# Constraints on low-energy flavor from LFV and $(g\text{-}2)_\mu$

Oscar Vives

Flasy 2022, Lisboa, 27 June - 1 July

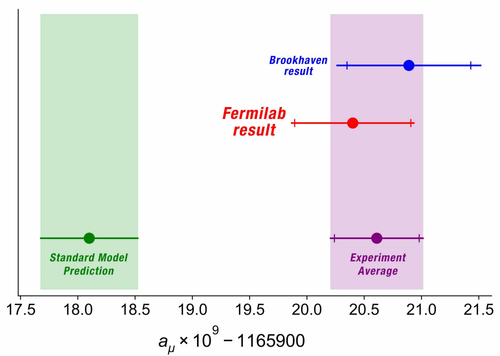
*L Calibbi, M.L. López-Ibáñez, A. Melis and O.V., JHEP* **06** *(2020), 087*

*L Calibbi, M.L. López-Ibáñez, A. Melis and O.V., Eur. Phys. J. C* **81** *(2021) no.10, 929*

*M. L. López-Ibáñez, A. Melis, M. J. Pérez, M. H. Rahat and O. V., Phys. Rev. D* **105** *(2022) 035021*

# First results from the Muon g-2 at Fermilab

Run1 in *Muon g-2* confirmed Brookhaven discrepancy:




$$a_\mu(\mathrm{Exp}) - a_\mu(\mathrm{SM}) = (251 \pm 59) \times 10^{-11} \quad \Rightarrow \quad \boxed{4.2\ \sigma}$$

Very large discrepancy, compared with EW contribution,

$$a_\mu^{\rm EW} = 153.6\,(1.0) \times 10^{-11}$$

and at one loop ...

$$\begin{aligned} a_\mu^{\rm EW(1)} &= \frac{G_F}{\sqrt{2}} \frac{m_\mu^2}{8\pi^2} \left[ \frac{5}{3} + \frac{1}{3}(1 - 4\sin^2\theta_W) \right] \\ &= \frac{\alpha}{4\pi \sin^2\theta_W} \frac{1.67\ m_\mu^2}{4\ M_w^2} = 194.79\,(1) \times 10^{-11} \end{aligned}$$

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any “natural” new physics with “minimal” flavor structure

$$a_\mu^{\rm NP} \simeq \frac{\alpha}{4\pi\sin^2\theta_W}\frac{m_\mu^2}{M_X^2} = 251 \times 10^{-11}$$

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$$a_\mu^{\rm NP} \simeq \frac{\alpha}{4\pi\sin^2\theta_W}\,\frac{m_\mu^2}{M_X^2} = 251 \times 10^{-11}$$

$\Longrightarrow M_X = 106$ GeV. So ... **Where is New Physics??**

Additional enhancing factors in NP contributions are possible ...

$$a_\mu^{\mathrm{NP}} \quad \simeq \quad \frac{\alpha}{4\pi \sin^2\theta_W} \frac{m_\mu^2}{M_X^2}\ \mathsf{k}$$

- In SUSY: $\mathsf{k} = \tan\beta$, Scalar Leptoquarks: $\mathsf{k} = m_t/m_\mu \simeq 1600$

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But ... Large enhancements in anomalous magnetic moments, produce enormous contributions to the fermion mass !!

$$\Delta a_\mu^{\rm NP} \simeq \frac{\alpha}{4\pi \sin^2\theta_W} \frac{m_\mu^2}{M_X^2}\, \mathsf{k} \;\Rightarrow\; \Delta m_\mu^{\rm NP} \simeq m_\mu \frac{\alpha\, \mathsf{k}}{4\pi \sin^2\theta_W}$$

$$\Downarrow$$

**Absence of fine-tuning requires** $\mathsf{k} \lesssim 4\pi \sin^2\theta_W/\alpha \simeq 380$

**largest possible (fully radiative** $m_\mu$**)** $M_X \lesssim 2$ TeV !!!

## Low-energy flavor symmetries

Flavour symmetry explains masses and mixings in Yukawas.

Small couplings generated in Froggatt-Nielsen, as function of small vevs, $Y_{ij} = \left( \left( \frac{\langle \theta \rangle}{M} \right) \ll 1 \right)^n$ .

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Magnetic moments and Yukawas have identical flavor charges.
$\Rightarrow$ identical flavons enter the Dipole and Yukawa matrices !!

$$Y_\ell \; = \; y_{33} \begin{pmatrix} \lambda^{5+a} & \lambda^{2+b} & \lambda^c \\ \lambda^{2+d} & \lambda^{2+e} & \lambda^f \\ \lambda^g & \lambda^h & 1 \end{pmatrix}$$

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with $\lambda = 0.2$ and $a, b, c, d, e, f, g, h \geq 0$.

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with $\lambda = 0.2$ and $a, b, c, d, e, f, g, h \geq 0$.

For low $\Lambda_f$ , lepton flavor violation, $\mu \to e\gamma$, require nearly diagonal charged-lepton Yukawa matrices, $a, e = 0,\ b, d \geq 6.6,\ g, h, c, f \geq 2.7$.

## $T_{13}$ Model

$T_{13} = Z_{13} \rtimes Z_3$, O(39) finite subgroup of $SU(3)$, with two ineq. complex triplets, $3_1, 3_2$.

| Fields | $\bar{L}$ | $l$ | $H$ | $\varphi_{33}$ | $\varphi_{22}$ | $\varphi_{13}$ | $\Delta$ | $\chi$ | $\chi'$ |
|---|---|---|---|---|---|---|---|---|---|
| $SU(2)_L$ | 2 | 1 | 2 | 1 | 1 | 1 | 1 | 1 | 1 |
| $T_{13}$ | $3_1$ | $3_1$ | 1 | $\bar{3}_1$ | $\bar{3}_1$ | $3_1$ | $\bar{3}_1$ | 1 | 1 |
| $Z_4$ | $\eta^1$ | 1 | $\eta^1$ | $\eta^1$ | $\eta^3$ | $\eta^2$ | $\eta^2$ | $\eta^3$ | $\eta^1$ |

Diagonal and off-diagonal in different $T_{13}$ representations.

$$\begin{pmatrix} \bar{L}_1 \\ \bar{L}_2 \\ \bar{L}_3 \end{pmatrix}_{\mathbf{3_1}} \otimes \begin{pmatrix} l_1 \\ l_2 \\ l_3 \end{pmatrix}_{\mathbf{3_1}} = \begin{pmatrix} \bar{L}_1 l_1 \\ \bar{L}_2 l_2 \\ \bar{L}_3 l_3 \end{pmatrix}_{\mathbf{3_2}} \oplus \begin{pmatrix} \bar{L}_2 l_3 \\ \bar{L}_3 l_1 \\ \bar{L}_1 l_2 \end{pmatrix}_{\mathbf{\bar{3}_1}} \oplus \begin{pmatrix} \bar{L}_3 l_2 \\ \bar{L}_1 l_3 \\ \bar{L}_2 l_1 \end{pmatrix}_{\mathbf{\bar{3}_1}} .$$

Three flavons: $\varphi_{33} \sim \bar{3}_1,\ \varphi_{22} \sim \bar{3}_1,\ \varphi_{13} \sim 3_1,$

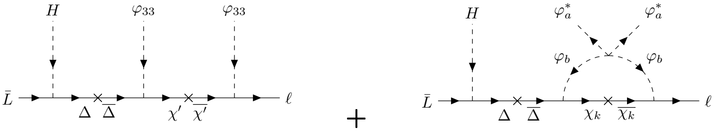
$$\langle \varphi_{33} \rangle = \epsilon\,(0,0,1)\,\Lambda_f, \qquad \langle \varphi_{22} \rangle = \epsilon^2\,(0,1,0)\,\Lambda_f,$$

$$\langle \varphi_{13} \rangle = \epsilon^{5/2}\,(0,1,0)\,\Lambda_f,$$

Yukawas Langrangian:

$$L_Y^e = \bar{L} l H \left[ \frac{1}{\Lambda_f} \varphi_{13} + \frac{1}{\Lambda_f^2} \varphi_{22}\varphi_{22} + \frac{1}{\Lambda_f^2} \varphi_{33}\varphi_{33} \right].$$

Tree-level + loop Yukawas:



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Tree-level + loop Yukawas:

$$Y_l/\epsilon^2 = \begin{pmatrix} 0 & 0 & \epsilon^{5/2} \\ 0 & \epsilon^2 & 0 \\ \epsilon^{5/2} & 0 & y_1 \end{pmatrix} + \frac{f_1(x_\varphi^2)}{32\,\pi^2} \begin{pmatrix} 0 & 0 & 2\,y_2\,y_4\,\epsilon^{5/2} \\ 0 & \beta_2\,y_2\epsilon^2 & 0 \\ 2\,y_2\,y_4\,\epsilon^{5/2} & 0 & y_1\,\beta_3\,y_3 \end{pmatrix}$$

## But, Dipole matrix similar structure

$$C_l = \frac{f_2(x_\varphi^2)}{16}\,\frac{\epsilon^2}{\Lambda_f^2}\left(\begin{array}{ccc} 0 & 0 & 2\,y_2\,y_4\,\epsilon^{5/2} \\ 0 & \beta_2\,y_2\epsilon^2 & 0 \\ 2\,y_2\,y_4\,\epsilon^{5/2} & 0 & y_1\,\beta_3\,y_3 \end{array}\right),$$

Can we consistently reproduce $(g-2)_\mu$?

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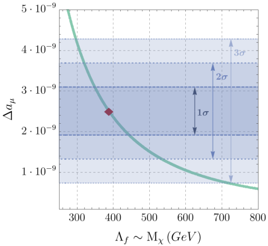


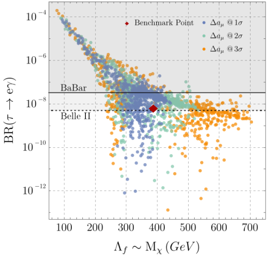

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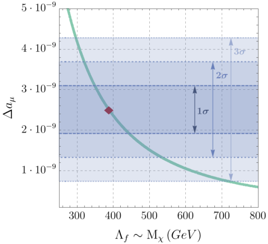
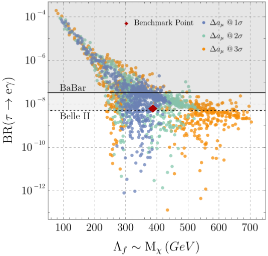



| $y_1$ | $y_2$ | $y_3$ | $y_4$ | $\beta_2$ | $\beta_3$ | $\beta_4$ | $\epsilon$ | $x_\varphi$ |
|---|---|---|---|---|---|---|---|---|
| 0.50 | 0.50 | 3.50 | 0.70 | 0.25 | 1.56 | −6.24 | 0.15 | 0.08 |

## $A_5$ Model

Even permitations of 5 objects, O(60), with singlet, two triplets, tetraplet and pentaplet $(1, 3, 3', 4, 5)$.

Flavor symmetry broken into different residual symmetries in charged lepton and neutrino sector $\Rightarrow$ Golden ratio

| Fields | $\bar{L}$ | $l$ | $H$ | $\varphi_3^e$ | $\varphi_{3'}^e$ | $\varphi_5^e$ | $\xi$ |
|---|---|---|---|---|---|---|---|
| $SU(2)_L$ | 2 | 1 | 2 | 1 | 1 | 1 | 1 |
| $A_5$ | 3 | 3 | 1 | 3 | $3'$ | 5 | 1 |
| $Z_{12}$ | $\rho^1$ | $\rho^1$ | $\rho^4$ | $\rho^6$ | $\rho^3$ | $\rho^3$ | $\rho^6$ |

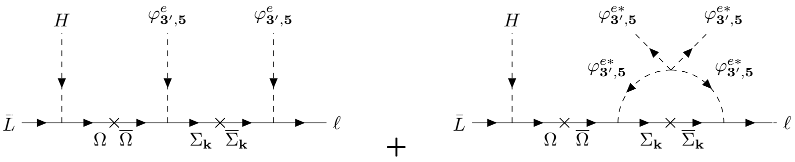
$Z_5$ residual symmetry in charged leptons.

$$\langle \varphi_3^e \rangle = (\epsilon_3, 0, 0)\,\Lambda_f, \quad \langle \varphi_{3'}^e \rangle = (\epsilon_{3'}, 0, 0)\,\Lambda_f, \quad \langle \varphi_5 \rangle = (\epsilon_5, 0, 0, 0, 0)\,\Lambda_f$$

Yukawas Langrangian:

$$L_Y^e = \bar{L} l H \left[ \frac{1}{\Lambda_f} (\xi + \varphi_3^e) + \frac{1}{\Lambda_f^2} (\varphi_{3'}^e \varphi_{3'}^e + \varphi_5^e \varphi_5^e) \right].$$

Tree-level + loop Yukawas:



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Tree-level + loop Yukawas:

$$Y_\ell = \begin{pmatrix} \epsilon_\xi + 3\,\epsilon_{3'}^2 + 7\,\epsilon_5^2 & 0 & 0 \\ 0 & \epsilon_\xi + \epsilon_3 + 3\,\epsilon_{3'}^2 - 8\,\epsilon_5^2 & 0 \\ 0 & 0 & \epsilon_\xi - \epsilon_3 + 3\,\epsilon_{3'}^2 - 8\,\epsilon_5^2 \end{pmatrix}$$
$$+ \frac{f_1(x_\varphi)}{32\pi^2} \begin{pmatrix} 3\,\epsilon_{3'}^2\,\beta_{3'}\,y_3 + 7\,\epsilon_5^2\,\beta_5\,y_5 & 0 & 0 \\ 0 & 3\,\epsilon_{3'}^2\,\beta_{3'}\,y_3 - 8\,\epsilon_5^2\,\beta_5\,y_5 & 0 \\ 0 & 0 & 3\,\epsilon_{3'}^2\,\beta_{3'}\,y_3 - 8\,\epsilon_5^2\,\beta_5\,y_5 \end{pmatrix},$$

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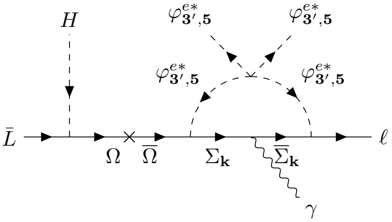

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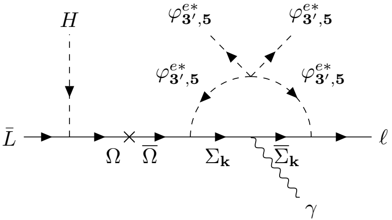


to reproduce $(g-2)_\mu$: $M_\chi \sim \Lambda_f = \frac{1}{4\pi}\sqrt{\frac{15\,\epsilon_5^2\,\beta_5\,v\,m_e\,m_\mu}{2(\,m_e\Delta a_\mu + m_\mu\Delta a_e)}f_2(x_{\varphi_5})}\,.$

$\Longrightarrow$ $\Lambda_f \leq 844$ GeV at $1\sigma$ and $\Lambda_f \leq 1354$ GeV at $3\sigma$.

## LHC searches

- Vector-like fermions with muon quantum numbers required with $M_x \lesssim 1$ TeV (in absence of fine-tuning) .
- Weak-production at LHC: $p\,p \to \chi^+\chi^-$
- Large couplings, $\mathcal{O}(1)$, to muon and scalar flavons
- Flavons, $\phi$, decay to heavier fermions, $\mu^+\mu^-$ ($\tau$ with three generations)

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Typical signature

$$p\, p \to \chi^+\chi^- \to \phi_1(\to \mu^+\mu^-)\,\ell^+\,\phi_1(\to \mu^+\mu^-)\,\ell^-, \qquad \text{with} \quad \ell = e, \mu$$

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$\Rightarrow$ **New Physics associated with $(g-2)_\mu$ should be visible at LHC.**

# Conclusions

- Low scale New Physics required to explain the muon (g-2) anomaly.
- Absence of fine-tuning requires electroweak-charged particles with $M_X \lesssim 1$ TeV
- Low-energy flavor symmetries can explain the muon (and electron anomalies).
- Realistic low-energy $T_{13}$, $A_5$ (others...) possible to explain $(g-2)_\mu$.
- Vector-like fermions with muon quantum numbers and new scalars at reach in LHC.

# Backup

# LFV Constraints

$$\mathcal{L} \supset \frac{e\, v}{8\,\pi^2}\, C_{\ell\ell'} \left(\bar{\ell}\sigma_{\mu\nu} P_R \ell'\right)\, F^{\mu\nu} \;+\; \mathrm{h.c.} \qquad \ell, \ell' = e, \mu, \tau$$

- $\Delta a_\ell = \frac{m_\ell v}{2\pi^2} \mathrm{Re}(C_{\ell\ell}),\ \mathrm{B}_{\ell\to\ell'\gamma} = \frac{3\alpha}{\sqrt{2}\pi\, G_F^3\, m_\ell^2} \left(|C_{\ell\ell'}|^2 + |C_{\ell'\ell}|^2\right) \mathrm{B}_{\ell\to\ell'\nu\bar{\nu}'}$

| | $M_X = 520\sqrt{\kappa}$ GeV | |
|---|---|---|
| $\mathrm{Re}(C_{\mu\mu})$ | $[1.5,\ 2.4] \times 10^{-9}$ GeV$^{-2}$ | $\lambda^2$ |
| $\mathrm{Re}(C_{ee})$ | $[-1.9,\ 1.7] \times 10^{-10}$ GeV$^{-2}$ | $\lambda^5$ |
| $\mathrm{Re}(C_{\tau\tau})$ | $[-3.7,\ 1.7] \times 10^{-4}$ GeV$^{-2}$ | 1 |
| $\lvert C_{e\mu}\rvert,\ \lvert C_{\mu e}\rvert$ | $\lesssim 3.9 \times 10^{-14}$ GeV$^{-2}$ | $\lambda^{\geq 8.6}$ |
| $\lvert C_{\tau\mu}\rvert,\ \lvert C_{\mu\tau}\rvert$ | $\lesssim 5.0 \times 10^{-10}$ GeV$^{-2}$ | $\lambda^{\geq 2.6}$ |
| $\lvert C_{\tau e}\rvert,\ \lvert C_{e\tau}\rvert$ | $\lesssim 4.3 \times 10^{-10}$ GeV$^{-2}$ | $\lambda^{\geq 2.7}$ |

with $\kappa \simeq 20$ if mass is fully radiative