Automatic Nelson-Barr from Gauged Flavor

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Based on "*Automatic Nelson-Barr from Gauged Flavor*"

2207.XXXXX to appear

in collaboration with B. Gavela and B. Grinstein

The Strong CP problem

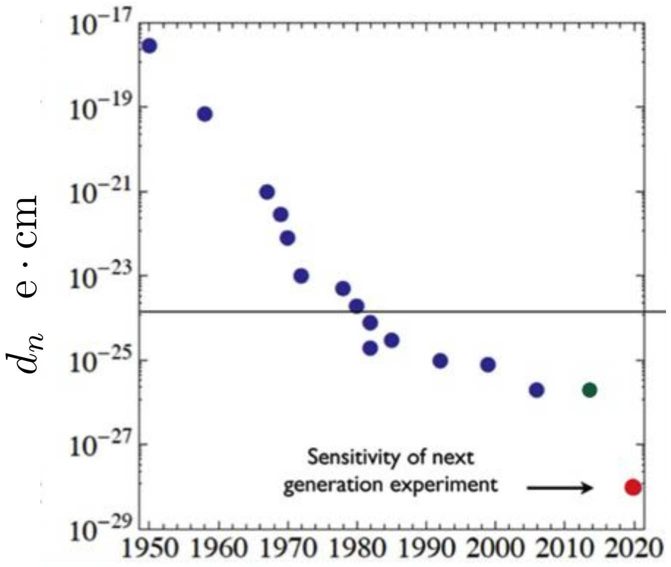
$$\mathcal{L}_{QCD} \supset \mathcal{L}_{\cancel{CP}} = -\bar{q}me^{i\beta}q + \theta_{QCD}\frac{\alpha_s}{8\pi}G^a_{\mu\nu}\tilde{G}^{a\mu\nu}$$

One physical CPV phase: $\bar{\theta} = \theta_{QCD} + \arg\det M$

$$d_n \approx 10^{-16}|\bar{\theta}|\mathrm{e}\cdot\mathrm{cm} \qquad \bar{\theta} \lesssim 10^{-10}$$

Why is it so small?

Neutron EDM (Electric Dipole Moment)



[B.Fillipone via diLuzio]

The Strong CP ~~problem~~ hint

Neutron EDM (Electric Dipole Moment)

$\mathcal{L}_{QCD}$

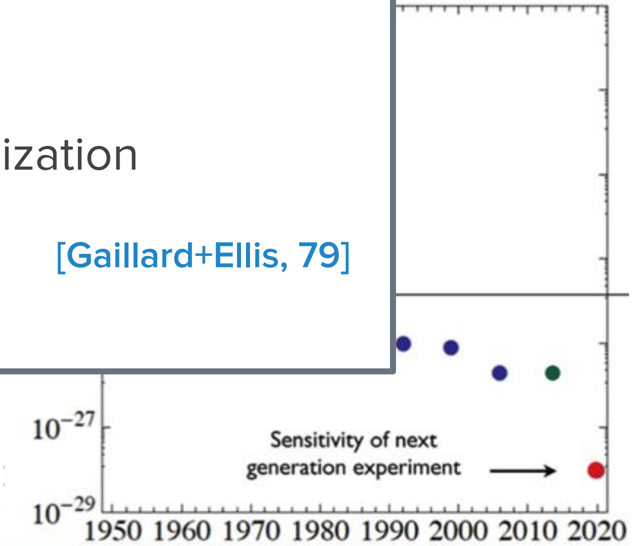
→ Fine-tuning

→ $\bar{\theta} = 0$ harmless under renormalization

[Gaillard+Ellis, 79]

One physi

$d_n \approx 10^{-16} |\theta| \mathrm{e} \cdot \mathrm{cm}$ $\qquad \theta \lesssim 10^{-10}$



[B.Fillipone via diLuzio]

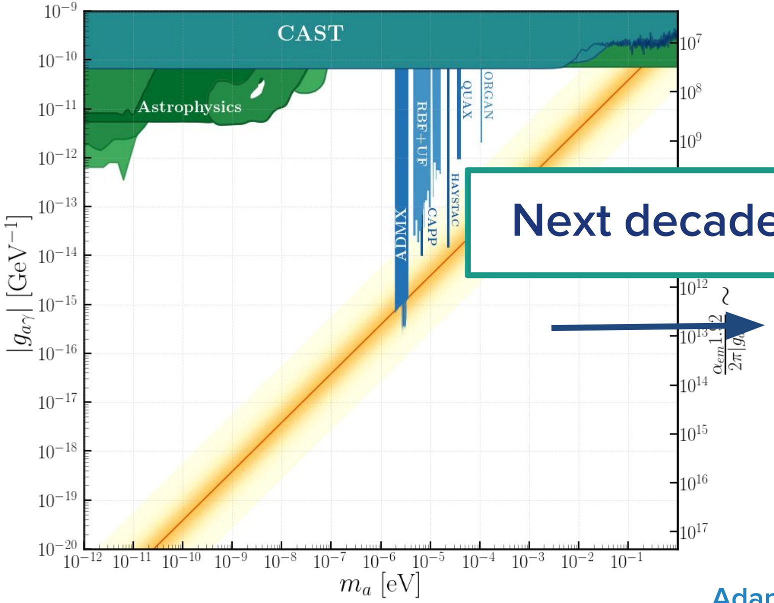
Why is it so small?

The QCD axion

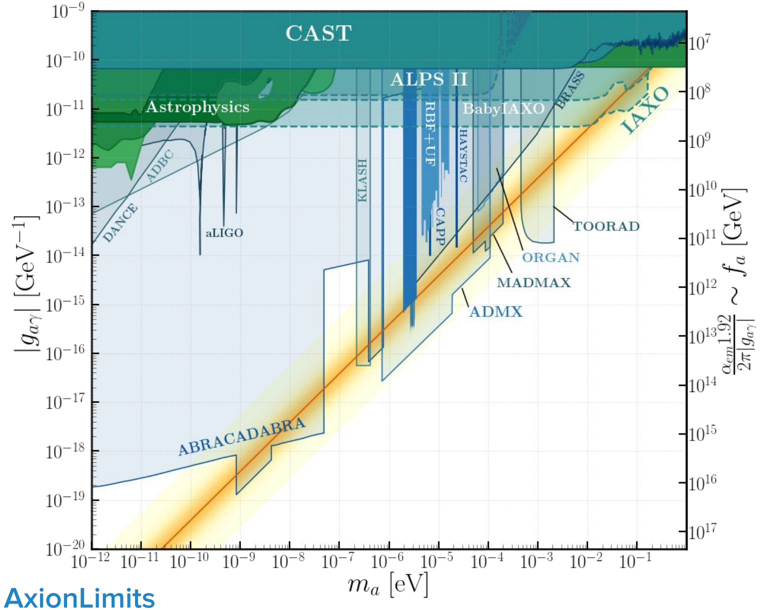
- ➔ Solves the Strong CP problem
- ➔ Excellent Dark Matter candidate

[Peccei+Quinn 77]
[Weinberg, 78]
[Wilczek, 78]

[Abbot+Sikivie, 83]
[Dine and W. Fischler, 83]
[Preskil et al, 91]



Next decades



Adapted from AxionLimits
[Ciaran O'hare, 20]

The Nelson-Barr mechanism

[Nelson, 84]
[Barr, 84]

➔ CP is conserved exactly in the UV => **NO PHASES**

$$\cancel{\theta_{QCD}\frac{\alpha_s}{8\pi}G\tilde{G}} \qquad \theta_{QCD} = 0,\ \arg\det M = 0 \text{ and } \delta_{KM} = 0$$

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- ➔ At low energies CP is spontaneously broken $\langle\eta\rangle$ is complex

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➔ The fermion mass matrices pattern only generates δ_{KM} but no θ

Ex. BBP model $\mathcal{L}\supset -\mu\bar{q}_L q_R - \lambda\eta\,\bar{q}_L D_R - y\Phi\bar{Q}_L D_R$

[Bento, Branco, Parada, 91]

$$M=\begin{pmatrix}\mu & \lambda\langle\eta\rangle\\ 0 & m_d\end{pmatrix}$$

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$$M=\begin{array}{c} \text{REAL}\ \ \text{COMPLEX} \\ \begin{pmatrix} \boxed{\mu} & \boxed{\lambda\langle\eta\rangle} \\ 0 & \boxed{m_d} \end{pmatrix} \\ \text{REAL} \end{array} \implies \arg\det M\big|_{\text{tree level}}=0 \implies \bar{\theta}\big|_{\text{tree level}}=0$$

The Nelson-Barr mechanism

[Nelson, 84]
[Barr, 84]

➔ CP is conserved exactly in the UV => **NO PHASES**

Spontaneous CP + Particular fermion mass pattern $\Longrightarrow$ **Strong CP problem solved**

$$\bar{\theta}\big|_{\text{tree level}} = 0$$

$$M = \begin{pmatrix} \boxed{\mu} & \boxed{\lambda\langle\eta\rangle} \\ 0 & \boxed{m_d} \end{pmatrix} \Longrightarrow \arg\det M\big|_{\text{tree level}} = 0 \Longrightarrow \bar{\theta}\big|_{\text{tree level}} = 0$$

REAL

Challenges of the NB mechanism

❌ 1. Enforcing the NB structure

[Dine+Draper, 15]

WHY?

$$M = \begin{pmatrix} \mu & \lambda\langle\eta\rangle \\ \boxed{0} & m_d \end{pmatrix}$$

Ex. $\cancel{\eta \bar{q}_L q_R}$

Challenges of the NB mechanism

[Dine+Draper, 15]

1. Enforcing the NB structure

WHY? $$M = \begin{pmatrix} \mu & \lambda\langle\eta\rangle \\ \boxed{0} & m_d \end{pmatrix}$$ **Ex.** $\cancel{\eta \bar{q}_L q_R}$

2. Higher dimensional operators

$$\frac{1}{\Lambda}\eta\,\eta\,\bar{q}_L q_R$$

$$\bar{\theta} \lesssim 10^{-10}$$

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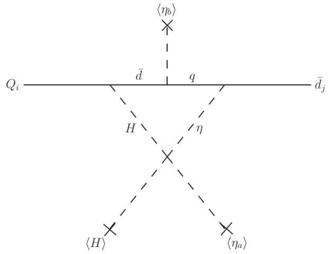

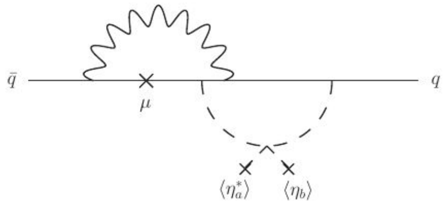

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Automatic Nelson-Barr from Gauged Flavor

B. Gavela, B. Grinstein and ***P. Q.***

To appear

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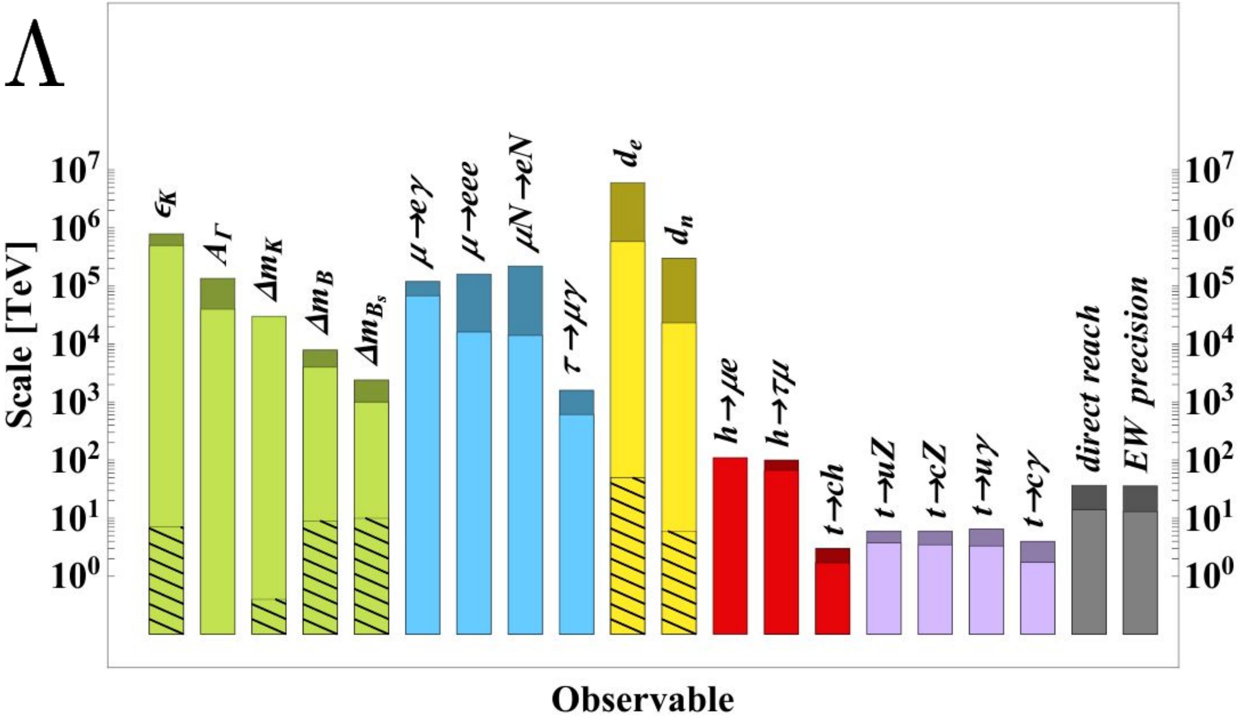
BSM reach of Flavor Physics



$$\mathcal{L} \supset \frac{1}{\Lambda^{n-4}} \mathcal{O}_n$$

Hatched bars: MFV
Darker colors: midterm prospects

[European Strategy for Particle Physics, 19]
by Aloni, Dery+Gavela+Nir

Gauged flavor symmetries

[Chakrabarti, 79]
[Barr, Zee, 78]
[Wilczek, Zee, 78]
[Georgi, Chivukula, 87]

- Gauging the non abelian **quark flavor symmetry**

$$G_f = SU(3)_{Q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R}$$

[Grinstein+Redi+Villadoro, 10]

- Gauging the non abelian **lepton flavor symmetry**

$$SU(3)_\ell \times SU(3)_e \qquad SU(3)_\ell \times SU(3)_e \times SO(3)_N$$

[Alonso, Fernandez-Martinez, Gavela, Grinstein, Merlo, PQ, 16]

- The Yukawas $\mathcal{Y}_u, \mathcal{Y}_d$ are promoted to dynamical fields whose vevs break spontaneously the SM flavor symmetry

Gauged quark Flavor symmetry

Largest non-abelian symmetry in the massless limit?

$$G_f = SU(3)_{Q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R}$$

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Gauged quark Flavor symmetry

Largest non-abelian symmetry in the massless limit?

$$G_f = SU(3)_{Q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R}$$ [Grinstein+Redi+Villadoro, 10]

ANOMALOUS

Exotic fermions are needed: $\Psi_{u_R}, \Psi_{d_R}, \Psi_{u_L}, \Psi_{d_L}$

Gauged quark Flavor symmetry

$$G_f = SU(3)_{Q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R}$$

[Grinstein+Redi+Villadoro, 10]

- Most general renormalizable Lagrangian

$$\begin{aligned}-\mathcal{L}_{int} &= \lambda_u \overline{Q}_L \tilde{H} \Psi_{u_R} + \lambda'_u \overline{\Psi}_{u_L} \mathcal{Y}_u \Psi_{u_R} + M_u \overline{\Psi}_{u_L} U_R \\ &+ \lambda_d \overline{Q}_L H \Psi_{d_R} + \lambda'_d \overline{\Psi}_{d_L} \mathcal{Y}_d \Psi_{d_R} + M_d \overline{\Psi}_{d_L} D_R + h.c.\end{aligned}$$

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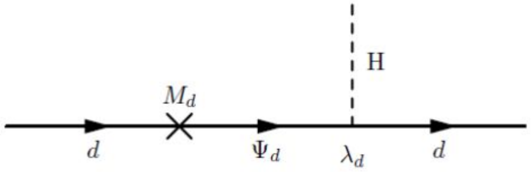
See-saw mechanism for all fermions

$$\begin{pmatrix} \overline{U}_L & \overline{\Psi}_{u_L} \end{pmatrix} \begin{pmatrix} 0 & \lambda_u \frac{v}{\sqrt{2}} \\ M_u & \lambda'_u \hat{\mathcal{Y}}_u V \end{pmatrix} \begin{pmatrix} U_R \\ \Psi_{U_R} \end{pmatrix} ; \qquad \begin{pmatrix} \overline{D}_L & \overline{\Psi}_{d_L} \end{pmatrix} \begin{pmatrix} 0 & \lambda_d \frac{v}{\sqrt{2}} \\ M_d & \lambda'_d \hat{\mathcal{Y}}_d \end{pmatrix} \begin{pmatrix} D_R \\ \Psi_{D_R} \end{pmatrix}$$



$$m_d \propto \frac{1}{\mathcal{Y}_d} \propto \frac{1}{m'_d}$$

NB from Gauged flavor

- Let's impose CP in the UV
- CP arises spontaneously through the vevs $\langle \mathcal{Y}_{u,d} \rangle$

NB from Gauged flavor

- Let's impose CP in the UV
- CP arises spontaneously through the vevs $\langle \mathcal{Y}_{u,d} \rangle$
- Seesaw-like mass matrices already present BN structure!!

$$\begin{pmatrix} \overline{U}_L & \overline{\Psi}_{u_L} \end{pmatrix} \begin{pmatrix} 0 & \lambda_u \frac{v}{\sqrt{2}} \\ M_u & \lambda'_u \hat{\mathcal{Y}}_u V \end{pmatrix} \begin{pmatrix} U_R \\ \Psi_{U_R} \end{pmatrix}; \qquad \begin{pmatrix} \overline{D}_L & \overline{\Psi}_{d_L} \end{pmatrix} \begin{pmatrix} 0 & \lambda_d \frac{v}{\sqrt{2}} \\ M_d & \lambda'_d \hat{\mathcal{Y}}_d \end{pmatrix} \begin{pmatrix} D_R \\ \Psi_{D_R} \end{pmatrix}$$

$$\bar{\theta}_{\mathrm{QCD}}\Big|_{tree} = \theta_{\mathrm{QCD}} + \arg\left(\det\left(\mathcal{M}_u \mathcal{M}_d\right)\right) = 0$$

- Intuitive interpretation:

$$\bar{\theta}_{\mathrm{QCD}} = \theta_{\mathrm{QCD}} + \arg\left(\det\left(m_u\, m_{u'}\, m_d\, m_{d'}\right)\right) =$$
$$\arg\left(\det\left(\frac{1}{\hat{\mathcal{Y}}_u}\right)\right) + \arg\left(\det\left(\hat{\mathcal{Y}}_u\right)\right) + \arg\left(\det\left(\frac{1}{\hat{\mathcal{Y}}_d}\right)\right) + \arg\left(\det\left(\hat{\mathcal{Y}}_d\right)\right) = 0$$

Solved challenges in NB from gauged flav.

✓ 1. Enforcing the NB structure

Enforced by the gauged flavor symmetry

WHY? $$\mathcal{M}_u = \begin{pmatrix} 0 & \lambda_u \frac{v}{\sqrt{2}} \\ M_u & \lambda'_u \hat{\mathcal{Y}}_u V \end{pmatrix} ; \qquad \mathcal{M}_d = \begin{pmatrix} 0 & \lambda_d \frac{v}{\sqrt{2}} \\ M_d & \lambda'_d \hat{\mathcal{Y}}_d \end{pmatrix}$$

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2. Higher dimensional operators

Higher dimensional operators

$$\mathcal{O}_{d,A} = \frac{1}{\Lambda^{\alpha+1}}\overline{Q}_L H\, f_\alpha(H_u, H_d)\mathcal{Y}_d^\dagger\, u_R\,, \qquad \mathcal{O}_{d,B} = \frac{1}{\Lambda^{\alpha}}\overline{Q}_L\, H\, f_\alpha(H_u, H_d)\,\Psi_{d_R}$$

$$\mathcal{O}_{d,C} = \frac{1}{\Lambda^{\alpha+1}}\overline{\Psi}_{d_L}\,\mathcal{Y}_d f_\alpha(H_u, H_d)\mathcal{Y}_d^\dagger\, u_R\,, \quad \mathcal{O}_{d,D} = \frac{1}{\Lambda^{\alpha}}\overline{\Psi}_{d_L}\,\mathcal{Y}_d f_\alpha(H_u, H_d)\,\Psi_{d_R}$$

$$f_\alpha(H_u, H_d) = H_u^p H_d^q H_u^r H_d^s \ldots\,, \qquad H_u = \mathcal{Y}_u^\dagger \mathcal{Y}_u \qquad H_d = \mathcal{Y}_d^\dagger \mathcal{Y}_d\,,$$

$$\bar{\theta} \sim \mathrm{Im}\,\mathrm{Tr}\left[H_u^p H_d^q H_u^r H_d^s \ldots\right]$$

$$\bar{\theta} \sim \mathrm{Im}\,\mathrm{Tr}\left[H_u H_d H_u^2 H_d^2\right]$$

Higher dimensional operators

Relevant contribution ∝ Jarlskog invariant

$$\operatorname{Im}\operatorname{Tr}\left[H_u H_d H_u^2 H_d^2\right] \propto \frac{M_d^6 M_u^6 v^{12}}{\left(\det h_u h_d\right)^2} \operatorname{Im}\operatorname{Tr}\left[h_u h_d h_u^2 h_d^2\right]$$

$$\propto \left(m_t^2 - m_c^2\right)\left(m_t^2 - m_u^2\right)\left(m_c^2 - m_u^2\right)\left(m_b^2 - m_s^2\right)\left(m_s^2 - m_d^2\right)\left(m_b^2 - m_d^2\right)$$

$$\times\, c_{12} c_{23} c_{13}^2 s_{12} s_{23} s_{13} \sin\delta_{\mathrm{KM}}$$

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WHY? $$\mathcal{M}_u = \begin{pmatrix} 0 & \lambda_u \frac{v}{\sqrt{2}} \\ M_u & \lambda'_u \hat{\mathcal{Y}}_u V \end{pmatrix} \; ; \qquad \mathcal{M}_d = \begin{pmatrix} 0 & \lambda_d \frac{v}{\sqrt{2}} \\ M_d & \lambda'_d \hat{\mathcal{Y}}_d \end{pmatrix}$$

✓ 2. Higher dimensional operators

Enforced by the gauged flavor symmetry

$$\bar{\theta} \sim \mathrm{Im}\,\mathrm{Tr}\left[H_u H_d H_u^2 H_d^2\right] \ll 10^{-10}$$

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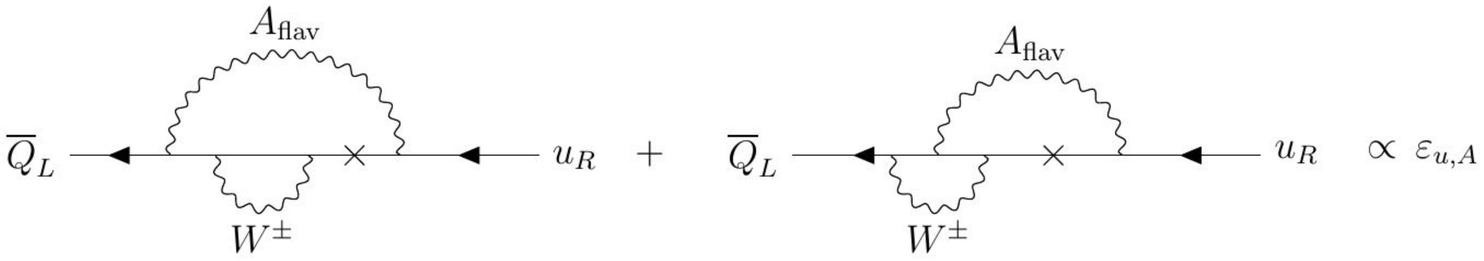
3. Loop corrections

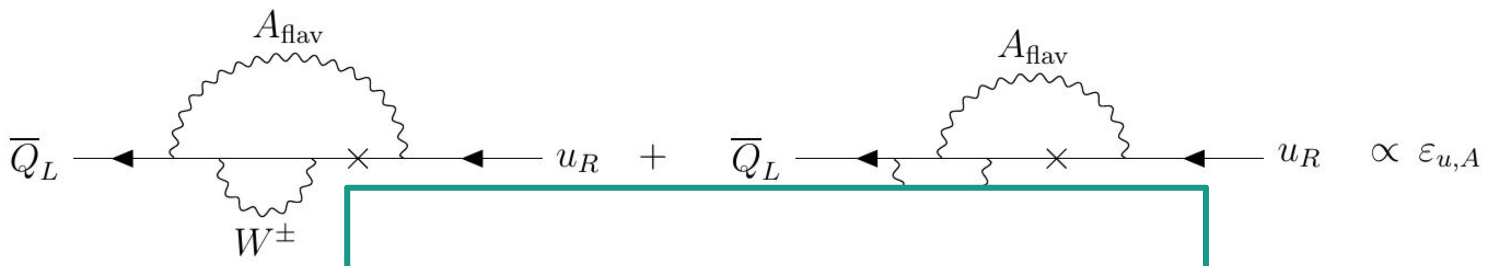
Loop corrections to $\bar{\theta}$



$$\bar{\theta}|_{\mathrm{loop}} = \mathrm{Im}\,\mathrm{Tr}\,[m_U^{-1}\,\varepsilon_{u,A}] \supset \mathrm{Im}\,\mathrm{Tr}\,[m_U^{-1}\lambda_U^a\, m_U\, f(h_u)\,\lambda_L^b\, g(h_d)\,]\left(\mathcal{M}_A^{-2}\right)_{ab}$$

A bit complicated… but with Caley-Hamilton Th. one can get rid of the gell-mann matrices and obtain polynomial invariants of the basic flavons

Loop corrections to $\bar{\theta}$



$\bar{\theta}|_{\text{loop}} = \text{Im}\,\text{T}$ ... $(\mathcal{M}_A^{-2})_{ab}$

A bit comp ... get rid of the

gell-mann ... he basic flavons

Preliminary results:

Extremely suppressed

$$\bar{\theta}\Big|_{2-\text{loop}} \ll 10^{-10}$$

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1. Enforcing the NB structure

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Conclusions

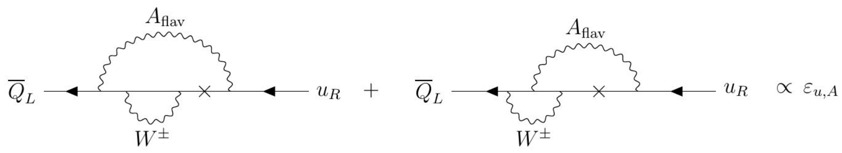
- ➔ The strong CP problem can be solved automatically à la Nelson-Barr in the context of gauged flavor models.
- ➔ The flavour gauge symmetry solves also common issues of NB constructions
  - ◆ NB structure automatically enforced
  - ◆ Proteccion against higher dim. operators
  - ◆ Loop contributions under control

- ➔ There is life beyond the axion!

Thank you

Backup slides

Loop corrections to $\bar{\theta}$



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A bit complicated... but with Caley-Hamilton Theorem

$$\mathcal{M}_A^{-2} = \frac{1}{\det\left[\mathcal{M}_A^2\right]}\operatorname{adj}\left[\mathcal{M}_A^2\right]$$

$$\operatorname{adj}(\mathbf{A}) = \sum_{s=0}^{n-1}\mathbf{A}^s \sum_{k_1,k_2,\ldots,k_{n-1}}\prod_{\ell=1}^{n-1}\frac{(-1)^{k_\ell+1}}{\ell^{k_\ell}k_\ell!}\operatorname{tr}\left(\mathbf{A}^\ell\right)^{k_\ell}$$

$$\operatorname{adj}(\mathbf{A}) = \frac{1}{2}\mathbf{I}_3\left((\operatorname{tr}\mathbf{A})^2 - \operatorname{tr}\mathbf{A}^2\right) - \mathbf{A}(\operatorname{tr}\mathbf{A}) + \mathbf{A}^2$$

One can get rid of the gell-mann matrices and obtain polynomial invariants of the basic flavons

Gauged flavor

Flavor gauge bosons

$$\mathcal{L}_{mass} = Tr\left[\left(D_\mu \mathcal{Y}_u\right)^\dagger \left(D^\mu \mathcal{Y}_u\right)\right] + Tr\left[\left(D_\mu \mathcal{Y}_d\right)^\dagger \left(D^\mu \mathcal{Y}_d\right)\right] \tag{1.12}$$

$$\supset Tr\left|g_Q \mathcal{Y}_u A_Q^\mu - g_U A_U^\mu \mathcal{Y}_u\right|^2 + Tr\left|g_Q \mathcal{Y}_d A_Q^\mu - g_D A_D^\mu \mathcal{Y}_d\right|^2 \tag{1.13}$$

$$= \frac{1}{2}\chi_\mu^T \left(\mathcal{M}_A^2\right) \chi^\mu \tag{1.14}$$

where $\chi_\mu^T = \left(A_{Q\,\mu}^1, ..., A_{Q\,\mu}^8, A_{U\,\mu}^1, ..., A_{U\,\mu}^8, A_{D\,\mu}^1, ..., A_{D\,\mu}^8\right)$ and $\mathcal{M}_A^2$ is a 24×24 matrix:

$$\mathcal{M}_A^2 = \begin{pmatrix} M_{QQ}^2 & M_{QU}^2 & M_{QD}^2 \\ M_{UQ}^2 & M_{UU}^2 & 0 \\ M_{DQ}^2 & 0 & M_{DD}^2 \end{pmatrix} \tag{1.15}$$

Gauged flavor

$$(M^2_{QQ})_{ab} = \frac{1}{4} g_Q^2 \, Tr \left[\mathcal{Y}_u \{\lambda^a, \lambda^b\} \mathcal{Y}_u^\dagger + \mathcal{Y}_d \{\lambda^a, \lambda^b\} \mathcal{Y}_d^\dagger \right]$$

$$(M^2_{UU})_{ab} = \frac{1}{4} g_U^2 \, Tr \left[\mathcal{Y}_u^\dagger \{\lambda^a, \lambda^b\} \mathcal{Y}_u \right]$$

$$(M^2_{DD})_{ab} = \frac{1}{4} g_D^2 \, Tr \left[\mathcal{Y}_d^\dagger \{\lambda^a, \lambda^b\} \mathcal{Y}_d \right]$$

$$(M^2_{QU})_{ab} = (M^2_{UQ})_{ba} = -\frac{1}{2} g_Q g_U \, Tr \left[\lambda^a \mathcal{Y}_u^\dagger \lambda^b \mathcal{Y}_u \right]$$

$$(M^2_{QD})_{ab} = (M^2_{DQ})_{ba} = -\frac{1}{2} g_Q g_D \, Tr \left[\lambda^a \mathcal{Y}_d^\dagger \lambda^b \mathcal{Y}_d \right]$$

Gauged flavor