Discrete Goldstone Bosons

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for

FLASY 2022

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arXiv:2205.09131

GBs from continuous symmetries

Exact discrete symmetry

SSB

Goldstone Bosons

Explicit breaking

$\boldsymbol{m^2 \propto \Lambda^2}$

Can m^2 be produced without explicit breaking???

What about *discrete* symmetries?

Discrete non-abelian symmetries, when non-linearly realized, offer new venues to address this problem.

[Das, Hook, 2006.10767]

- Allow contributions to $V(\phi)$ that generate a mass for the GBs
- Remove $\propto \Lambda^2$ radiative corrections from $V(\phi)$
- Coefficients in $V(\phi)$ go as $c_n \propto \epsilon^n$ → m^2 supressed with $\epsilon \ll 1$

Φ, a triplet of A_4

Φ in a triplet of A_4, we impose $\mathbf{\Phi}^T\mathbf{\Phi} = \phi_1^2 + \phi_2^2 + \phi_3^2 = f^2$, where $f \sim 4\pi\,\Lambda$

$\Rightarrow$ **non-linearly realized**

Three independent invariants allowed by symmetry:

~~$\mathcal{J}_2 = \phi_1^2 + \phi_2^2 + \phi_3^2$,~~

$$\mathcal{J}_3 = \phi_1\phi_2\phi_3,$$

$$\mathcal{J}_4 = \phi_1^4 + \phi_2^4 + \phi_3^4.$$

$\boldsymbol{\mathcal{J}_2 = f^2 = cte}$

disappears from V(*Φ*)

$$V_{dGB}(\Phi) = c_3\Lambda\,\mathcal{J}_3 + c_4\mathcal{J}_4 + \cdots$$

$\Longrightarrow$ **Absence of $\propto \Lambda^2$ corrections**

$\Longrightarrow$ $\mathrm{m}^2 \ll \Lambda^2$

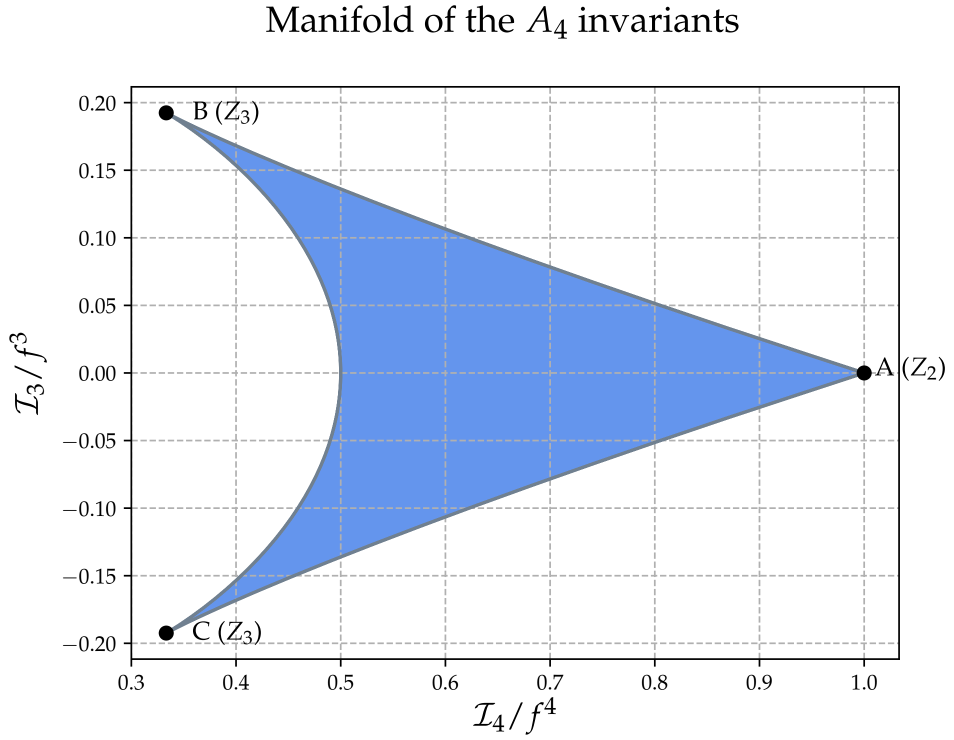
Natural minima & remnant symmetries

Natural minima defined by $\frac{\partial \mathcal{I}_j}{\partial \phi_i} = 0.$

$$\Rightarrow \frac{\partial V(\Phi)}{\partial \phi_i} = \sum_j \frac{\partial V}{\partial \mathcal{I}_j}\frac{\partial \mathcal{I}_j}{\partial \phi_i} = 0$$

⇒ Do not depend on the details of $V(\Phi)$

⇒ In the low-energy potential, **remnant explicit symmetry.**



Manifold of the A_4 invariants

E.g. around C, $A_4 \to Z_3$ and

Enforced by Z_3 symmetry

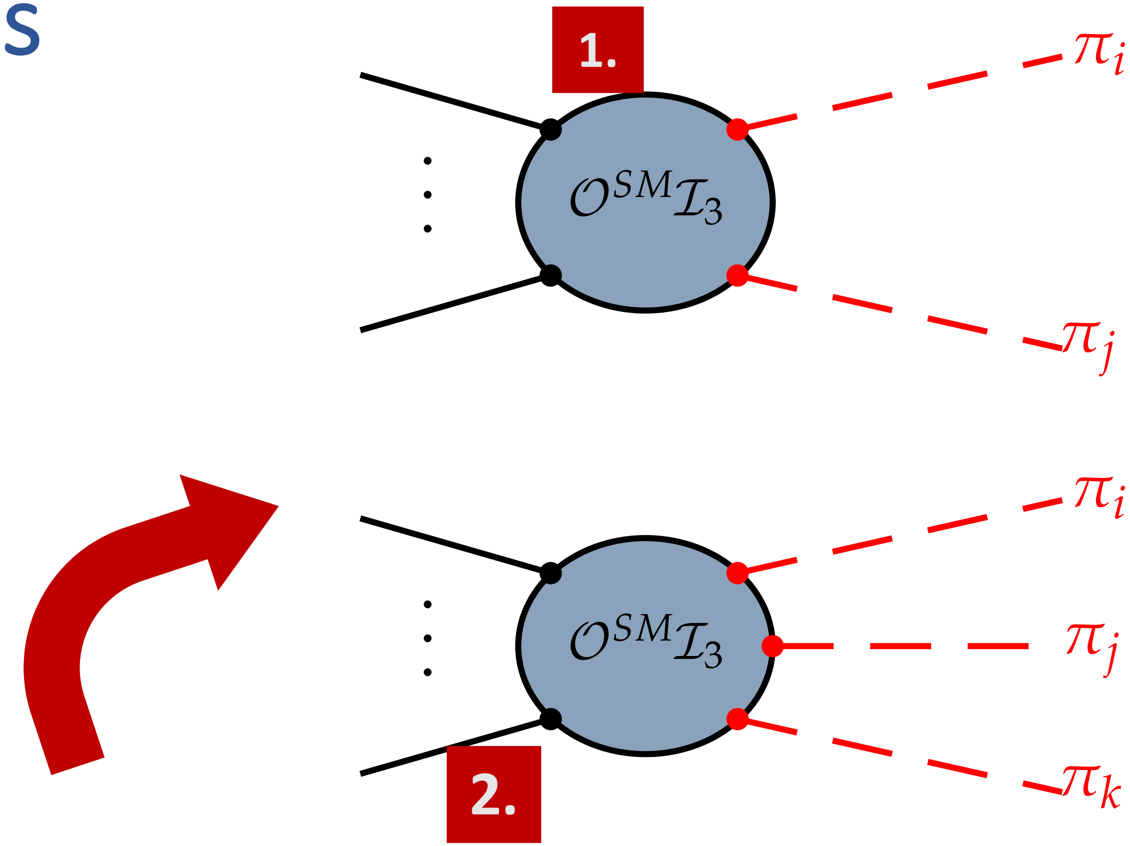
$$\mathcal{I}_3 = \frac{f}{\sqrt{3}}\left[-\frac{f^3}{3} + (\pi_1^2+\pi_2^2) - \frac{1}{3\sqrt{2}f}(\pi_1^3 - 3\pi_1\pi_2^2) - \frac{17}{24f^2}(\pi_1^2+\pi_2^2)^2\right] + \cdots$$

Phenomenological signals

Tell-tale experimental signals are brought in by the remnant symmetry:

- **Degeneracy of the masses &**
- **Simultaneous production** from the SM

Assuming interaction: $\mathcal{L} = \mathcal{O}_{SM}\mathcal{J}_3$
(SM singlet of A_4)



$$\mathcal{J}_3 = \frac{f}{\sqrt{3}}\left[-\frac{f^3}{3} + \underset{1.}{\boxed{(\pi_1^2 + \pi_2^2)}} - \frac{1}{3\sqrt{2}f}\overset{2.}{\boxed{(\pi_1^3 - 3\pi_1\pi_2^2)}} - \frac{17}{24f^2}(\pi_1^2 + \pi_2^2)^2\right] + \cdots$$

Phenomenological signals

Tell-tale experimental signals are brought in by the remnant symmetry:

- **Definite production rates** from the SM

  Assuming the interaction $\mathcal{L} = \mathcal{O}_{SM}\mathcal{J}_3$

Assumming:

- SM singlet of A_4
- $m_{\pi_i}^2 \ll f^2$

Production rates are fixed by the lowest-order:

$$\frac{\sigma(SM \to 2\pi)}{\sigma(SM \to 3\pi)} = 64\pi^2 \frac{f^2}{E_{CM}^2} \qquad \frac{\sigma(SM \to 3\pi)}{\sigma(SM \to 4\pi)} = \frac{6(24\pi)^2}{19(17)^2}\frac{f^2}{E_{CM}^2}$$

Discrete vs continuous

Provided that the coefficients are small, discrete ⟹ approximate continuous

SO(3)-invariant

$$V_{dGB}^{A_4}(\Phi) = (\mu^2\Phi^T\Phi + \lambda(\Phi^T\Phi)^2) + (c_3\Lambda\mathcal{J}_3 + c_4\mathcal{J}_4) + \cdots$$

A_4-invariant

$$(\mathcal{J}_2 = \Phi^T\Phi)$$

$c_3, c_4 \ll 1$

Other cases with even better suppresion??? ⟹ continuous more exact

⟹ more suppression to masses

Discrete vs continuous

The SO(3)-breaking invariants arise at higher orders:

SO(3)-invariant

$$V_{dGB}^{A_4}(\Phi) = (\mu^2\Phi^T\Phi + \lambda(\Phi^T\Phi)^2) + (c_6\,\frac{\mathcal{J}_6}{\Lambda^2} + c_{10}\,\frac{\mathcal{J}_{10}}{\Lambda^6}) + \cdots$$

$$(\mathcal{J}_2 = \Phi^T\Phi)$$

A_5-invariant

$c_6, c_{10} \ll 1$

$\Longrightarrow$ Masses much more suppressed than for A_4

Other examples

Discrete subgroups of SO(3):

- Alternating group A_4: in its **3** representation.
- Symmetric group S_4: in its two triplet **3** and **3’** reps.
- Alternating group A_5: in its **3**, **3’**, **4**(?) and **5**(?).
- …

Also as subgroups of larger continuous symmetries:

- A_5 as a subgroup of SO(4)
- …

Final remarks

- Non-linearly realized discrete symmetries open **new and suggestive model-building venues.**

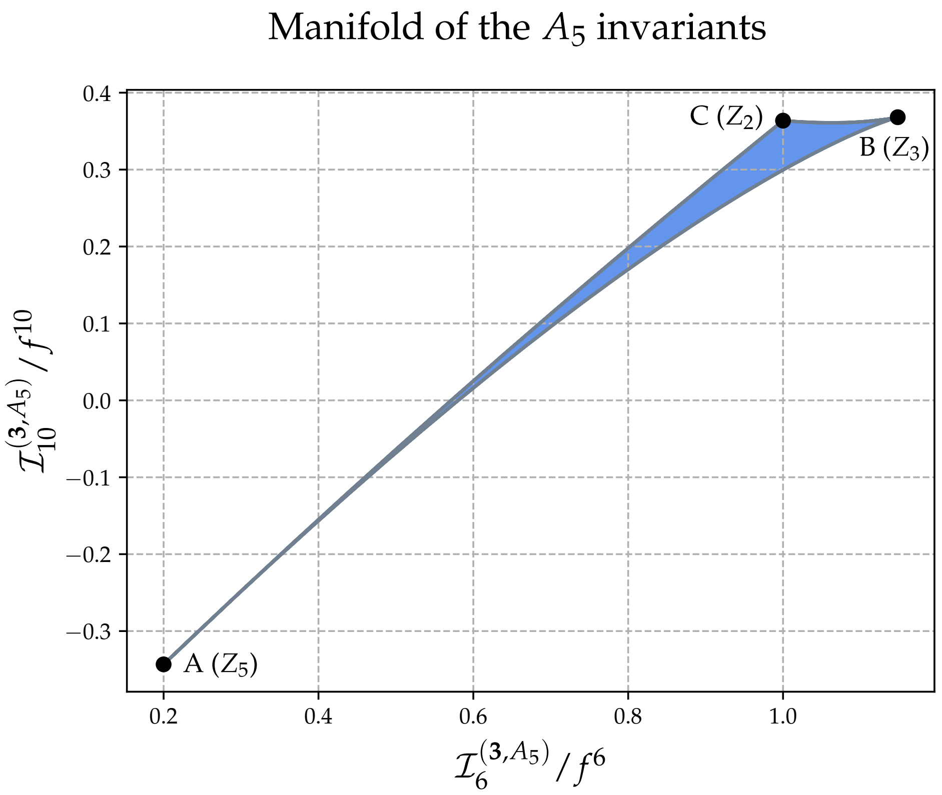
- They offer
  - ✓ **Ameliorated UV-insensitivity**
  - ✓ **Definite and distinct phenomenology**

- Many interesting **examples considered:**
  - ✓ Triplet of A_4
  - ✓ Triplets of S_4
  - ✓ Triplet & quadruplet of A_5
  - ✓ …

For more, see our paper arXiv:2205.09131…

Thank you very much

Remnant symmetries



Manifold of the A_5 invariants

The **natural minima** A & B retain two different subgroups à la Wigner, Z_5 and Z_3 resp.

Two different low-energy structures for $\mathcal{J}_6$:

Explicit Z_3 symmetry:

$$\mathcal{J}_6 = \frac{32}{9} f^4 \left[\frac{31}{96} f^2 - (\pi_1^2 + \pi_2^2) + \frac{10\sqrt{2}}{24f}(\pi_1^3 - 3\pi_1\pi_2^2) + \frac{\sqrt{30}}{4f}(\pi_2^3 - 3\pi_1^2\pi_2) + \frac{31}{12f^2}(\pi_1^2 + \pi_2^2)^2\right] + \cdots$$

Explicit Z_5 symmetry:

$$\mathcal{J}_6 = \frac{32}{5}\left[\frac{f^2}{32} + (\pi_1^2 + \pi_2^2) - \frac{31}{12f^2}(\pi_1^2 + \pi_2^2)^2 - \frac{1}{4f^3}(\pi_1^5 - 10\pi_1^3\pi_2^2 + 5\pi_1\pi_2^4)\right] + \cdots$$

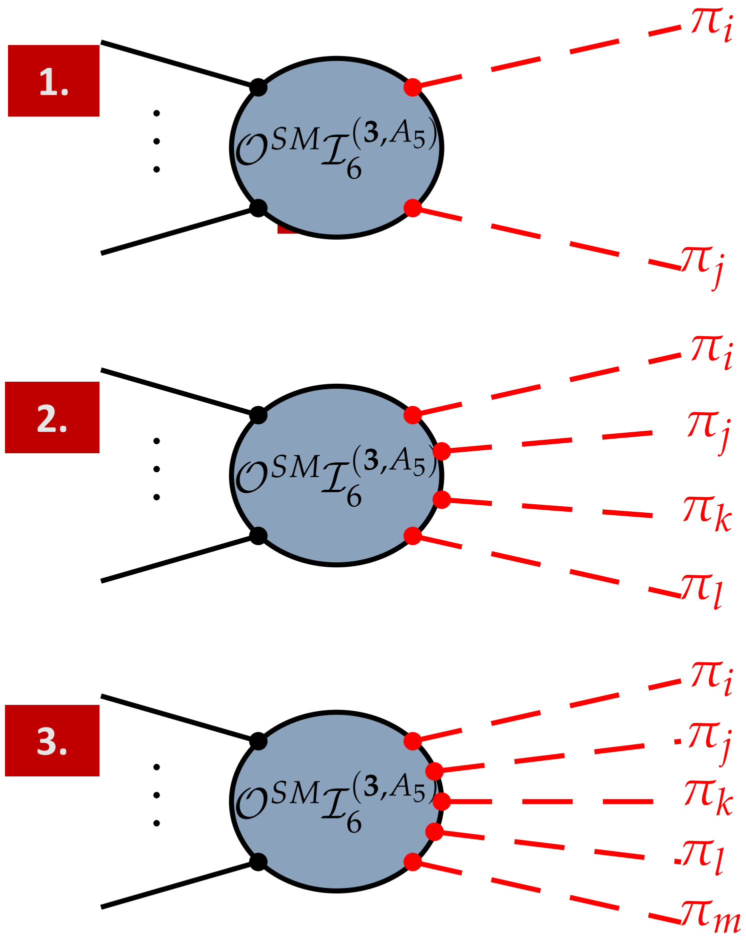
Phenomenological signals

Tell-tale experimental signals are brought in by the remnant symmetry:

- **Degeneracy of the masses &**
- **Simultaneous production**
- **Definite production rates**

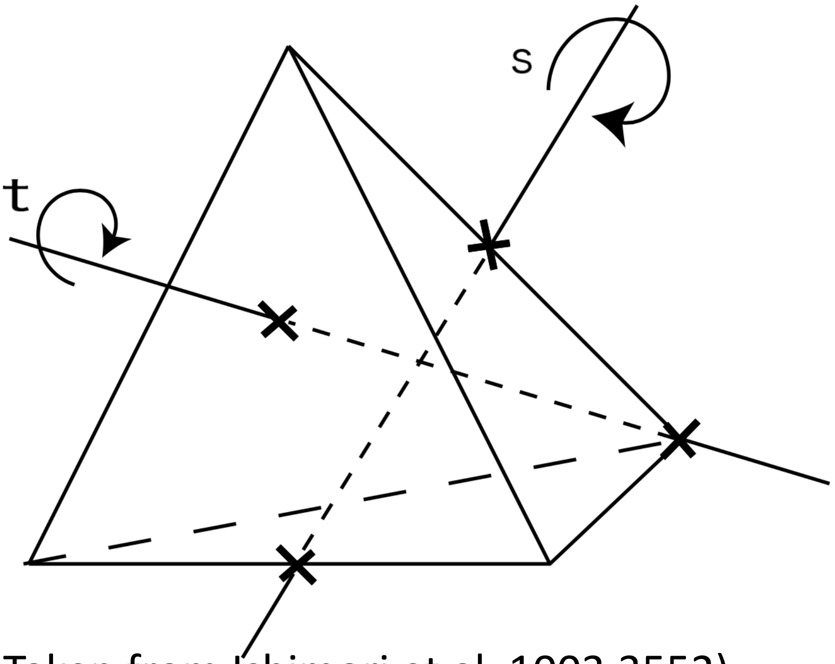
$$\frac{\sigma(SM \to 2\pi)}{\sigma(SM \to 4\pi)} = \frac{216\,(4\pi)^4}{19(31)^2}\frac{f^4}{E_{CM}^4}$$

$$\frac{\sigma(SM \to 4\pi)}{\sigma(SM \to 5\pi)} = \frac{19(31)^2(8\pi)^2}{45^2}\frac{f^2}{E_{CM}^2}$$



$$\mathcal{J}_6 = \frac{32}{5}\left[\frac{f^2}{32} + \underbrace{(\pi_1^2 + \pi_2^2)}_{1.} - \frac{31}{12f^2}\overbrace{(\pi_1^2 + \pi_2^2)^2}^{2.} - \frac{1}{4f^3}\underbrace{(\pi_1^5 - 10\pi_1^3\pi_2^2 + 5\pi_1\pi_2^4)}_{3.}\right] + \cdots$$

$\boldsymbol{\Phi}$, a triplet of $\boldsymbol{A_4}$



(Taken from Ishimori et al, 1003.3552)