Dark matter in three-Higgs-doublet models with S_3 symmetry

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In collaboration with: **O. M. Øgreid**, **P. Osland**, **M. N. Rebelo**

Based on **[2108.07026]** and **[2204.05684]**

$$\mathcal{V}_{\text{2HDM}} = m_{11}^2 h_{11} + m_{22}^2 h_{22} - \left(m_{12}^2 h_{12} + \text{h.c.} \right) \\ + \frac{1}{2}\lambda_1 h_{11}^2 + \frac{1}{2}\lambda_2 h_{22}^2 + \lambda_3 h_{11} h_{22} + \lambda_4 h_{12} h_{21} \\ + \left\{ \frac{1}{2}\lambda_5 h_{12}^2 + \lambda_6 h_{11} h_{12} + \lambda_7 h_{22} h_{12} + \text{h.c.} \right\} .$$

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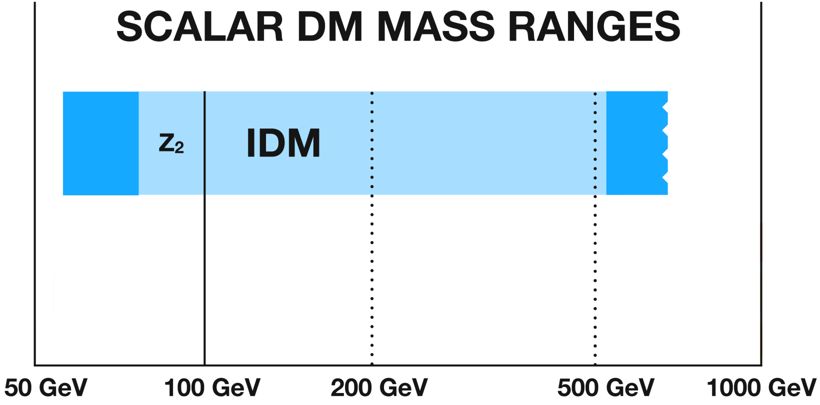
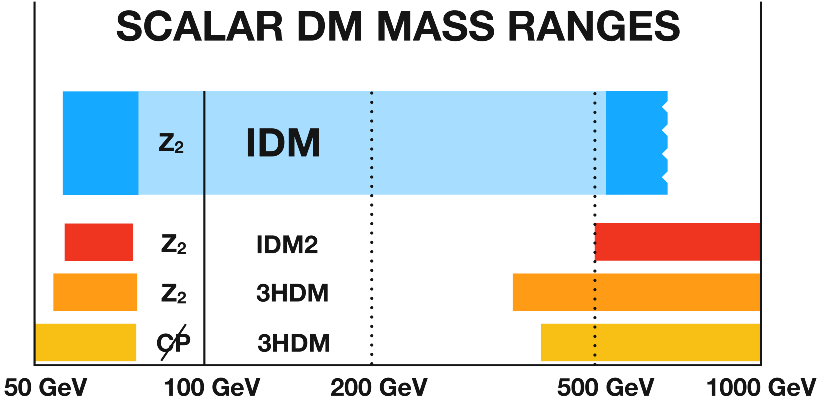
$$\mathbb{Z}_2 : \begin{cases} h_1 \to h_1, \\ h_2 \to -h_2, \end{cases} \qquad \text{vacuum:} \begin{cases} \langle 0|h_1|0\rangle \neq 0, \\ \langle 0|h_2|0\rangle = 0. \end{cases}$$



IDM: **[1612.00511]**, **[1809.07712]**;
IDM2 (one inert doublet): **[1911.06477]**;

(Two inert doublets)
3HDM: **[1407.7859]**, **[1507.08433]**, **[1712.09598]**;
~~CP~~-3HDM: **[1608.01673]**;



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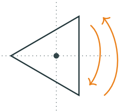
Possible permutations: $(1)\,(2)\,(3)$, $(1\,2)\,(3)\ldots$, $(1\,3\,2)\ldots$

S_3-Symmetric Three-Higgs-Doublet Models: Generalities

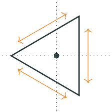
Possible permutations: (1) (2) (3), (1 2) (3) . . . , (1 3 2) . . .

Transformations of an equilateral triangle:

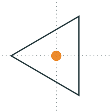
- 2 rotations



- 3 reflections



- Identity



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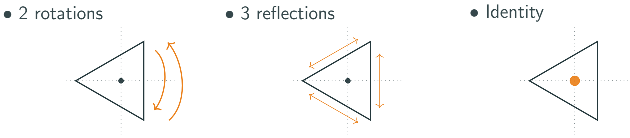

S_3 irreducible representation: $\chi_{\mathbf{1}} \oplus \chi_{\mathbf{1'}} \oplus \chi_{\mathbf{2}}$.

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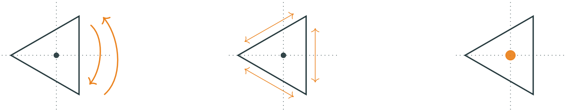

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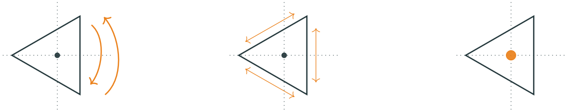

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Symmetries reduce free parameters: NHDM $\xrightarrow{\text{3HDM}}$ (54) $\xrightarrow{S_3}$ (12) $\xrightarrow{\mathbb{R}\mathrm{e}}$ 10.

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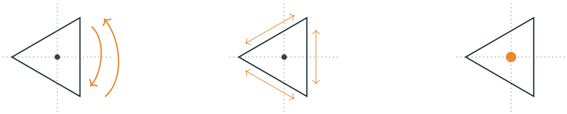

S_3 irreducible representation: $\chi_{\mathbf{1}} \oplus \chi_{\mathbf{1}'} \oplus \chi_{\mathbf{2}}$. Assume $(h_S)_{\mathbf{1}} \oplus \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}_{\mathbf{2}}$.

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S_3-3HDM models were classified in **[1601.04654]**:

$$\text{vacuum:} \begin{cases} 11 \text{ real } \left(w_1,\, w_2,\, w_S\right), \\ 18 \text{ complex } \left(\hat{w}_1 e^{i\sigma_1},\, \hat{w}_2 e^{i\sigma_2},\, \hat{w}_S\right). \end{cases}$$

Whenever $w_S \neq 0$ we can construct a trivial Yukawa sector:

$$\mathcal{M}_u = \frac{1}{\sqrt{2}} \left(y^u_{ij} \right) w^*_S, \qquad \mathcal{M}_d = \ldots$$

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Fermions can transform non-trivially under S_3:

$$\mathbf{2} : \left(Q_1 \; Q_2 \right)^{\mathrm{T}}, \left(u_{1R} \; u_{2R} \right)^{\mathrm{T}}, \left(d_{1R} \; d_{2R} \right)^{\mathrm{T}} \quad \text{and} \quad \mathbf{1} : Q_3, \; u_{3R}, \; d_{3R},$$

$$\mathcal{M}_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^u w_S^* + y_2^u w_2^* & y_2^u w_1^* & y_4^u w_1^* \\ y_2^u w_1^* & y_1^u w_S^* - y_2^u w_2^* & y_4^u w_2^* \\ y_5^u w_1^* & y_5^u w_2^* & y_3^u w_S^* \end{pmatrix}, \qquad \mathcal{M}_d = \dots$$

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$$\mathcal{M}_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^u w_S^* + y_2^u w_2^* & y_2^u w_1^* & y_4^u w_1^* \\ y_2^u w_1^* & y_1^u w_S^* - y_2^u w_2^* & y_4^u w_2^* \\ y_5^u w_1^* & y_5^u w_2^* & y_3^u w_S^* \end{pmatrix}, \qquad \mathcal{M}_d = \dots$$

Suppose that $w_1 = 0$:

$$\mathcal{M}_u = \frac{1}{\sqrt{2}} \begin{pmatrix} y_1^u w_S^* + y_2^u w_2^* & 0 & 0 \\ 0 & y_1^u w_S^* - y_2^u w_2^* & y_4^u w_2^* \\ 0 & y_5^u w_2^* & y_3^u w_S^* \end{pmatrix}.$$

S_3-Symmetric Three-Higgs-Doublet Models: Dark Matter Models

Vacuum	vevs	λ_4	symmetry	# massless states	fermions under S_3
R-I-1	$(0, 0, w_S)$	$\checkmark$	$S_3,\ h_1 \to -h_1$	none	trivial
R-I-2a	$(w, 0, 0)$	$\checkmark$	S_2	none	non-trivial
R-I-2b,2c	$(w, \pm\sqrt{3}w, 0)$	$\checkmark$	S_2	none	non-trivial
R-II-1a	$(0, w_2, w_S)$	$\checkmark$	$S_2,\ h_1 \to -h_1$	none	trivial
R-II-2	$(0, w, 0)$	0	$h_1 \to -h_1, h_S \to -h_S$	1	non-trivial
R-II-3	$(w_1, w_2, 0)$	0	$h_S \to -h_S$	1	non-trivial
R-III-s	$(w_1, 0, w_S)$	0	$h_2 \to -h_2$	1	trivial
C-I-a	$(\hat{w}_1, \pm i\hat{w}_1, 0)$	$\checkmark$	cyclic $\mathbb{Z}_3$	none	non-trivial
C-III-a	$(0, \hat{w}_2 e^{i\sigma_2}, \hat{w}_S)$	$\checkmark$	$S_2,\ h_1 \to -h_1$	none	trivial
C-III-b	$(\pm i\hat{w}_1, 0, \hat{w}_S)$	0	$h_2 \to -h_2$	1	trivial
C-III-c	$(\hat{w}_1 e^{i\sigma_1}, \hat{w}_2 e^{i\sigma_2}, 0)$	0	$h_S \to -h_S$	2	non-trivial
C-IV-a	$(\hat{w}_1 e^{i\sigma_1}, 0, \hat{w}_S)$	0	$h_2 \to -h_2$	2	trivial

Possible DM candidates: 3 (exact S_3) and 8 (softly broken S_3) solutions.

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R-II-2	$(0, w, 0)$	0	$h_1 \to -h_1, h_S \to -h_S$	1	non-trivial
R-II-3	$(w_1, w_2, 0)$	0	$h_S \to -h_S$	1	non-trivial
R-III-s	$(w_1, 0, w_S)$	0	$h_2 \to -h_2$	1	trivial
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Physical Spectrum and Constraints

	R-II-1a	C-III-a
Vacuum	$\{0,\ w_2,\ w_S\}$	$\{0,\ \hat{w}_2 e^{i\sigma},\ \hat{w}_S\}$ CP-violation

Physical Spectrum and Constraints

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Inert states	$\{h^{\pm},\ \eta,\ \chi\}$ 2 DM candidates	$\{h^{\pm},\ (\varphi_1 - \varphi_2)\}$

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Active states	$\{H^\pm,\ (h - H)\ ,\ A\}$	$\{H^\pm,\ (H_1 - H_2 - H_3)\}$

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Models are analysed using 8 input parameters.

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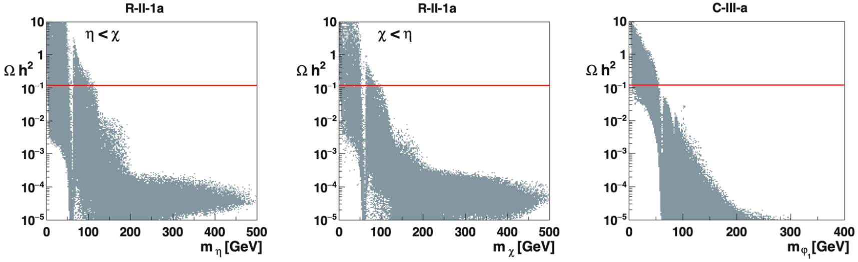
Both theoretical and experimental constraints, at 3-σ, are evaluated:

- Cut 1: perturbativity, stability, unitarity checks, LEP constraints;
- Cut 2: $h \to \{VV,\ FF\}$, S and T, $\bar{B} \to X(s)\gamma$;
- Cut 3: $h \to \{\text{invisible},\ \gamma\gamma\}$, $\Omega_{\rm CDM} h^2$, direct searches;

Results: Relic Density



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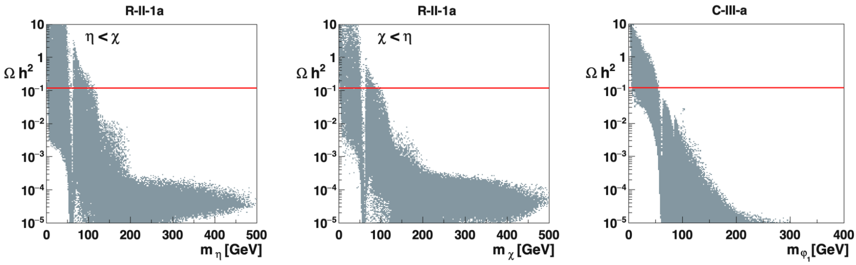
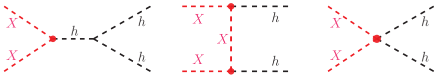
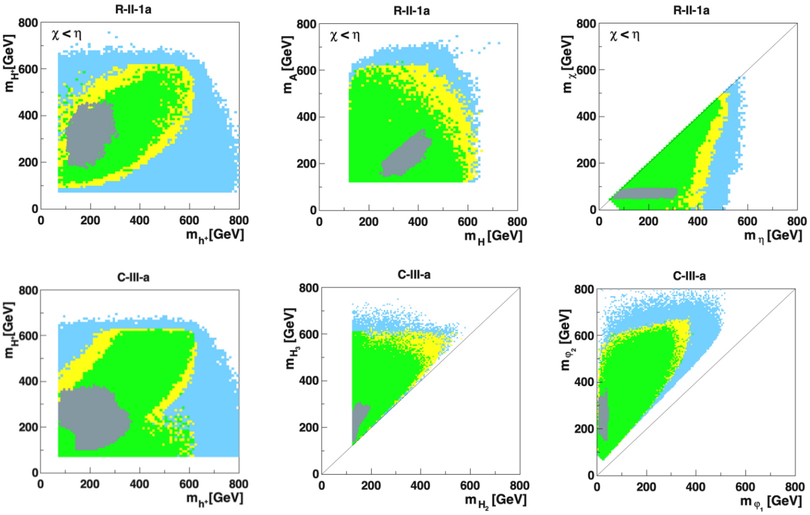


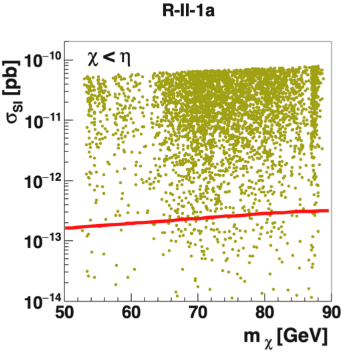


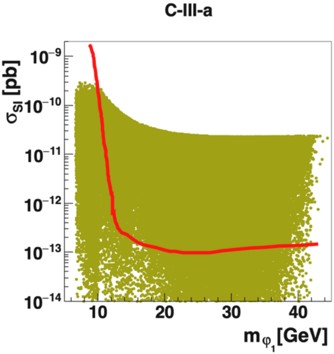
Trilinear and quartic couplings are not tuneable!

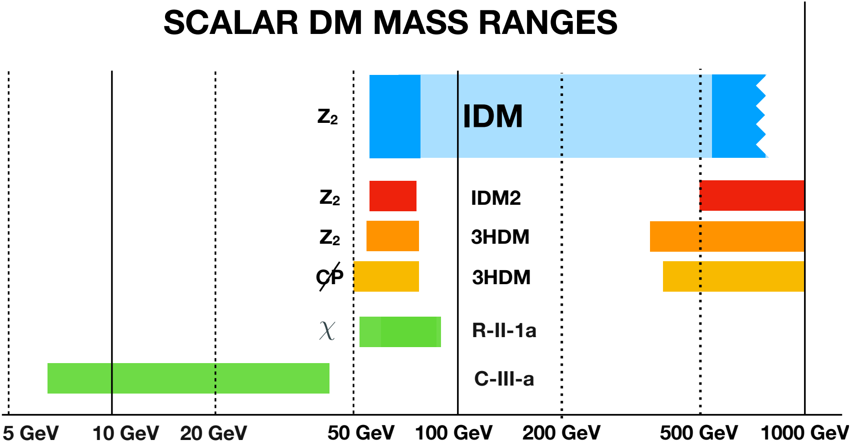
R-II-1a: $\left.\frac{g(XXh)}{v}\right|_{\rm SM} = g(XXhh)\big|_{\rm SM} = \frac{1}{v^2}\left[m_h^2 + 2m_X^2\right].$

Results: Scalar Masses




R-II-1a
χ < η
σ_{SI} [pb]
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
50
60
70
80
90
m_χ [GeV]


C-III-a
σ_{SI} [pb]
10^{-9}
10^{-10}
10^{-11}
10^{-12}
10^{-13}
10^{-14}
10
20
30
40
m_{φ_1} [GeV]


SCALAR DM MASS RANGES
Z2
IDM
Z2
IDM2
Z2
3HDM
CP
3HDM
χ
R-II-1a
C-III-a
5 GeV
10 GeV
20 GeV
50 GeV
100 GeV
200 GeV
500 GeV
1000 GeV

Conclusions

- Multi-Higgs-doublet models are phenomenologically rich and can accommodate a dark matter candidate;
- Possible dark matter candidates were identified within S_3-3HDM;
- Viable dark matter regions: R-II-1a $[52.5, 89]$ GeV, C-III-a $[6.5, 44.5]$ GeV;
- "*It is incidentally suggested that when the theory is perfected it may be possible to determine the amount of dark matter from its gravitational effect*", J. C. Kapteyn, 1922.

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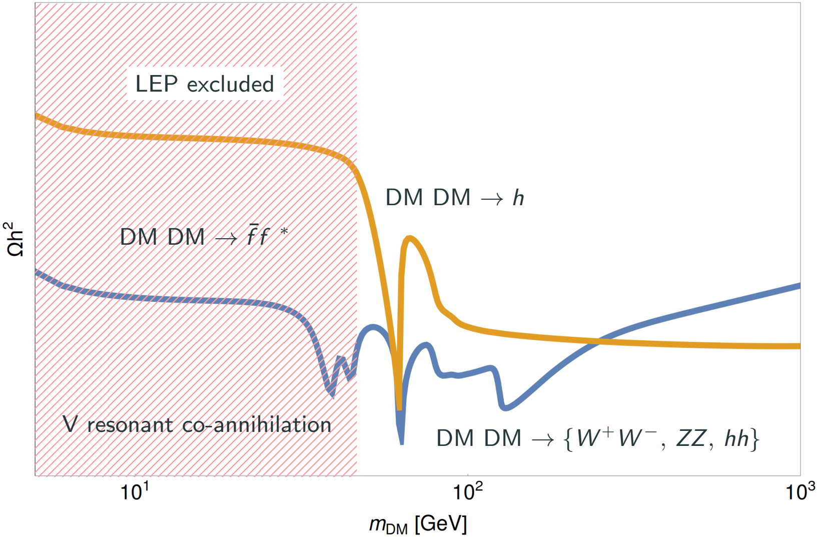
Appendix: Inert Doublet Model Profile



Appendix: Scalar Potential

$$\begin{aligned}\mathcal{V}_{\rm 3HDM} =& \mu_1^2 \left(h_{11}+h_{22}\right) + \mu_0^2 h_{SS} \\ &+ \lambda_1 \left(h_{11}+h_{22}\right)^2 + \lambda_2 \left(h_{12}-h_{21}\right)^2 + \lambda_3 \left[\left(h_{11}-h_{22}\right)^2 + \left(h_{12}+h_{21}\right)^2\right] \\ &+ \left\{\lambda_4 \left[h_{S1}\left(h_{12}+h_{21}\right) + h_{S2}\left(h_{11}-h_{22}\right)\right] + \text{h.c.}\right\} + \lambda_5 \left[h_{SS}\left(h_{11}+h_{22}\right)\right] \\ &+ \lambda_6 \left[h_{1S}h_{S1} + h_{2S}h_{S2}\right] + \left\{\lambda_7 \left[h_{S1}^2 + h_{S2}^2\right] + \text{h.c.}\right\} + \lambda_8 h_{SS}^2 .\end{aligned}$$

$$\begin{aligned}&\mathbf{1}: \; [2\otimes 2]_1,\ [1\otimes 1]_1,\ [1'\otimes 1']_1; \\ &\mathbf{1}': \; [2\otimes 2]_{1'},\ [1\otimes 1']_{1'},\ [1'\otimes 1]_{1'}; \\ &\mathbf{2}: \; [2\otimes 2]_2,\ [1\otimes 2]_2,\ [2\otimes 1]_2,\ [1'\otimes 2]_2,\ [2\otimes 1']_2;\end{aligned}$$

$$\begin{aligned}\mathcal{V}_{\rm 3HDM} =& \mu_1^2 [2\otimes 2]_1 + \mu_0^2 [1\otimes 1]_1 \\ &+ \lambda_1 \Big([2\otimes 2]_1 \otimes [2\otimes 2]_1\Big) + \lambda_2 \Big([2\otimes 2]_{1'} \otimes [2\otimes 2]_{1'}\Big) + \lambda_3 \Big([2\otimes 2]_2 \otimes [2\otimes 2]_2\Big) \\ &+ \lambda_4 \left\{ \Big([2\otimes 2]_2 \otimes [1\otimes 2]_2\Big) + \xleftrightarrow{\text{sym}} \right\} + \lambda_5 \Big([2\otimes 2]_1 \otimes [1\otimes 1]_1\Big) + \lambda_6 \Big([1\otimes 2]_2 \otimes [2\otimes 1]_2\Big) \\ &+ \lambda_7 \left\{ \Big([1\otimes 2]_2 \otimes [1\otimes 2]_2\Big) + \xleftrightarrow{\text{sym}} \right\} + \lambda_8 \Big([1\otimes 1]_1 \otimes [1\otimes 1]_1\Big).\end{aligned}$$

Massless state:

$$\mathcal{V}\left(Uh\right) = \mathcal{V}\left(h\right),$$
$$\langle 0|\left(Uh\right)|0\rangle \neq \langle 0|h|0\rangle\,.$$

Results of **[2001.01994]**:

Constraints	Continuous symmetries	# of massless states
$[\lambda_4 = 0]$	O(2)	1
$\cdots + [\lambda_7 = 0]$	$\mathrm{O(2)}\otimes\mathrm{U(1)}_{h_S}$	2
$\cdots + [\lambda_2 + \lambda_3 = 0]$	SU(2) [$\mathrm{O(2)}\otimes\mathrm{U(1)}_{h_1}\otimes\mathrm{U(1)}_{h_2}\otimes\mathrm{U(1)}_{h_S}$]	3

Appendix: SU(2) Doublets in Terms of the Mass Eigenstates

R-II-1a:

$$h_1 = \begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}}\left(\eta + i\chi\right) \end{pmatrix},$$

$$h_2 = \begin{pmatrix} \sin\beta\, G^+ - \cos\beta\, H^+ \\ \frac{1}{\sqrt{2}}\left(\sin\beta\, v + \cos\alpha\, h - \sin\alpha\, H + i\left(\sin\beta\, G^0 - \cos\beta\, A\right)\right) \end{pmatrix},$$

$$h_S = \begin{pmatrix} \cos\beta\, G^+ + \sin\beta\, H^+ \\ \frac{1}{\sqrt{2}}\left(\cos\beta\, v + \sin\alpha\, h + \cos\alpha\, H + i\left(\cos\beta\, G^0 + \sin\beta\, A\right)\right) \end{pmatrix}.$$

C-III-a:

$$h_1 = e^{i\gamma}\begin{pmatrix} h^+ \\ \frac{1}{\sqrt{2}}\left(\varphi_1 + i\varphi_2\right) \end{pmatrix},$$

$$h_2 = e^{i\sigma}\begin{pmatrix} \sin\beta\, G^+ - \cos\beta\, H^+ \\ \frac{1}{\sqrt{2}}\left(\sin\beta\, v + i\sin\beta\, G^0 + \sum_{i=1}^{3}\left[\sin\beta\, \mathcal{R}^0_{i1} - \cos\beta\, \left(\mathcal{R}^0_{i2} + i\mathcal{R}^0_{i3}\right)\right] H_i\right) \end{pmatrix},$$

$$h_S = \begin{pmatrix} \cos\beta\, G^+ + \sin\beta\, H^+ \\ \frac{1}{\sqrt{2}}\left(\cos\beta\, v + i\cos\beta\, G^0 + \sum_{i=1}^{3}\left[\cos\beta\, \mathcal{R}^0_{i1} + \sin\beta\, \left(\mathcal{R}^0_{i2} + i\mathcal{R}^0_{i3}\right)\right] H_i\right) \end{pmatrix}.$$

Appendix: Input Parameters

R-II-1a (2 angles + 6 masses):

- Diagonalisation angles $\{\beta,\, \alpha\}$;
- Charged masses $m_{\varphi_i^\pm} \in [0.07,\ 1]$ TeV;
- Inert masses $m_{\varphi_i} \in [0,\ 1]$ TeV;
- Active masses $\{m_H,\ m_A\} \in [m_h,\ 1\ \mathrm{TeV}]$;

C-III-a (5 angles + 3 masses):

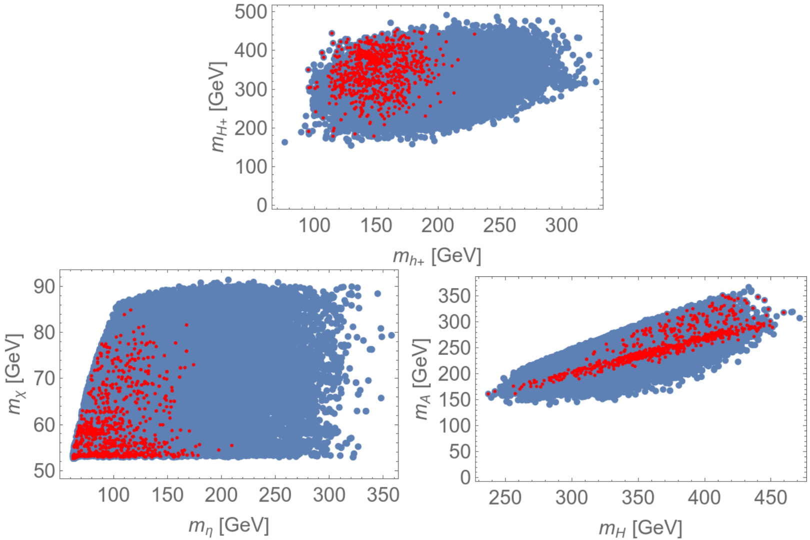
- Diagonalisation angles $\{\beta,\, \gamma,\, \theta_2,\, \theta_3\}$ and phase σ;
- Charged masses $\{m_{h^+},\ m_{H^+}\} \in [0.07,\ 1]$ TeV;
- DM mass $m_{\varphi_1} \in [0,\ 1]$ TeV;

Appendix: R-II-1a Benchmark Points

Parameter	BP 1	BP 2	BP 3	BP 4	BP 5	BP6	BP7	BP8	BP9
DM (χ) mass [GeV]	52.6	56.1	59.6	63.02	65.7	70.3	75.0	82.2	88.6
η mass [GeV]	62.7	203.8	270.4	169.38	150.5	157.7	202.8	127.8	210.7
h^+ mass [GeV]	115.4	167.4	273.6	188.6	214.1	170.5	232.0	151.8	243.0
H^+ mass [GeV]	192.6	369.5	367.4	246.6	265.5	405.8	319.8	410.6	311.9
H mass [GeV]	263.9	349.3	352.9	276.3	298.2	402.0	368.5	405.2	317.6
A mass [GeV]	179.2	208.0	190.7	173.9	205.2	255.3	251.3	330.0	247.0
β/π	0.162	-0.204	-0.201	-0.165	0.163	0.220	0.203	-0.218	0.183
α/π	0.252	0.763	0.765	0.752	0.254	0.225	0.239	0.769	0.238
$\sigma_{\rm SI}$ [10^{-11} pb]	0.029	1.456	4.928	0.176	5.326	1.341	2.711	8.553	4.491
$\eta \to \chi q\bar{q}$ [%]	63.27				54.38	54.35		53.95	
$\eta \to \chi b\bar{b}$ [%]	0.48				14.80	14.85		13.90	
$\eta \to \chi \nu\bar{\nu}$ [%]	24.62				20.48	20.46		20.72	
$\eta \to \chi l\bar{l}$ [%]	11.61				10.33	10.33		11.42	
$\eta \to \chi Z$ [%]		99.98	53.09	100			100		100
$\eta \to \chi A$ [%]			46.91						
$h^+ \to \chi W^+$ [%]		100	100	99.98	99.89	99.99	99.99		99.99
$h^+ \to \eta q\bar{q}$ [%]	20.18							0.30	
$h^+ \to \eta \nu\bar{l}$ [%]	9.88							0.16	
$h^+ \to \chi q\bar{q}$ [%]	46.94							66.82	
$h^+ \to \chi \nu\bar{l}$ [%]	22.99							32.71	
$H^+ \to t\bar{b}$ [%]	9.07	43.69	58.23	95.09	95.78	30.95	96.25	31.54	93.59
$H^+ \to AW^+$ [%]		20.56	35.74	0.29	0.06	8.66	0.05	0.05	0.05
$H^+ \to hW^+$ [%]		1.94	2.67	4.46	4.00	1.23	2.86	1.15	6.20
$H^+ \to h^+\eta$ [%]	85.9					43.74		61.68	
$H^+ \to h^+\chi$ [%]	5.0	33.74	3.26			15.36	0.68	5.53	
$H \to \chi\chi$ [%]	0.15	0.03	0.07	0.87	15.03		11.34	7.63	63.75
$H \to \eta\eta$ [%]	89.9					24.89		25.31	
$H \to hh$ [%]	3.07	2.64	9.40	34.59	33.53	1.33	13.43	0.88	14.72
$H \to AZ$ [%]	0.09	13.55	70.93	13.91	2.87	7.61	22.78		0.07
$H \to W^+W^-$ [%]	4.06	3.13	10.40	34.98	33.35	1.89	16.32	1.26	14.70
$H \to ZZ$ [%]	1.75	1.43	4.77	15.29	14.82	0.88	7.53	0.59	6.62
$H \to h^+h^-$ [%]	0.8	78.59				52.94		56.33	
$H \to q\bar{q}$ [%]		0.62	4.40	0.32	0.34	10.43	28.52	8.00	0.12
$A \to \eta\chi$ [%]	99.97					99.32		99.01	
$A \to b\bar{b}$ [%]	0.02	79.78	84.15	84.63	75.28	0.07	8.84	0.02	4.76
$A \to q\bar{q}$ [%]		3.56	3.75	3.77	3.36		0.39		0.21
$A \to \tau^+\tau^-$ [%]		9.85	10.19	10.00	9.24		1.13		0.61
$A \to hZ$ [%]		6.81	1.87	1.55	12.08	0.6	89.63	0.96	94.42

Appendix: C-III-a Benchmark Points

Parameter	BP 1	BP 2	BP 3	BP 4	BP 5	BP6	BP7	BP8	BP9
DM (φ_1) mass [GeV]	6.85	11.55	16.24	20.82	25.50	30.36	35.13	39.73	44.24
φ_2 mass [GeV]	192.43	247.91	294.06	224.63	223.13	171.54	153.74	268.90	265.78
h^+ mass [GeV]	183.55	273.87	314.66	150.90	238.64	196.77	143.47	200.65	193.85
H^+ mass [GeV]	290.50	152.52	202.09	317.17	145.92	124.49	180.35	259.35	285.91
H_2 mass [GeV]	126.49	142.01	156.26	164.17	143.09	128.72	128.29	138.87	149.83
H_3 mass [GeV]	244.54	216.75	244.67	259.36	205.77	178.37	182.78	195.88	222.07
σ/π	0.365	0.633	-0.370	-0.622	-0.615	-0.590	0.564	-0.538	-0.541
β/π	0.167	0.146	0.160	0.191	0.139	0.128	0.138	0.152	0.150
$\sigma_{\rm SI}$ [10^{-11} pb]	9.23	1.55	1.45	0.01	0.10	1.65	1.23	0.67	3.09
$\varphi_2 \to \varphi_1 H_1$ [%]	0.88	0.15	1.28	3.26	0.80	0.07		3.77	2.71
$\varphi_2 \to \varphi_1 H_2$ [%]	7.49	0.44	2.88			0.07		64.02	60.25
$\varphi_2 \to \varphi_1 H_3$ [%]		24.80	21.13						
$\varphi_2 \to \varphi_1 Z$ [%]	91.63	74.61	74.70	96.73	99.20	99.85	100	32.21	37.04
$h^+ \to \varphi_1 H^+$ [%]		63.84	44.92		60.98	65.40			
$h^+ \to \varphi_1 W^+$ [%]	100	36.16	55.08	100	39.02	34.60	100	100	100
$H^+ \to h^+ \varphi_1$ [%]	33.91			45.61			72.07	16.74	33.83
$H^+ \to H_1 W^+$ [%]	2.26			3.10				2.25	2.50
$H^+ \to H_2 W^+$ [%]	15.19			9.34				10.73	10.55
$H^+ \to t\bar{b}$ [%]	48.56		99.78	41.88			27.68	70.15	53.03
$H^+ \to q\bar{q}$ [%]	0.08	29.32	0.17	0.06	29.49	30.14		0.10	0.08
$H^+ \to \nu\bar{l}$ [%]	0.08	70.68	0.05		70.51	69.86	0.15		
$H_2 \to \varphi_1\varphi_1$ [%]	99.96	99.99	99.99	99.36	99.99	99.99	99.96	99.94	99.95
$H_2 \to W^+W^-$ [%]				0.60					
$H_2 \to q\bar{q}$ [%]	0.03			0.03		0.01	0.04	0.06	0.04
$H_3 \to \varphi_1\varphi_1$ [%]	81.99	96.04	79.32	83.49	98.17	99.93	99.90	98.08	96.95
$H_3 \to \varphi_1\varphi_2$ [%]	9.10			7.57					
$H_3 \to H_1 H_1$ [%]				0.08					
$H_3 \to H_1 Z$ [%]	1.20		15.82	2.57					0.01
$H_3 \to H_2 Z$ [%]	7.67			0.40					
$H_3 \to W^+W^-$ [%]		2.64	3.18	4.10	1.26	0.04	0.08	1.44	2.17
$H_3 \to ZZ$ [%]		1.05	1.34	1.76	0.47			0.48	0.87
$H_3 \to b\bar{b}$ [%]	0.03	0.27	0.34		0.08	0.02	0.01		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