Scotogenic Majorana neutrino masses in an A_4 orbifold theory of flavour

Omar Medina (In collaboration with...)

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AHEP - IFIC - CSIC - UV

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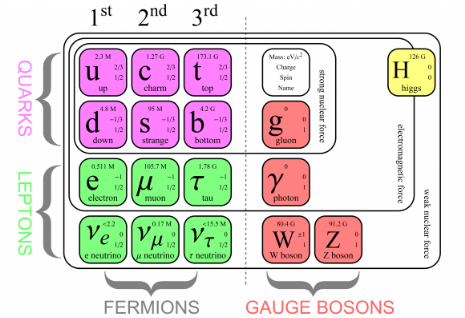
Outline

- Motivation
- Aim of this Model
- Elements & Properties
- Phenomenology
- Conclusions

Fundamental Particles And Interactions



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$$SU(3)_c \otimes \underbrace{SU(2)_L \otimes U(1)_Y}_{\text{Electroweak Sector}}.$$

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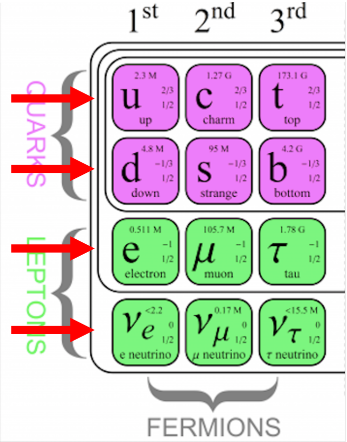

- The $\mathcal{L}_{\text{SM}}$ is **built** gauge invariant under

$$SU(3)_c \otimes \underbrace{SU(2)_L \otimes U(1)_Y}_{\text{Electroweak Sector}}.$$

- The SM gauge group is **generation blind**. Preserves **full flavour symmetry.**

Yukawa interaction

- Yukawa interaction is **not** based on the gauge principle and in the SM **breaks** the flavour symmetry e.g.

$$-\underline{Y_e^{ij}}\overline{L_i}_L^I \Phi e_{jR}^I, \qquad Y_e = \begin{pmatrix} Y_e^{ee} & Y_e^{e\mu} & Y_e^{e\tau} \\ Y_e^{\mu e} & Y_e^{\mu\mu} & Y_e^{\mu\tau} \\ Y_e^{\tau e} & Y_e^{\tau\mu} & Y_e^{\tau\tau} \end{pmatrix} \tag{1}$$

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Flavour Basis

- Gauge Interaction

$$\delta^{ij}\frac{g}{\sqrt{2}}\overline{u}^I_{iL}\not{W}^- d^I_{jL},$$

- Yukawa Interaction

$$-\underline{Y_d^{ij}}\,\overline{Q}^I_{iL}\Phi d^I_{jR}. \tag{2}$$

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	• Gauge Interaction	• Yukawa Interaction
Flavour Basis	$\delta^{ij}\frac{g}{\sqrt{2}}\overline{u}^I_{i\,L}\not{W}^- d^I_{j\,L},$	$-\underline{Y_d^{ij}}\,\overline{Q}^I_{i\,L}\Phi d^I_{j\,R}.$ (2)
Mass Basis	$\frac{g}{\sqrt{2}}\overline{u}_{i\,L}\underline{V^{ij}_{\text{CKM}}}\not{W}^- d_{j\,L},$	$-D_d^{ij}\overline{Q}_{i\,L}\Phi d_{j\,R}.$ (3)

The CKM Matrix

Relates **down-type** quark **Flavour-** and **Mass-eigenstates**. Using the **PDG** Parametrization:

$$\{\theta^{q}_{12}, \theta^{q}_{13}, \theta^{q}_{23}, \delta^{q}\}, \tag{4}$$

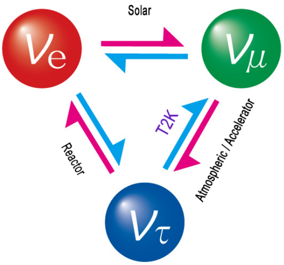
$$V_{\text{CKM}}=\begin{pmatrix} c^{q}_{12}c^{q}_{13} & s^{q}_{12}c^{q}_{13} & s^{q}_{13}e^{-i\delta^{q}} \\ -s^{q}_{12}c^{q}_{23}-c^{q}_{12}s^{q}_{13}s^{q}_{23}e^{i\delta^{q}} & c^{q}_{12}c^{q}_{23}-s^{q}_{12}s^{q}_{13}s^{q}_{23}e^{i\delta^{q}} & c^{q}_{13}s^{q}_{23} \\ s^{q}_{12}s^{q}_{23}-c^{q}_{12}s^{q}_{13}c^{q}_{23}e^{i\delta^{q}} & -c^{q}_{12}s^{q}_{23}-s^{q}_{12}s^{q}_{13}c^{q}_{23}e^{i\delta^{q}} & c^{q}_{13}c^{q}_{23} \end{pmatrix},$$

where

$$c^{q}_{ij} = \cos\theta^{q}_{ij}, \qquad s^{q}_{ij} = \sin\theta^{q}_{ij}. \tag{5}$$

Neutrino Oscillations

- Neutrinos suffer flavor changing while they propagate (**Neutrino Oscillations**):



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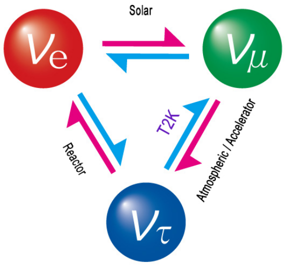

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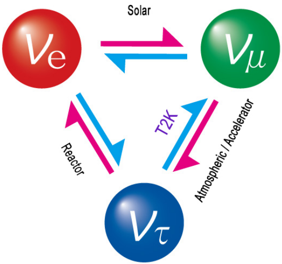
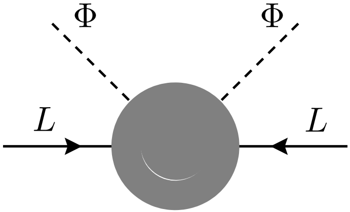


Neutrino oscillation between three generations

- **Interaction-** and **Mass-** eigenbasis are **Misaligned**
- Therefore neutrinos have a mass.
- Neutrinos at least $\mathcal{O}(10^{-6})$ lighter than the e^-. This strongly suggests a **different mass mechanism**

Neutrino Mass



- Majorana neutrino mass from Weinberg operator [PRL 43 (1979), 1566]

$$\sim \frac{c}{\Lambda} \Phi \overline{L}^c \Phi L \qquad (6)$$

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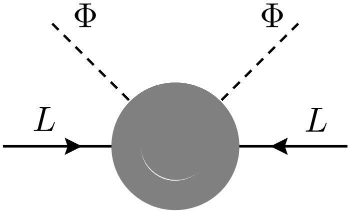

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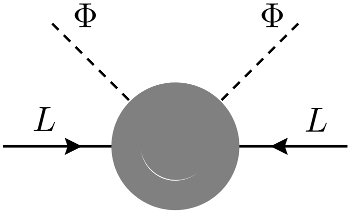

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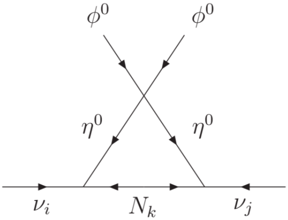
- **Seesaw** mechanisms, (Tree Level Neutrino mass)
- **Radiative** (Neutrino mass only in the QFT)

Radiative Neutrino Mass



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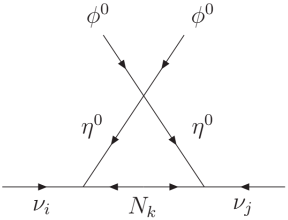
Radiative Neutrino Mass



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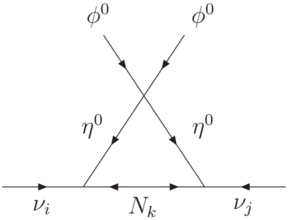


- **Scotogenic**: Additional N_k (sterile), and a scalar η in the field content + $\mathbb{Z}_2$ Symmetry **[E. Ma, PRD 73, 077301 (2006)]** .
- Mediator mass is **suppressed** by the loop.
- $\mathbb{Z}_2$ symmetry stabilizes the lightest "Dark Particle" giving a **DM candidate**.

The Lepton Mixing Matrix

Relates **neutrino** **Flavour-** and **Mass-eigenstates**.

$$|\nu^{\alpha}\rangle = \sum_i U_{\alpha i} |\nu^{i}\rangle , \qquad \alpha = e, \mu, \tau. \qquad i = 1, 2, 3, \tag{7}$$

Symmetric parametrization **[Rodejohann, and J. W. F. Valle, PRD 84, 073011 (2011)]** :

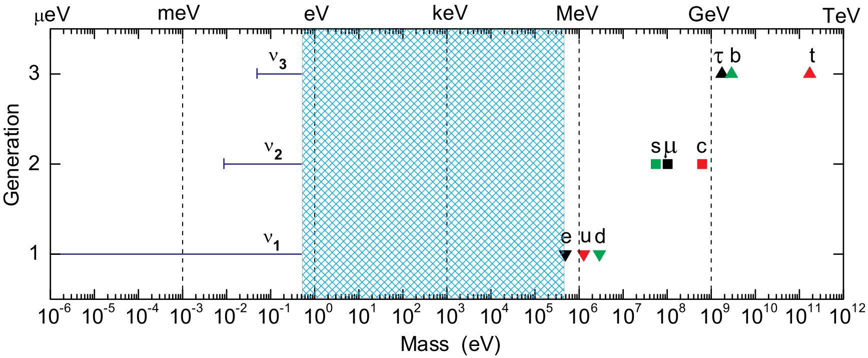
$$\left\{\theta^l_{12}, \theta^l_{13}, \theta^l_{23}, \phi_{12}, \phi_{13}, \phi_{23}\right\}, \quad \text{with} \quad \delta^l = \phi_{13} - \phi_{12} - \phi_{23}. \tag{8}$$

$$\begin{bmatrix} c^l_{12}c^l_{13} & s^l_{12}c^l_{13}e^{-i\phi_{12}} & s^l_{13}e^{-i\phi_{13}} \\ -s^l_{12}c^l_{23}e^{i\phi_{12}} - c^l_{12}s^l_{13}s^l_{23}e^{-i(\phi_{23}-\phi_{13})} & c^{\mathrm{l}}_{12}c^{\mathrm{q}}_{23} - s^l_{12}s^l_{13}s^l_{23}e^{-i(\phi_{23}+\phi_{12}-\phi_{13})} & c^l_{13}s^l_{23}e^{-i\phi_{23}} \\ s^l_{12}s^l_{23}e^{i(\phi_{23}+\phi_{12})} - c^l_{12}s^l_{13}c^l_{23}e^{i\phi_{13}} & -c^l_{12}s^l_{23}e^{i\phi_{23}} - s^{\mathrm{l}}_{12}s^{\mathrm{l}}_{13}c^{\mathrm{l}}_{23}e^{-i(\phi_{12}-\phi_{13})} & c^l_{13}c^l_{23} \end{bmatrix}. \tag{9}$$

The Flavour Puzzle

Flavour symmetry breaking has consequences...

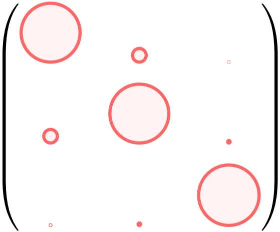
- **Fermion masses, and their hierachy** [Int.J.Mod.Phys.A 29 (2014) 1430067] .



The Flavour Puzzle

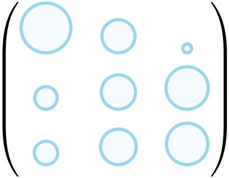
- **Mixing Patterns**

$$\begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} \sim$$



,

$$\begin{pmatrix} |U_{e1}| & |U_{e2}| & |U_{e3}| \\ |U_{\mu 1}| & |U_{\mu 2}| & |U_{\mu 3}| \\ |U_{\tau 1}| & |U_{\tau 2}| & |U_{\tau 3}| \end{pmatrix} \sim$$



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- There are 5 parameters related to **gauge symmetry** and its breaking:

$$\{g_e,\, \theta_W,\, g_s,\, v_h,\, m_h\} \tag{10}$$

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- The Standard Model + massive **Majorana** neutrinos is a model with **27 Input Parameters**
- From this model it's possible to obtain accurately **several hundreds of observables**. See **PDG**.
- There are 5 parameters related to **gauge symmetry** and its breaking:

$$\{g_e,\, \theta_W,\, g_s,\, v_h,\, m_h\} \tag{10}$$

- There are 22 parameters related to **Yukawa interactions**:

$$\{m_e,\, m_\mu,\, m_\tau,\, m_{\nu_1},\, m_{\nu_2},\, m_{\nu_3},\, m_d,\, m_s,\, m_b,\, m_u,\, m_c,\, m_b\} \tag{11}$$

$$\{\theta^l_{12},\, \theta^l_{13},\, \theta^l_{23},\, \delta^l,\, \phi_{12},\, \phi_{13},\, \theta^q_{12},\, \theta^q_{13},\, \theta^q_{23},\, \delta^q\} \tag{12}$$

Flavour Symmetry

- **Flavour symmetry** at high-energy regime.

$$SU(3)_c \otimes SU(2)_L \otimes U(1)_Y \underbrace{\otimes G}_{\text{Flavour}}. \tag{13}$$

- Constraining, or relating the Yukawa coupling structure

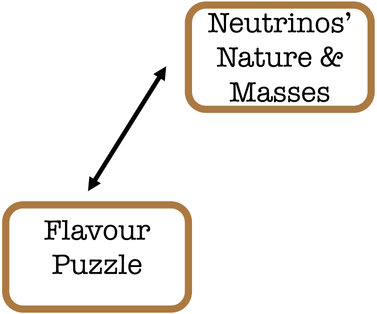
$$\underbrace{G}_{\text{Symmetry}} \xrightarrow{\text{SSB}} \underbrace{V_{\text{CKM}},\ U,\ \text{Mass Hierarchy}}_{\text{Flavour Observables}}. \tag{14}$$

- An appealling option are Discrete and Non-Abelian Groups

Bottom-Up: Extending the Standard Model

Neutrinos'
Nature &
Masses

Bottom-Up: Extending the Standard Model



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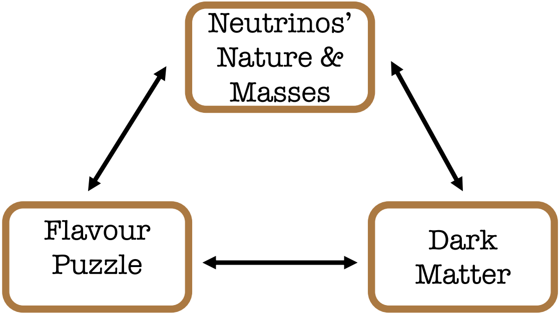
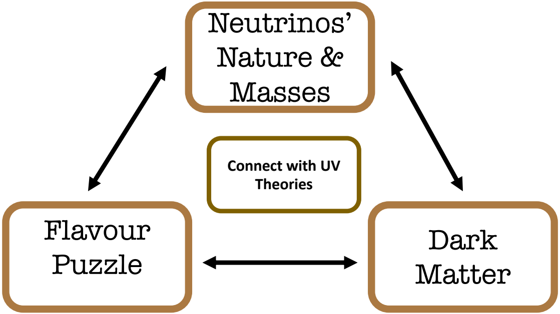


Bottom-Up: Extending the Standard Model



The Spacetime Manifold

- **Build upon**: 6-dimensional spacetime,

$$\mathcal{M} = \mathbb{M}^4 \times \left(\mathbb{T}^2/\mathbb{Z}_2\right)\,, \tag{15}$$

- Identifications **[Altarelli et al, NPB 775 (2007) 31–44]**

$$T_1\ :\ z \sim z+1, \qquad T_2\ :\ z \sim z+\omega, \qquad \mathbb{Z}_2\ :\ z \sim -z. \tag{16}$$

$$\textbf{Orbifold} \qquad \mathcal{O} = \mathbb{T}^2/\mathbb{Z}_2 \tag{17}$$

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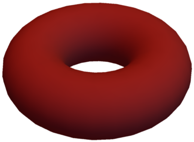
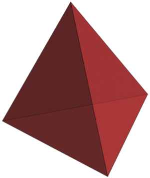
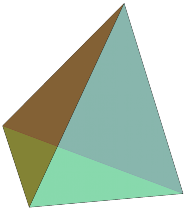
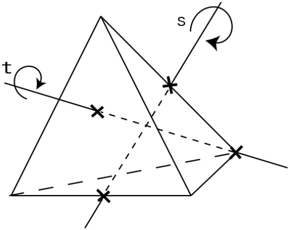




- Flat and with **four singular points**, the location of **4-Branes**

The Spacetime A_4 Symmetry





- **Tetrahedron Symmetries**:
  - Vertex permutation

$$S \equiv (2143) \ , \quad T \equiv (1423). \tag{18}$$

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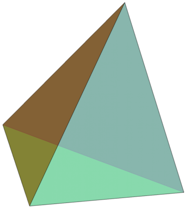

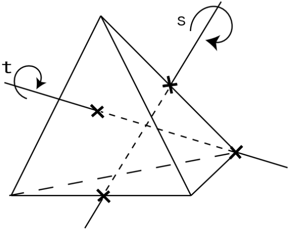

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- Described by the $\boldsymbol{A_4}$ group

The A_4 Group

- **The A_4 presentation**

$$A_4 \simeq \left\{ S, T \mid S^2 = T^3 = (ST)^2 = \mathbf{1} \right\}, \tag{19}$$

Class	n	h	χ_1	χ_2	χ_3	χ_4
C_1	1	1	1	1	1	3
C_2	4	3	1	ω	ω^2	0
C_3	4	3	1	ω^2	ω	0
C_4	3	2	1	1	1	-1

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Four Irreps.

$$\mathbf{1}, \mathbf{1}', \mathbf{1}'', \mathbf{3}. \quad (20)$$

$$\omega \equiv e^{\frac{2\pi i}{3}}. \quad (21)$$

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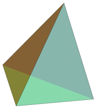
The A_4 Flavour Symmetry



- **Three generations** defined over **different branes**

Field Content and Quantum Numbers

Field	$SU(3)_C$	$SU(2)_{\rm L}$	$U(1)_{\rm Y}$	$\mathbb{Z}_4$	Location	A_4
L	**1**	**2**	-1	1	Brane	**3**
d^c	**3**	**1**	$2/3$	1	Brane	**3**
e^c	**1**	**1**	2	1	Brane	**3**
Q	**3**	**2**	$1/3$	1	Brane	**3**
$u^c_{1,2,3}$	**3**	**1**	$-4/3$	-1	Bulk	$\mathbf{1''}, \mathbf{1'}, \mathbf{1}$
F	**1**	**1**	0	i	Brane	**3**
H_u	**1**	**2**	1	-1	Brane	**3**
H_d	**1**	**2**	-1	1	Brane	**3**
η	**1**	**2**	1	$-i$	Brane	**1**
σ	**1**	**1**	0	-1	Bulk	**3**

Effective A_4 Invariant Yukawa Lagrangian

- This A_4 Flavour Symmetry Constraints the Yukawa couplings from first principles

$$\mathcal{L}^{\text{Model}}_{\text{Yukawa}} = y^d_1 \left(Qd^c H_d\right)_{\mathbf{1_1}} + y^d_2 \left(Qd^c H_d\right)_{\mathbf{1_2}} + y^e_1 \left(Le^c H_d\right)_{\mathbf{1_1}} + y^e_2 \left(Le^c H_d\right)$$
$$+ y^u_1 \left(QH_u\right)_{\mathbf{1'}} u^c_1 + y^u_2 \left(QH_u\right)_{\mathbf{1''}} u^c_2 + y^u_1 \left(QH_u\right)_{\mathbf{1}} u^c_3$$
$$+\boldsymbol{y^{\sigma}} \left(\boldsymbol{F^T F \sigma}\right)_{\mathbf{1_1}} + \text{H.c.}$$
$$+\boldsymbol{y^{\eta}_1} \left(\boldsymbol{L \eta F}\right)_{\mathbf{1}} + \text{H.c.} \tag{22}$$

- **Down-Type Quarks**

$$M_d = v_d \begin{pmatrix} 0 & y^d_1 \epsilon^d_1 & y^d_2 \epsilon^d_2 \\ y^d_2 \epsilon^d_1 & 0 & y^d_1 \\ y^d_1 \epsilon^d_2 & y^d_2 & 0 \end{pmatrix},$$

- **Up-Type Quarks**

$$M_u = v_u \begin{pmatrix} y^u_1 \epsilon^u_1 & y^u_1 \epsilon^u_2 \omega^2 & y^u_1 \omega \\ y^u_2 \epsilon^u_1 & y^u_2 \epsilon^u_2 & y^u_2 \omega^2 \\ y^u_3 \epsilon^u_1 & y^u_3 \epsilon^u_2 & y^u_3 \end{pmatrix}.$$

Dark Matter Candidate

- **Flavour Symmetry Breaking**

$$\boldsymbol{y^{\sigma}} \left(\boldsymbol{F^{T} F \langle\sigma\rangle}\right)_{\mathbf{1_1}} + \mathrm{H.c.} \tag{23}$$

VEV assumed to be fixed by Scherk-Schwarz **boundary conditions** from the extra dimensions, the most general $\langle\sigma\rangle$ parametrization

$$\langle\sigma\rangle = v_{\sigma} \begin{pmatrix} \epsilon_1^{\sigma}\, e^{i\varphi} \\ \epsilon_2^{\sigma} \\ 1 \end{pmatrix}, \quad \text{with} \quad \epsilon_1^{\sigma}, \epsilon_2^{\sigma} \in \mathbb{R} \quad \text{and} \quad 0 \leq \varphi < \pi. \tag{24}$$

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$$\boldsymbol{y^{\sigma}}\left(\boldsymbol{F^{T}F}\langle\boldsymbol{\sigma}\rangle\right)_{\mathbf{1_1}}+\text{H.c.} \tag{23}$$

VEV assumed to be fixed by Scherk-Schwarz **boundary conditions** from the extra dimensions, the most general $\langle\sigma\rangle$ parametrization

$$\langle\sigma\rangle = v_\sigma \begin{pmatrix} \epsilon_1^\sigma\, e^{i\varphi} \\ \epsilon_2^\sigma \\ 1 \end{pmatrix}, \quad \text{with} \quad \epsilon_1^\sigma, \epsilon_2^\sigma \in \mathbb{R} \quad \text{and} \quad 0 \le \varphi < \pi. \tag{24}$$

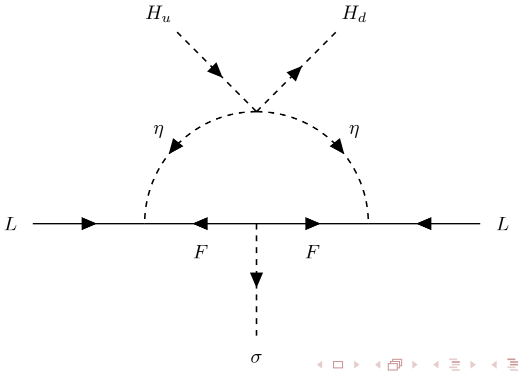
- This $\langle\sigma\rangle$ determines the “Dark Fermion” F Masses, these together with η are WIMP-DM candidates due to the “Dark” $\mathbb{Z}_2$.

Neutrino Mass

- This $\mathbb{Z}_2$ Symmetry is **Assumed** to be conserved, therefore:

$$\langle \eta \rangle = 0. \quad \longrightarrow \quad +y_1^{\eta} (L\eta F)_1 + \text{H.c.} \tag{25}$$

Flavour symmetrical one-loop Majorana mass model



Neutrino Mass Matrix

- The Neutrino Mass Matrix

$$\boxed{(M_\nu)_{ij} = \sum_k^3 \frac{h_{ik}(h^T)_{kj}}{32\pi^2} S(m_{F_k}),} \tag{26}$$

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where

$$S(m_{F_k}) = m_{F_k}\left(\frac{m_R^2}{m_R^2 - m_{F_k}^2}\ln\frac{m_R^2}{m_{F_k}^2} - \frac{m_I^2}{m_I^2 - m_{F_k}^2}\ln\frac{m_I^2}{m_{F_k}^2}\right), \tag{27}$$

with $m_R = m(\operatorname{Re}\eta^0)$, $m_I = m(\operatorname{Im}\eta^0)$ and

$$m_R^2 - m_I^2 \equiv= 2\lambda_5\langle(H_u\rangle_{\mathbf{3}}\langle H_d\rangle_{\mathbf{3}})_{\mathbf{1}} \tag{28}$$

Model's Parameters

- General parameterization, Assuming CP preserved at the high energy scale.

$$\langle H_u \rangle = v_u \begin{pmatrix} \epsilon_1^u \\ \epsilon_2^u \\ 1 \end{pmatrix}, \quad \langle H_d \rangle = v_d \begin{pmatrix} 1 \\ \epsilon_2^d \\ \epsilon_1^d \end{pmatrix}, \quad \langle \sigma \rangle = v_\sigma \begin{pmatrix} \epsilon_1^\sigma \, e^{i\varphi} \\ \epsilon_2^\sigma \\ 1 \end{pmatrix}. \tag{29}$$

Working with a quasi-aligned VEV for H_u, H_d [Morisi et al, PRD 84, 036003 (2011)] .

$$\epsilon_1^u, \epsilon_2^u, \epsilon_1^d, \epsilon_2^d << 1. \tag{30}$$

- **16 Independet Parameters:**

$$\{y_1^\eta, y_{1,2}^{e,d} v_d, y_{1,2,3}^u v_u, y^\sigma v_\sigma, \epsilon_{1,2}^{u,d}, \epsilon_{1,2}^\sigma, e^{i\sigma}\}. \tag{31}$$

Mass Matrices

- **Dark Fermions**

$$M_F = y_\sigma v_\sigma \begin{pmatrix} 0 & 1 & \epsilon_2^\sigma \\ 1 & 0 & \epsilon_1^\sigma e^{i\varphi} \\ \epsilon_2^\sigma & \epsilon_1^\sigma e^{i\varphi} & 0 \end{pmatrix} .$$

- **Up-Type Quarks**

$$M_u = v_u \begin{pmatrix} y_1^u \epsilon_1^u & y_1^u \epsilon_2^u \omega^2 & y_1^u \omega \\ y_2^u \epsilon_1^u & y_2^u \epsilon_2^u & y_2^u \omega^2 \\ y_3^u \epsilon_1^u & y_3^u \epsilon_2^u & y_3^u \end{pmatrix} .$$

- **Charged Leptons**

$$M_e = v_d \begin{pmatrix} 0 & y_1^e \epsilon_1^d & y_2^e \epsilon_2^d \\ y_2^e \epsilon_1^d & 0 & y_1^e \\ y_1^e \epsilon_2^d & y_2^e & 0 \end{pmatrix} ,$$

- **Down-Type Quarks**

$$M_d = v_d \begin{pmatrix} 0 & y_1^d \epsilon_1^d & y_2^d \epsilon_2^d \\ y_2^d \epsilon_1^d & 0 & y_1^d \\ y_1^d \epsilon_2^d & y_2^d & 0 \end{pmatrix} ,$$

Mass Matrices

- **Neutrinos** (Scotogenic mechanism).

$$(M_\nu)_{ij} = \sum_{k}^{3} \frac{h_{ik}(h^T)_{kj}}{32\pi^2} S(m_{F_k}),$$

- **22 Flavour Standard Model Parameters**, thus correlations are expected.

$$\{\theta^l_{12}, \theta^l_{13}, \theta^l_{23}, \phi_{12}, \phi_{13},\ \phi_{23}, \Delta m^2_{21}, \Delta m^2_{31}, m_{u,c,t,d,s,b,e,\mu,\tau}, \theta^q_{12}, \theta^q_{13}, \theta^q_{23}, \delta^q\}. \tag{32}$$

Some Interesting Results

- **Golden** Quark Lepton Mass Relation. From M_e, M_d structure [Morisi et al, PRD 84, 036003 (2011)] .

$$\frac{m_\tau}{\sqrt{m_\mu m_e}} \approx \frac{m_b}{\sqrt{m_s m_d}}. \tag{33}$$

- prediction for the neutrino mass scale

$$m^\nu_{\text{lightest}} \gtrapprox 10^{-2} eV \tag{34}$$

- We considered **Normal Ordering for neutrino masses.** The CP phases

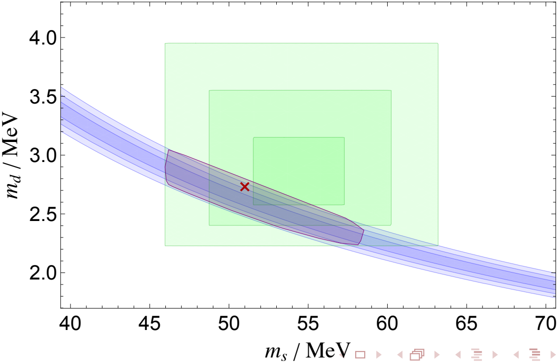
$$\delta^l = \phi_{13} - \phi_{12} - \phi_{23} \approx 192\ ^\circ. \tag{35}$$

$$\omega, \varphi \overset{\text{SSB}}{\Longrightarrow} \left\{\delta^{\text{q}}, \phi_{12}, \phi_{13}, \phi_{23}\right\}. \tag{36}$$

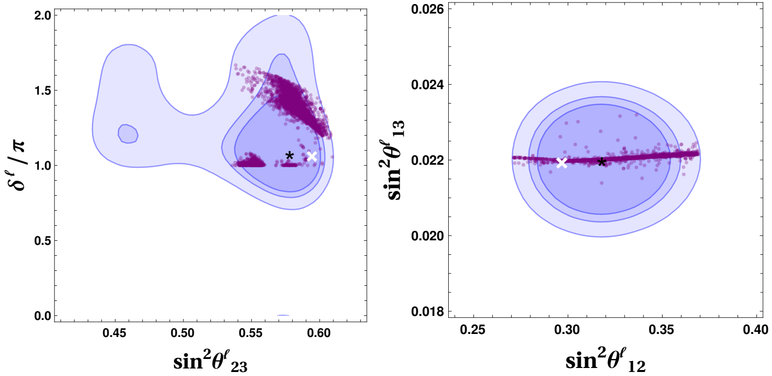
Golden Quark-Lepton Relation

- Prediction from A_4 and H_u, $H_d \sim \mathbf{3}$ [de Salas et al, JHEP 2021, 71 (2021)]

$$\frac{m_\tau}{\sqrt{m_\mu m_e}} \approx \frac{m_b}{\sqrt{m_s m_d}}. \tag{37}$$



Leptonic CP Phase

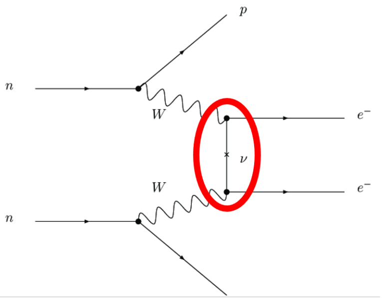


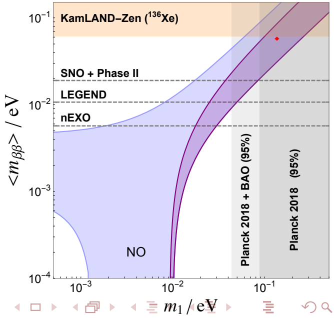
- [de Anda et al, PLB 801 (2020) 135195]

Neutrinoless Double Beta Decay

- For testing the Majorana nature of neutrinos [Schechter and Valle, PRD, 25 (1982), p. 2951] .

$$\langle m_{\beta\beta} \rangle = \left| \sum_{j=1}^{3} \left(U_{ej}\right)^2 m_j \right| = 58.08\text{meV}. \tag{38}$$





Conclusions

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- It also has testable predictions

$$\frac{m_\tau}{\sqrt{m_\mu m_e}} \approx \frac{m_b}{\sqrt{m_s m_d}}, \quad m^\nu_{\text{lightest}} \gtrsim 10^{-2} eV, \quad \theta^l_{23} > 45, \quad \langle m_{\beta\beta}\rangle.$$