \theta, \phi) = \frac{\rho_0}{1 + e^{(r - R(\theta, \phi))/a_0}}$$

$$R(\theta, \phi) = R_0 \left(1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^3 \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^4 \alpha_{4,m} Y_{4,m} \right)$$



Triaxial spheroid: $a \neq b \neq c$.

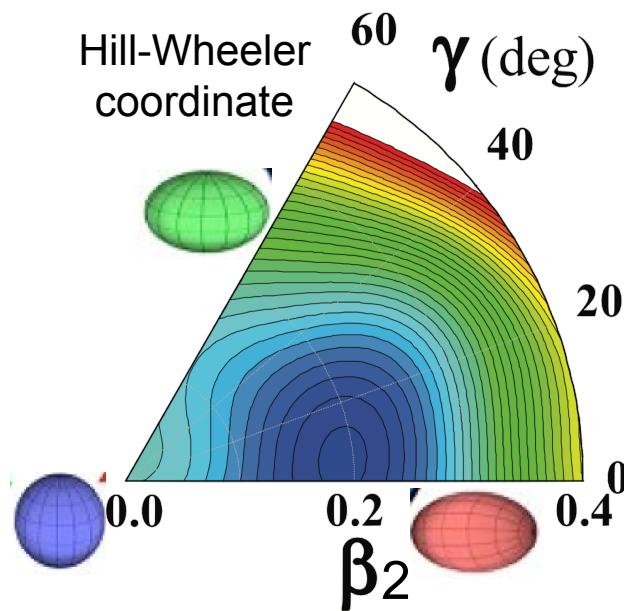
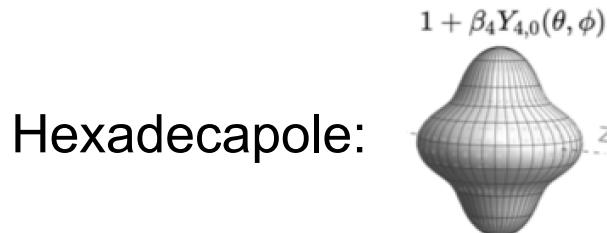
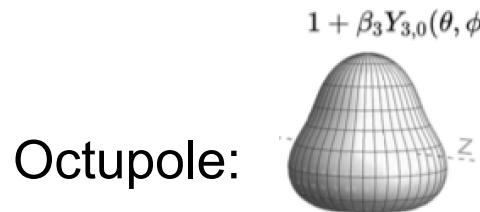
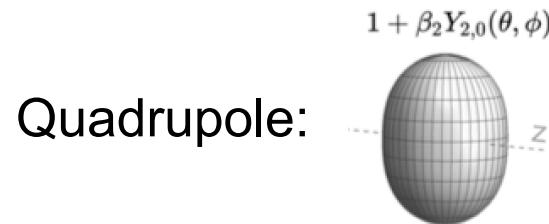


Prolate: $a=b < c \rightarrow \beta_2, \gamma=0$
 Oblate: $a < b=c \rightarrow \beta_2, \gamma=\pi/3$ or $-\beta_2, \gamma=0$

Shape of nuclei

$$\rho(r, \theta, \phi) = \frac{\rho_0}{1 + e^{(r - R(\theta, \phi))/a_0}}$$

$$R(\theta, \phi) = R_0 \left(1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] + \beta_3 \sum_{m=-3}^3 \alpha_{3,m} Y_{3,m} + \beta_4 \sum_{m=-4}^4 \alpha_{4,m} Y_{4,m} \right)$$

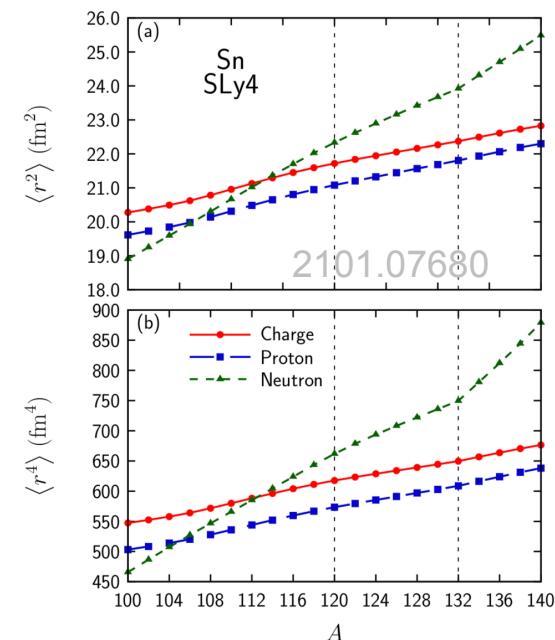
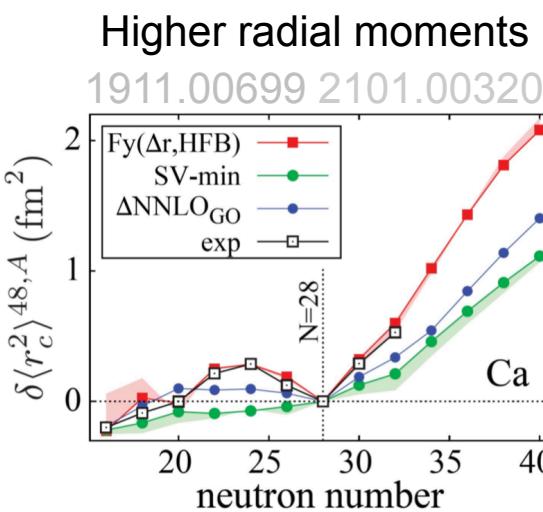
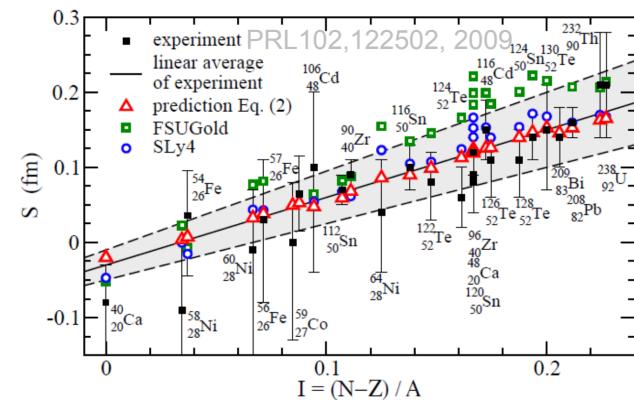
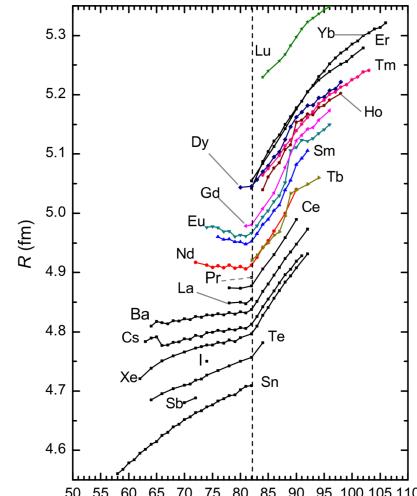
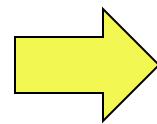
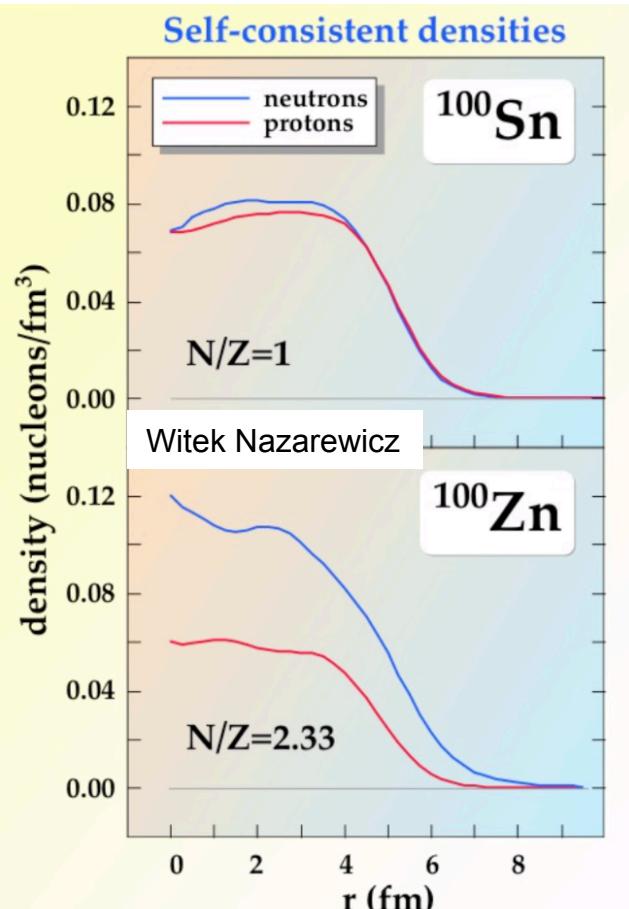


Radial structure of nuclei

$$\rho = \frac{\rho_0}{1 + e^{(r - R_0(1 + \sum_n \beta_n Y_n^0(\theta, \phi))) / a_0}}$$

R_0

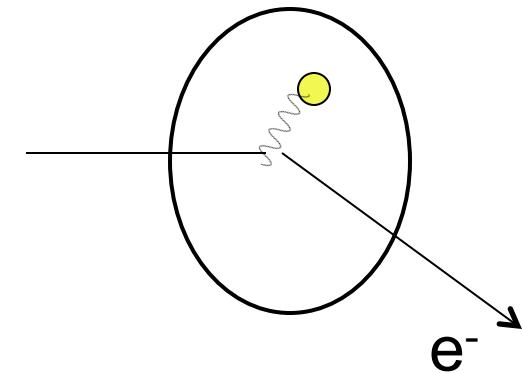
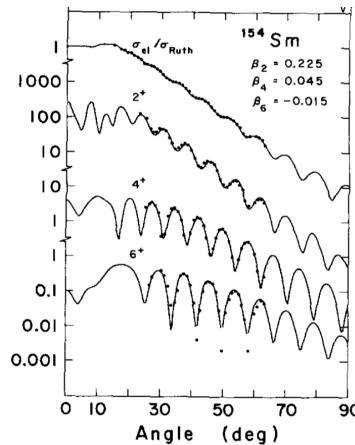
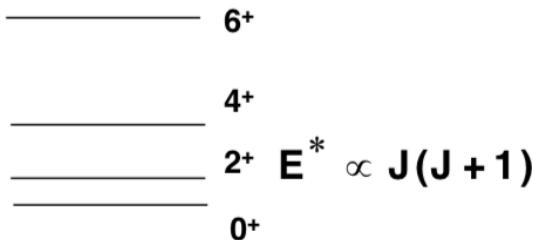
$\Delta r_{np} = R_n - R_p$



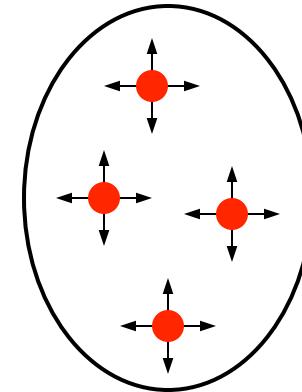
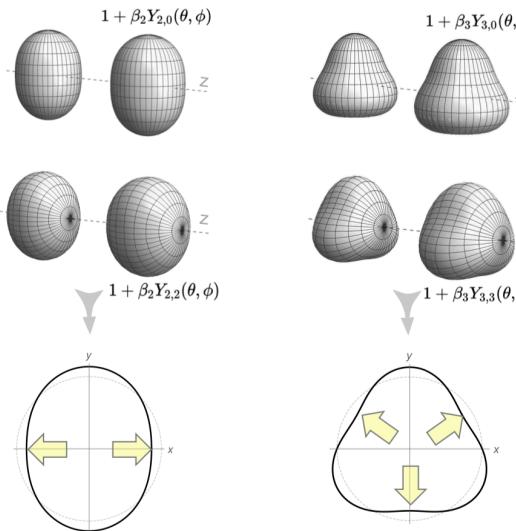
Nuclear structure vs HI method

- Shape from $B(En)$, radial profile from $e+A$ or ion-A scattering

«rotational» spectrum



- Probe entire mass distribution: multi-point correlations



$$\begin{aligned} S(\mathbf{s}_1, \mathbf{s}_2) &\equiv \langle \delta\rho(\mathbf{s}_1)\delta\rho(\mathbf{s}_2) \rangle \\ &= \langle \rho(\mathbf{s}_1)\rho(\mathbf{s}_2) \rangle - \langle \rho(\mathbf{s}_1) \rangle \langle \rho(\mathbf{s}_2) \rangle. \end{aligned}$$

nuclear shape imaging via collective flow response

Evidence of deformation in U+U vs Au+Au

7

<https://indico.cern.ch/event/854124/contributions/4135480/>

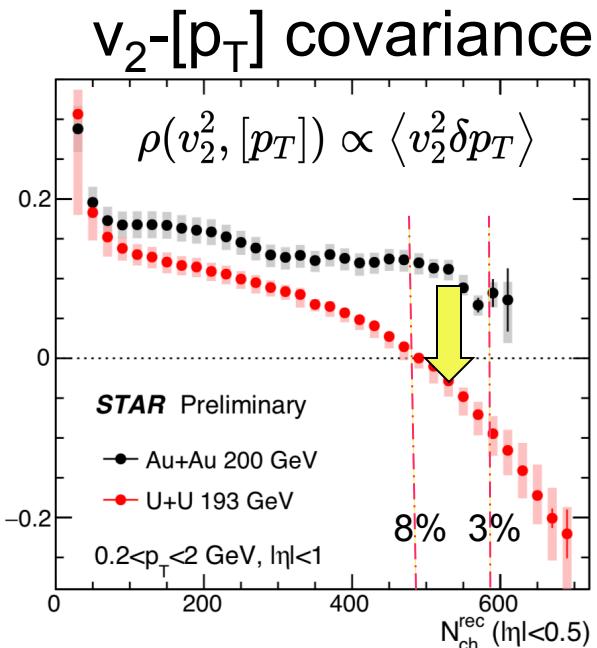
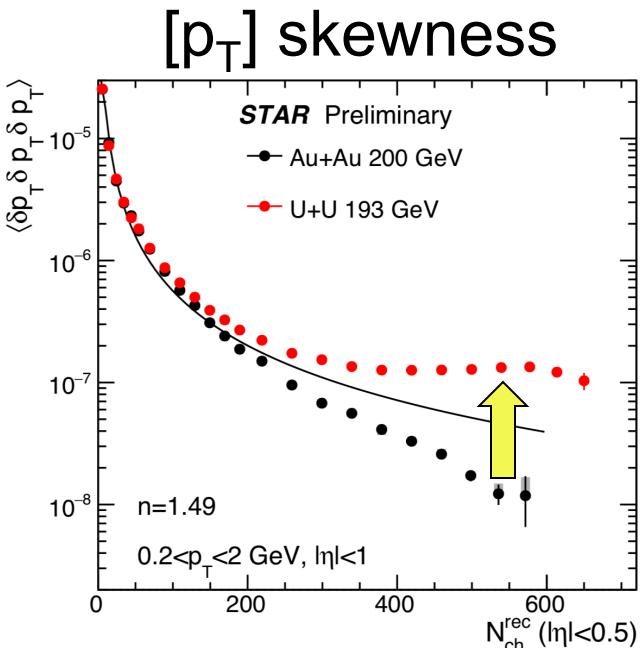
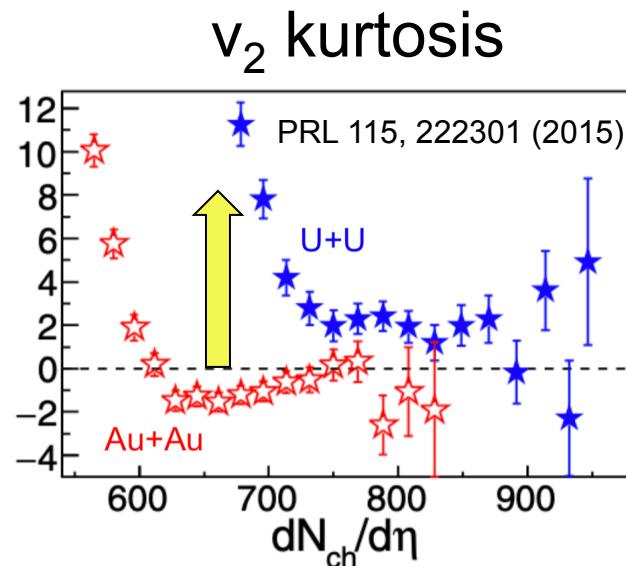
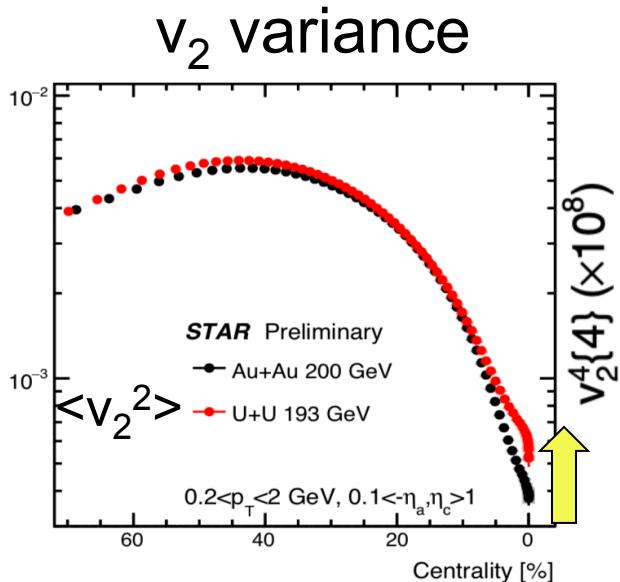
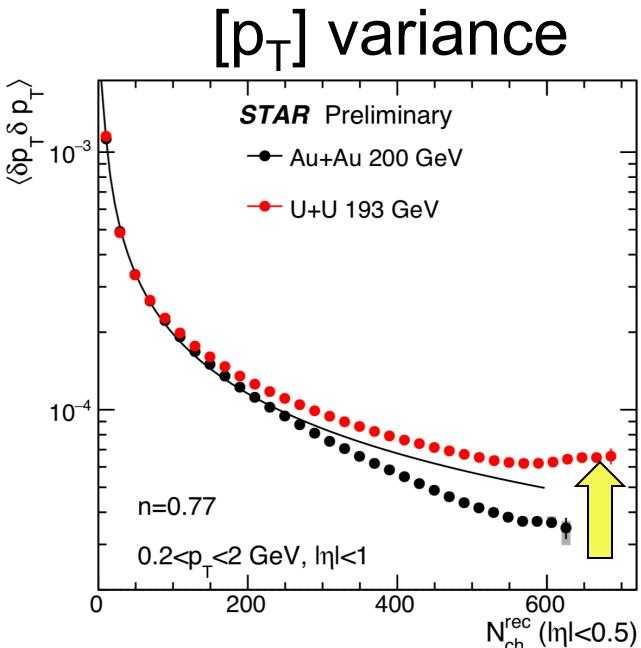
Collisions at $\sqrt{s_{NN}}=193\text{-}200 \text{ GeV}$

Large deformation in ^{238}U
relative to ^{197}Au strongly
influence flow signals

$$\beta_{2\text{U}} \sim 0.28$$

$$\beta_{2\text{Au}} \sim -0.13?$$

Working to turn these into
a quantitative tool !



Parametric dependence

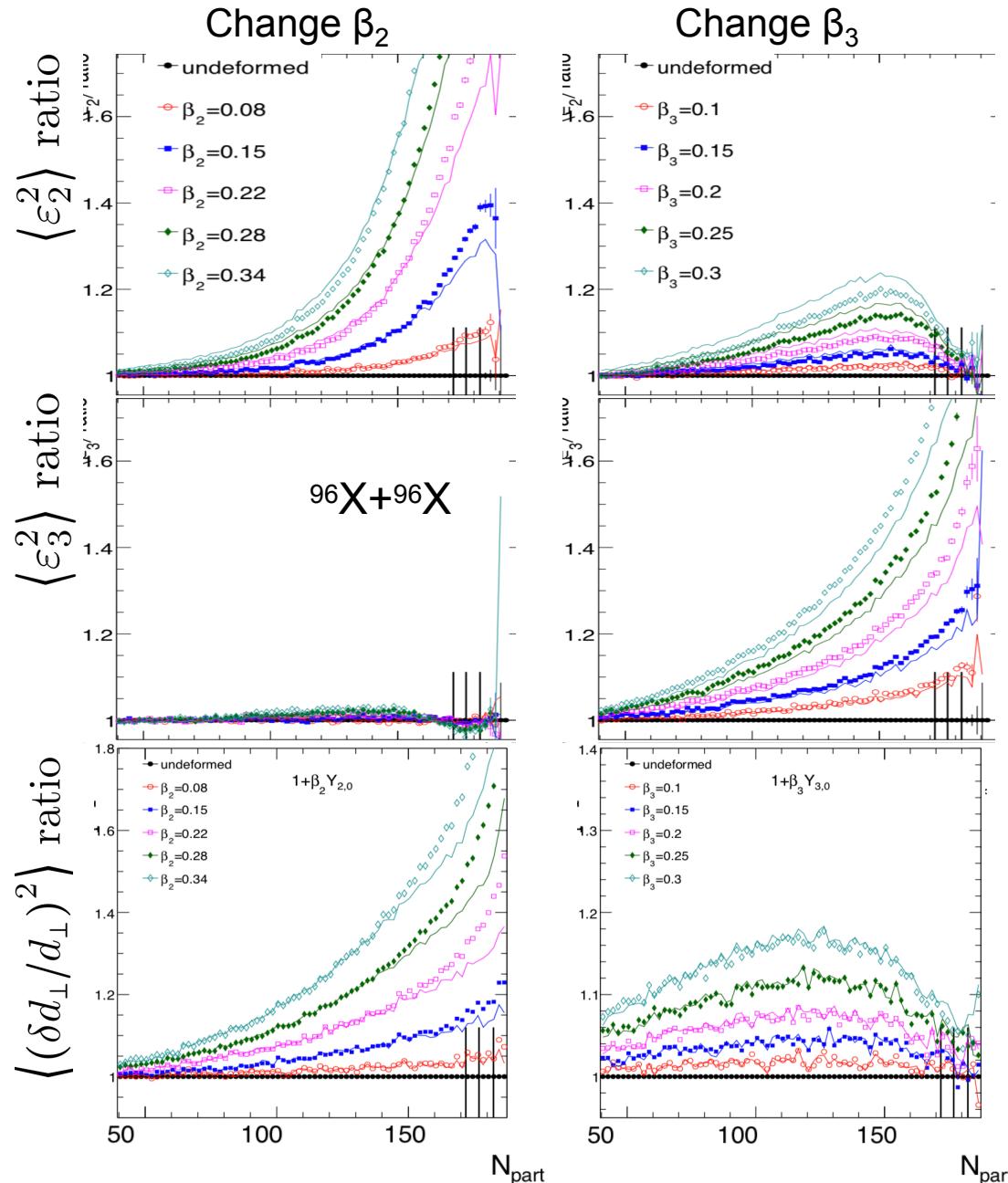
- ϵ_n has the form $\epsilon_n = \epsilon_{n;0} + \sum_{m=2}^4 \underbrace{p_{n;m}(\Omega_1, \Omega_2)}_{\text{undeformed}} \beta_m + \mathcal{O}(\beta^2)$
 γ only appear here, in the form of $\cos 3\gamma, \cos 6\gamma, \dots$
- $R_\perp^2 = \langle x^2 \rangle + \langle y^2 \rangle$ has the form $\delta d_\perp/d_\perp = \delta_d + \sum_{m=2}^4 p_{0;m}(\Omega_1, \Omega_2) \beta_m + \mathcal{O}(\beta^2)$
 $d_\perp \equiv 1/R_\perp$
- Two particle correlation

$$\langle \varepsilon_n^2 \rangle \approx \langle \varepsilon_{n;0}^2 \rangle + \sum_m \langle p_{n;m} p_{n;m}^* \rangle \beta_m^2 \quad \left\langle \left(\frac{\delta d_\perp}{d_\perp} \right)^2 \right\rangle \approx \langle \delta_d^2 \rangle + \sum_m \langle p_{0;m}^2 \rangle \beta_m^2$$

- Consider also the influence of R_0 and a $\frac{\rho_0}{1 + e^{(r - \mathbf{R}_0(1 + \sum_n \beta_n Y_n^0(\theta, \phi))) / a_0}}$
- Linear response to relate to final state: $v_n \propto \varepsilon_n \frac{\delta[p_T]}{[p_T]} \propto \frac{\delta d_\perp}{d_\perp}$

Parametric dependence

See 2106.08768

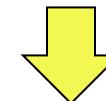


medium size system:

$$\varepsilon_2^2 = a'_2 + b'_2 \beta_2^2 + b'_{2,3} \beta_3^2$$

$$\varepsilon_3^2 = a'_3 + b'_3 \beta_3^2$$

$$(\delta d_\perp/d_\perp)^2 = a'_0 + b'_0 \beta_2^2 + b'_{0,3} \beta_3^2$$



$$v_2^2 = a_2 + b_2 \beta_2^2 + b_{2,3} \beta_3^2$$

$$v_3^2 = a_3 + b_3 \beta_3^2$$

$$(\delta p_T/p_T)^2 = a_0 + b_0 \beta_2^2 + b_{0,3} \beta_3^2$$

Isobar collisions as precision tool

- Unique running mode of RHIC and STAR to minimize systematics
 - 0.4% precision is achieved in ratio of many observables between two isobar systems → precision imaging tool

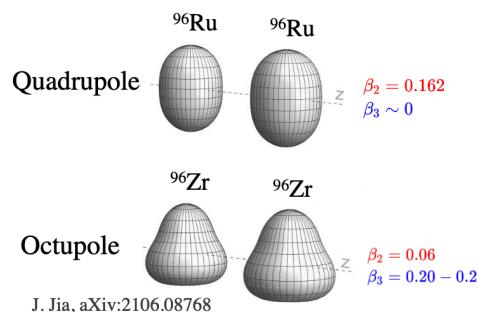
A key question for any
HI observable \mathcal{O}

$$\frac{\mathcal{O}_{X+X}}{\mathcal{O}_{Y+Y}} = ?$$

${}^A\text{X} + {}^A\text{X}$ vs ${}^A\text{Y} + {}^A\text{Y}$

Deviation from 1 must have its origin in the nuclear structure, which is reflected by the initial state and then survives the final state. A precision tool to study initial state and final state responses

■ Expectation



$$\mathcal{O} \approx b_0 + b_1 \beta_2^2 + b_2 \beta_3^2 + b_3 (R_0 - R_{0,\text{ref}}) + b_4 (a - a_{\text{ref}})$$

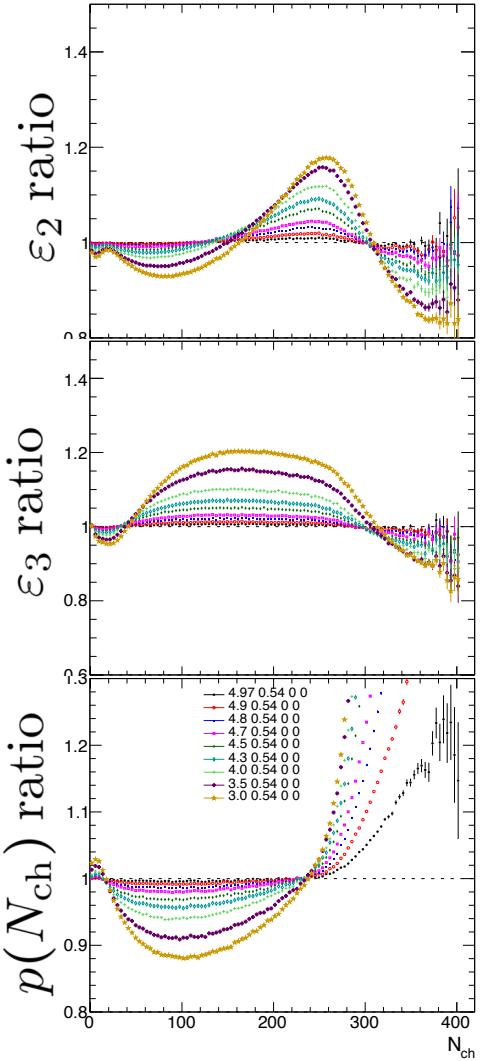
$$R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\text{Ru}}}{\mathcal{O}_{\text{Zr}}} \approx 1 + c_1 \Delta \beta_2^2 + c_2 \Delta \beta_3^2 + c_3 \Delta R_0 + c_4 \Delta a$$

Species	β_2	β_3	a_0	R_0
Ru	0.162	0	0.46 fm	5.09 fm
Zr	0.06	0.20	0.52 fm	5.02 fm
difference	$\Delta \beta_2^2$	$\Delta \beta_3^2$	Δa_0	ΔR_0
	0.0226	-0.04	-0.06 fm	0.07 fm

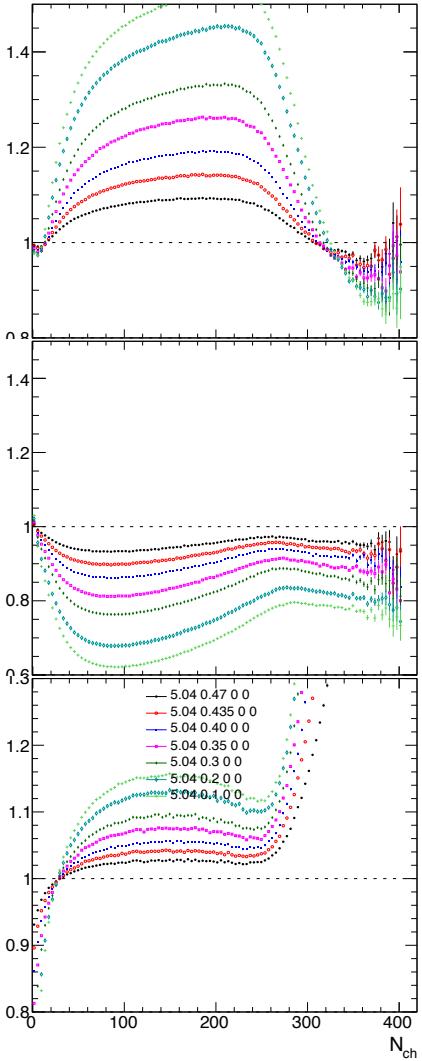
Valid for most single- and two-particle observable: $v_2, v_3, p(N), \langle p_T \rangle, \langle \delta p_T^2 \rangle \dots$

Glauber results: N_{ch} dep

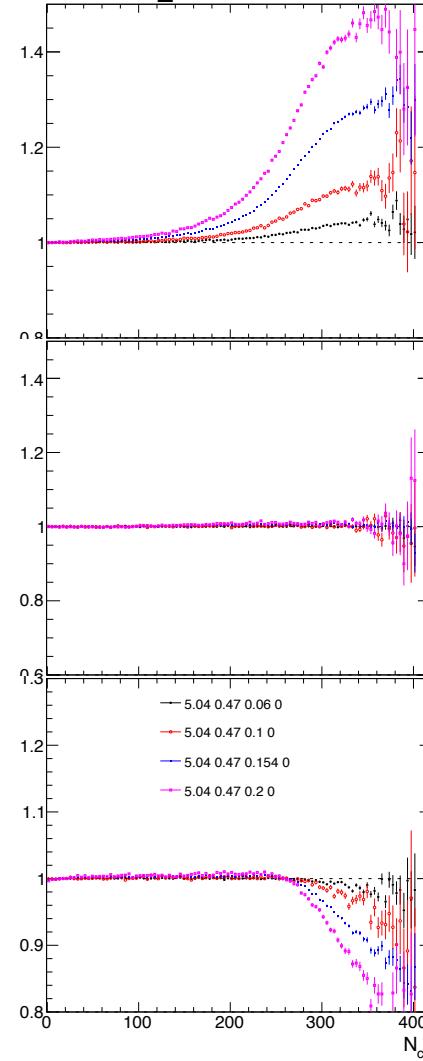
ΔR dep
5.04fm \rightarrow 3.0fm



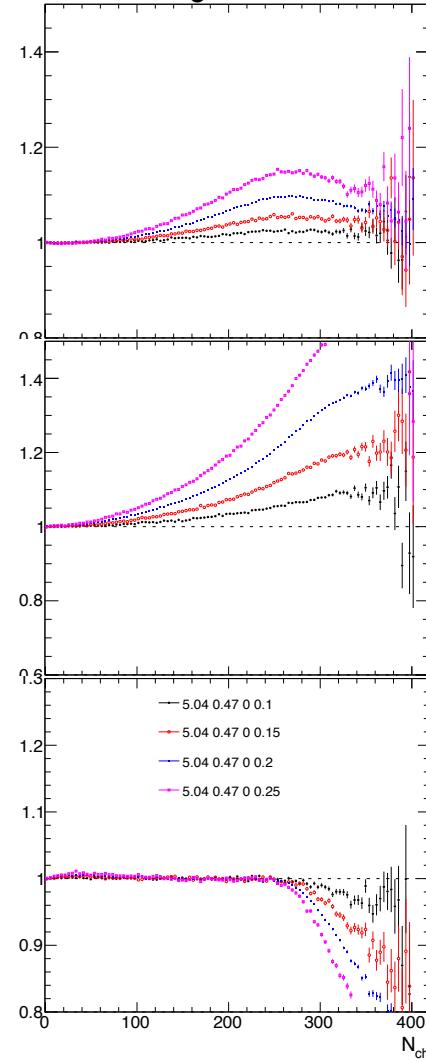
Δa dep
0.54fm \rightarrow 0.1fm



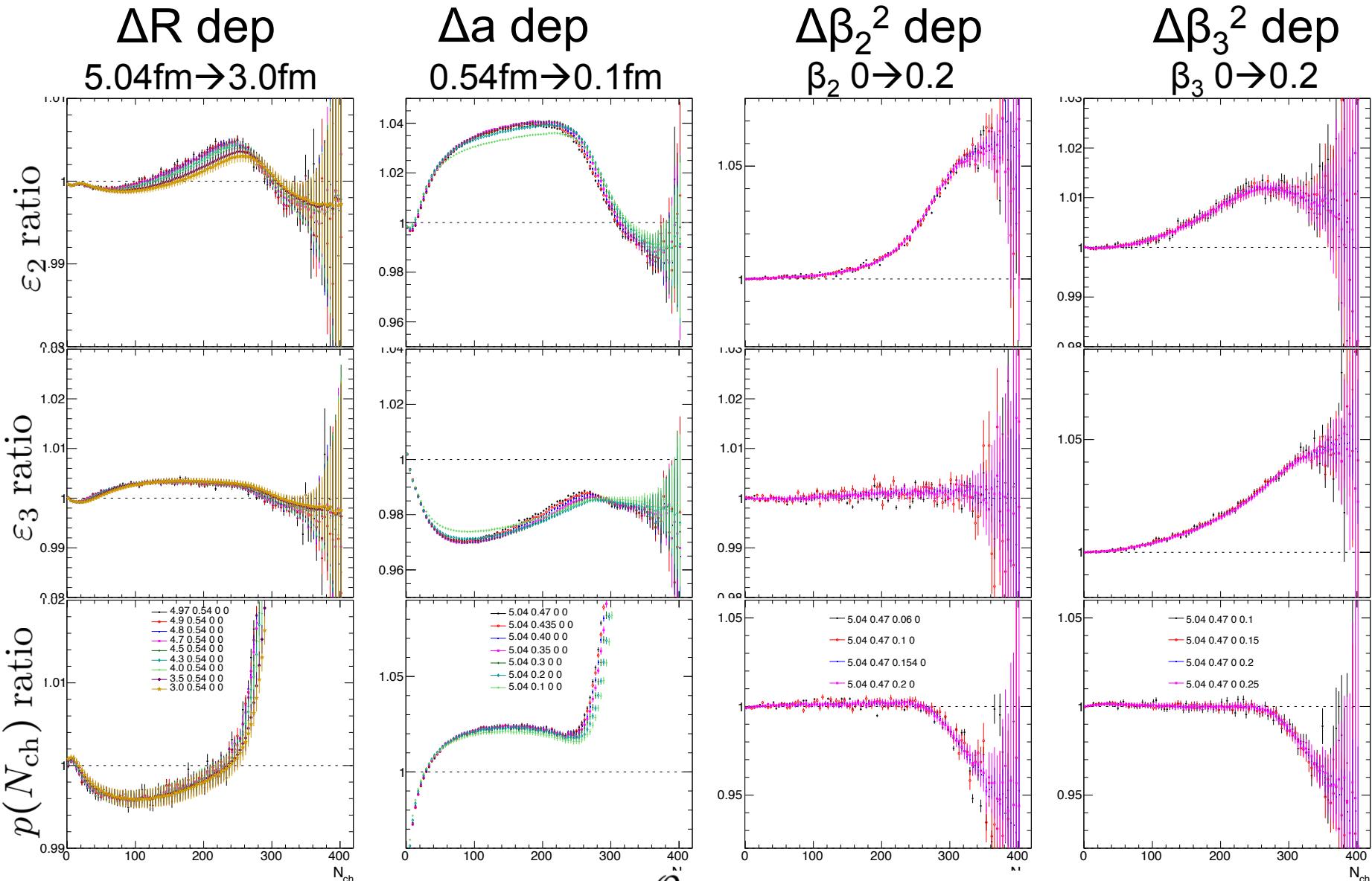
$\Delta \beta_2^2$ dep
 β_2 0 \rightarrow 0.2



$\Delta \beta_3^2$ dep
 β_3 0 \rightarrow 0.2

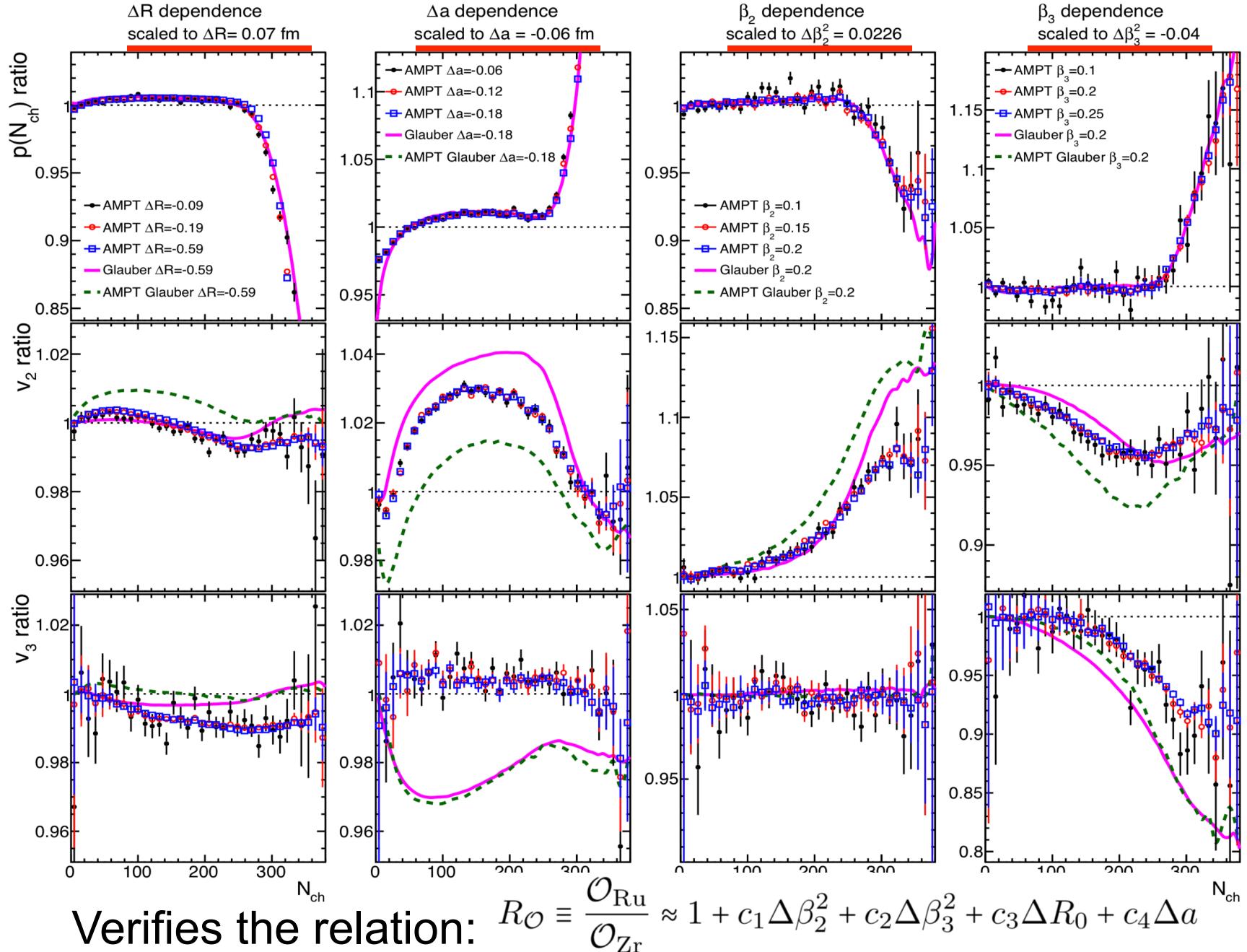


Glauber results scaled



Verifies the relation: $R_{\mathcal{O}} \equiv \frac{\mathcal{O}_{\text{Ru}}}{\mathcal{O}_{\text{Zr}}} \approx 1 + c_1 \Delta \beta_2^2 + c_2 \Delta \beta_3^2 + c_3 \Delta R_0 + c_4 \Delta a$

AMPT results: scaled



Scaling approach to nuclear structure

14

arXiv:2111.15559

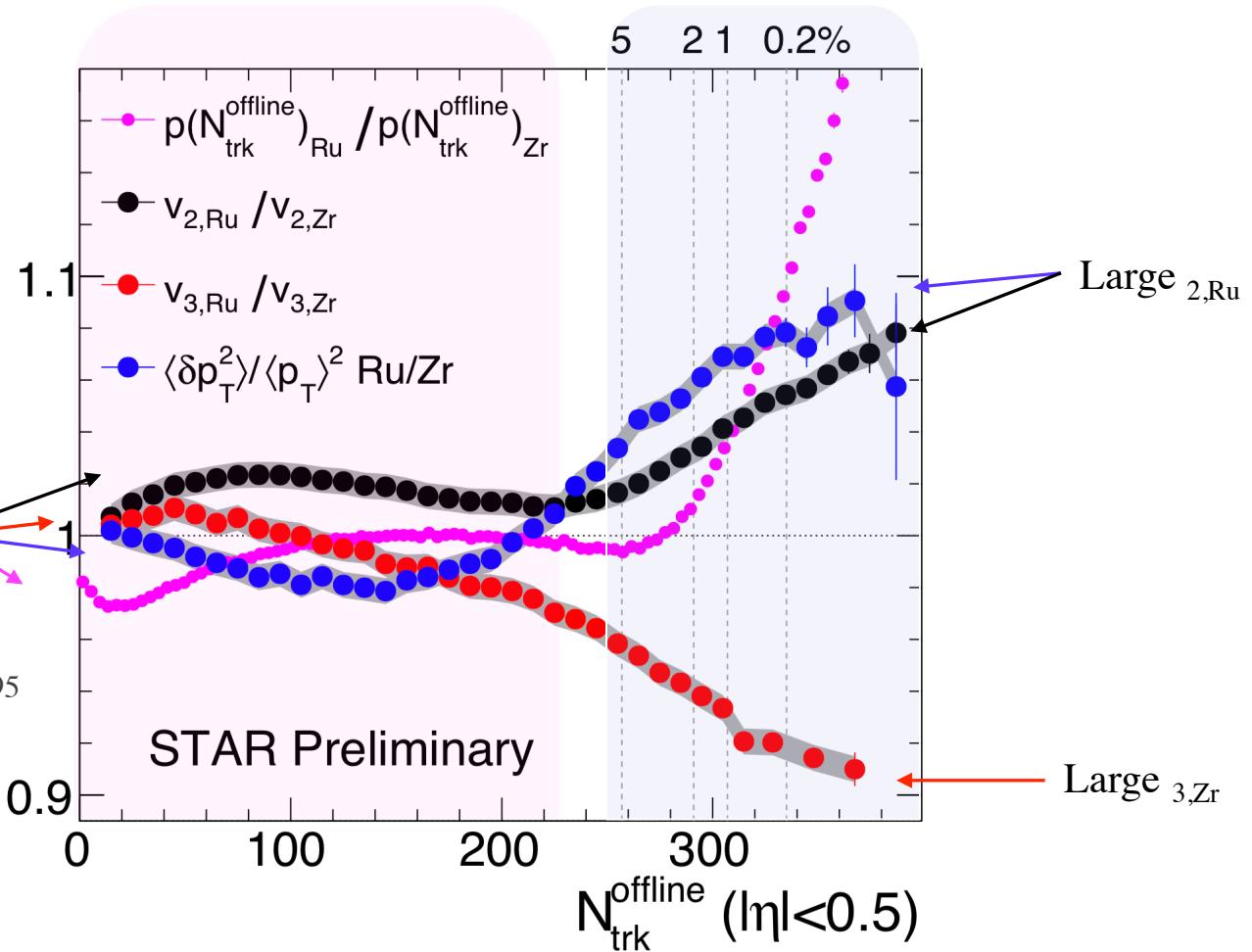
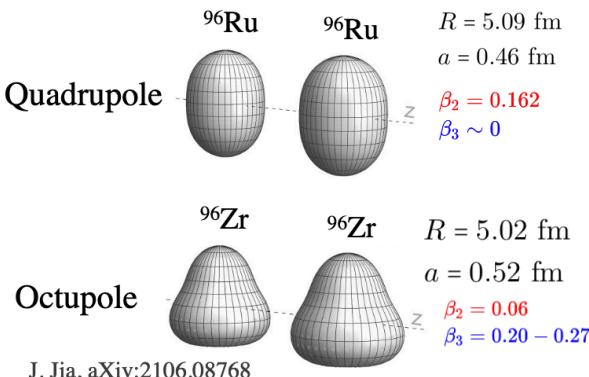
$$\frac{\mathcal{O}_{\text{Ru}}}{\mathcal{O}_{\text{Zr}}} \approx 1 + c_1 \Delta \beta_2^2 + c_2 \Delta \beta_3^2 + c_3 \Delta R_0 + c_4 \Delta a$$

Valid for most single- and two-particle observable: $v2, v3, p(N), \langle p_T \rangle, \langle \delta p_T^2 \rangle \dots$

- Determine c_n once, and predict ratios for other parameter values.
- Constrain parameters via χ^2 analysis or Bayesian inference.
- Generalize to multi-particle observables\dots

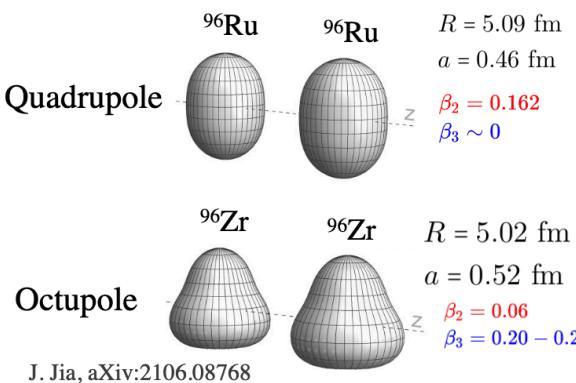
Compare with isobar data

see Chunjian Zhang talk next

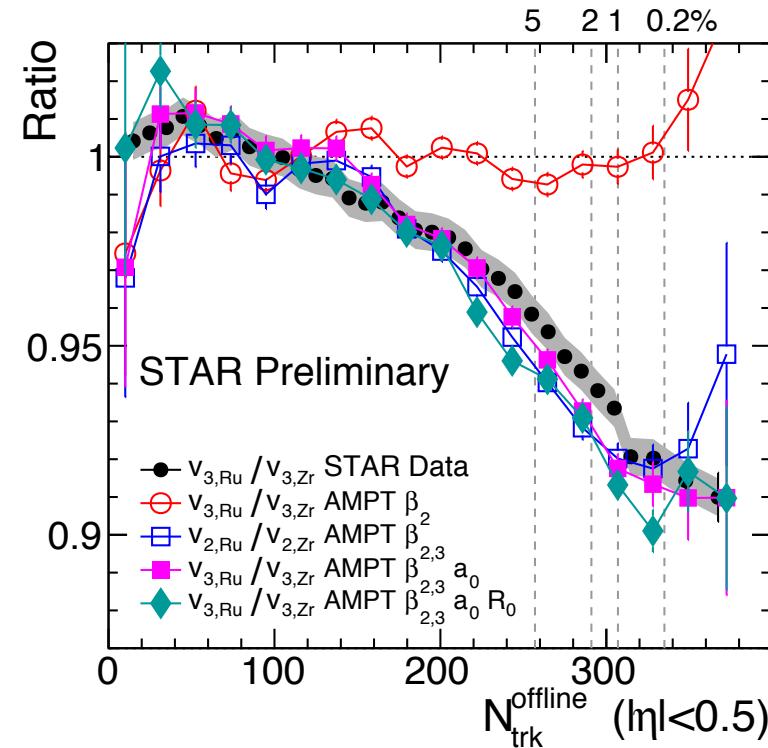
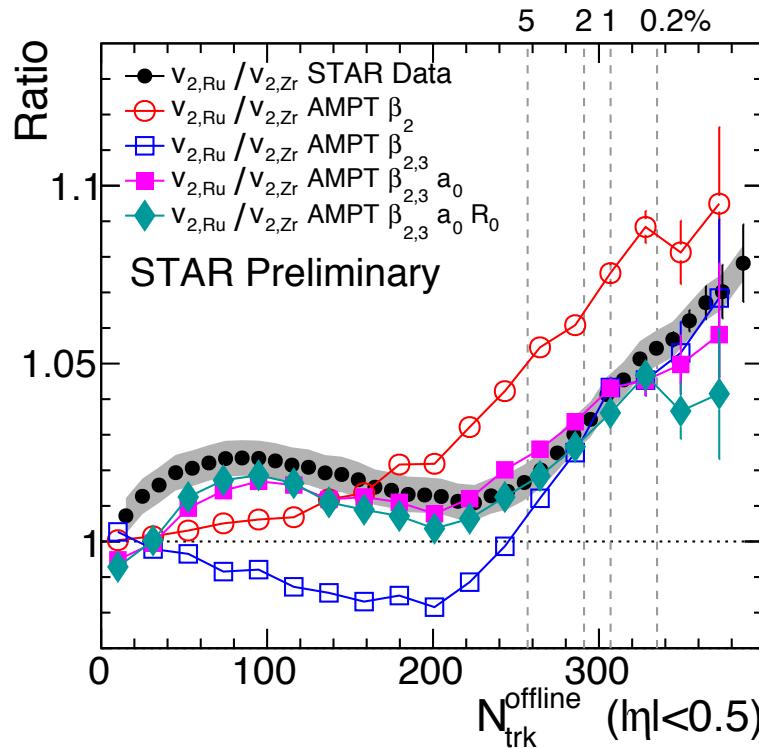


Use these ratios to probe shape and radial structure of nuclei.

Nuclear structure via v_n -ratio

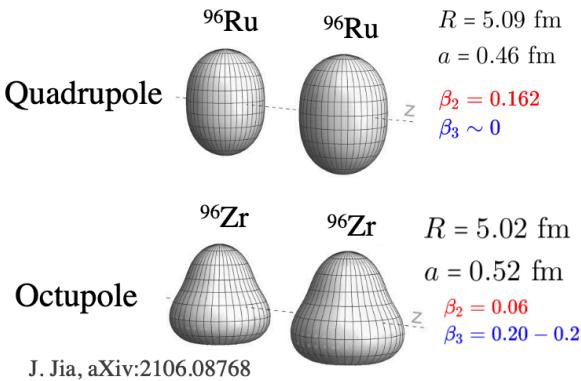


- $\beta_{2\text{Ru}} \sim 0.16$ increase v_2 , no influence on v_3 ratio
- $\beta_{3\text{Zr}} \sim 0.2$ decrease v_2 in mid-central, decrease v_3 ratio
- diffu. $\Delta a_0 = -0.06 \text{ fm}$ increase v_2 mid-central, no influe. on v_3 .
 - Similar study by Haojie et.al.
- Radius $\Delta R_0 = 0.07 \text{ fm}$ only slightly affects v_2 and v_3 ratio.

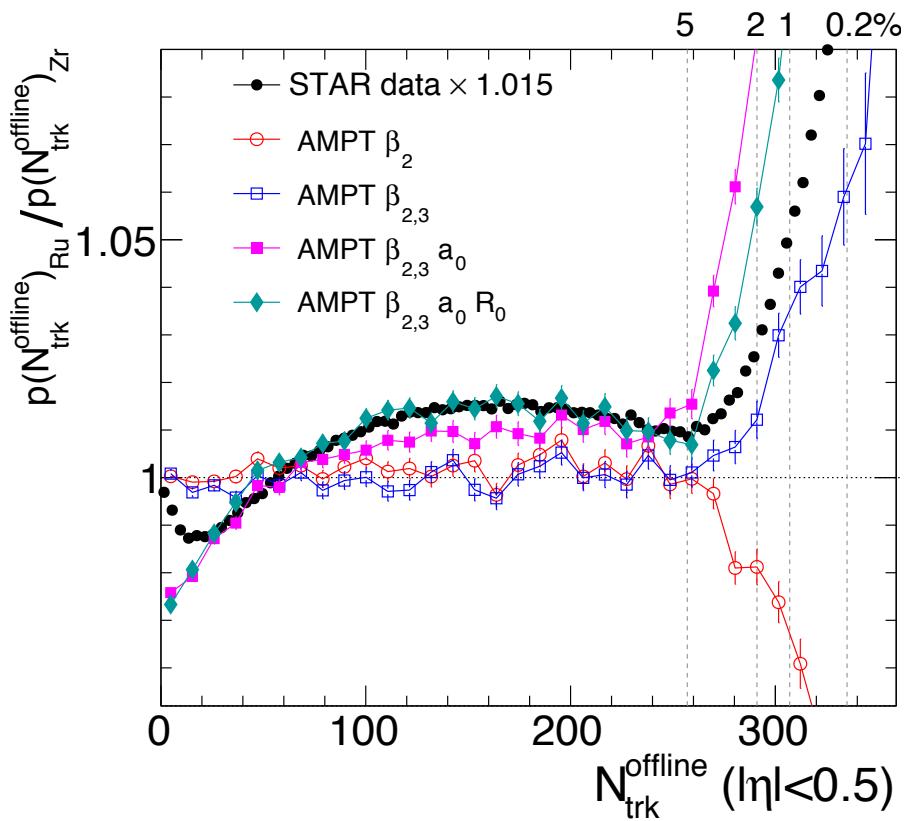


Simultaneously constrain these parameters using different N_{ch} regions

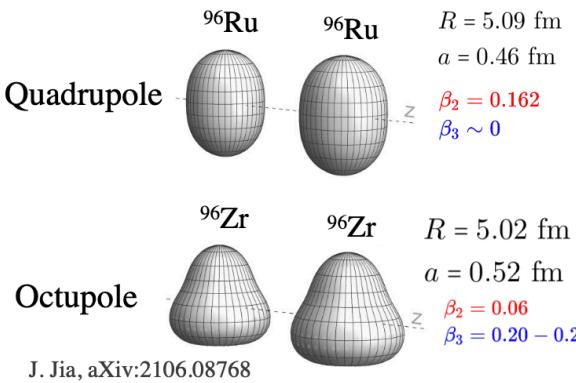
Nuclear structure via p(N_{ch})-ratio



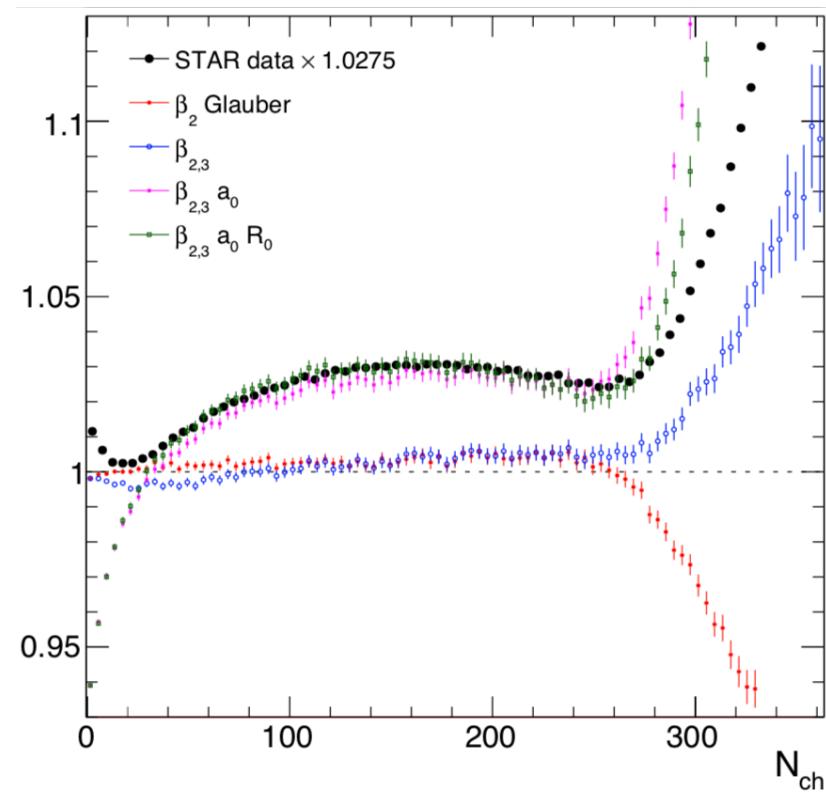
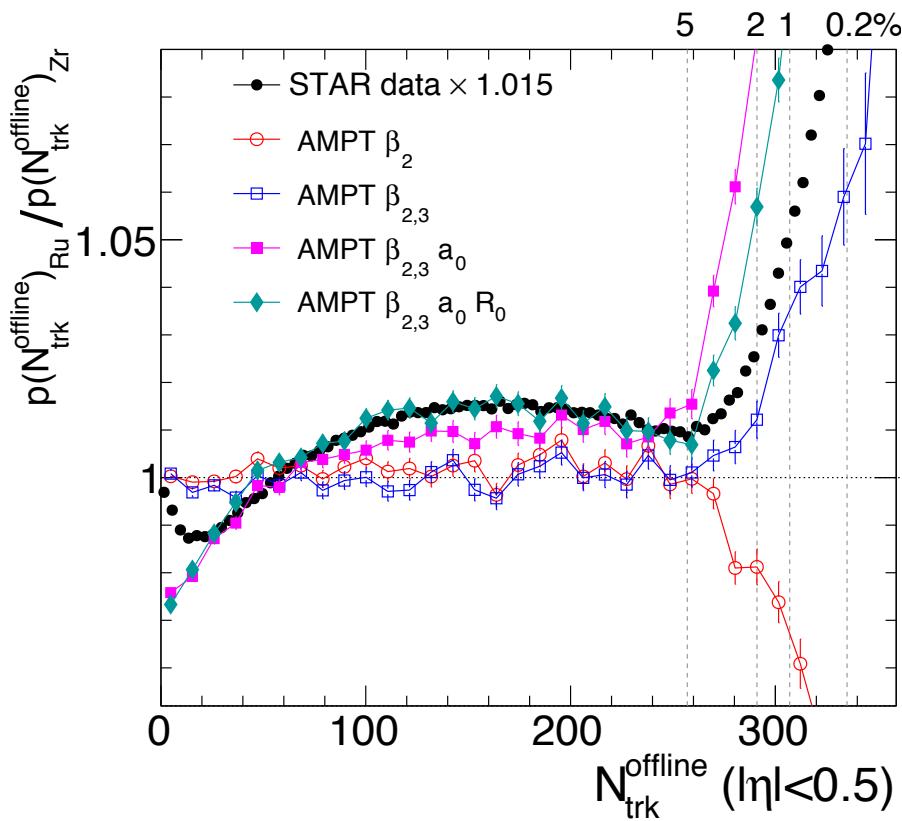
- $\beta_{2\text{Ru}} \sim 0.16$ decrease ratio, increase after considering $\beta_{3\text{Zr}} \sim 0.2$
- The bump structure in non-central region mostly sensitive to differences in surface diffuseness Δa_0 and radius ΔR_0



Nuclear structure via p(N_{ch})-ratio



- $\beta_{2\text{Ru}} \sim 0.16$ decrease ratio, increase after considering $\beta_{3\text{Zr}} \sim 0.2$
- The bump structure in non-central region mostly sensitive to differences in surface diffuseness Δa_0 and radius ΔR_0
- All these trends are quantitatively reproduced by Glauber
 - Note the normalization is sensitive to trigger efficiency!!
 - See related study by Haojie et.al.

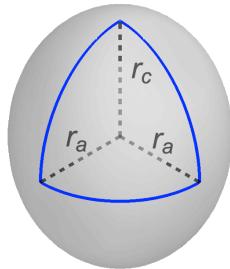


Triaxiality γ :

$$R(\theta, \phi) = R_0 \left(1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] \right)$$

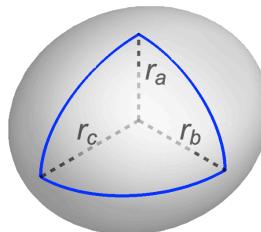
Prolate

$$\beta_2 = 0.25, \cos(3\gamma) = 1$$



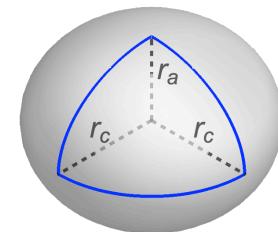
Triaxial

$$\beta_2 = 0.25, \cos(3\gamma) = 0$$

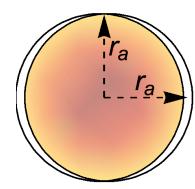


Oblate

$$\beta_2 = 0.25, \cos(3\gamma) = -1$$

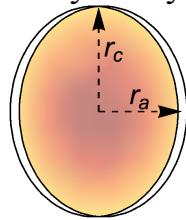


tip+tip



$$\epsilon_2 \downarrow, R_\perp \downarrow$$

body+body



$$\epsilon_2 \uparrow, R_\perp \uparrow$$

$$\text{cov}(\epsilon_2^2, R_\perp) > 0$$

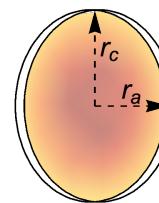
$$\text{cov}(\epsilon_2^2, d_\perp) < 0$$

$$[p_T] \sim 1/R_\perp = d_\perp$$

$$\text{cov}(\epsilon_2^2, R_\perp) = 0$$

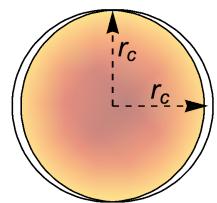
$$\text{cov}(\epsilon_2^2, d_\perp) = 0$$

body+body



$$\epsilon_2 \uparrow, R_\perp \downarrow$$

tip+tip



$$\epsilon_2 \downarrow, R_\perp \uparrow$$

$$\text{cov}(\epsilon_2^2, R_\perp) < 0$$

$$\text{cov}(\epsilon_2^2, d_\perp) > 0$$

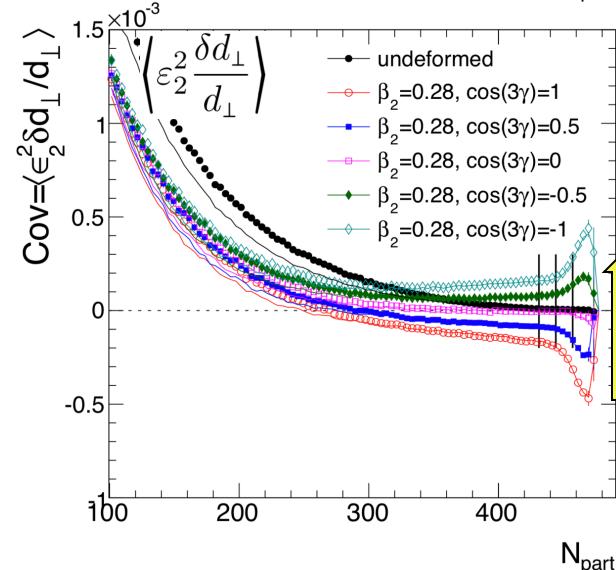
ϵ_2, R_\perp no linear correlation

Influence of triaxiality on initial state

Skewness super sensitive to γ ☺

Described by

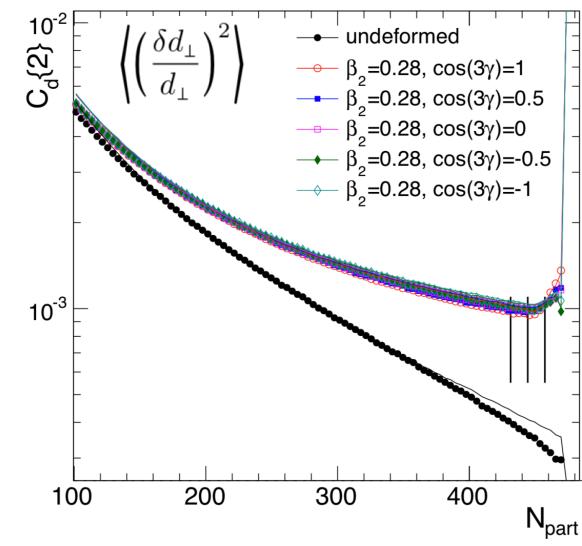
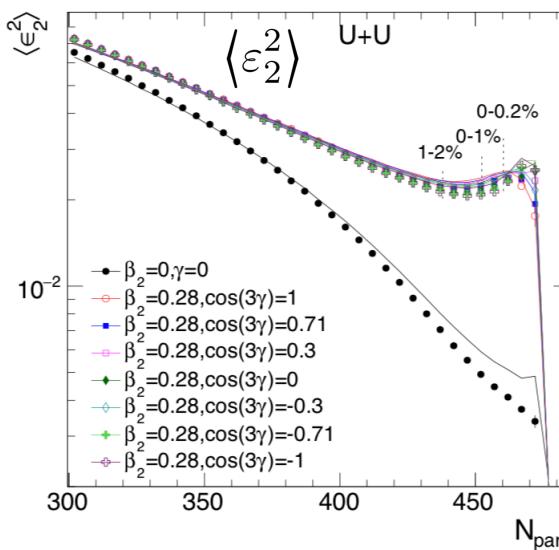
$$a' + (b' + c' \cos(3\gamma)) \beta_2^3$$



variances insensitive to γ

Only a function of β_2 in central

$$a' + b' \beta_2^2$$



Use variance to constrain β_2 , use skewness to constrain γ

(β_2, γ) diagram in heavy-ion collisions

The (β_2, γ) dependence in 0-1% U+U Glauber model can be approximated by:

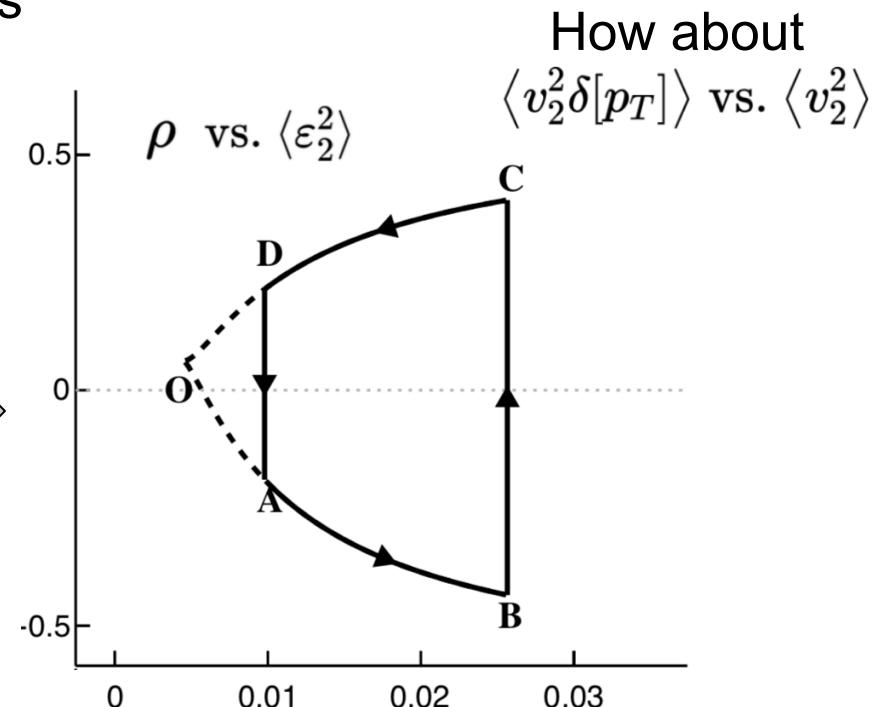
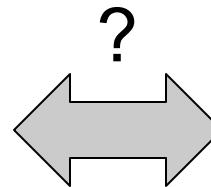
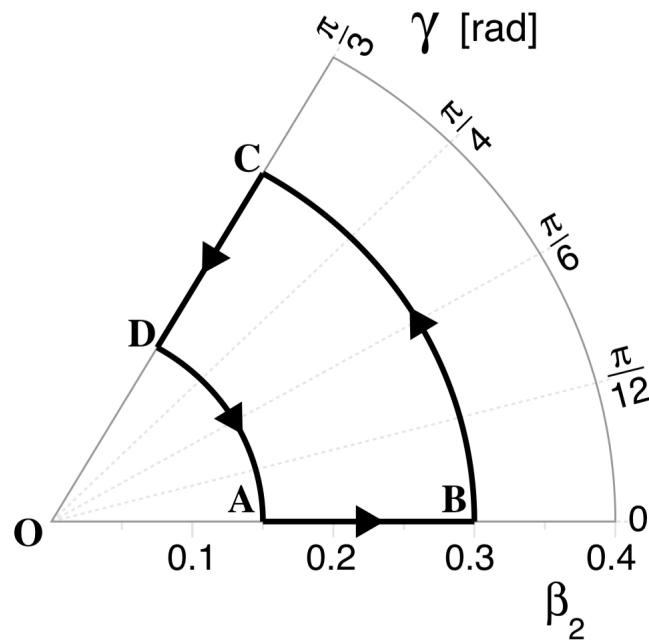
$$\langle \varepsilon_2^2 \rangle \approx [0.02 + \beta_2^2] \times 0.235$$

$$\langle (\delta d_\perp/d_\perp)^2 \rangle \approx [0.035 + \beta_2^2] \times 0.0093$$

$$\langle \varepsilon_2^2 \delta d_\perp/d_\perp \rangle \approx [0.0005 - (0.07 + 1.36 \cos(3\gamma)) \beta_2^3] \times 10^{-2}$$

$$\rho = \frac{\langle \varepsilon_2^2 \delta d_\perp \rangle}{\langle \varepsilon_2^2 \rangle \sqrt{\langle (\delta d_\perp)^2 \rangle}}$$

Map from (β_2, γ) plane to HI observables

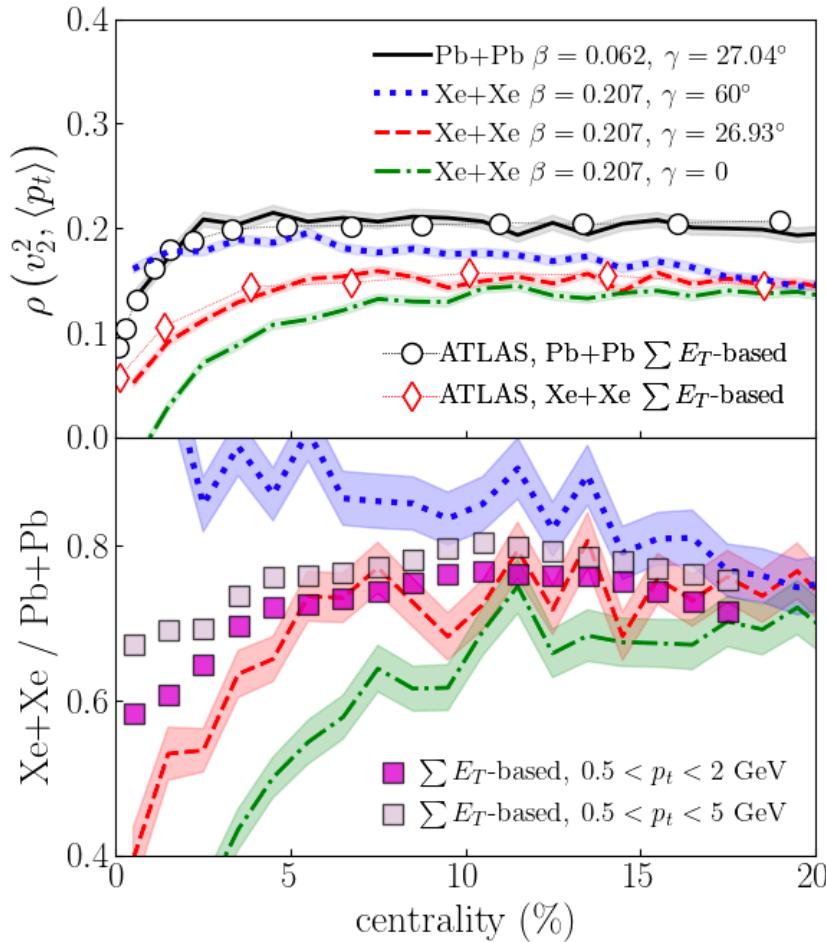


Collision system scan to map out this trajectory: calib. coefficients with species with known β, γ , then predict for species of interest.

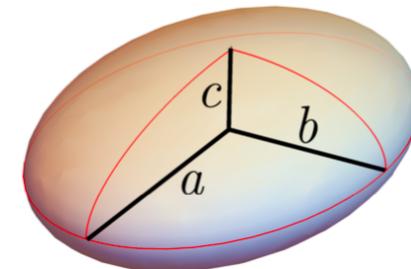
Triaxiality from v_2^2 - p_T correlation at LHC

22

B Bally, M Bender, G Giacalone, V Somà 2108.09578



$$R(\theta, \phi) = R_0 \left(1 + \beta_2 [\cos \gamma Y_{2,0} + \sin \gamma Y_{2,2}] \right)$$



- Clear sensitivity to the triaxiality of 129Xe.

Outlook

- Precision of hydrodynamics allow imaging of nuclear structure
- Great opportunity for possible (isobaric) system scans.
 - Larger systems have better statistical sensitivity e.g. ^{136}Xe vs ^{136}Ce
 - Small systems to disentangle geometry from initial momentum anisotropy
 - Profit from larger multiply/acceptance from LHC and low energy at NICA
 - Manifestation of nuclear structure are \sqrt{s} and rapidity dependent!
- Model study to explore the connection to nuclear structure and potential for heavy ion physics.

We're only limited by our imagination ...

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RIKEN BNL Research Center

Physics Opportunities from the RHIC Isobar Run

This workshop will be held virtually.

January 25–28, 2022

<https://www.bnl.gov/porir2022/index.php>



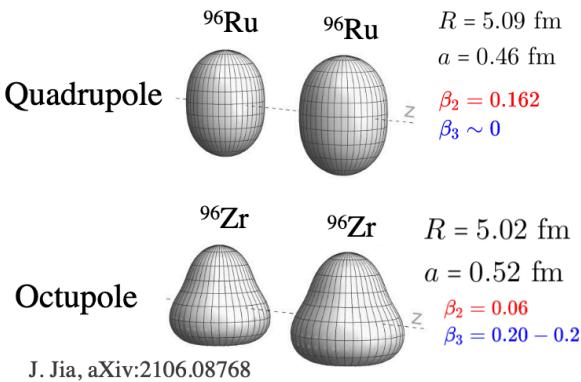
INSTITUTE for
NUCLEAR THEORY

**“Intersection of nuclear structure and
high-energy nuclear collisions”**

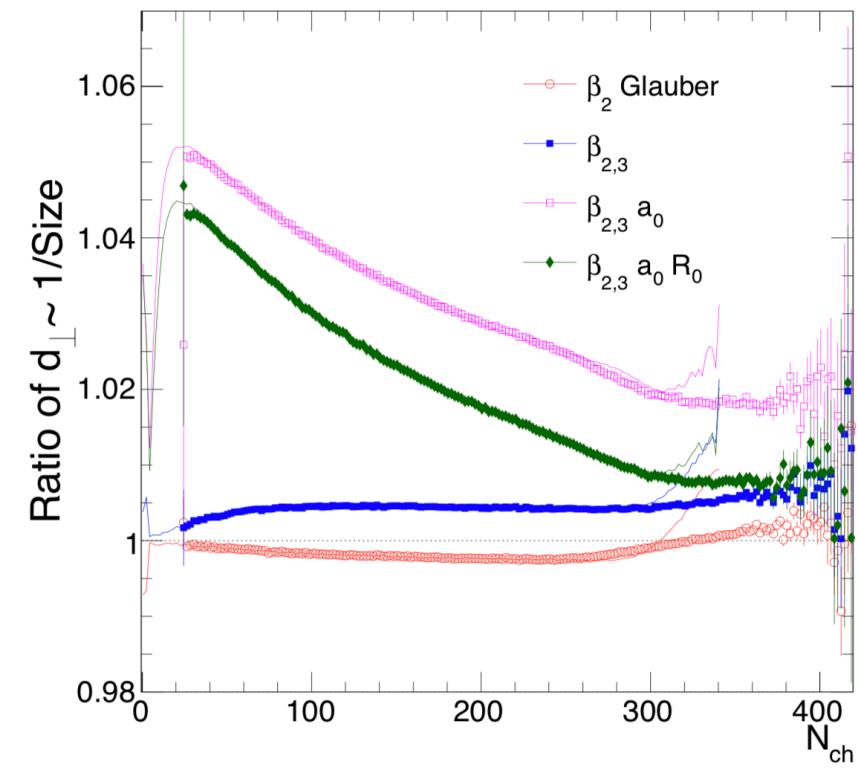
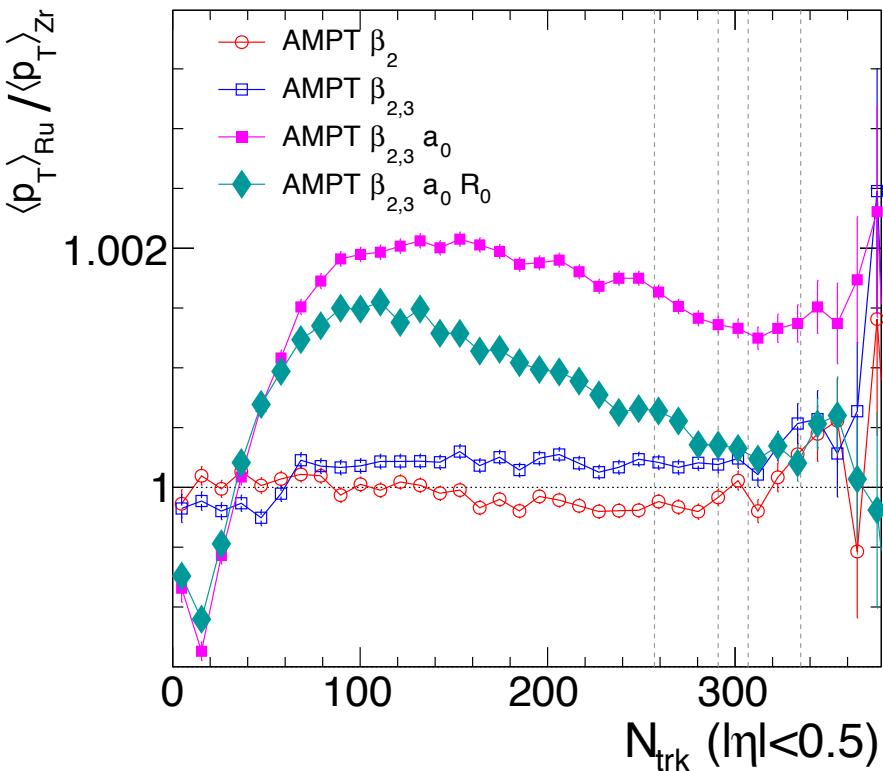
January 23 – February 24, 2023

Giuliano Giacalone, Jiangyong Jia, Dean Lee,
Matthew Luzum, Jaki Noronha-Hostler , Fuqiang Wang

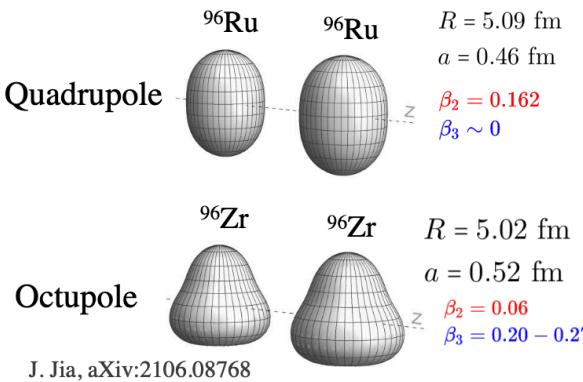
Nuclear structure via $\langle p_T \rangle$



- AMPT underestimate response by x3, so trust trends only
- β_2, β_3 small impact in noncentral, but some increase in UCC
- Enhancement dominated by surface diffuseness
- Radius difference leads to stronger N_{ch} dependence
- Glauber model also describe the trends..



Nuclear structure via p_T fluctuations



- Glauber model is used by assuming $\frac{\delta[p_T]}{[p_T]} \propto \frac{\delta d_\perp}{d_\perp}$
- $\beta_{2\text{Ru}} \sim 0.16$ increase ratio while $\beta_{3\text{Zr}} \sim 0.2$ decrease it
- AMPT has wrong $\langle p_T \rangle$ responses (see 2109.00604), but..

