NOISE GATES

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Classical simulations of quantum computers

Advantages of classical simulations of a quantum computation:

- **Errors propagation** in quantum devices can be analyzed before actual quantum hardware implementations.
- **Efficiency test of quantum algorithms** and their criticality.

Errors analysis: Standard approach

To date, the main approach in errors analysis is to schematize them as quantum channels that act before and/or after each gate (Breuer et al. 2002).

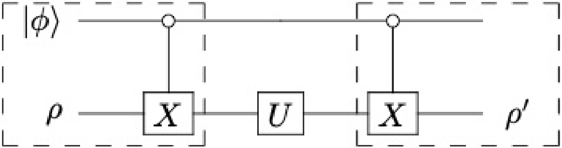

Figure: Generic single-qubit gate $\hat{U}$ with bit-flip channel before and after.

- Computation is **not** affected by errors **during** the unitary evolution of the system but only **before** and/or **after** it.
- **Matrix density** $\hat{\rho}(t)$ formalism.

Disadvantages of the standard approach.

- The behavior of the gates is unrealistic.
  Real errors can also take place **during** the unitary evolution of the system. (Sun et al. 2020).

- Solving for the density matrix $\hat{\rho}(t)$ scales **quadratically** with the number of degrees of freedom: 2^{2N}.
  Challenging computational task.

Noise gates approach

- Master equation for open quantum systems

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -\frac{i}{\hbar}[\hat{\mathcal{H}}_t, \hat{\rho}(t)] + \sum_k \gamma_k \left[\hat{L}_k \hat{\rho}(t) \hat{L}^\dagger{}_k - \frac{1}{2}\{\hat{L}^\dagger{}_k \hat{L}_k, \hat{\rho}(t)\}\right], \tag{1}$$

$\hat{\mathcal{H}}_t$: Hamiltonian of the system; $\hat{L}_k$: Lindblad operators.

- Eq. (1) can be obtained through the following **stochastic unravelling**:

$$\mathrm{d}|\psi_t\rangle = \left[-\frac{i}{\hbar}\hat{\mathcal{H}}_t \mathrm{d}t + \sum_k \left(i\sqrt{\gamma_k}\hat{L}_k \mathrm{d}W_k(t) - \frac{\gamma_k}{2}\hat{L}^\dagger{}_k \hat{L}_k \mathrm{d}t \right)\right] |\psi_t\rangle \tag{2}$$

Noise gates approach

The master equation (1) and the stochastic differential equation (SDE) (2) are **statistically equivalent**:

$$\hat{\rho}(t) = \mathbb{E}[|\psi_t\rangle \langle\psi_t|]. \tag{3}$$

Eq. (2) is **linear** thus the solution can be written as

$$|\psi_t\rangle = \hat{U}^N(t, t_0) |\psi_{t_0}\rangle , \tag{4}$$

$\hat{U}^N(t, t_0)$ is a linear operator $\implies$ **noise gate**

Figure: Generic single-qubit noise gate

Advantages of the noise gates approach

- The gates behavior is more realistic. The noise takes place during the entire evolution of the system and it is integrated in the gates themselves (Bassi et al. 2008; Guerreschi et al. 2020, INTEL)

- The solution of SDE for the state $|\psi_t\rangle$ scales **linearly** with the number of degree of freedom: 2^N. **Faster** simulation thanks to the state vector formalism.

Noisy algorithms

By using the **noise gates**, once a set of **universal gates** are identified, it is possible to implement **"noisy" algorithms** without adding further qubits to simulate the environment.

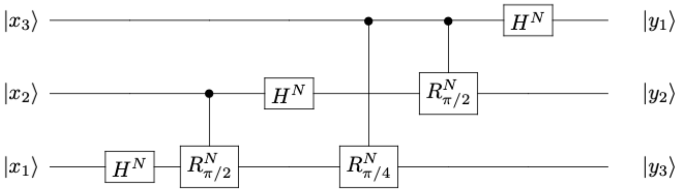

Figure: Three-qubit Quantum Fourier Transform using noise gates instead of standard unitary gates.