A flavor model for cobimaximal neutrino mixing



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June 28, 2022

Outline

Introduction:

 \ast Historically, neutrino oscillation data indicates that the lepton mixing matrix can be described by various mixing schemes at the zeroth order.

* Most of the proposed mixing schemes are outdated by data.

 \ast Cobimaximal is still a viable option for recent fits to the neutrino oscillation data.

• The set-up:

- * particle contents and symmetries
- * light neutrino mass generated by Type-I see-saw
- * parameters involved and their correlation

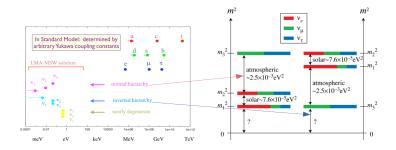
Leptogenesis:

 $\label{eq:Baryon asymmetry can be realized with NLO \ contribution \ to \ the neutrino \ Yukawa \ coupling$

Conclusions

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Neutrino parameters and the known unknowns



	Normal Ordering (best fit)		Inverted Ordering $(\Delta \chi^2 = 2.6)$		
	bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range	
$\sin^2 \theta_{12}$	$0.304^{+0.013}_{-0.012}$	$0.269 \rightarrow 0.343$	$0.304^{+0.012}_{-0.012}$	$0.269 \rightarrow 0.343$	
$\theta_{12}/^{\circ}$	$33.44^{+0.77}_{-0.74}$	$31.27 \rightarrow 35.86$	$33.45_{-0.74}^{+0.77}$	$31.27 \rightarrow 35.87$	
$\sin^2 \theta_{23}$	$0.573^{+0.018}_{-0.023}$	$0.405 \rightarrow 0.620$	$0.578^{+0.017}_{-0.021}$	$0.410 \rightarrow 0.623$	
$\theta_{23}/^{\circ}$	$49.2^{+1.0}_{-1.3}$	$39.5 \rightarrow 52.0$	$49.5^{+1.0}_{-1.2}$	$39.8 \rightarrow 52.1$	
$\sin^2 \theta_{13}$	$0.02220^{+0.00068}_{-0.00062}$	$0.02034 \to 0.02430$	$0.02238\substack{+0.00064\\-0.00062}$	$0.02053 \rightarrow 0.02434$	
$\theta_{13}/^{\circ}$	$8.57^{+0.13}_{-0.12}$	$8.20 \rightarrow 8.97$	$8.60^{+0.12}_{-0.12}$	$8.24 \rightarrow 8.98$	
$\delta_{CP}/^{\circ}$	194^{+52}_{-25}	$105 \to 405$	287^{+27}_{-32}	$192 \to 361$	
$\frac{\Delta m^2_{21}}{10^{-5}~{\rm eV}^2}$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	$7.42^{+0.21}_{-0.20}$	$6.82 \rightarrow 8.04$	
$\frac{\Delta m^2_{3\ell}}{10^{-3}~{\rm eV}^2}$	$+2.515^{+0.028}_{-0.028}$	$+2.431 \rightarrow +2.599$	$-2.498\substack{+0.028\\-0.029}$	$-2.584 \rightarrow -2.413$	

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A flavor model for cobimaximal mixing

Flavor symmetries, why?

• Using the diagonalization relation

$$m_{\nu} = U_0^{\star} \operatorname{diag}(m_1, m_2, m_3) U_0^{\dagger},$$

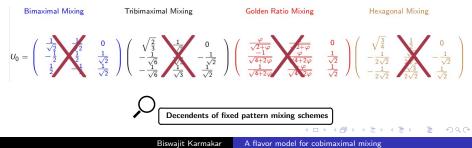
such a mixing matrices can easily diagonalize a $\mu - \tau$ symmetric (transformations $\nu_e \rightarrow \nu_e$, $\nu_\mu \rightarrow \nu_\tau$, $\nu_\tau \rightarrow \nu_\mu$ under which the neutrino mass term remains unchanged) neutrino mass matrix of the form

$$m_{\nu} = \left(\begin{array}{ccc} A & B & B \\ B & C & D \\ B & D & C \end{array}\right),$$

With A + B = C + D this matrix yields tribimaximal mixing pattern where $s_{12} = 1/\sqrt{3}$ i.e., $\theta_{12} = 35.26^{\circ}$

* Tribimaximal mixing pattern can be easily realize incorporating various discrete groups Ma,Rajasekharan PRD64; Babu,Valle,Ma PLB512 Altarelli,Feruglio NPB741, Varzielas,King,Ross PLB648Manv more...

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Non-zero θ_{13} : Decendents of tribimaximal mixing

$$U_{TBM} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1^{\sqrt{2}}}{\sqrt{2}} \end{pmatrix},$$

$$|U_{\mathrm{TM}_1}| = \begin{pmatrix} \frac{2}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \\ \frac{1}{\sqrt{6}} & * & * \end{pmatrix} \qquad \qquad |U_{\mathrm{TM}_2}| = \begin{pmatrix} * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \\ * & \frac{1}{\sqrt{3}} & * \end{pmatrix},$$

 $U_{\mathrm{TM}_{1}} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{c_{\theta}}{\sqrt{3}} & \frac{s_{\theta}}{\sqrt{2}}e^{-i\gamma} \\ -\frac{1}{\sqrt{6}} & \frac{c_{\theta}}{\sqrt{3}} - \frac{s_{\theta}}{\sqrt{2}}e^{i\gamma} & -\frac{s_{\theta}}{\sqrt{3}}e^{-i\gamma} - \frac{c_{\theta}}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{c_{\theta}}{\sqrt{3}} - \frac{s_{\theta}}{\sqrt{2}}e^{i\gamma} & -\frac{s_{\theta}}{\sqrt{3}}e^{-i\gamma} + \frac{c_{\theta}}{\sqrt{2}} \end{pmatrix}, U_{\mathrm{TM}_{2}} = \begin{pmatrix} \frac{2c_{\theta}}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{2s_{\theta}}{\sqrt{6}}e^{-i\gamma} \\ -\frac{c_{\theta}}{\sqrt{6}} + \frac{s}{\sqrt{2}}e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_{\theta}}{\sqrt{3}}e^{-i\gamma} - \frac{c_{\theta}}{\sqrt{2}} \\ -\frac{c_{\theta}}{\sqrt{6}} + \frac{s}{\sqrt{2}}e^{i\gamma} & \frac{1}{\sqrt{3}} & -\frac{s_{\theta}}{\sqrt{3}}e^{-i\gamma} + \frac{c_{\theta}}{\sqrt{2}} \end{pmatrix}$

Cobimaximal Mixing

- Predicts specific values for the atmospheric mixing angle $\theta_{23} = 45^{\circ}$ and Dirac CP phase $\delta = -90^{\circ}$.
- Still remains as good approximation for the observed neutrino oscillation data.
- The mixing matrix satisfy the condition :

$$|U_{\mu i}| = |U_{\tau i}|$$
 with $i = 1, 2, 3$.

the mass matrix leading to the above mixing matrix can be written as

$$m_
u = egin{pmatrix} \mathbf{a} & \mathbf{b} & \mathbf{b}^\star \ \mathbf{b} & \mathbf{c} & \mathbf{d} \ \mathbf{b}^\star & \mathbf{d} & \mathbf{c}^\star \end{pmatrix},$$

where b and c are in general complex while c and d remain real.

Fukuura, Miura, Takasugi, Yoshimura PRD 99; Miura, Takasugi, Yoshimura PRD01; Harrison, Scott PLB02; Grimus, Lavoura PLB04 Babu, Ma, Valle, PLB03.....

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Neutrino Mass Generation

• Dirac Mass: $m_D \overline{\nu_R} \nu_L + h.c.$

• Majorana Mass: $\frac{1}{2}m_L\nu_L^T C^\dagger \nu_L + \frac{1}{2}M_R\nu_R^T C^\dagger \nu_R + h.c.$

Type-I see-saw

$$\mathcal{L} = \frac{1}{2} \psi_L^T \mathcal{C}^{\dagger} M_{\nu} \psi_L + \text{h.c.},$$

here $\psi_L = \begin{pmatrix} \nu_L \\ \nu_R^C \end{pmatrix} = \begin{pmatrix} \nu_L \\ \mathcal{C} \overline{\nu_R} \\ T \end{pmatrix}$

Mass matrix:

$$M_{\nu} = \begin{bmatrix} m_L & m_D^T \\ m_D & M_R \end{bmatrix}$$

with $m_L = 0$ and $m_D << M_R$,

$$m_{\nu} = -m_D^T M_R^{-1} m_D$$

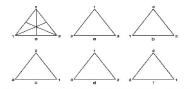
- For three generation case m_D , M_R are all 3X3 matrices.
- Non-Abelian discrete groups have enough potential to explore these substructures.

• Non-Abelian discrete Groups:

 A_4 , A_5 , S_3 , S_4 , $\Delta(27)$ etc.

Altarelli, Feruglio RMP10; Xing PR19; Grimus, Ludl JPA 12; King, Luhn RPP13; Ishimori, Kobayashi, Shimizu, Okada, Tanimoto PTPS10

Example 1- S_3 : Permutation group of three objects. It has 3! = 6 elements.



An equilateral triangle can reproduce six elements of S_3 . *e* is the identity element. Reflection transformations of *e* are given by *a*, *b* and *c*. *d* and *f* are from rotations. **Example 2**- S_4 :

- Permutation group of four object.
- Geometrically S_4 is symmetry group of a tetrahedron.
- It has 4! = 24 elements.

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Example 3- A₄ :

- Even permutation group of four object.
- It has 4!/2 = 12 elements.
- It is a subgroup of S₄.

A4 group multiplication [arXiv:1002:0211]

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• Discrete group A_4 has three 1D representation 1, 1', 1'' and a irreducible 3D representation 3. Product of the singlet and triplets are given by

$$\begin{split} 1\otimes 1 &= 1, \ 1'\otimes 1' = 1'', \\ 1'\otimes 1'' &= 1, \\ 1''\otimes 1'' &= 1, \\ &\otimes 3 &= 1\oplus 1'\oplus 1''\oplus 3_a\oplus 3_s \end{split}$$

where subscripts a and s stands for "asymmetric" and "symmetric" respectively. If we have two triplets (a_1, a_2, a_3) and (b_1, b_2, b_3) , their products are given by

$$\begin{array}{rcl} (A \times B)_1 & \backsim & a_1b_1 + a_2b_2 + a_3b_3, \\ (A \times B)_{1'} & \backsim & a_1b_1 + \omega^2 a_2b_2 + \omega a_3b_3, \\ (A \times B)_{1''} & \backsim & a_1b_1 + \omega a_2b_2 + \omega^2 a_3b_3, \\ (A \times B)_{3_5} & \backsim & (a_2b_3 + a_3b_2, a_3b_1 + a_1b_3, a_1b_2 + a_2b_1), \\ (A \times B)_{3_6} & \backsim & (a_2b_3 - a_3b_2, a_3b_1 - a_1b_3, a_1b_2 - a_2b_1) \end{array}$$

here ω (= $e^{2i\pi/3}$) is the cube root of unity.

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Cobimaximal : A flavor model

Compatible mass matrix can take the form
$$m_{\nu} \sim \begin{pmatrix} a & b & b^{\star} \\ b & c & d \\ b^{\star} & d & c^{\star} \end{pmatrix}$$

Can you use discrete symmetry to reproduce this mixing scheme?

- Yes !! Δ(27) Herńandez, Varzielas PLB20
- Can we use A₄ ?

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Symmetries and Particle Content: Soon to be appear on arXiv

	$\ell_{e,\mu,\tau}$	e_R, μ_R, τ_R	Н	N _R	<i>φ</i> _{1,2,3}	ξ	ϕ_S
A4	1, 1', 1''	1, 1'', 1'	1	3	3	1	3
Z3	1	1	1	ω^2	ω	ω^2	ω^2
Z_4	-i,-1,i	i,-1,- i	1	1	i,-1,- i	1	1

Charged Leptons:

$$-\mathcal{L}_{CL} = y_e(\bar{\ell}_e)_1 H(e_R)_1 + y_\mu(\bar{\ell}_\mu)_{1'} H(\mu_R)_{1''} + y_\tau(\bar{\ell}_\tau)_{1''} H(\mu_R)_{1'} + h.c.,$$

Neutrinos:

$$\begin{aligned} -\mathcal{L}_{\nu} &= \frac{y_1}{\Lambda} \left(\bar{\ell_e} \right)_1 \tilde{H} \left(N_R \phi_1 \right)_1 + \frac{y_2}{\Lambda} \left(\bar{\ell_{\mu}} \right)_{1'} \tilde{H} \left(N_R \phi_2 \right)_{1''} + \frac{y_3}{\Lambda} \left(\bar{\ell_{\tau}} \right)_{1''} \tilde{H} \left(N_R \phi_3 \right)_{1'} \\ &+ (y_x \xi + y_\phi \phi_S) \overline{N_E^c} N_R + h.c. \end{aligned}$$

Considered VEV alignment:

$$\langle \phi_1 \rangle = \mathsf{v}_1(1,0,0), \ \langle \phi_2 \rangle = \mathsf{v}_1(0,1,0), \ \langle \phi_3 \rangle = \mathsf{v}_1(0,0,\omega), \ \langle \xi \rangle = \mathsf{v}_\xi \text{ and } \langle \phi_5 \rangle = (\mathsf{v}_2,\mathsf{v}_3,\mathsf{v}_3).$$

Cobimaximal : A flavor model

Charged Leptons

Dirac Neutrinos

Majorana Neutrinos

$$m_{\ell} = \frac{v}{\sqrt{2}} \begin{pmatrix} y_{e} & 0 & 0\\ 0 & y_{\mu} & 0\\ 0 & 0 & y_{\tau} \end{pmatrix}; \ m_{D} = \frac{yvv_{1}}{\sqrt{2}\Lambda} \begin{pmatrix} 1 & 0 & 0\\ 0 & \omega & 0\\ 0 & 0 & \omega^{2} \end{pmatrix} = vY_{\nu}^{0}; \ M = \begin{pmatrix} p & q & q\\ q & p & s\\ q & s & p \end{pmatrix} = p \begin{pmatrix} 1 & \kappa_{2} & \kappa_{2} \\ \kappa_{2} & 1 & \kappa_{1} \\ \kappa_{2} & \kappa_{1} & 1 \end{pmatrix}$$

where we have considered $y_1 = y_2 = y_3 = y$ (say) and $p = 2y_x v_\xi$, $s = 2y_\phi v_2$, $q = 2y_\phi v_3$. Here $\kappa_1 = s/p$ and $\kappa_2 = q/p$ are two dimensionless quantities.

Light neutrino mass via type-I seesaw:

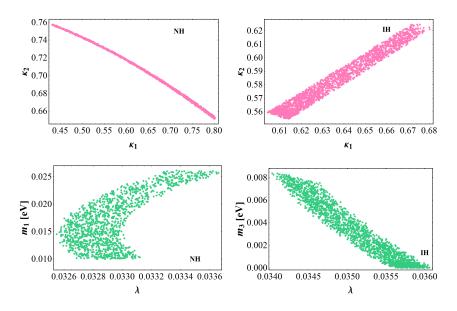
$$\begin{split} m_{\nu} &\sim -m_D M^{-1} m_D^T \\ &\sim \lambda \begin{pmatrix} 1 - \kappa_1^2 & (\kappa_1 \kappa_2 - \kappa_2) \omega & (\kappa_1 \kappa_2 - \kappa_2) \omega^2 \\ (\kappa_1 \kappa_2 - \kappa_2) \omega & (1 - \kappa_2^2) \omega^2 & \kappa_2^2 - \kappa_1 \\ (\kappa_1 \kappa_2 - \kappa_2) \omega^2 & \kappa_2^2 - \kappa_1 & (1 - \kappa_2^2) \omega \end{pmatrix} \end{split}$$

Diagonalization

 $m_{\nu} = U^{\star} \operatorname{diag}(m_1, m_2, m_3) U^{\dagger}$

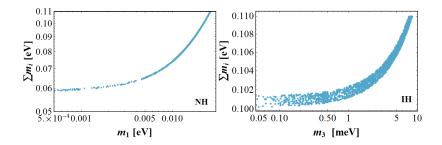
$$\begin{split} & \mathcal{U} = \begin{pmatrix} \mathcal{U}_{11} & \mathcal{U}_{12} & \mathcal{U}_{13} \\ \mathcal{U}_{21} & \mathcal{U}_{22} & \mathcal{U}_{23} \\ \mathcal{U}_{31} & \mathcal{U}_{32} & \mathcal{U}_{33} \end{pmatrix} = \begin{pmatrix} \cos \vartheta_{12} \cos \vartheta_{12} \sin \vartheta_{13} & -\sin \vartheta_{12} \sin \vartheta_{13} & -\sin \vartheta_{13} \\ \sin \vartheta_{12} - i \cos \vartheta_{12} \sin \vartheta_{13} & \cos \vartheta_{12} + i \sin \vartheta_{13} & \frac{\cos \vartheta_{12} + i \sin \vartheta_{13}}{\sqrt{2}} \\ \frac{\sin \vartheta_{12} + i \cos \vartheta_{12} \sin \vartheta_{13}}{\sqrt{2}} & \frac{\cos \vartheta_{12} - i \sin \vartheta_{12} \sin \vartheta_{13}}{\sqrt{2}} & \frac{-i \cos \vartheta_{13}}{\sqrt{2}} \\ \delta = \arcsin \left[\frac{\operatorname{Im}[\mathcal{U}_{23}\mathcal{U}_{13}^*\mathcal{U}_{12}\mathcal{U}_{22}^*]}{\sin \vartheta_{13} \cos \vartheta_{13}} \right] = -\pi/2; \quad \sin^2 \vartheta_{23} = \frac{|\mathcal{U}_{23}|^2}{1 - |\mathcal{U}_{13}|^2} = \frac{1}{2} \end{split}$$

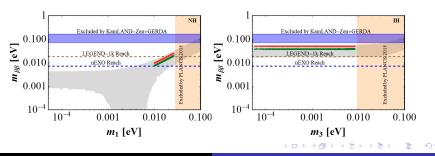
• Allowed regions for model parameters :



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 The presence of right handed neutrinos in the seesaw realization of light neutrino mass provides an opportunity to study leptogenesis from the CP-violating out-of-equilibrium decay of RH neutrinos into lepton and Higgs doublets in the early universe.
 Fukugita, Yanagida, 1986; Covi, Roulet, Vissani 9605319

$$\begin{split} \epsilon_{i}^{\alpha} &= \frac{\Gamma(N_{i} \rightarrow \ell_{\alpha} H) - \Gamma(N_{i} \rightarrow \overline{\ell}_{\alpha} \overline{H})}{\Gamma(N_{i} \rightarrow \ell_{\alpha} H) + \Gamma(N_{i} \rightarrow \overline{\ell}_{\alpha} \overline{H})} \\ &= \frac{1}{8\pi} \sum_{j \neq i} \frac{\mathrm{Im} \left[\left((\hat{Y}_{\nu}^{\dagger} \hat{Y}_{\nu})_{ij} \right)^{2} \right]}{(\hat{Y}_{\nu}^{\dagger} \hat{Y}_{\nu})_{ii}} f\left(\frac{m_{i}^{2}}{m_{j}^{2}} \right) \end{split}$$

where the loop factor f(x) in the above expression has been defined as follows

$$f(x) = \sqrt{x} \left[\frac{2-x}{1-x} - (1-x) \ln \left(1 + \frac{1}{x} \right) \right]$$

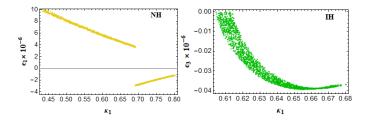
with $x = m_i^2 / m_j^2$.

- CP asymmetry $\epsilon_i \propto \operatorname{Im}[(\hat{Y}^{\dagger}_{\nu}\,\hat{Y}_{\nu})^2_{ij}] \Longrightarrow Y^{\dagger}_{\nu}Y_{\nu} \propto I \Longrightarrow \epsilon_i \propto 0$
- NLO contribution in Dirac Yukawa $\Longrightarrow \ell_{\alpha} H \alpha_R \phi_i f f^{\dagger} / \Lambda^3 \Longrightarrow Y_{\nu} = Y_{\nu}^0 + \delta Y_{\nu}$

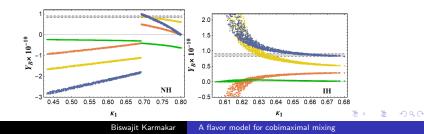
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Leptogenesis

• Estimation of CP asymmetry :



• Estimation of Baryon asymmetry :



- Guided by the neutrino data : Various mixing schemes are proposed
- Neutrino oscillation data entered in the precision era.
- Cobimaximal mixing can accommodate current data.
- An predictive flavor symmetric set-up based on A₄ discrete symmetry
- Tiny neutrino mass is explained by type-I seesaw mechanism while matter-antimatter asymmetry is also addressed via leptogenesis.
- Leptogenesis becomes viable once higher order contribution to the Dirac Yukawa is considered.



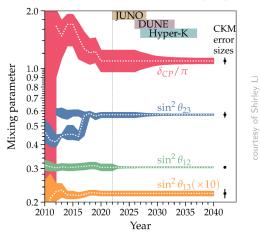
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Current and future precision of neutrino experiments

Song, Li, Argüelles, Bustamante & Vincent [2021]





$$\sin^2 \theta_{13} = |U_{13}|^2 = \sin^2 \vartheta_{13} \tag{2}$$

$$\sin^2 \theta_{12} = \frac{|U_{12}|^2}{1 - |U_{13}|^2} = \sin^2 \vartheta_{12} \tag{3}$$

$$\sin^2 \theta_{23} = \frac{|U_{23}|^2}{1 - |U_{13}|^2} = \frac{1}{2}$$
(4)

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$$Y_B \approx \sum Y_{Bi}$$
 (5)

where

$$Y_{Bi} \simeq -1.48 \times 10^{-3} \epsilon_i \eta_{ii}. \tag{6}$$

 Y_{Bi} 's are coming from decay of each RH neutrinos and η_{ii} stands for efficiency factor [hep-ph/0310123] when $M_i < 10^{14}$ GeV,

$$\frac{1}{\eta_{ii}} \approx \frac{3.3 \times 10^{-3} \text{ eV}}{\tilde{m}_i} + \left(\frac{\tilde{m}_i}{0.55 \times 10^{-3} \text{ eV}}\right)^{1.16},\tag{7}$$

with washout mass parameter, $\tilde{m_i} = \frac{(\hat{Y}_{\nu}^{\dagger} \, \hat{Y}_{\nu})_{ii} v_u^2}{M_i}$.