Motivations and precision targets for an accurate luminosity determination at the LHC

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Abstract

We present a pedagogical introduction to the physics implications of a precise knowledge of the LHC luminosity, defining the goals and some benchmark accuracy targets.

INTRODUCTION

The cross section for the production of a general final state O at the LHC is given by the following formula:

$$\sigma(pp \to O + X) = \int dx_1\, dx_2 \times \sum_{i,j} f_i(x_1, Q) f_j(x_2, Q) \hat{\sigma}_{(ij\to O)}(M_O, g_{ijO}, \ldots)\,. \quad (1)$$

Here, $f_i(x, Q)$ is the density of partons (PDF) of type i (quarks of different flavours or gluons) inside the proton, carrying a fraction x of the proton momentum at a resolution scale Q. Theory predicts the PDFs to be independent of O. $\hat{\sigma}_{(ij\to O)}$ is the *partonic* cross section to produce the final state O in the collisions of partons i and j. It depends on properties of the final state (e.g. the mass of O, M_O, the momenta of the various particles involved, etc), and on the the nature of the interactions involved in the process (for example the strength, g_{ijO}, of the coupling between i, j and O). Parameters like M_O and g_{ijO} are therefore what defines the underlying theory, and extracting their value as accurately as possible is the ultimate goal of an experimental measurement. For example, M_Z and the weak-force mixing angle $\sin^2\theta_W$ were determined at LEP/SLC by measuring the shape and normalization of the e^+e^- cross sections as a function of $\sqrt{S}$.

The precision of the extraction of these parameters is determined by:

- The precision of the calculation of $\hat{\sigma}_{ij\to O}$, as a function of M_O, g_{ijO}, etc.. This is a theoretical issue. Inclusion of higher and higher orders of perturbation theory makes the prediction more accurate.

- The precision of the knowledge of the PDFs. This touches on both theory and experiment. For example, experimental data from measurements such as deep inelastic scattering (DIS) are necessary to extract the PDFs, by fitting these data to the DIS equivalent of an equation like (1). Likewise, one expects to use the LHC data themselves to improve our knowledge of the PDFs, by including the functions f_i to the list of parameters in eq. (1) that are allowed to float in order to fit the data.

- The precision of the measured cross section, $\sigma_{exp}(O)$, determined by the quantities on the right-hand-side of the following relation: $\sigma_{exp}(O) = N_{events}(O)/Luminosity$. Here N_{events} depends on the accurate knowledge of signal and background acceptances and efficiencies, and *Luminosity* is what we are discussing at this Workshop.

The target of the programme of precision measurements is therefore to bring to the same level the accuracy all elements in the following relation:

$$\sigma(pp \to O) = \frac{N_{events}(O)}{Luminosity}\,. \quad (2)$$

The target is driven by the level of accuracy of the available, required, inputs. At LEP/SLC, the theoretical precision in the cross section calculations was very high, setting tough requirements on the determination of the luminosity and of the experimental accuracy, in order to allow a precise extraction of the fundamental theory parameters. At the LHC, the theoretical calculations for hadronic processes are less precise than at LEP/SLC. On one side the calculations are harder, since perturbation theory for strong interactions is less effective. On the other, the limited knowledge of PDFs introduces a new uncertainty. The possibility to have a very accurate absolute luminosity determination, and therefore very accurate experimental cross section measurements, allows to develop a physics programme in which this precise information can help improve, at the same time, the theoretical calculations, the PDF knowledge, and ultimately the measurement of the theory parameters such as masses and couplings. In this review we shall now provide a few examples of the interplay between these different ingredients.

EXAMPLES

Indirect luminosity measurement

When a certain process is known with great theoretical precision, the relation (2) can be used in its inverted form, as a means of determining the luminosity:

$$\text{Luminosity} = \frac{N_{events}(O)}{\sigma(pp \to O)}\,. \quad (3)$$

This is what we refer to as "indirect" luminosity measurement. By using these measurements to set the scale of the luminosity we typically give up the possibility of using them for physics. The assumption is that we know them well enough already that we have nothing to gain from a comparison of the calculated cross section to an absolute cross sections measurement. This was the approach used at

LEP, where elastic ("Bhabha") e^+e^- scattering was the reference process. The only input parameters for the Bhabha rates are the electron mass and α_{em}, which are known well enough that no realistic effort towards a direct luminosity measurement could possibly help improving their knowledge.

At the LHC, examples of this type include some processes of electromagnetic nature, which are expected to be calculable from first principles with precision better than 1%, on the basis of the knowledge of the EM proton form factor. Since, for these processes, we do not anticipate to learn new physics from a possible precise comparison of data and theory, we can afford using them as a luminosity-setting standard candle. The simplest example is the mesurement of the elastic pp cross section at very small angle, where the impact parameter is so large that only the well known Coulomb interaction between protons is relevant [1]. This is the programme of ATLAS's ALFA spectrometer [2]. Other examples that have been considered include the reactions $pp \to pp\mu^+\mu^-$ (pair production via $\gamma\gamma$ collisions) and $pp \to pp\gamma$, and have been discussed in the presentation by V. Khoze at this Workshop [1]. The main challenge in these cases is of experimental nature. In the case of the $\mu^+\mu^-$ final states, which are discussed in the LHCb presentation [3], the measured cross section varies very strongly with the p_T detection threshold of muons, introducing a potentially large systematics. In the case the $pp\gamma$ reactions, dedicated photon detectors at large rapidities are required [4]. In both cases, the requirement that no other particles are produced (which is a precondition to the validity of the calculations for these purely semi-elastic processes) sets a further source of uncertainty, since no detector is fully hermetic, and the real precision of the MC modeling of the final states is still not known [1]. It is however expected that the current MCs will be improved and validated using LHC data, particularly from the detectors with acceptance in the forward regions.

Another concrete example, which forms the basis of the programme of the TOTEM experiment [5], is the use of the optical theorem, together with the measurement of the elastic and inelastic rates. This allows a simultaneous determination of the luminosity and of the total pp cross section [1]:

$$\sigma_{tot}^{pp} = \frac{16\pi}{1+\rho^2} \frac{\frac{dN_{el}}{dt}|_{t=0}}{N_{el}+N_{inel}} \quad (4)$$

$$L = \frac{1+\rho^2}{16\pi} \frac{(N_{el}+N_{inel})^2}{\frac{dN_{el}}{dt}|_{t=0}} \quad (5)$$

where N_{el} and N_{inel} are the number of elastic and inelastic events in a given data sample, $dN_{el}/dt|_{t=0}$ is the differential elastic rate extrapolated at zero scattering angle, and ρ is the ratio of real and imaginary part of the forward elastic scattering amplitude. The uncertainty on $\rho \sim 0.12 \pm 0.03$ leads to a 1% uncertainty in eqs. (4) and (5). A detailed account of the systematics due to the modeling of the extrapolation of the elastic and inelastic rates to uninstrumented detector regions, was given in [1, 5].

The W and Z cross sections can also be calculated with good precision, and in the past have been proposed as possible channels for indirect luminosity measurements. We discuss these prospects in more detail in the following.

Indirect mass measurements

The mass of a particle can be reconstructed *directly* from the full reconstruction of its decay particles, using $M_O^2 = (\sum_{decays} p_i)^2$. Alternatively, one can use the mass-dependence of the production cross section. This *indirect* measurement is the technique used, for example, in the Z mass measurement at LEP/SLC [6], where M_Z is one of the parameters in the fit of the Z line shape, namely the energy dependence of the e^+e^- cross section. The knowledge of the absolute luminosity, and thus of the absolute cross section, has a small impact on the systematics, since it is mostly the shape that counts. The leading systematics here is therefore the theoretical knowledge of the mass-dependence of the cross setion, and the absolute scale of the beam energy. In hadron collisions the beam energy is practically unknown, since the relevant initial state is made of quarks or gluons, and the energy determination from the reconstructed final states has limited precision. The precision of the indirect mass measurement is therefore limited by the theoretical knowledge of the cross section, and of the experimental determination of the absolute cross section, which includes both the experimental systematics (acceptance and reconstruction efficiency) and the beam luminosity.

A concrete example is given by the top quark mass, m_t. The relative m_t dependence of the production cross section for top quark pairs is known very accurately, and is given by the relation $\delta m_t/m_t = 0.21\delta\sigma/\sigma$. The measurement and theoretical determination of the cross section with a total precision of 5% will therefore lead to an indirect determination of m_t with a 1% precision, or about 1.5 GeV. This would provide a significant measurement of m_t, with a systematics totally independent of that botained in the direct reconstruction of the top mass form its decay products, whose ultimate accuracy at the Tevatron and LHC is expected to reach 1 GeV.

At this time, the theoretical calculations have, for a fixed m_t value, an uncertainty of $\pm 8\%$(NLL)$\pm 3\%$(PDF) (see e.g. [7]). The first uncertainty is due to the incomplete knowledge of higher-order perturbative corrections beyond the fixed next-to-leading-order (NLO) and the next-to-leading logarithms (NLL) which dominate at all orders of pertubation theory. The second uncertainty comes from the uncertainties in the quark and gluon PDFs. It is expected that, within a couple of years, the combined theoretical uncertainty will be reduced to an overall $\pm 3-5\%$. The experimental systematics could be reduced to the percent level. A luminosity uncertainty at the level of 3% would therefore provide a firm ground on which to stimulate further progress on the theoretical side, to achieve the desired

± 1.5 GeV precision on m_t.

W and Z production cross sections

The production of W and Z gauge bosons is the process with the best theoretical precision. For a fixed PDF choice, the cross sections $\sigma_{W,Z}$ are known to within $1-2\%$ [8, 9]. A larger uncertainty comes from the knowledge of the PDFs. As of today, the envelope of the predictions for the W and Z cross sections, obtained by using the various PDF fits in the literature [10], is at the level of $\pm 5\%$, as shown in Fig. 1. The ultimate experimental accuracy on

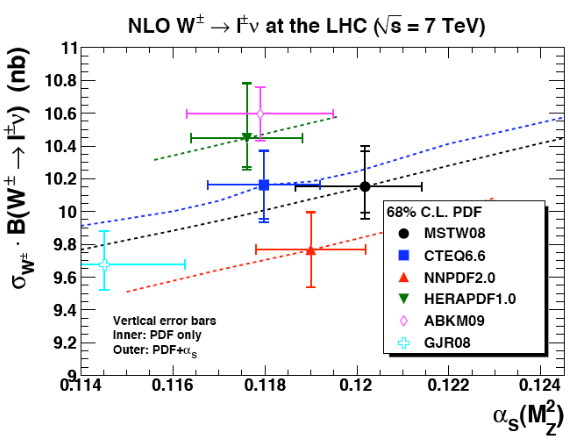

Figure 1: Predictions of the W cross section, at the LHC, as obtained with different sets of PDFs [10].

this observable is at the level of percent, or below, as documented in the current analyses of LHC data [11, 12] and as discussed at this Workshop [13, 14]. The systematics will therefore soon be dominated by the luminosity. When this will be known to 5%, the first compelling tests of the theoretical predictions for $\sigma_{W,Z}$ will be possible. Below the 5% threshold one will start gaining new information, capable of selecting among different sets of PDFs. The ultimate precision is set, today, by the theoretical uncertainty on the partonic cross section $\hat{\sigma}_W$, namely 2%.

Of course one could anticipate an improvement in the knowledge of the protons PDFs, using input from the LHC, as discussed in the next Section, and use the prediction of $\sigma_{W,Z}$ to obtain an indirect determination of the LHC luminosity, at a level of the order of few percent. For example, the ratio of W and Z cross sections, a quantity which is not subject to luminosity uncertainty, shows some degree of correlation with the PDFs, and a potential to improve the PDF knowledge. This is shown in Fig. 2 [17], where the correlation coefficients between the prediction for σ_W/σ_Z with various parton distributions is shown. Notice, however, that the correlation is much stronger with the absolute determination of σ_W (Fig. 3). Therefore, given the physics interest in the comparison of the data and theory for the absolute rates, and the possibility of using the measurement

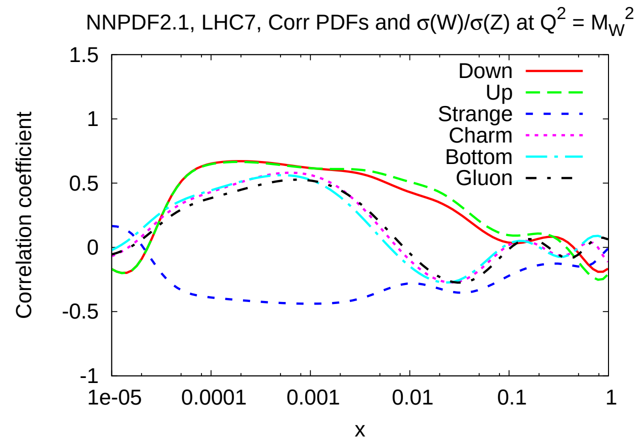

Figure 2: Correlation between various PDFs and ratio of W and Z cross sections [17].

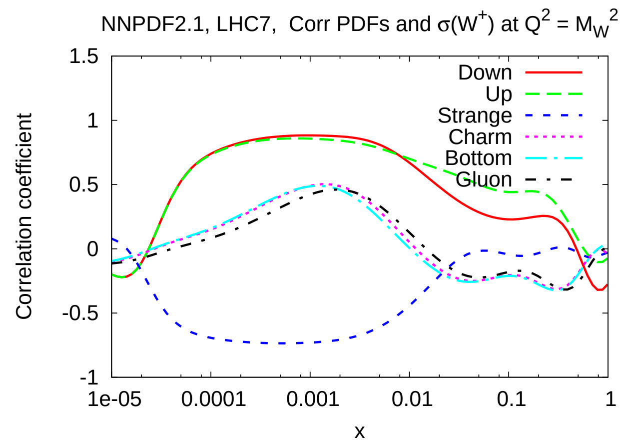

Figure 3: Correlation between various PDFs and σ_W [17].

as a way of pinning down with the greatest possible accuracy the PDFs, it would be a loss to sacrifice this physics channel for an indirect luminosity measurement.

W and Z distributions

In addition to the measurement of the total W and Z cross sections, input on the proton PDFs will also arise from their rapidity distributions. These, in fact, probe the spectrum of quarks and gluons at different values of x, providing important constraints on the PDFs. For example, the forward acceptance of the LHCb detector selects initial states where one of the two partons has large $x \sim 0.1$, and the other has $x \ll 1$. Furthermore, at the LHC the difference between up-quark (u) and down-quark (d) distributions is directly reflected in the production asymmetry between W^+ and W^-. The convolution of the $W^\pm$ production spectra and of their decay angular distributions, leads to a characteristic shape of the ratio of the positive and negative lepton rapidity spectra, which is larger than one for lepton rapidity $y \lesssim 3$, and smaller than one for $y \gtrsim 3$. These asymmetries are already being measuremed, with increasing precision, by the LHC experiments [3, 13, 14, 15, 16].

The precise interplay of the measurement of the absolute production rates and of the shapes of rapidity spectra in improving our knowledge of PDFs, is the subject of ongoing work [10, 17]. I present here some preliminary results, as an illustration of some possible outcomes of these analyses.

The correlation between $\sigma_{W,Z}$ and the W rapidity distribution is shown in Fig. 4. The continuous (red) curve, in

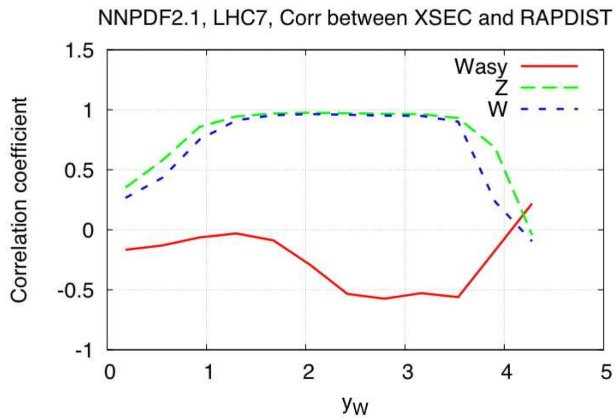

Figure 4: Correlations between W and Z observables. The continuos (red) line is the correlation between σ_W and the W^+/W^- rapidity asymmetry. The long-dashed (green) line is the correlation between σ_Z and $d\sigma/dy_Z$. The short-dashed (blue) line is the correlation between σ_W and $d\sigma/dy_W$.

particular, represents the correlation between σ_W and the W asymmetry. The correlation is weak in the central region (ATLAS/CMS), but becomes more sizable in the forward region (LHCb). This shows that a measurement of σ_W adds a different amount of information if combined with the W asymmetries measured in the central or in the forward region. More explicitly, the impact of these data is highlighted in Figs. 5 and 6, which show the improvements in the determination of the strange quark PDF, when including the information that will arise from 1fb^{-1} of LHC data at 7 TeV [17]. These two figures consider the future knowledge arising from the W asymmetry, as well as from the measurement of $\sigma_{W,Z}$ or of the Z boson rapidity spectrum. Improvements by factors up to 3 are expected for $x \lesssim 10^{-2}$. The figures only consider the impact of future data from ATLAS and CMS, at central rapidity; greater improvements will come, in the region $x \sim 0.1$, from the inclusion of data from LHCb, which are directly sensitive to the large-x range, due to the forward kinematics.

CONCLUSIONS

The main physics driver for the precision of the LHC luminosity determination is the determination of the cross

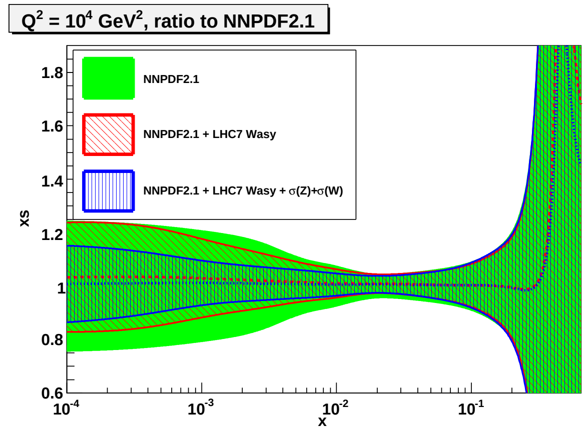

Figure 5: Improvements in the relative uncertainty on the knowledge of the strange quark PDF, when including the information that will arise with 1fb^{-1} of data from the LHC at 7 TeV. The outer (green) band shows the current uncertainty. The lightly-shaded (red) region is the improvement obtained using the W asymmetry measurement from the ATLAS and CMS experiments. The dark-shaded (blue) region is the further improvement obtained with the addition of the σ_W and σ_Z measurements. Figure from Ref. [17].

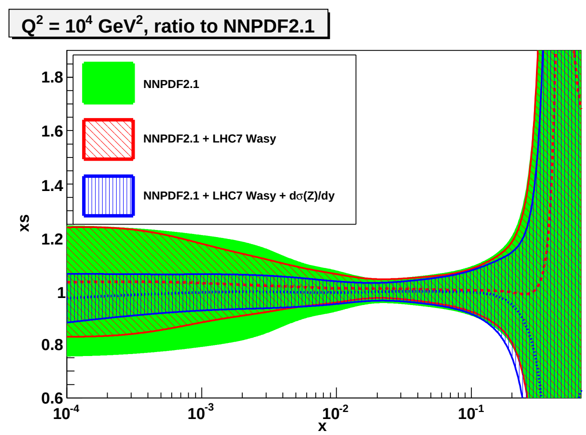

Figure 6: Same as the previous figure, but the last step (dark band) arising from the addition of the Z boson rapidity measurement. From Ref. [17].

section of W and Z bosons. With $\Delta L/L \sim 5\%$, the experimental measure will match the current uncertainty of the theoretical calculation, which is dominated by the PDF uncertanties. Below 5% one will start extracting new information on the proton PDF. Further improvements down to the level of $\Delta L/L \sim 2\%$ are justified by the current level of precision of the partonic cross section. Overall, the improved determination of PDFs will have a direct impact on the systematics of several crucial precision measurements. For example, it is known from the Tevatron experience that the PDF uncertainty must be reduced in order to improve the precision of the W mass measurement. An improved knowledge of the PDFs will also be required to extract the value of the electroweak coupling $\sin^2\theta_W$ from the Z forward-backward asymmetry [18].

Of course there are many other studies that will benefit from the reduction of $\Delta L/L$ to the few percent level. For example, $\Delta L/L \sim 3\%$ is a necessary ingredient for the indirect measurement of m_t at the level of 1.5-2 GeV.

An interesting observation was made during the discussion following this presentation (W. Krasny): it may be desirabile to monitor the relative $\Delta L/L$ at different beam energies (2.76, 7 and 14 TeV) to higher precision, possibly at the per mille level, in order to achieve high-precision determinations of cross-section ratios at various energies. The theoretical and experimental uncertainty on ratios of cross-sections for the same process at different energies is much better than for individual energies. More work is required to quantify these statements in some concrete cases of interest.

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