Precision of Model Predictions and Modular Flavor Symmetries

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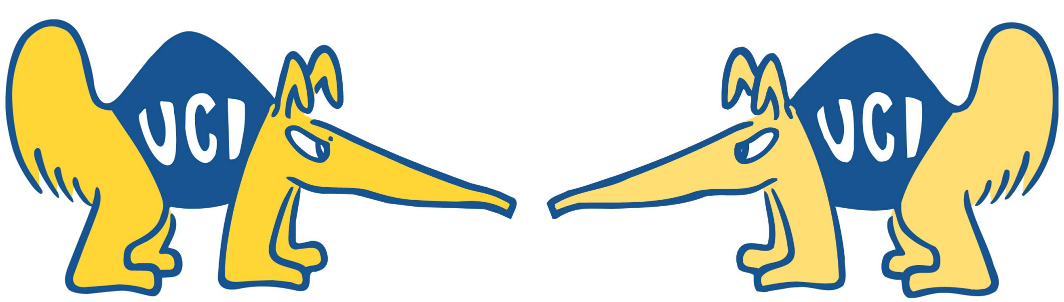
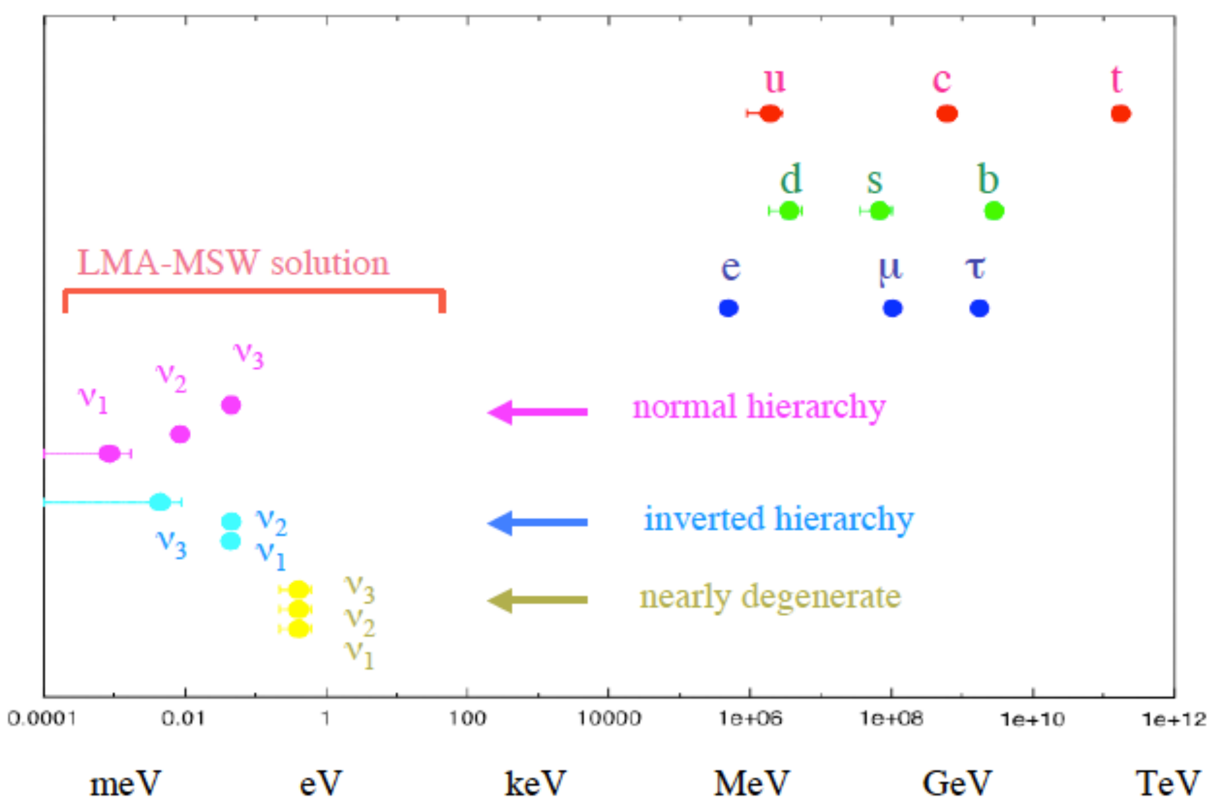


FLASY 2022, Lisbon, June 28, 2022

Open Questions - Theoretical

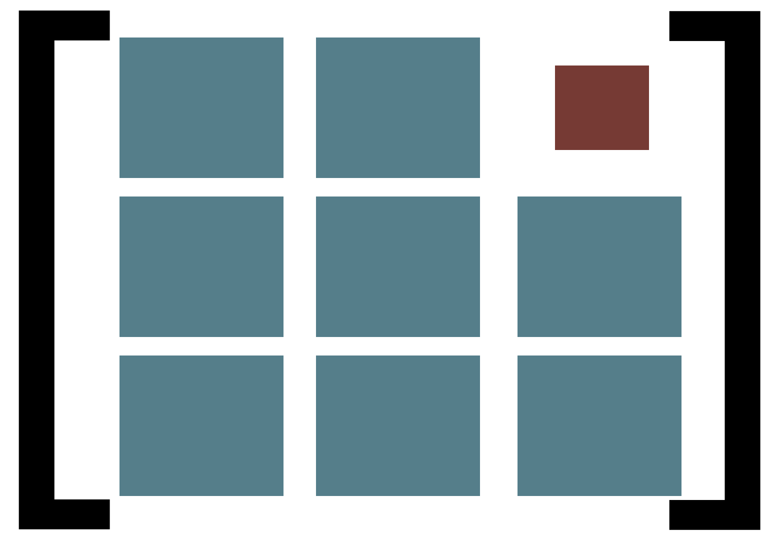
☞ **Smallness of neutrino mass:**

$$m_\nu \ll m_{e,\,u,\,d}$$

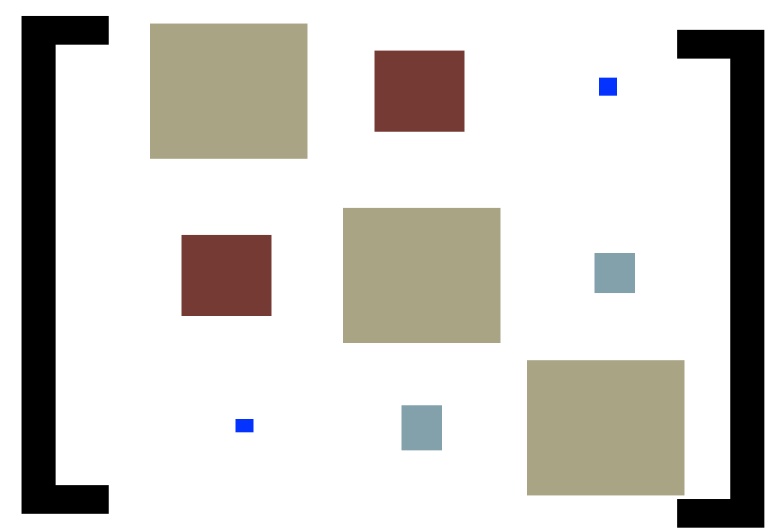


Fermion mass and hierarchy problem ➠ Dominant fraction (22 out of 28) of free parameters in SM

☞ **Flavor structure:**



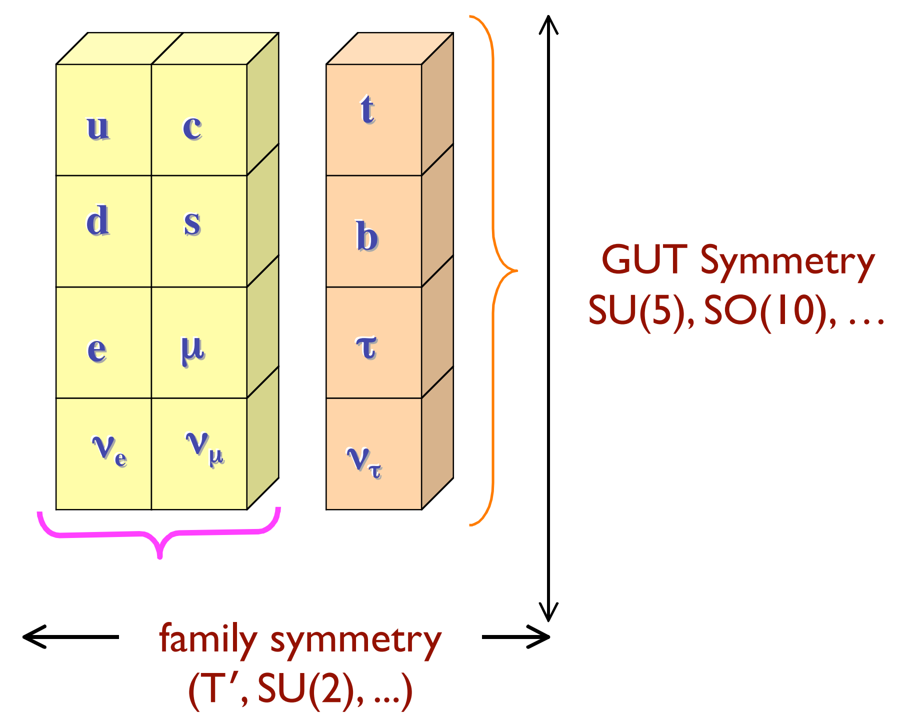
leptonic mixing

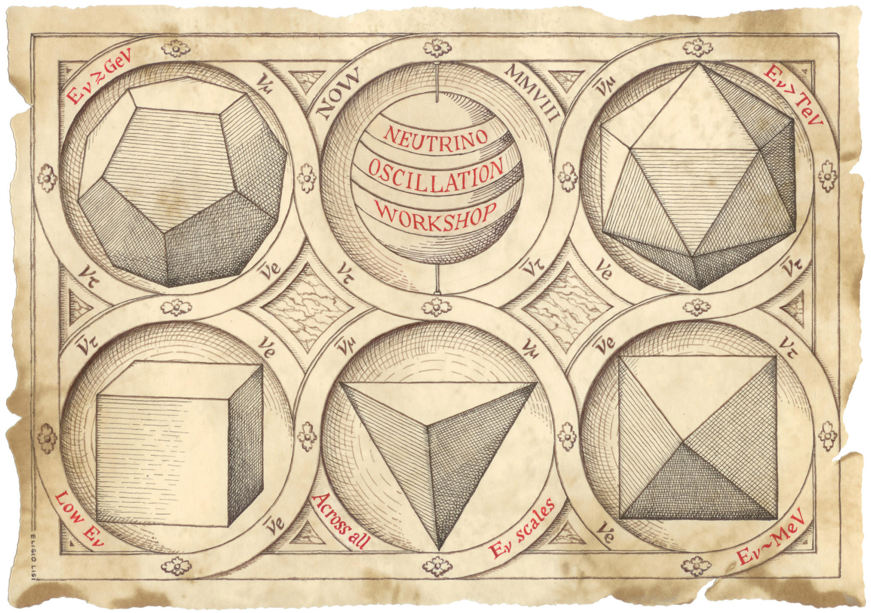


quark mixing

Non-Abelian Discrete Flavor Symmetries

- Large neutrino mixing motivates discrete flavor symmetries
  - A_4 (tetrahedron)
  - T´ (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_6
  - .....



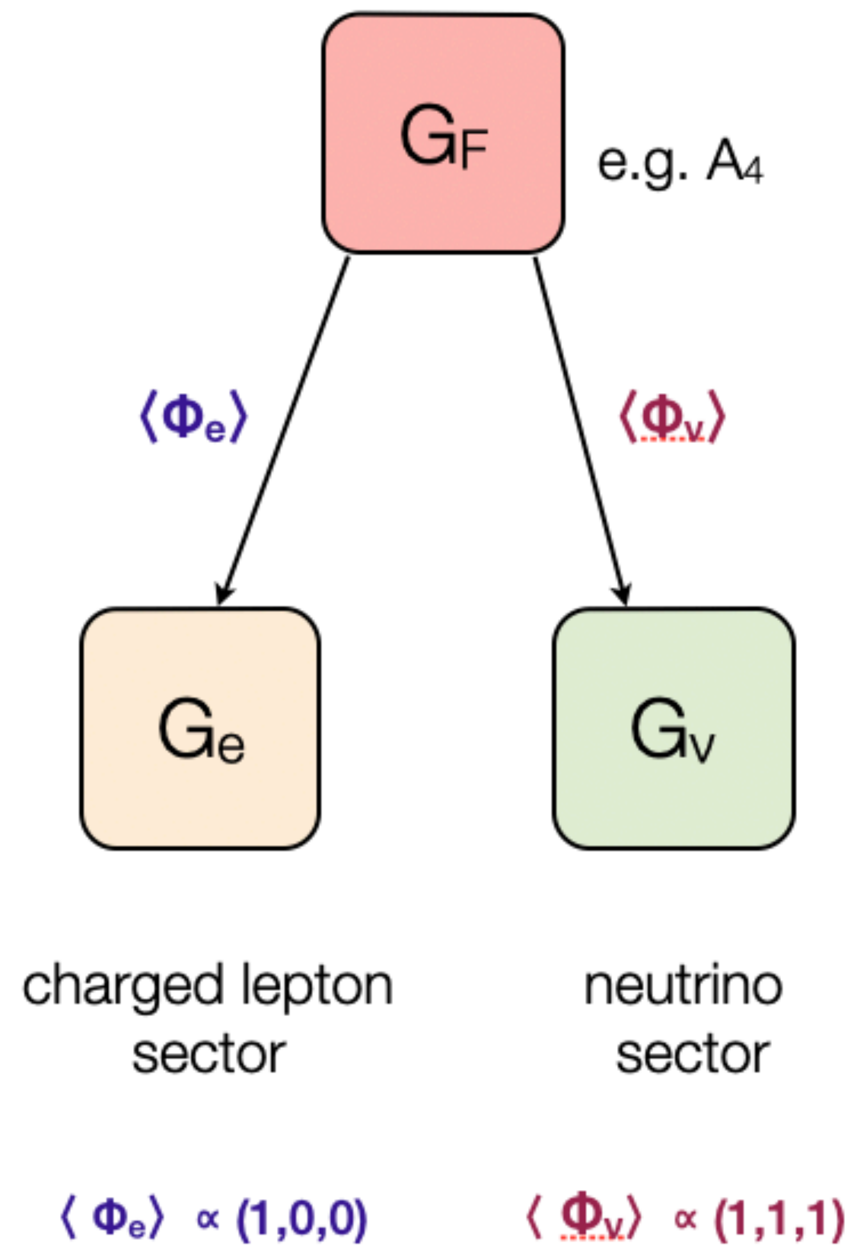


[Eligio Lisi for NOW2008]

Neutrino Mass Matrix from A4

Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); Altarelli, Feruglio (2005)

- Imposing A4 flavor symmetry on the Lagrangian
- A4 spontaneously broken by flavon fields



Neutrino Mass Matrix from A4

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- Imposing A4 flavor symmetry on the Lagrangian
- A4 spontaneously broken by flavon fields

2 free parameters

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

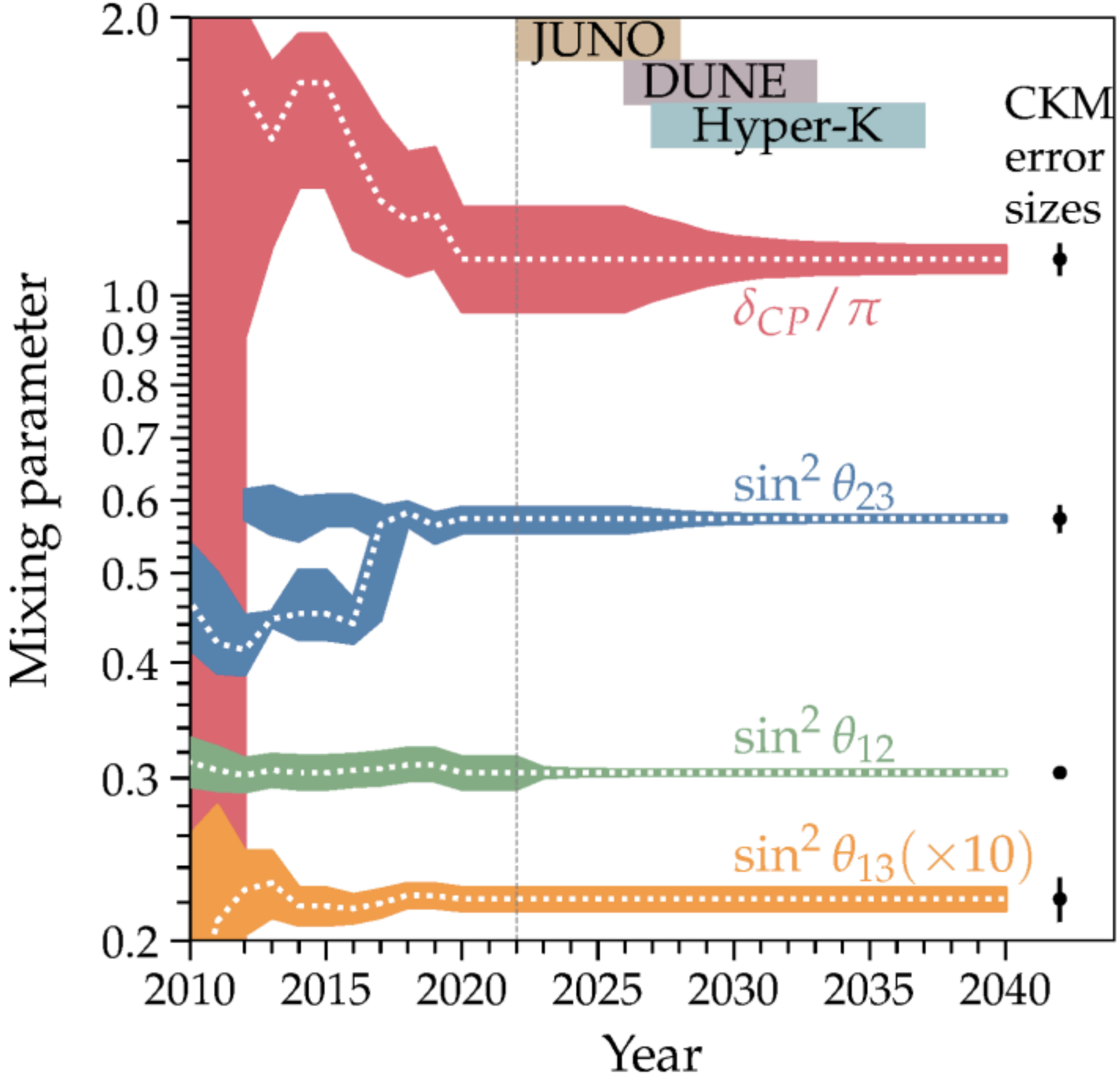
relative strengths $\Rightarrow$ CG's

- always diagonalized by TBM matrix, independent of the two free parameters

$$U_{\rm TBM} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

Neutrino Mixing Angles from Group Theory

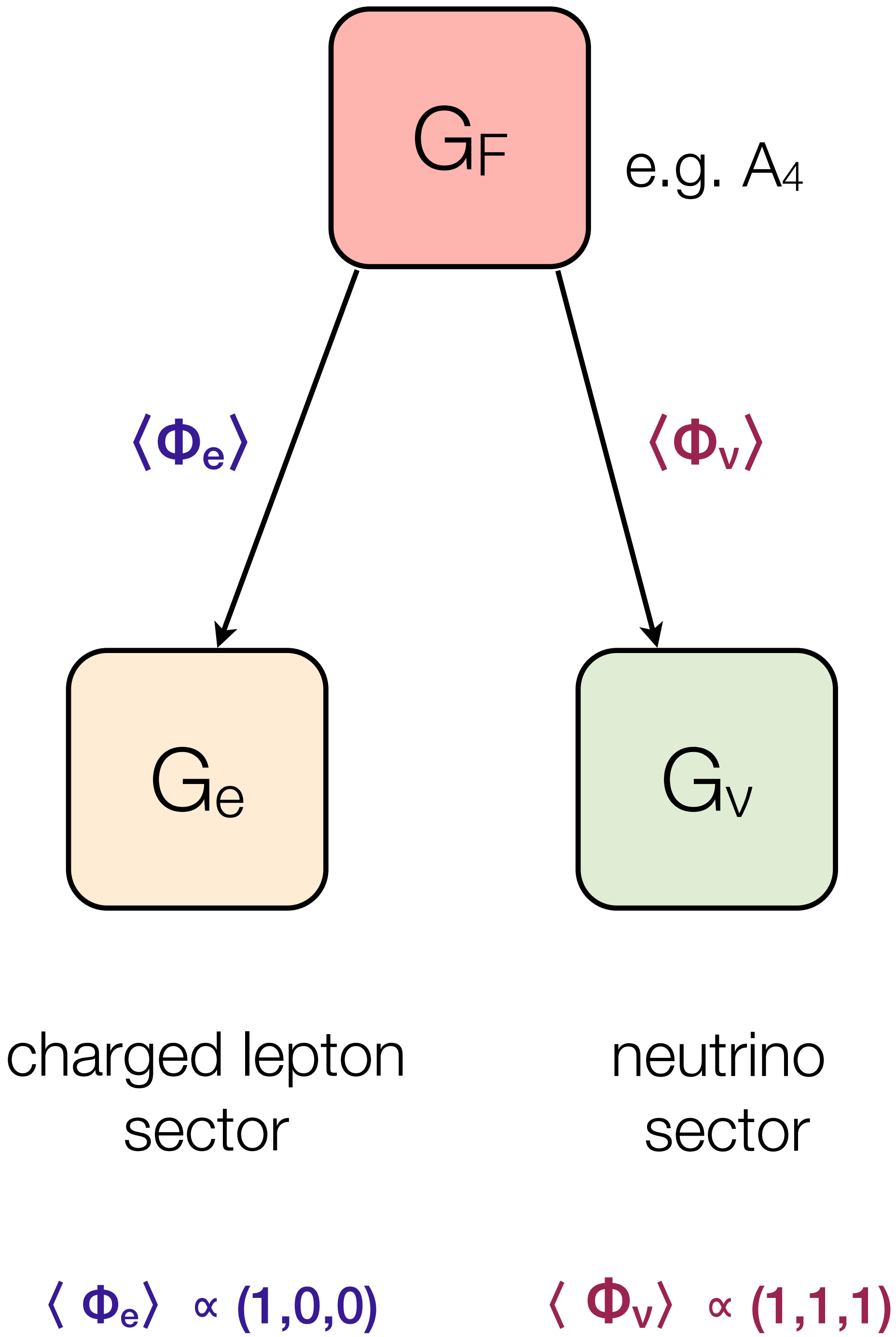
Experimental Precision



Are precisions in model predictions compatible with experimental precisions?

Figure from Song, Li, Argüelles, Bustamante, Vincent (2020)

Flavor Model Structure: A_4



- interplay between the symmetry breaking patterns in two sectors lead to lepton mixing (BM, TBM, ...)
- symmetry breaking achieved through flavon VEVs
- each sector preserves different residual symmetry
- full Lagrangian does not have these residual symmetries
- general approach: include high order terms in holomorphic superpotential
- possible to construct models where higher order holomorphic superpotential terms vanish to ALL orders

Corrections to Kinetic Terms

- Corrections to the kinetic terms induced by family symmetry breaking generically are present, should be properly included Leurer, Nir, Seiberg (1993); Dudas, Pokorski, Savoy (1995); Dreiner, Thomeier (2003)
  - can be along different directions than RG corrections
  - dominate over RG corrections (no loop suppression, copious heavy states)
  - could be sizable for neutrino mass models based on discrete family symmetries, e.g. A_4 M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)
    - nontrivial flavor structure can be induced
    - non-zero CP phase can be induced
    - Presence of additional undetermined parameters

Kähler Corrections

M.-C.C., Fallbacher, Ratz, Staudt (2012)

- Superpotential: holomorphic

$$\mathscr{W}_{\text{leading}} = \frac{1}{\Lambda}(\Phi_e)_{gf}\, L^g\, R^f\, H_d + \frac{1}{\Lambda\,\Lambda_\nu}(\Phi_\nu)_{gf}\, L^g\, H_u\, L^f\, H_u$$

$$\longrightarrow \mathscr{W}_{\text{eff}} = (Y_e)_{gf}\, L^g\, R^f\, H_d + \frac{1}{4}\kappa_{gf}\, L^g\, H_u\, L^f\, H_u$$

order parameter
<flavon vev> / Λ ~ θc

- Kähler potential: non-holomorphic

$$K = K_{\text{canonical}} + \Delta K$$

- Canonical Kähler potential

$$K_{\text{canonical}} \supset \left(L^f\right)^\dagger \delta_{fg}\, L^g + \left(R^f\right)^\dagger \delta_{fg}\, R^g$$

- Correction

$$\Delta K = \left(L^f\right)^\dagger (\Delta K_L)_{fg}\, L^g + \left(R^f\right)^\dagger (\Delta K_R)_{fg}\, R^g$$

- can be induced by flavon VEVs
- important for order parameter ~ θc
- can lead to non-trivial mixing

Kähler Corrections

M.-C.C., Fallbacher, Ratz, Staudt (2012)

- Consider infinitesimal change, x :

$$K = K_{\text{canonical}} + \Delta K = L^{\dagger}\,(1 - 2x\,P)\,L$$

- rotate to canonically normalized L′:

$$L \;\to\; L' = (1 - x\,P)\,L$$

$\Rightarrow$ corrections to neutrino mass matrix

$$\begin{aligned}\mathcal{W}_\nu &= \frac{1}{2}(L\cdot H_u)^T\kappa_\nu(L\cdot H_u)\\ &\simeq \frac{1}{2}[(\mathbb{1}+xP)L'\cdot H_u]^T\kappa_\nu[(\mathbb{1}+xP)L'\cdot H_u]\\ &\simeq \frac{1}{2}(L'\cdot H_u)^T\kappa_\nu L'\cdot H_u + x(L'\cdot H_u)^T(P^T\kappa_\nu+\kappa_\nu P)L'\cdot H_u\end{aligned}$$

with

$$\kappa \cdot v_u^2 = 2m_\nu$$

Kähler Corrections

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

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$$K = K_{\text{canonical}} + \Delta K = L^\dagger (1 - 2x\, P)\, L$$

- rotate to canonically normalized L′:

$$L \to L' = (1 - x\, P)\, L$$

⇒ corrections to neutrino mass matrix

$$m_\nu(x) \simeq m_\nu + x\, P^T m_\nu + x\, m_\nu\, P$$

⇒ differential equation

$$\frac{\mathrm{d}m_\nu}{\mathrm{d}x} = P^T m_\nu + m_\nu\, P$$

- same structure as the RG evolutions for neutrino mass operator
- size of Kähler corrections can be substantially larger (no loop suppression)

Back to A_4 Example

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Kähler corrections due to flavon field:
  - linear in flavon:

$$\Delta K_{\text{linear}} = \sum_{i \in \{\text{a,s}\}} \left(\frac{\kappa^{(i)}_{\Phi_\nu}}{\Lambda} \Delta K^{(i)}_{L^\dagger\,(L\otimes\Phi_\nu)_{\mathbf{3}_i}} + \frac{\kappa^{(i)}_{\Phi_e}}{\Lambda} \Delta K^{(i)}_{L^\dagger\,(L\otimes\Phi_e)_{\mathbf{3}_i}} \right) + \frac{\kappa_\xi}{\Lambda} \Delta K_{\xi L^\dagger L} + \text{h.c.}$$

  - possible to forbid these terms with additional symmetries

Back to A_4 Example

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Kähler corrections due to flavon field:
  - quadratic in flavon

$$\Delta K^{\text{quadratic}}_{\phi^{(\prime)}} \supset \frac{1}{\Lambda^2} \sum_{X}^{6} \kappa^{X}_{\phi^{(\prime)},\text{quadratic}} \left(L\phi^{(\prime)}\right)^{\dagger}_{X} \left(L\phi^{(\prime)}\right)_{X} + \text{h.c.}$$

$$(L\Phi_\nu)^\dagger (L\Phi_\nu) \quad \text{and} \quad (L\Phi_e)^\dagger (L\Phi_e)$$

  - such terms cannot be forbidden by any (conventional) symmetry
  - Kähler corrections once flavon fields attain VEVs
  - additional parameters $\kappa^{X}_{\phi^{(\prime)}}$ diminish predictivity of the scheme
  - possible to forbid all contributions from RH sector as well as $(L\Phi_\nu)^\dagger (L\Phi_e)$ with additional symmetries in the particular A_4 model considered

Back to A_4 Example

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Kähler corrections due to flavon field χ :
  - six possible non-trivial contractions:

$$\Delta K \supset \sum_{i=1}^{6} \kappa^{(i)}\, \Delta K^{(i)}_{(L\chi)^\dagger_{\boldsymbol{x}}(L\chi)_{\boldsymbol{x}}} + \text{h.c.}$$

$$\begin{aligned}
\Delta K^{(1)}_{(L\chi)^\dagger_{\mathbf{1}}(L\chi)_{\mathbf{1}}} &= (L_1^\dagger\,\chi_1^\dagger + L_2^\dagger\,\chi_3^\dagger + L_3^\dagger\,\chi_2^\dagger)(L_1\,\chi_1 + L_2\,\chi_3 + L_3\,\chi_2)\ ,\\
\Delta K^{(2)}_{(L\chi)^\dagger_{\mathbf{1}'}(L\chi)_{\mathbf{1}'}} &= (L_3^\dagger\,\chi_3^\dagger + L_1^\dagger\,\chi_2^\dagger + L_2^\dagger\,\chi_1^\dagger)(L_3\,\chi_3 + L_1\,\chi_2 + L_2\,\chi_1)\ ,\\
\Delta K^{(3)}_{(L\chi)^\dagger_{\mathbf{1}''}(L\chi)_{\mathbf{1}''}} &= (L_2^\dagger\,\chi_2^\dagger + L_1^\dagger\,\chi_3^\dagger + L_3^\dagger\,\chi_1^\dagger)(L_2\,\chi_2 + L_1\,\chi_3 + L_3\,\chi_1)\ ,\\
\Delta K^{(4)}_{(L\chi)^\dagger_{\mathbf{3}_1}(L\chi)_{\mathbf{3}_1}} &= (L_1^\dagger\,\chi_1^\dagger + \omega^2\,L_2^\dagger\,\chi_3^\dagger + \omega\,L_3^\dagger\,\chi_2^\dagger)(L_1\,\chi_1 + \omega\,L_2\,\chi_3 + \omega^2\,L_3\,\chi_2)\\
&+ (L_3^\dagger\,\chi_3^\dagger + \omega^2\,L_1^\dagger\,\chi_2^\dagger + \omega\,L_2^\dagger\,\chi_1^\dagger)(L_3\,\chi_3 + \omega\,L_1\,\chi_2 + \omega^2\,L_2\,\chi_1)\\
&+ (L_2^\dagger\,\chi_2^\dagger + \omega^2\,L_1^\dagger\,\chi_3^\dagger + \omega\,L_3^\dagger\,\chi_1^\dagger)(L_2\,\chi_2 + \omega\,L_1\,\chi_3 + \omega^2\,L_3\,\chi_1)\\
\Delta K^{(5)}_{(L\chi)^\dagger_{\mathbf{3}_2}(L\chi)_{\mathbf{3}_2}} &= (L_1^\dagger\,\chi_1^\dagger + \omega\,L_2^\dagger\,\chi_3^\dagger + \omega^2\,L_3^\dagger\,\chi_2^\dagger)(L_1\,\chi_1 + \omega^2\,L_2\,\chi_3 + \omega\,L_3\,\chi_2)\\
&+ (L_3^\dagger\,\chi_3^\dagger + \omega\,L_1^\dagger\,\chi_2^\dagger + \omega^2\,L_2^\dagger\,\chi_1^\dagger)(L_3\,\chi_3 + \omega^2\,L_1\,\chi_2 + \omega\,L_2\,\chi_1)\\
&+ (L_2^\dagger\,\chi_2^\dagger + \omega\,L_1^\dagger\,\chi_3^\dagger + \omega^2\,L_3^\dagger\,\chi_1^\dagger)(L_2\,\chi_2 + \omega^2\,L_1\,\chi_3 + \omega\,L_3\,\chi_1)\\
\Delta K^{(6)}_{(L\chi)^\dagger_{\mathbf{3}_1}(L\chi)_{\mathbf{3}_2}} &= (L_1^\dagger\,\chi_1^\dagger + \omega^2\,L_2^\dagger\,\chi_3^\dagger + \omega\,L_3^\dagger\,\chi_2^\dagger)(L_1\,\chi_1 + \omega^2\,L_2\,\chi_3 + \omega\,L_3\,\chi_2)\\
&+ (L_3^\dagger\,\chi_3^\dagger + \omega^2\,L_1^\dagger\,\chi_2^\dagger + \omega\,L_2^\dagger\,\chi_1^\dagger)(L_3\,\chi_3 + \omega^2\,L_1\,\chi_2 + \omega\,L_2\,\chi_1)\\
&+ (L_2^\dagger\,\chi_2^\dagger + \omega^2\,L_1^\dagger\,\chi_3^\dagger + \omega\,L_3^\dagger\,\chi_1^\dagger)(L_2\,\chi_2 + \omega^2\,L_1\,\chi_3 + \omega\,L_3\,\chi_1)
\end{aligned}$$

Back to A_4 Example

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)

- Contributions from Flavon VEVs (1,0,0) and (1,1,1)
  - five independent "basis" matrices

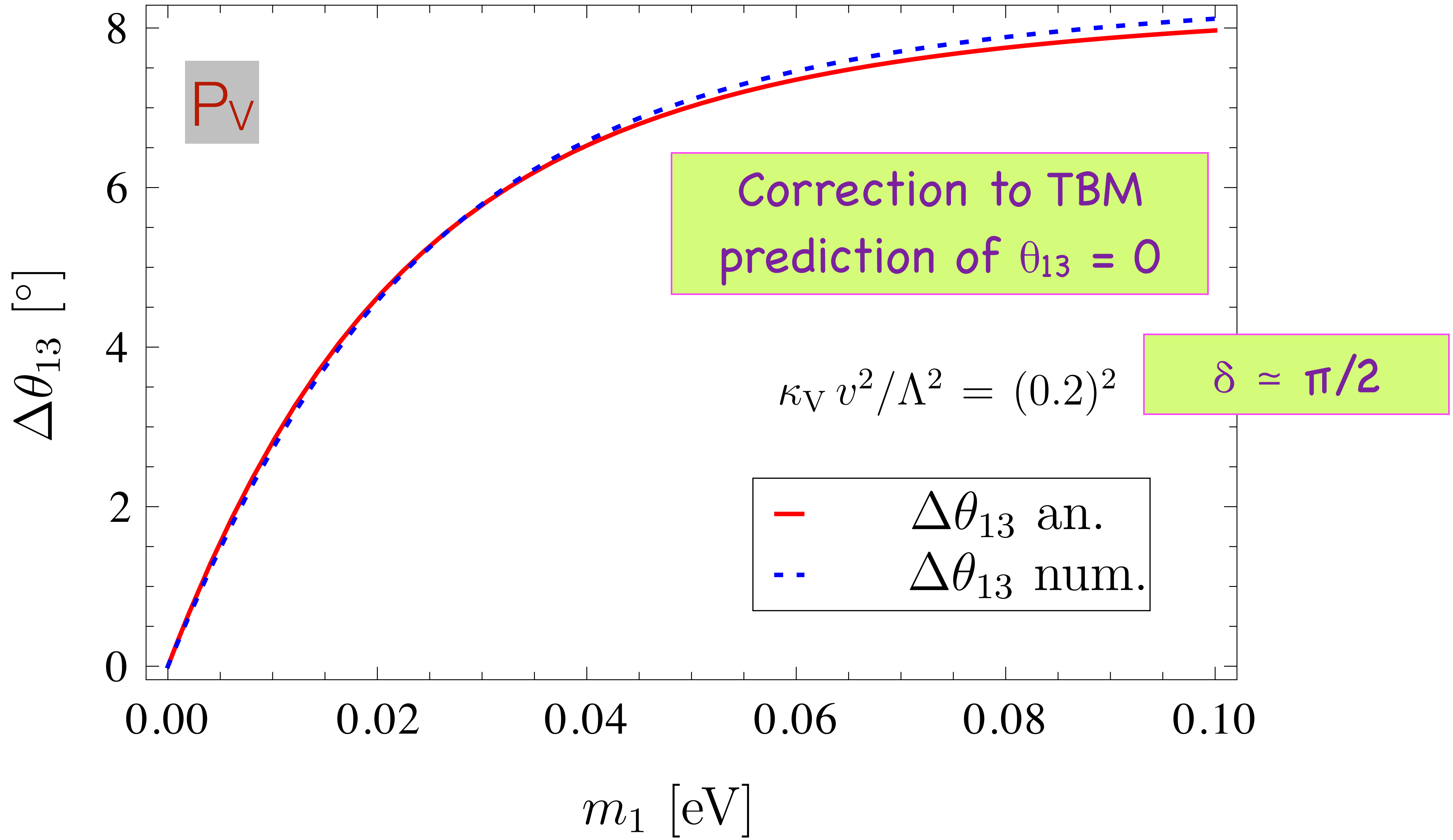
$$P_{\mathrm{I}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\mathrm{II}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad P_{\mathrm{III}} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$P_{\mathrm{IV}} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}, \quad P_{\mathrm{V}} = \begin{pmatrix} 0 & \mathrm{i} & -\mathrm{i} \\ -\mathrm{i} & 0 & \mathrm{i} \\ \mathrm{i} & -\mathrm{i} & 0 \end{pmatrix}$$

- RG correction: essentially along P_{III} = diag(0,0,1) direction due to y_τ dominance

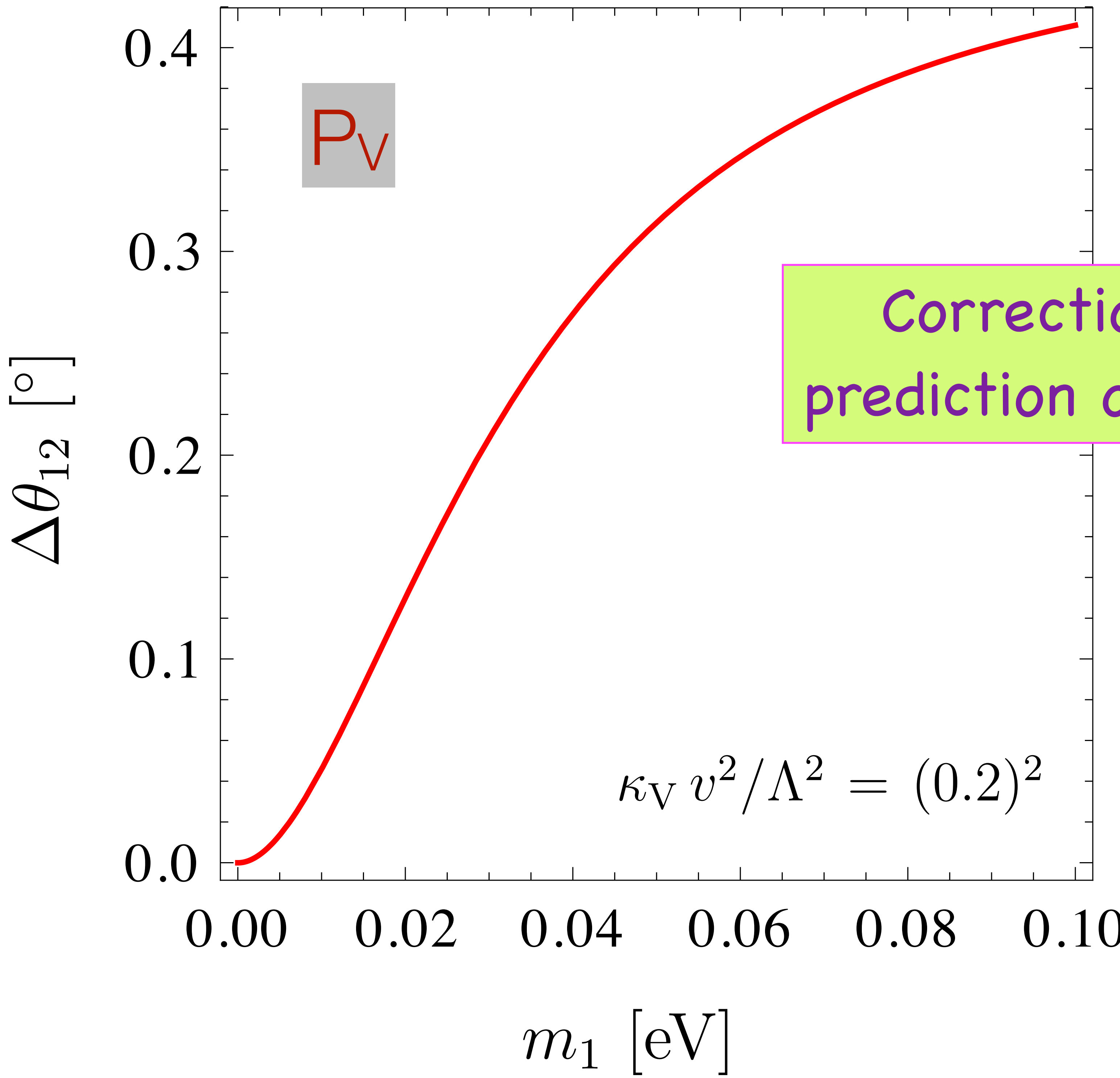
An Example: Enhanced θ_{13} in A_4

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Corresponding Change in θ_{12}

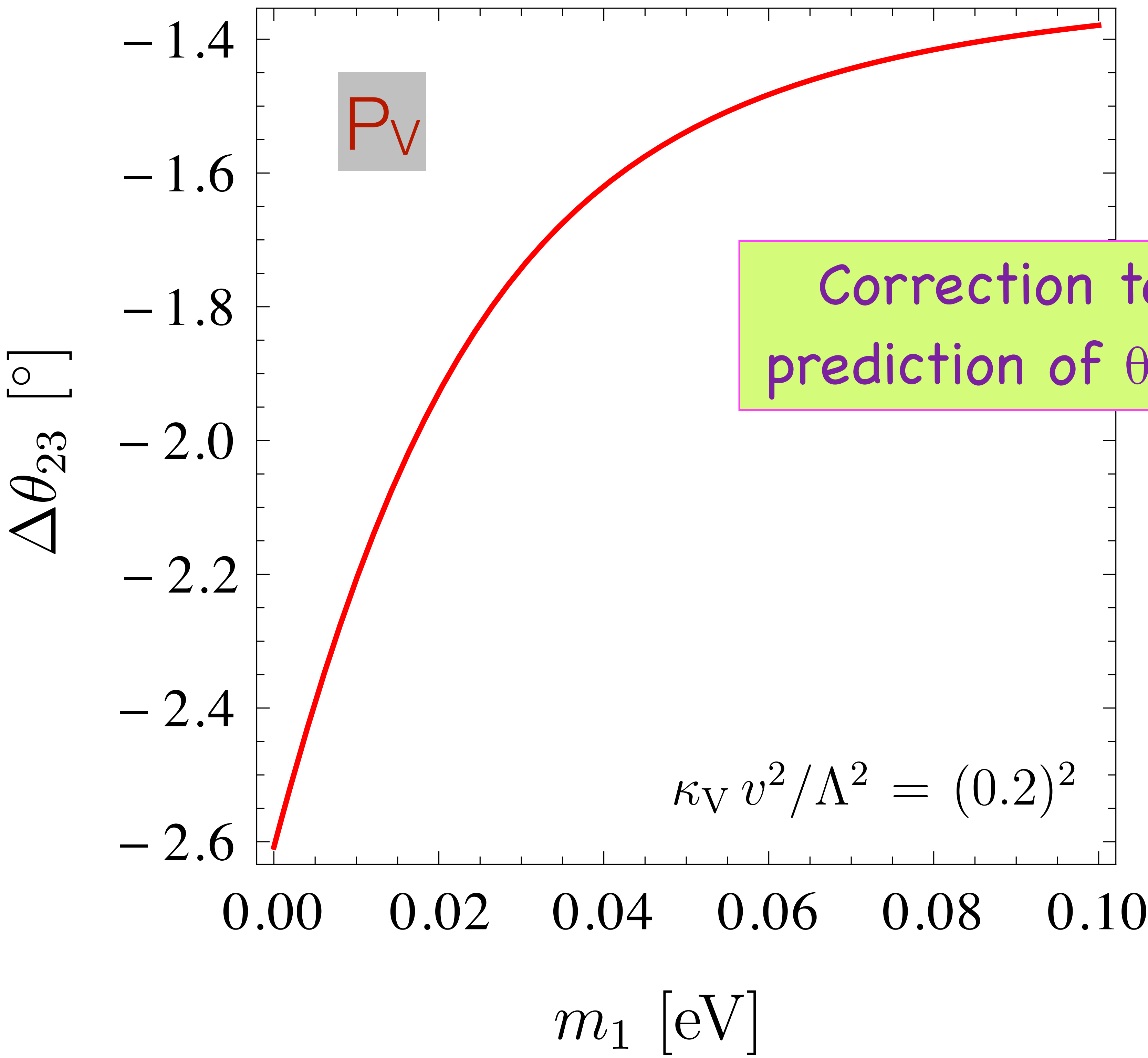
M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Correction to TBM prediction of $\theta_{12} = 35.3°$

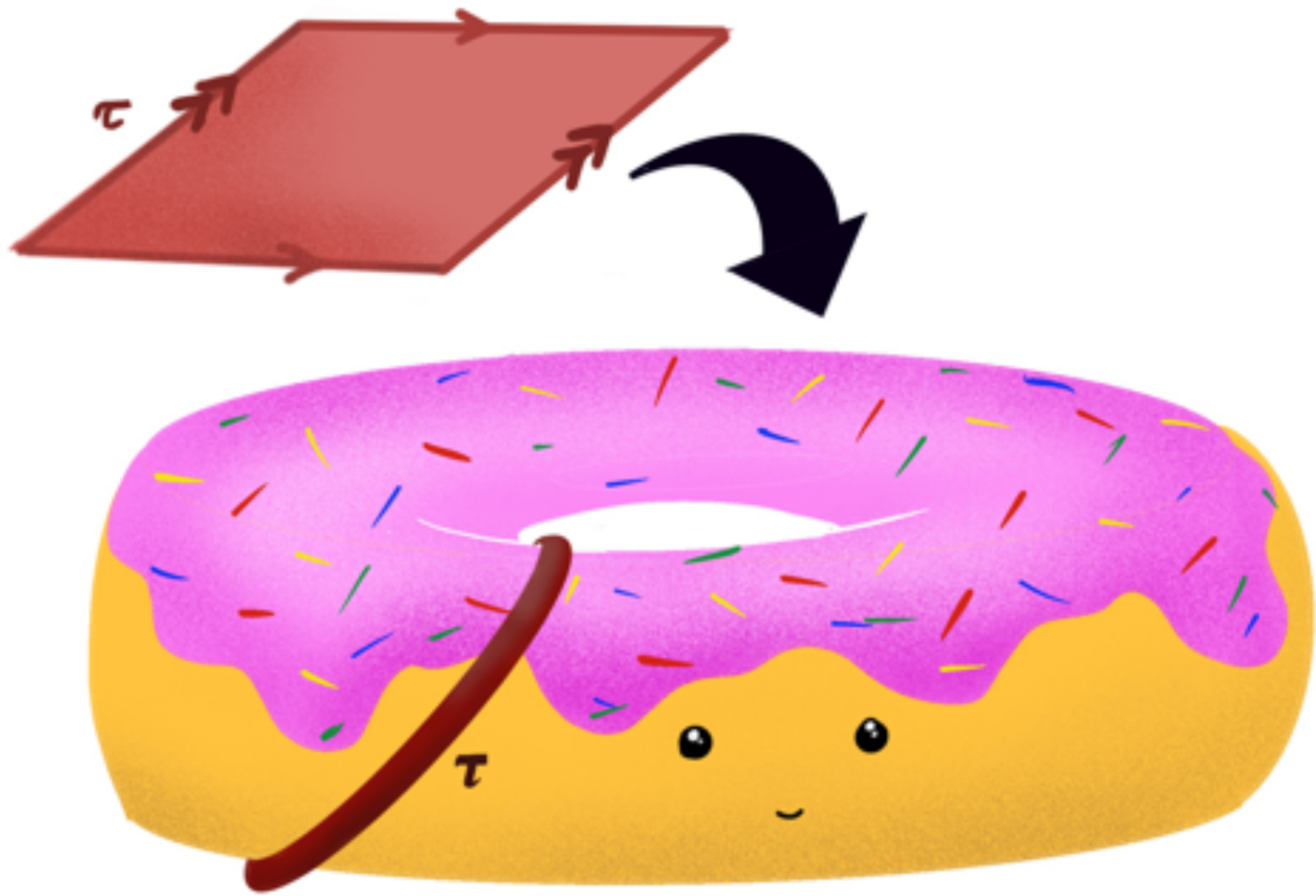
Corresponding Change in θ_{23}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Correction to TBM prediction of θ_{23} = 45°

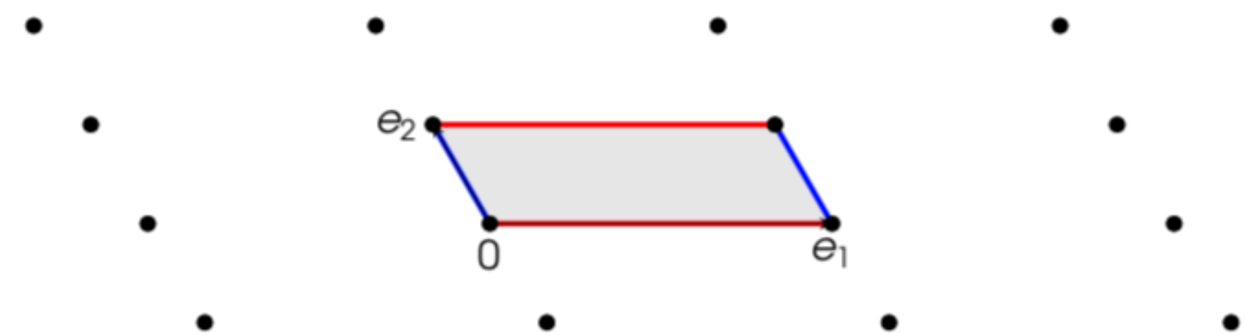
Modular Flavor Symmetries



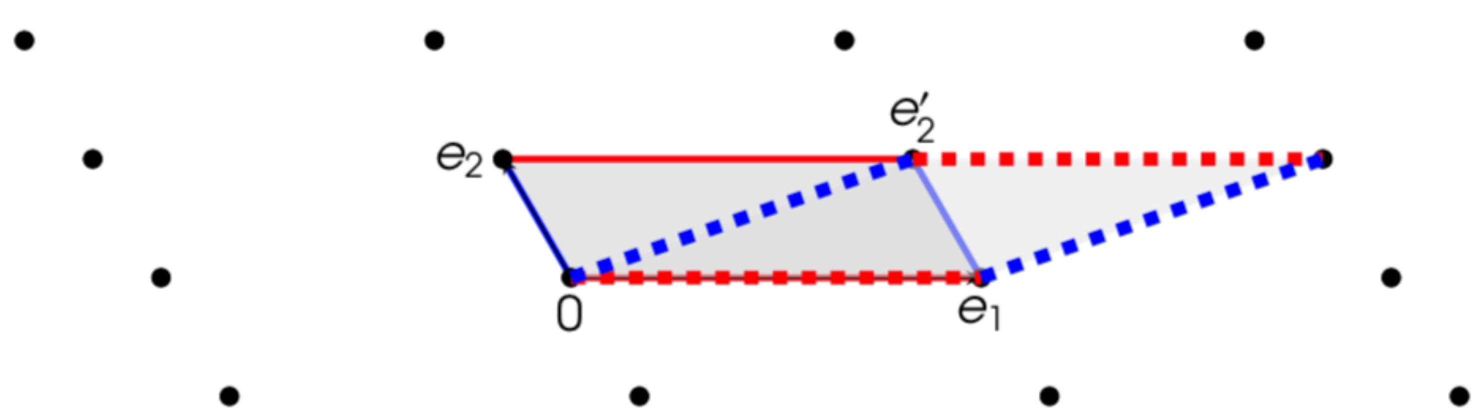
Artwork by Shreya Shukla

Modular Symmetries

edges ⇒ lattice basis vectors



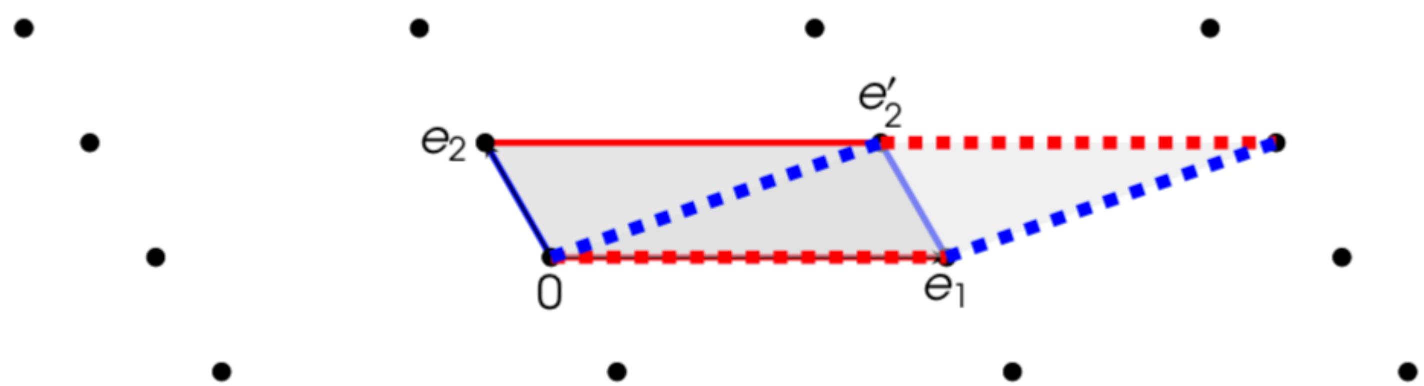
points in plane identified if differ by a lattice translation



Equivalent TORI related by Modular Symmetries

Modular Symmetries

- TORI: fundamental domain not unique



- Basis Vectors are related: $\begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \overset{\gamma}{\mapsto} \begin{pmatrix} e'_2 \\ e'_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_2 \\ e_1 \end{pmatrix} =: \gamma \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$

$a, b, c, d \in \mathbb{Z}$

- Volume of fundamental domain the same $\Rightarrow$ $\det \gamma = 1$

Modular Symmetries

- **Finite Modular Group** (quotient group): $\Gamma_N := \Gamma/\Gamma(N)$ where principal congruence group $\Gamma(N)$ is

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2 ; \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mod N \right\}$$

Γ

- Generators of the quotient group Γ_N satisfy

$$S^2 = 1, \quad (ST)^3 = 1, \quad T^N = 1$$

- Some examples

$$\Gamma_2 \simeq S_3, \quad \Gamma_3 \simeq A_4, \quad \Gamma_4 \simeq S_4, \quad \Gamma_5 \simeq A_5$$

Modular Symmetries

Feruglio (2017)

- Imposing modular symmetry Γ on the Lagrangian:

$$\mathscr{L} \supset \sum Y_{i_1,\, i_2,\, \ldots\ldots\, i_n} \Phi_{i_1} \Phi_{i_2} \cdots \Phi_{i_n}$$

$$\tau \xrightarrow{\gamma} \gamma\tau := \frac{a\tau + b}{c\tau + d}\,,$$

$$\Phi_j \xrightarrow{\gamma} (c\tau + d)^{k_j} \rho_{r_j}(\gamma)\, \Phi_j\,, \quad \text{where } \gamma := \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

k_i : integers

representation matrix of Γ_N

- Yukawa Couplings = Modular Forms at level "N" w/ weight "k"

$$f_i(\gamma\tau) = (c\tau + d)^{-k} [\rho_N(\gamma)]_{ij} f_j(\tau)$$

$k = k_{i_1} + k_{i_2} + \ldots + k_{i_n}$

representation matrix of Γ_N

A Toy Modular A_4 Model

Feruglio (2017)

- Weinberg Operator $$\mathscr{W}_\nu = \frac{1}{\Lambda}[(H_u \cdot L)\; Y\; (H_u \cdot L)]_1$$
- Traditional A4 Flavor Symmetry
  - Yukawa Coupling Y $\rightarrow$ Flavon VEVs (A_4 triplet, 6 real parameters)

$$Y \rightarrow \langle\phi\rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \Longrightarrow \quad m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2a & -c & -b \\ -c & 2b & -a \\ -b & -a & 2c \end{pmatrix}$$

- Modular A4 Flavor Symmetry
  - Yukawa Coupling Y $\rightarrow$ Modular Forms (A4 triplet, 2 real parameters)

$$Y \rightarrow \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \quad \Longrightarrow \quad m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

Modular Forms

Feruglio (2017)

- Level (N) = 3, Weight (k) = 2, in terms of Dedekind eta-function

$$
\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}-\frac{27\eta'(3\tau)}{\eta(3\tau)}\right] \\
Y_2(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega^2\,\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega\,\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] \\
Y_3(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega\,\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega^2\,\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] \quad .
\end{aligned}
$$

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n) \qquad\qquad q \equiv e^{i2\pi\tau}$$

A Toy Modular A_4 Model

Feruglio (2017)

- Input Parameters:

$$\tau = 0.0111 + 0.9946i$$

$$v_u^2/\Lambda$$

- Predictions:

$$\frac{\Delta m^2_{sol}}{|\Delta m^2_{atm}|} = 0.0292$$

$$\sin^2\theta_{12} = 0.295 \qquad \sin^2\theta_{13} = 0.0447 \qquad \sin^2\theta_{23} = 0.651$$

$$\frac{\delta_{CP}}{\pi} = 1.55 \qquad \frac{\alpha_{21}}{\pi} = 0.22 \qquad \frac{\alpha_{31}}{\pi} = 1.80 \quad .$$

$$m_1 = 4.998 \times 10^{-2}\ eV \qquad m_2 = 5.071 \times 10^{-2}\ eV \qquad m_3 = 7.338 \times 10^{-4}\ eV$$

Kähler Corrections in Modular A4 Model

- Particle Content

Feruglio (2017)

	(E_1^c, E_2^c, E_3^c)	L	H_d	H_u	φ
$SU(2)_L \times U(1)_Y$	$\mathbf{1}_1$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{1/2}$	$\mathbf{1}_0$
Γ_3	$(\mathbf{1}, \mathbf{1}', \mathbf{1}'')$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3}$
k	$(k_{E_1}, k_{E_2}, k_{E_3})$	k_L	k_{H_d}	k_{H_u}	k_φ

- Weinberg Operator

$$\mathscr{W}_\nu = \frac{1}{\Lambda}[(H_u \cdot L)\; Y\; (H_u \cdot L)]_{\mathbf{1}}$$

- Superpotential for Charged Leptons: couple to φ $\Rightarrow$ diagonal mass matrix

Kähler Corrections in Modular A4 Model

- Minimal Kähler Potential for charged leptons

$$K_L = (-\mathrm{i}\,\tau + \mathrm{i}\,\bar{\tau})^{-1} L^\dagger L$$

- Additional terms allowed in Kähler Potential

MCC, Ramos-Sánchez, Ratz (2019)

$$K = \alpha_0 \left(-\mathrm{i}\,\tau + \mathrm{i}\,\bar{\tau}\right)^{-1} \left(\overline{L}\,L\right)_{\mathbf{1}} + \sum_{k=1}^{7} \alpha_k \left(-\mathrm{i}\,\tau + \mathrm{i}\,\bar{\tau}\right) \left(Y\,L\,\overline{Y}\,\overline{L}\right)_{\mathbf{1},\,k} + \dots\,.$$

$$\Delta K = \alpha_1 \left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(1)}} \left(Y\,L\right)_{\mathbf{3}^{(1)}} + \alpha_2 \left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(2)}} \left(Y\,L\right)_{\mathbf{3}^{(2)}}$$
$$+ \alpha_3 \left[\left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(1)}} \left(Y\,L\right)_{\mathbf{3}^{(2)}} + \left(\overline{Y}\,\overline{L}\right)^T_{\mathbf{3}^{(2)}} \left(Y\,L\right)_{\mathbf{3}^{(1)}}\right] + \dots$$

- “Leading terms” vs “corrections” on equal footing

Kähler Corrections in Modular A4 Model

- Additional terms induced by flavon VEV

MCC, Ramos-Sánchez, Ratz (2019)

$$\Delta K \;=\; \sum_i \beta_i \left(-\mathrm{i}\,\tau + \mathrm{i}\,\bar{\tau}\right)^{-k_L - k_\varphi} \left(\varphi\, L\, \overline{\varphi}\, \overline{L}\right)_{\mathbf{1},\, i}$$

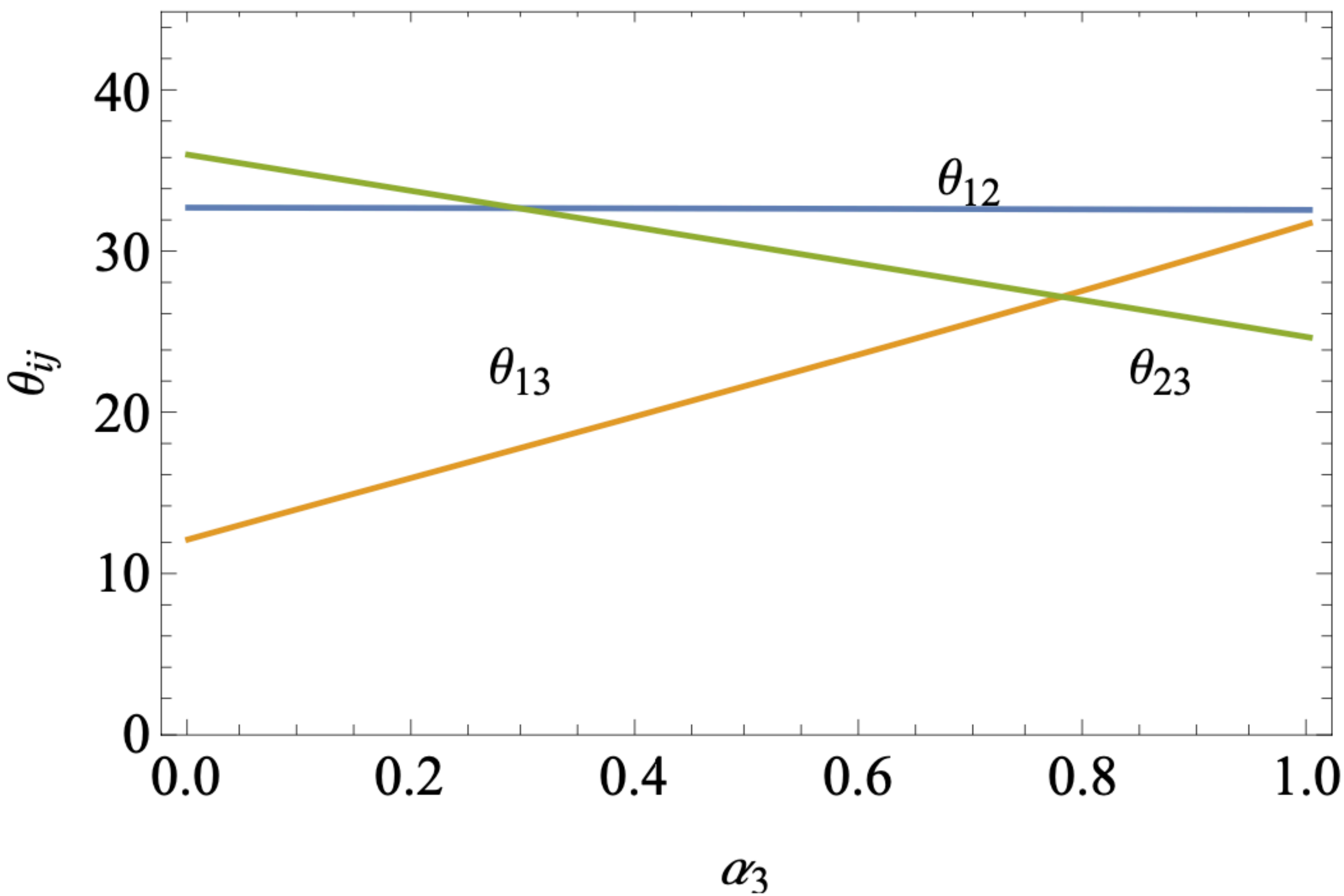
- Modifying Kähler metric

$$K_L^{i\bar{\jmath}} \;=\; \frac{\partial^2 K}{\partial L_i\, \partial \overline{L}_{\bar{\jmath}}}$$

- Back to Canonical Basis ➜ sizable corrections to mixing parameters

Kähler Corrections in Modular A4 Model

M.-C.C., Ramos-Sánchez, Ratz (2019)



Quasi-Eclectic Modular Symmetry

- Quasi-eclectic setup:

MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)

$$G_{\text{quasi-eclectic}} = G_{\text{traditional}} \times G_{\text{modular}} = A_4 \times \Gamma_3$$

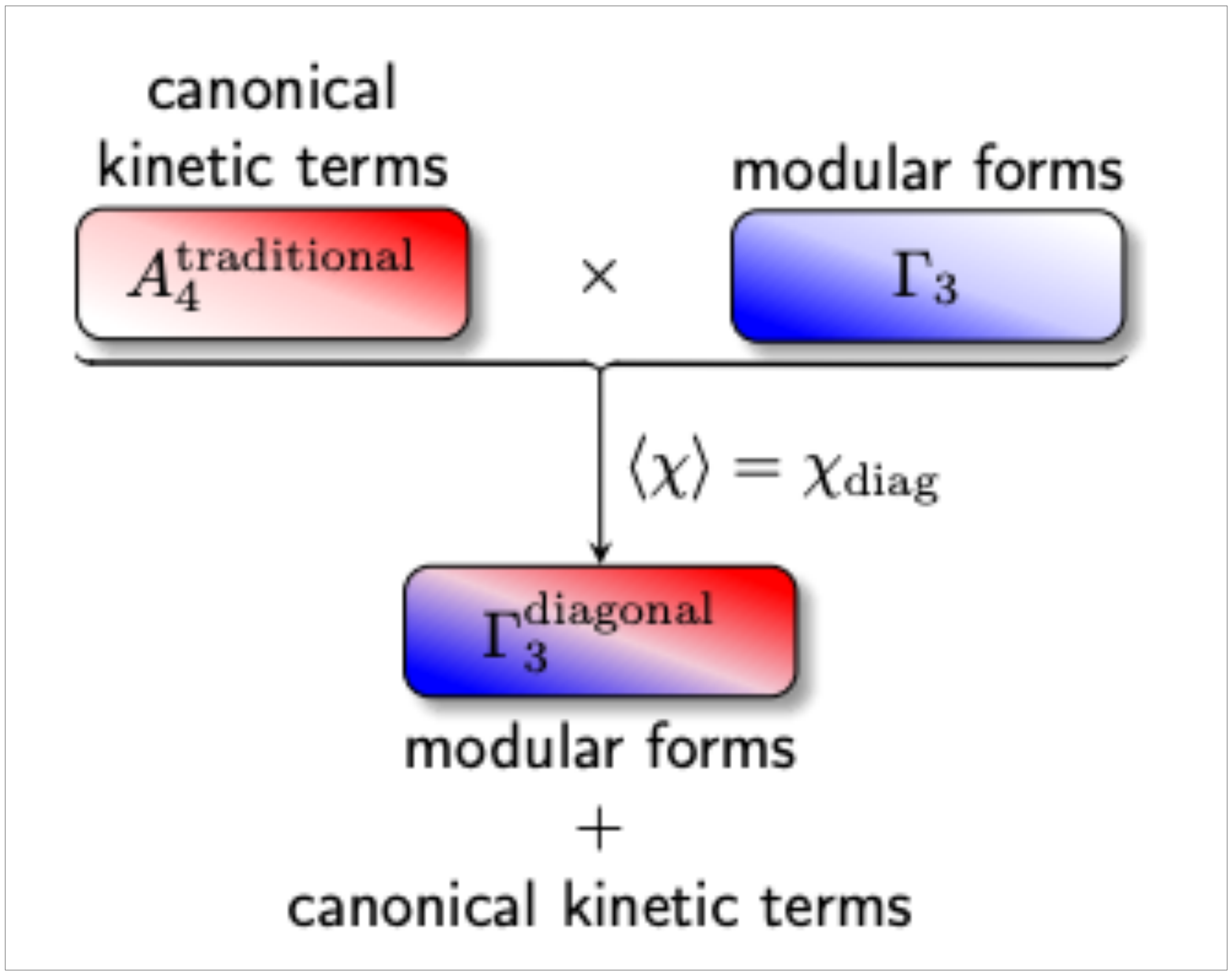
- Field Content:

	$(E_1^{\mathcal{C}}, E_2^{\mathcal{C}}, E_3^{\mathcal{C}})$	L	H_d	H_u	χ	φ	S_χ	S_φ	Y
$\mathrm{SU}(2)_\mathrm{L} \times \mathrm{U}(1)_Y$	$\mathbf{1}_1$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{-1/2}$	$\mathbf{2}_{1/2}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$
$A_4^{\text{traditional}}$	$(\mathbf{1}_0, \mathbf{1}_2, \mathbf{1}_1)$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$
$\mathbb{Z}_3^\chi$	0	0	0	1	1	0	1	0	0
$\mathbb{Z}_3^\varphi$	1	0	1	0	0	1	0	1	0
Γ_3	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{1}_0$	$\mathbf{3}$
k	$(k_{E_1}, k_{E_2}, k_{E_3})$	k_L	k_{H_d}	k_{H_u}	k_χ	k_φ	k_S	k_S	k_Y
modular weights	$(1, 1, 1)$	-1	0	0	0	0	0	0	2

Quasi-Eclectic Modular Symmetry

- Symmetry Breaking

MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)



- VEVs pattern resulting from vacuum alignment

$$\langle \chi_i^a \rangle = v_1 \, \mathbb{1}_3$$

$$\langle \varphi_i \rangle = v_2 \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

Quasi-Eclectic Modular Symmetry

MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)

- After Symmetry Breaking: diagonal Γ_3
  - Neutrino Sector: $\mathscr{W}_\nu = \frac{1}{\Lambda^2}\left[(H_u \cdot L)\ \chi\ (H_u \cdot L)\ Y\right]_{\mathbf{1}_0}$

$$\rightarrow \quad m_\nu = \frac{v_u^2 \varepsilon_1}{\sqrt{3}\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

  - Charged lepton sector:

$$\mathscr{W}_e = \frac{\widetilde{y}_e}{\Lambda} H_d (L \varphi E_1^{\mathcal{C}})_{\mathbf{1}_0} + \frac{\widetilde{y}_\tau}{\Lambda} H_d (L \varphi E_2^{\mathcal{C}})_{\mathbf{1}_0} + \frac{\widetilde{y}_\mu}{\Lambda} H_d (L \varphi E_3^{\mathcal{C}})_{\mathbf{1}_0}$$

$$\rightarrow \quad m_e = v_d \frac{v_2}{\Lambda} \operatorname{diag}\left(\widetilde{y}_e, \widetilde{y}_\tau, \widetilde{y}_\mu\right)$$

Quasi-Eclectic Modular Symmetry

MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)

- After Symmetry Breaking: diagonal Γ_3
  - Kähler Corrections:

$$K_L = L^\dagger L + \mathcal{O}(\varepsilon_1^2) + \mathcal{O}(\varepsilon_2^2)$$

  - Corrections involving only Y: absent to all orders, due to traditional A4 symmetry
  - Corrections involving flavon $\langle \varphi_i \rangle$: suppressed

$$\Delta K_L = \varepsilon_2^2 \left(C_1 \mathbb{1}_3 + \frac{2C_2}{3} \operatorname{diag}(2,-1,-1) + \frac{2C_3}{\sqrt{3}} \operatorname{diag}(0,1,-1) \right)$$

$$\varepsilon_2^2 = v_2^2/\Lambda^2 \gtrsim y_\tau^2$$

Quasi-Eclectic Modular Symmetry

- After Symmetry Breaking: diagonal Γ_3

MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)

  - Kähler Corrections:

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    - Corrections involving only Y: absent to all orders, due to traditional A4 symmetry
    - Corrections involving flavon $\langle \varphi_i \rangle$: suppressed

$$\varepsilon_2^2 = v_2^2/\Lambda^2 \gtrsim y_\tau^2$$

$$\Delta\theta_{12} \simeq C_i \left(\frac{\varepsilon_2}{0.03}\right)^2 \cdot \begin{cases} 0\,, & \text{if } i = 1\,, \\ -0.05\,, & \text{if } i = 2\,, \\ 0.01\,, & \text{if } i = 3\,. \end{cases}$$

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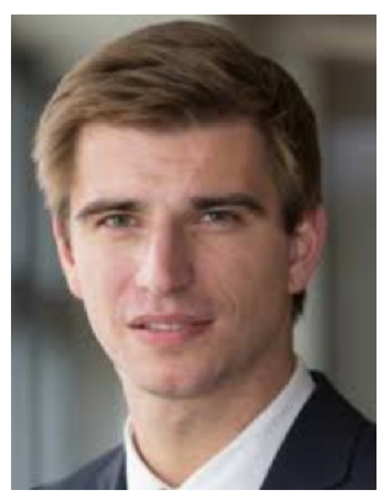

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Conclusion

- Modular Flavor Symmetries: Significant reduction of the number of parameters
- Kähler Corrections: worse compared to the case with traditional discrete flavor symmetries
- In quasi-eclectic setup: corrections can be greatly reduced to the level compatible with experiment uncertainty
- Ultimate goal: more economical scheme for realistic predictions, with highly suppressed/calculable Kähler corrections