# $B^0 - \overline{B}^0$ entanglement for an ideal experiment on the direct CP violation $\phi_3/\gamma$ phase

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June 2022

# Introduction on phases I

- The value of the quark charged current couplings are parametrized by the Cabibbo-Kobayashi-Maskawa (CKM) matrix $V$

$$\begin{aligned}\mathcal{L}_{CC} &= \frac{g}{\sqrt{2}} \begin{pmatrix} \overline{u} & \overline{c} & \overline{t} \end{pmatrix}_L \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \gamma_\mu \begin{pmatrix} d \\ s \\ b \end{pmatrix}_L W^{\mu\dagger} \\ &= \frac{g}{\sqrt{2}} \overline{U}_L V \gamma_\mu D_L W^{\mu\dagger}\end{aligned}$$

- We can eliminate 5 phases out of 9, $\left(q_L \longrightarrow e^{i\phi_q} q_L\right)$ :

$$V = \begin{pmatrix} |V_{ud}| & |V_{us}|\, e^{i\chi'} & |V_{ub}|\, e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|\, e^{-i\beta} & -|V_{ts}|\, e^{i\beta_s} & |V_{tb}| \end{pmatrix}$$

# Introduction on phases II

We can only consider 4 independent phases in the CKM matrix. These are the Aleksan, Kayser and London phases: $\beta, \gamma$ and $\beta_s$ and $\chi'$ (they even can be used as a CKM parametrization)

$$
\begin{aligned}
\beta &= \phi_1 = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right) \text{ (in } B_d^0 - \overline{B}_d^0 \text{ mixing)} \\
\gamma &= \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \text{ (in tree level } B^{\pm,0} \text{ decays)} \\
\beta_s &= \arg\left(-V_{ts}V_{tb}^*V_{cs}^*V_{cb}\right) \text{ (in } B_s^0 - \overline{B}_s^0 \text{ mixing)} \\
\chi' &= \arg\left(-V_{cd}V_{cs}^*V_{ud}^*V_{us}\right) \text{ (in direct CPV } D \to h^+h^-)
\end{aligned}
$$

- Because $\mathrm{Im}\left(V_{ij}V_{il}^*V_{kj}^*V_{kl}\right)$ for $i \neq k, j \neq l$ is universal (Chau, Keung, Jarlskog,..) up to a sign we have

$$\beta, \gamma \sim \mathcal{O}(1) \;;\; \beta_s \sim \lambda^2 \;;\; \chi' \sim \lambda^4$$

# The phase gamma I

- It appears in $V_{ub}$ defined by

$$
\begin{aligned}
\gamma &= \phi_3 = \arg\left(-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right) \\
\gamma &\sim \gamma + \chi' = \arg\left(+\frac{V_{us}V_{ub}^*}{V_{cs}V_{cb}^*}\right)
\end{aligned}
$$

In fact a minimal ingredient is the interference among $b \longrightarrow u\bar{c}s$ transition and $b \longrightarrow c\bar{u}s$ transitions.

$$
\begin{aligned}
A\left(b \longrightarrow u\bar{c}s\right) &= A\left(B^- \longrightarrow \overline{D}^0 K^-\right) \propto V_{ub}V_{cs}^* \\
A\left(b \longrightarrow c\bar{u}s\right) &= A\left(B^- \longrightarrow D^0 K^-\right) \propto V_{cb}V_{us}^*
\end{aligned}
$$

# The phase gamma II

But $(u\bar{c}) \equiv \overline{D}^0$ is different to $(c\bar{u}) \equiv D^0$ so we need a common final sate $f$ to have interference with

$$b \to \overline{D}^0 (\to f) + s \quad ; \quad B^- \to \overline{D}^0 (\to K^+K^-) + K^-$$
$$b \to D^0 (\to f) + s \quad ; \quad B^- \to D^0 (\to K^+K^-) + K^-$$

then both ways are open and we write

$$B^- \to D (\to f) + K^-$$

with more precision

$$B^- \to D^{\perp}_{\not\to f} + K^-$$

# The phase gamma III

where $D^{\perp}_{\nrightarrow f}$ is the state filtered by the final state $f$ that is the orthogonal to the state that does not decay to $f$ $D_{\nrightarrow f}$. This states can be shown to be

$$
\begin{aligned}
\left|D_{\nrightarrow f}\right\rangle &= \overline{c}_f \left|D^0\right\rangle - c_f \left|\overline{D}^0\right\rangle \\
\left|D^{\perp}_{\nrightarrow f}\right\rangle &= c_f^* \left|D^0\right\rangle + \overline{c}_f^* \left|\overline{D}^0\right\rangle
\end{aligned}
$$

with $\left|c_f\right|^2 + \left|\overline{c}_f\right|^2 = 1$ and fixed by the amplitudes

$$
\begin{aligned}
c_f &\propto A\left(D^0 \to f = K^+K^-\right) \propto V_{us}V_{cs}^* \\
\overline{c}_f &\propto A\left(\overline{D}^0 \to f = K^+K^-\right) \propto V_{us}^* V_{cs}
\end{aligned}
$$

# The phase gamma IV

at the end one has

$$\begin{aligned} A^- \equiv A\left(B^- \to D^{\perp}_{\not\to f} + K^-\right) = & \\ = \quad & c_f A\left(B^- \to D^0 K^-\right) + \overline{c}_f A\left(B^- \to \overline{D}^0 K^-\right) \\ \propto \quad & a V_{cb} V_{us}^* V_{us} V_{cs}^* + b V_{ub} V_{cs}^* V_{us}^* V_{cs} = \\ = \quad & a \left|V_{us}\right|^2 V_{cb} V_{cs}^* + b \left|V_{cs}\right|^2 V_{ub} V_{us}^* \end{aligned}$$

when constructing $CP$ violating observable one get

$$\left|A^-\right|^2 - \left|A^+\right|^2 \propto \mathrm{Im}\left(ab^*\right) \mathrm{Im}\left(V_{cb} V_{cs}^* V_{ub}^* V_{us}\right)$$

sensible to the interference phase

$$\arg\left(V_{cb} V_{cs}^* V_{ub}^* V_{us}\right) = \gamma + \chi' \sim \gamma$$

but needed of strong phases in $\mathrm{Im}\left(ab^*\right)$. This is the basis of many works by Gronau, London, Wyler (GLW), Atwood, Dunietz, Soni (ADS), Giri, Grossman, Soffer, Zupan (GGSZ) and many more.

# Using entanglement for gamma I

The use of the EPR correlation to study CP violation was proposed by Wolfenstein, Gavela et al, Falk and Petrov and Alvarez and Bernabeu among others for several decay channels in the B factories. The method for $\gamma$ consists in the observation of the coherent double decay to CP eigenstates. In it, the extraction of the phase is free from the essential strong phases contamination needed in charged B decays. The necessary interference between amplitudes containing the $V_{cd}V_{cb}^*$ and $V_{ud}V_{ub}^*$ sides of the unitarity triangle is automatic from the two terms of the entangled $B^0 - \overline{B}^0$ system (from $\Upsilon(4s)$):

$$|\Psi_0\rangle = \frac{1}{\sqrt{2}}\left(|B_d^0\rangle\left|\overline{B}_d^0\right\rangle - \left|\overline{B}_d^0\right\rangle|B_d^0\rangle\right) =$$
$$= \frac{1}{2\sqrt{2}pq}\left(|B_L\rangle|B_H\rangle - |B_H\rangle|B_L\rangle\right)$$

# Using entanglement for gamma II

with $B_H = pB^0 + q\overline{B}^0$ $B_L = pB^0 - q\overline{B}^0$, as can be seen in

$$\langle f, t_0; g, t_0 + t |\, \mathcal{T}\, |\Psi_0\rangle = \frac{e^{-i(\mu_L+\mu_H)t_0}}{2\sqrt{2}pq} \times$$
$$\times \left( e^{-i\mu_H t} A_L^f A_H^g - e^{-i\mu_L t} A_H^f A_L^g \right)$$

So the double decay rate to the state $f$ at $t_0$ and to the state $g$ at $t_0 + t$ integrated for $t_0$ is given by
$\Upsilon(4s) \to \left( B^0\overline{B}^0 - \overline{B}^0 B^0 \right) \to (f, t_0; g, t_0 + t)$

$$I(f, g; t) = \frac{e^{-\Gamma|t|}}{16\Gamma\,|pq|^2} \left| e^{-i\Delta M t/2} A_L^f A_H^g - e^{i\Delta M t/2} A_H^f A_L^g \right|^2$$

$$= \frac{e^{-\Gamma|t|}}{16\Gamma\,|pq|^2} \left| \begin{array}{c} \cos\left(\frac{\Delta M t}{2}\right) \left( A_L^f A_H^g - A_H^f A_L^g \right) \\ -i \sin\left(\frac{\Delta M t}{2}\right) \left( A_L^f A_H^g + A_H^f A_L^g \right) \end{array} \right|^2$$

# Using entanglement for gamma III

where $A^{f,g}_{L,H}$ are the decay amplitudes
$\left(A_f = \langle f|\,\mathcal{T}\,|B^0\rangle\ ;\ \overline{A}_f = \langle f|\,\mathcal{T}\,\left|\overline{B}^0\right\rangle\ ;\ \lambda = q\overline{A}_f / pA_f\right)$

$$
\begin{aligned}
A^f_H &= \langle f|\,\mathcal{T}\,|B_H\rangle = pA_f + q\overline{A}_f \\
A^f_L &= \langle f|\,\mathcal{T}\,|B_L\rangle = pA_f - q\overline{A}_f
\end{aligned}
$$

The double rate intensity to the $(f;g)$ and $(g;f)$ pairs of CP eigenstate decay products, with $f = J/\psi K_S, J/\psi K_L$ (in short $S$ or $L$) and $g = \pi^+\pi^-, \pi^0\pi^0, \rho^+_L\rho^-_L, \rho^0_L\rho^0_L$ , will do the job from CP-conserving and CP-violating transitions:

$$
\begin{array}{ccccc}
\Upsilon(4s) & (J/\psi K_{S,L})_B & (\pi\pi)_B & \left[\begin{array}{c}(J/\psi K_S)_B \\ (\pi\pi)_B\end{array}\right]_{\Upsilon(4s)} & \left[\begin{array}{c}(J/\psi K_L)_B \\ (\pi\pi)_B\end{array}\right]_{\Upsilon(4s)} \\
1^{--} & 0^{-+,-} & 0^{++} & 1^{++} & 1^{+-}
\end{array}
$$

# Using entanglement for gamma IV

Therefore

$$\Upsilon(4s) \to (J/\psi K_S)_B (\pi\pi)_B \quad ; \quad \text{is CP allowed}$$
$$\Upsilon(4s) \to (J/\psi K_L)_B (\pi\pi)_B \quad ; \quad \text{is CP forbidden}$$

so we have trivially

$$\widehat{I}(f,g;t) \equiv \frac{\Gamma}{\langle\Gamma_f\rangle \langle\Gamma_g\rangle} I(f,g;t) =$$
$$= \quad e^{-\Gamma|t|}\left[I_d^{fg}\cos^2\left(\frac{\Delta M t}{2}\right) + I_m^{fg}\sin^2\left(\frac{\Delta M t}{2}\right) + I_{od}^{fg}\sin(\Delta M t)\right]$$

# Important consistency properties I

- We have formally

$$\widehat{I}\left(f,g;t\right)=\widehat{I}\left(g,f;-t\right)$$

with the following implications

$$I_d^{fg}=I_d^{gf}\ ;\ I_m^{fg}=I_m^{gf}\ ;\ I_{od}^{fg}=-I_{od}^{gf}$$

- Because

$$\widehat{I}\left(g,S;t\right)=\left|\left\langle B^{\perp}_{\nrightarrow S}\middle|B_{\nrightarrow g}\left(t\right)\right\rangle\right|^2$$

and $\left|B^{\perp}_{\nrightarrow S}\right\rangle$ and $\left|B^{\perp}_{\nrightarrow L}\right\rangle$ are orthogonal

$$\widehat{I}\left(g,S;t\right)+\widehat{I}\left(g,L;t\right)=\left\langle B_{\nrightarrow g}\left(t\right)\middle|B_{\nrightarrow g}\left(t\right)\right\rangle=e^{-\Gamma t}$$

therefore we have

$$I_d^{Sg}+I_d^{Lg}=1\ ;\ I_m^{Sg}+I_m^{Lg}=1\ ;\ I_{od}^{Sg}+I_{od}^{Lg}=0$$

# Important consistency properties II

They provide a controlled connection between the CP forbidden and CP-allowed time-dependent transitions for any of the four decay products $g$. For practical purposes by adjusting the data sample to these constraints one can measure all three observables $I_{d,m,od}^{\overset{L}{S}g}$ for all the $(f, g)$ and $(g, f)$ channels $f = J/\psi K_S, J/\psi K_L$ and $g = \left(\rho_L^+ \rho_L^-\right), \left(\rho_L^0 \rho_L^0\right), \left(\pi^+ \pi^-\right), \left(\pi^0 \pi^0\right)$.

# The observables I

Our observables will be $I^{^L_S g}_{d,m,od}$ for each channel $g = \left(\rho_L^+ \rho_L^-\right), \left(\rho_L^0 \rho_L^0\right), \left(\pi^+\pi^-\right), \left(\pi^0\pi^0\right)$ in both time ordering making a total of 16 channels. We use

$$\left(\frac{q}{p}\right)_B = e^{-2i\phi_M} \; ; \; \frac{\overline{A}_g}{A_g} = \rho_g e^{-2i\phi_g} \; ; \; \left(\frac{\overline{A}_{(\pi\pi)_{I=2}}}{A_{(\pi\pi)_{I=2}}} = e^{-2i\gamma}\right)$$

$$\frac{\overline{A}_S}{A_S} = \left(\frac{p}{q}\right)_K \frac{V_{cs} V_{cb}^*}{V_{cs} V_{cb}^*} = -\left(\frac{V_{cd} V_{cb}^*}{V_{cd}^* V_{cb}}\right) e^{+2i\chi'} = -1$$

$$\frac{\overline{A}_S}{A_S} = -\frac{\overline{A}_L}{A_L} = -1 \; ; \; \lambda_L = -\lambda_S = -e^{-2i\phi_M} \; ; \; \phi_M = \beta$$

# The observables II

for the observable we get

$$
\begin{aligned}
I_d^{\substack{L\\S}g} &= \frac{\left|\left(\frac{\overline{A}_g}{A_{g-}} - \frac{\overline{A}_f}{A_{f-}}\right)\right|^2}{\left(1+|\lambda_f|^2\right)\left(1+|\lambda_g|^2\right)} \\
&= \frac{\left|\rho_g e^{-2i\phi_g} \mp 1\right|^2}{2\left(1+\rho_g^2\right)} = \frac{1}{2}\left[1 \mp \frac{2\rho_g \cos\left(2\phi_g\right)}{\left(1+\rho_g^2\right)}\right]
\end{aligned}
$$

it is present at $t=0$ it is CP forbidden for $f = J/\psi K_L$ and CP allowed for $f = J/\psi K_S$. It is sensible to the phases in $g$ and $f$. If there were not penguin pollution $\left(\rho_g = 1\right)$ in the decays $g = \left(\rho_L^+ \rho_L^-\right), \left(\rho_L^0 \rho_L^0\right), \left(\pi^+\pi^-\right), \left(\pi^0\pi^0\right)$ all $\phi_g$ would be $\phi_g = \gamma$ and we would have

$$
I_d^{Lg} = \sin^2 2\gamma \text{ and } I_d^{Sg} = \cos^2 2\gamma
$$

# The observables III

It is clear why we have named $I_d$ for **d**irect CP, because mixing is not needed.

For the other observables we get

$$I_m^{\overset{L}{S}g} = \frac{\left|(1-\lambda_g\lambda_f)\right|^2}{\left(1+|\lambda_f|^2\right)\left(1+|\lambda_g|^2\right)} = \frac{1}{2}\left[1 \mp \frac{2\rho_g \cos\left(4\phi_M + 2\phi_g\right)}{\left(1+\rho_g^2\right)}\right]$$

$$I_{od}^{\overset{L}{S}g} = \frac{2\,\mathrm{Im}\left[\left(\lambda_g^* - \lambda_f^*\right)(1-\lambda_g\lambda_f)\right]}{\left(1+|\lambda_f|^2\right)\left(1+|\lambda_g|^2\right)} = (\mp)\frac{\left(1-\rho_g^2\right)}{\left(1+\rho_g^2\right)}\sin\left(2\phi_M\right)$$

depending also on the mixing phase. The quantities to be extracted from $I_{d,m,od}^{\overset{L}{S}g}$, for each channel $g$, are $\phi_g, \rho_g$ and $\phi_M$. The one we are mainly interested is $\phi_g$ that if they were not penguin pollution in all the $g$ channels we will have $\phi_g = \gamma$ (and also $\rho_g = 1$). The way of getting $\gamma$ from $\phi_g$ is by the Gronau and London isospin analysis".

In general we will have for each $g$ channel a departure from the universal $\gamma$ value that we call

$$\epsilon_g = \gamma - \phi_g$$

The neutral and charged $B$ meson decays differ in the presence versus absence, respectively, of the penguin contribution to the amplitudes for each final $h = \pi, \rho_L$ system. The charged decay amplitudes $A_{+0} = A\left(B^+ \to h^+ h^0\right)$ and $\overline{A}_{+0} = A\left(B^- \to h^- h^0\right)$ have a final $\left(h^\pm h^0\right)$ isospin 2 state and, therefore, only the $\Delta I = 3/2$ tree-level amplitude contributes with the weak phase $\gamma$. It is convenient to define, with the same notation for both neutral decay channels $\pi\pi$and $\rho_L \rho_L$ and using $g = \pm$ or 00 for the corresponding decay charges, the quantities

$$a_g = \frac{A_g}{A_{+0}} \; ; \; \overline{a}_g = \frac{\overline{A}_g}{\overline{A}_{+0}}$$

# Isospin analysis II

in such a way that the double ratio fixes the penguin pollution parameters $\rho_g$ and $\epsilon_g$

$$\frac{\overline{a}_g}{a_g} = \rho_g e^{2i\epsilon_g}$$

The isospin triangular relations with these complex ratios are

$$\frac{1}{\sqrt{2}} a_{+-} = 1 - a_{00} \; ; \; \frac{1}{\sqrt{2}} \overline{a}_{+-} = 1 - \overline{a}_{00}$$

that allows to obtain $\mathrm{Re}\left(\overset{(-)}{a}_{+-}\right)$ and $\mathrm{Re}\left(\overset{(-)}{a}_{00}\right)$ in terms of $\left|\overset{(-)}{a}_{+-}\right|^2$ and $\left|\overset{(-)}{a}_{00}\right|^2$ and of course also $\mathrm{Im}\left(\overset{(-)}{a}_{+-}\right)$ and $\mathrm{Im}\left(\overset{(-)}{a}_{00}\right)$. Therefore we can get $a_g$ and $\overline{a}_g$ from the branching ratios of the processes

# Isospin analysis III

$B^{\pm} \to h^{\pm} h^0$ ;$B^0, \overline{B}^0 \to h^+ h^-, h^0 h^0$ fixing $\epsilon_g$ and $\rho_g$. The summary of our isospin analysis with the present PDG data is

| g | $\rho_g$ | $\epsilon_g$ |
|---|---|---|
| $\rho_L^+ \rho_L^-$ | $1.007 \pm 0.076$ | $0.008 \pm 0.091$ |
| $\rho_L^0 \rho_L^0$ | $0.972 \pm 0.241$ | $0.007 \pm 0.345$ |
| $\pi^+ \pi$ | $1.392 \pm 0.062$ | $\pm (0.307 \pm 0.170)$ |
| $\pi^0 \pi^0$ | $1.306 \pm 0.206$ | $\pm (0.427 \pm 0.172)$ |

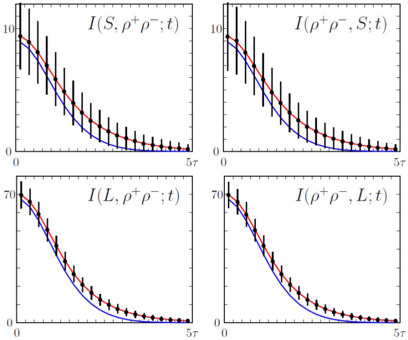
Because the $\rho_L^+ \rho_L^-$ is the one with largest branching ratio, $\delta\epsilon_{\rho_L^+ \rho_L^-} = 0.091 = 5.2^o$ gives us an estimate of the uncertainty, due to the present knowledge of the penguin pollution, in the determination of $\gamma/\phi_3$. (Important improvements are expected from Belle II and LHCb).

# Potential estimate of the method I

The intrinsic accuracy of the proposed method is controlled by the ability to extract $\phi_g$.
We generate events according to the double decay rate time distributions fixing all values as $\rho_g, \phi_g, \phi_M = \beta$ and the isopin analysis parameters.
Under the assumption that Belle II can collect 1000 $\rho_L^+\rho_L^-$ events in the categories $\left(L, \rho_L^+\rho_L^-\right)$, $\left(S, \rho_L^+\rho_L^-\right)$, $\left(\rho_L^+\rho_L^-, L\right)$, $\left(\rho_L^+\rho_L^-, S\right)$, 50 $\rho_L^0\rho_L^0$, 200 $\pi^+\pi^-$ and 50 $\pi^0\pi^0$
For each g, we generate values of t, the events, distributed according to the four double-decay intensities.
In order to incorporate the effect of experimental time resolution, each t is randomly displaced following a normal distribution with zero mean and $\sigma = 1ps$. Additional experimental effects such as efficiencies are not included. Generation proceeds until the chosen number of events. Events are binned.

# Potential estimate of the method II

The procedure is repeated in order to obtain mean values and standard deviations in each bin: these constitute our simulated data, as we illustrate with 20 bins. There are no significant differences if one considers, for example, 15 or 10 bins.



# Potential estimate of the method III

The fit to these simulations gives $I_d^{S\rho_L^+\rho_L^-} = 0.1170 \pm 0.0138$, $I_m^{S\rho_L^+\rho_L^-} = 0.1658 \pm 0.0456$ and $I_{od}^{S\rho_L^+\rho_L^-} = 0.0000 \pm 0.0198$. Together with the other fits it give rise to

| g | $\phi_g$ | $\rho_g$ |
|---|---|---|
| $\rho_L^+\rho_L^-$ | $1.222 \pm 0.020$ | $1.00 \pm 0.06$ |
| $\rho_L^0\rho_L^0$ | $1.22 \pm 0.09$ | $1.00 \pm 0.24$ |
| $\pi^+\pi$ | $1.57 \pm 0.12$ | $1.35 \pm 0.12$ |
| $\pi^0\pi^0$ | $1.57 \pm 0.18$ | $1.35 \pm 0.24$ |
| | $\phi_M = 0.384 \pm 0.031$ | |

We conclude that, since $\gamma = \phi_g + \epsilon_g$, the error $\delta\phi_{\rho_L^+\rho_L^-} = 0.020 = 1.1^o$ gives an idea of the intrinsic statistical limiting error we would expect in the determination of $\gamma$ for the assumed number of events.

# Conclusions I

With $B^0 - \overline{B}^0$ entanglement we consider the double decay rate Intensity to flavor-non-specific channels governed by the c- and u-quarks.

It offers a conceptual alternative to the decay of single $B^{\pm}$ mesons for the extraction of the direct CPV $\gamma/\phi_3$ phase.

The interference between two decay amplitudes is provided by the exchanged terms of the entangled state and no strong phases appear as essential ingredients.

The eight $(f, g)$ channels $f = J/\psi K_S, /\psi K_S$ and $g = (\pi\pi)^0, (\rho_L\rho_L)^0$ have a tree level common $\gamma$ phase. $\rho_L^+\rho_L^-$ is the benchmark channel.

Several constraining consistencies among the different intensities appear.

We find that an intrinsic accuracy of the order of 1 degree could be achievable for the relative phase of the f- and g-amplitudes. The present limitation of $\pm 5^o$, to be improved by the existing experimental facilities, comes from the phase of the penguin contribution in the g-amplitude, extracted from an isospin analysis to neutral and charged B decays.