

UV origin of modular flavor symmetries



Michael Ratz



June 27 2022



FLASY 22, Instituto Superior Técnico, Lisbon, Portugal

Based on:

Y. Almumin, M.-C. Chen, V. Knapp-Pérez, S. Ramos-Sánchez, M.R. & S. Shukla, JHEP 05 (2021) 078

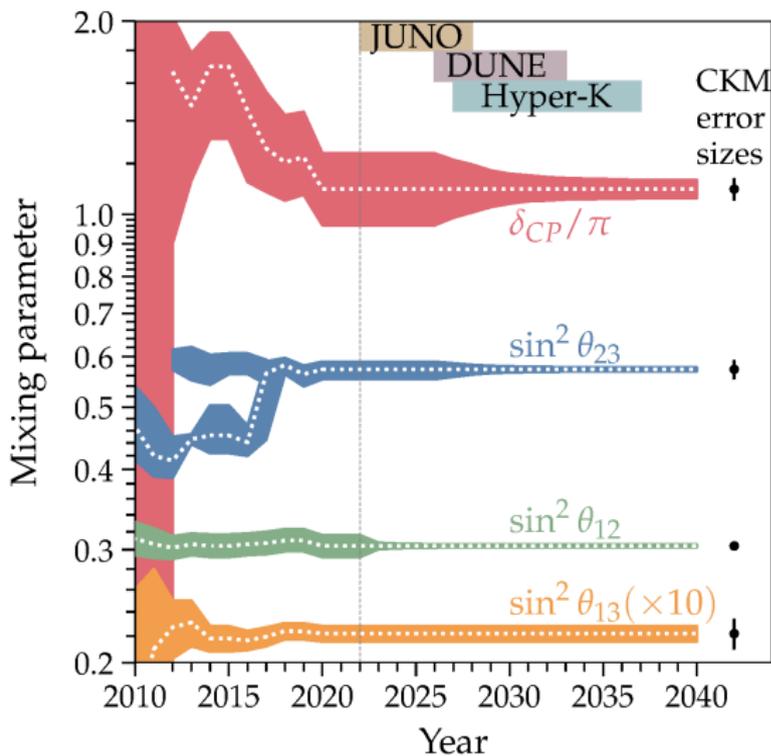
Outline

&

Disclaimers

Current and future precision of neutrino experiments

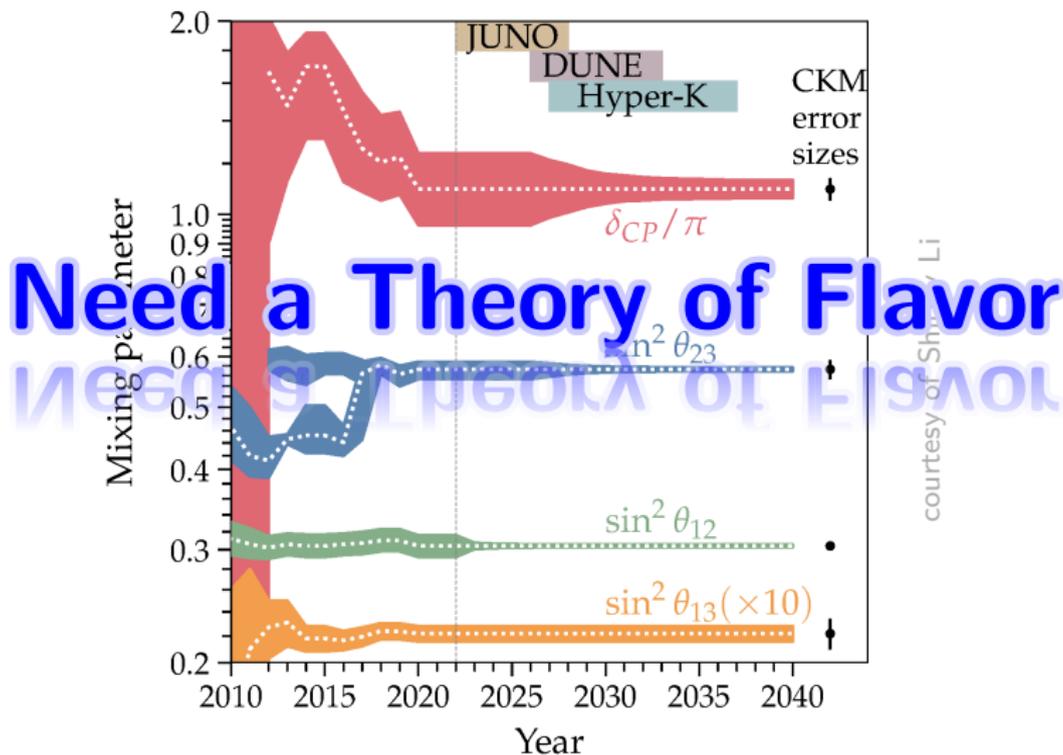
[Song, Li, Argüelles, Bustamante & Vincent \[2021\]](#)



courtesy of Shirley Li

Current and future precision of neutrino experiments

[Song, Li, Argüelles, Bustamante & Vincent \[2021\]](#)



Outline

👉 New game in town:

[🔗 Feruglio \[2017\]](#)
see talk by Ferruccio Feruglio

Modular Flavor Symmetries

MODULAR FLAVOR SYMMETRIES

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- 👉 Elegant and successful models have been constructed in the bottom-up approach

see talks by Mu-Chun Chen, Ferruccio Feruglio,
Serguei Petcov, Morimitsu Tanimoto, ...

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Apologies and disclaimers:

- 🙄 This talk will not have extensive references to model building activities that contribute to this exciting field, sorry!
- 👉 I will have to suppress many details which are not essential to get the big picture.

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- 👉 Elegant and successful models have been constructed in the bottom-up approach

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- 👉 Yet there are still some limitations and open questions:
 - No obvious (geometric or otherwise) interpretation of modular weights etc.
 - Assignment of modular weights etc. rather arbitrary

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This talk:

Derive modular flavor symmetries from tori/string theory to obtain additional insights

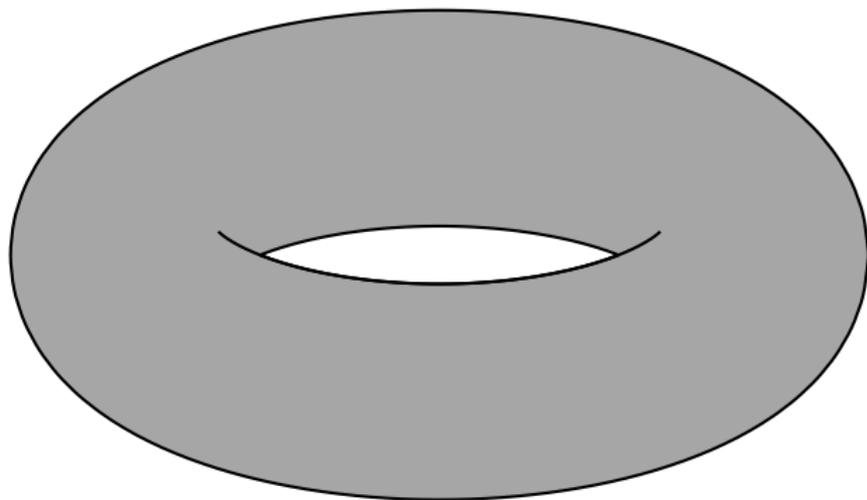
Modular
Modular

Flavor
Flavor

Symmetries
Symmetries

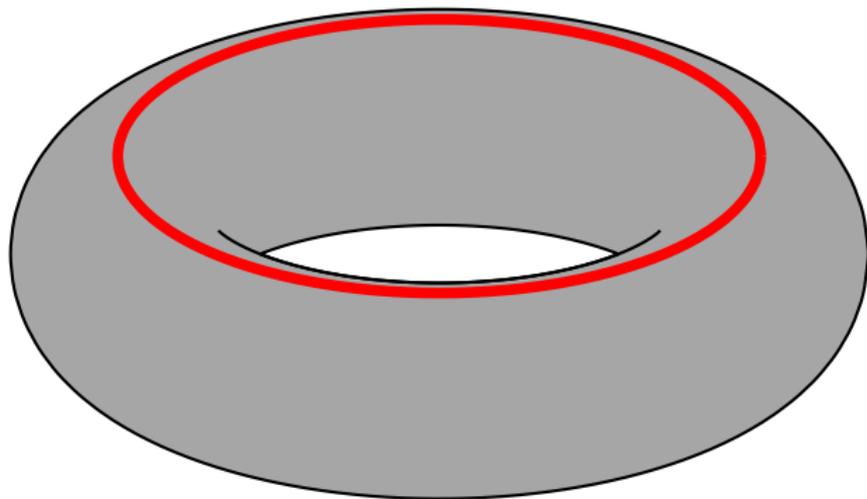
Tori

☞ Torus = donut



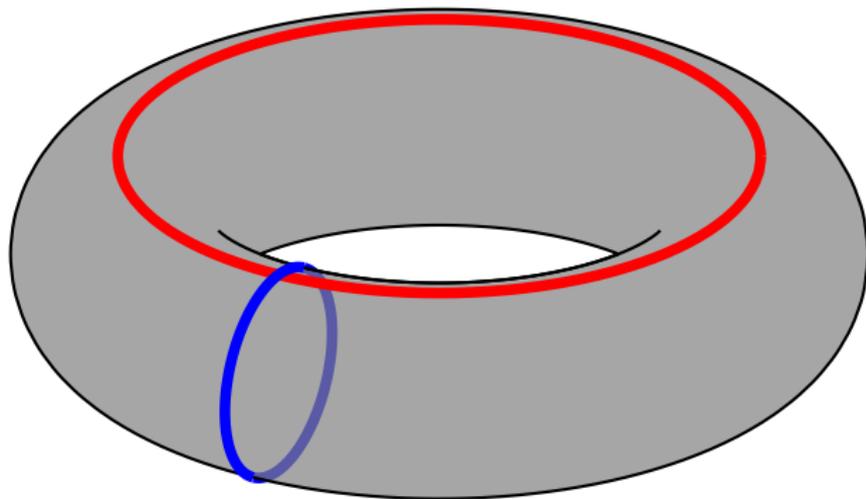
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- ☞ Torus = donut
- ☞ Two cycles



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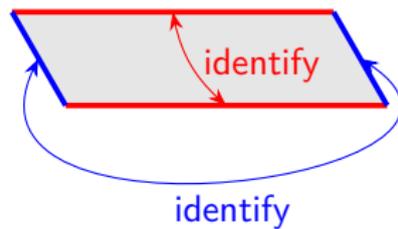


Tori



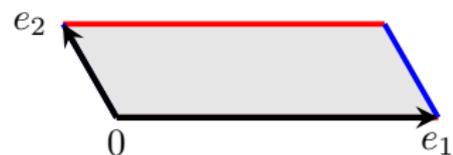
- Torus can be thought of as a parallelogram (which emerges by cutting the torus open along the red and blue cycles)

Tori



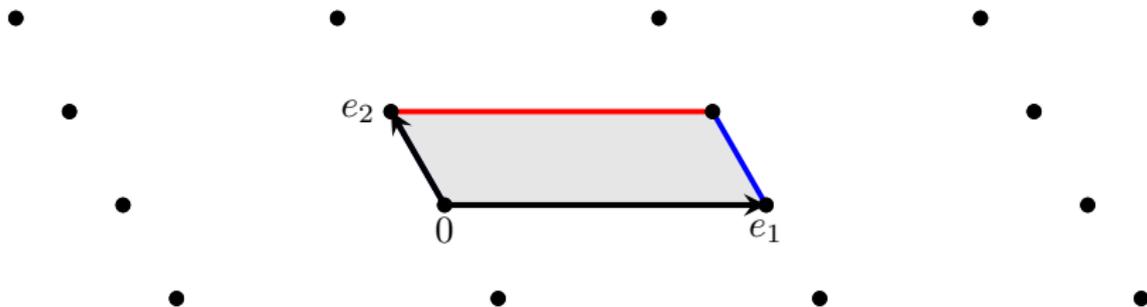
👉 Opposite edges get identified

Tori



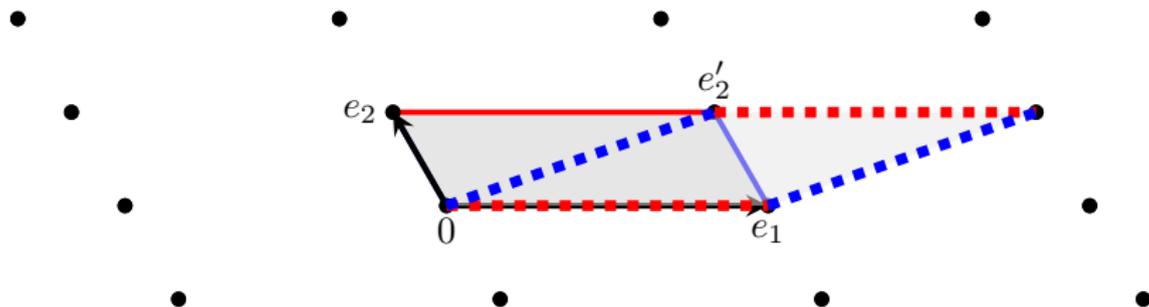
👉 Edges define basis vectors of a lattice

Tori



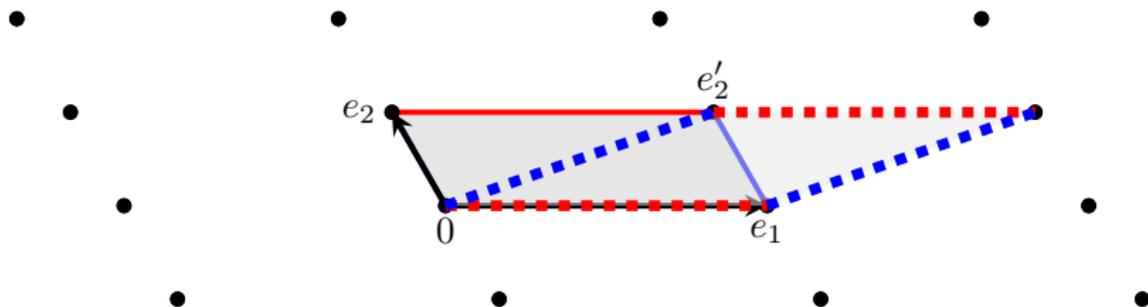
- ☞ Torus is $\mathbb{T}^2 = \mathbb{R}^2/\mathbb{Z}^2$: two points in the plane get identified if they differ by a lattice translation

Tori



👉 Fundamental domain is not unique

Tori

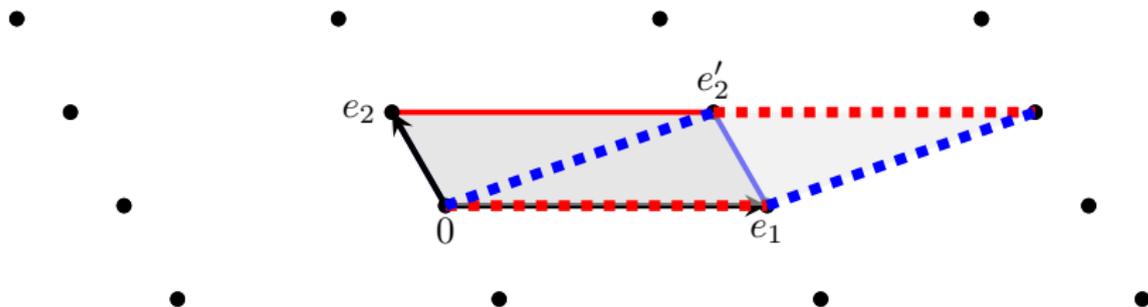


- 👉 Fundamental domain is not unique
- 👉 We can build linear combinations of the basis vectors

$$\begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \xrightarrow{\gamma} \begin{pmatrix} e'_2 \\ e'_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_2 \\ e_1 \end{pmatrix} =: \gamma \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$$

$$a, b, c, d \in \mathbb{Z}$$

Tori



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- 👉 Volume of fundamental domain stays the same $\Leftrightarrow \det \gamma = 1 \curvearrowright$
 $\gamma \in \text{SL}(2, \mathbb{Z})$ (there is a superfluous sign, so $\gamma \in \Gamma = \text{SL}(2, \mathbb{Z})/\mathbb{Z}_2$)

SL(2, \mathbb{Z})

👁 Two basic transformations

$$T : e_2 \xrightarrow{T} e'_2 = e_2 + e_1 \quad \curvearrowright \gamma = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} =: T$$

$$S : e_1 \xrightarrow{S} e'_1 = e_2 \quad \& \quad e_2 \mapsto e'_2 = -e_1 \quad \curvearrowright \gamma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} =: S$$

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☞ Complex structure modulus $\tau = e_2/e_1$ w/ $\text{Im } \tau > 0$

$$\tau \xrightarrow{T} \tau + 1 \quad \text{and} \quad \tau \xrightarrow{S} \frac{-1}{\tau}$$

SL(2, \mathbb{Z}) and modular flavor symmetries

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Modular flavor symmetries:

Identify finite groups with generators satisfying
 $\Gamma = \text{SL}(2, \mathbb{Z})/\mathbb{Z}_2$ relations

$$S^2 = (ST)^3 = \mathbb{1} \quad \& \quad \text{additional relations}$$

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Modular flavor symmetries

Finite subgroups $\Gamma_N := \Gamma/\Gamma(N)$ where

[Feruglio \[2017\]](#)

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \Gamma ; \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \right\}$$

level

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E.g. $\Gamma_3 \simeq A_4$ (symmetry of tetrahedron)

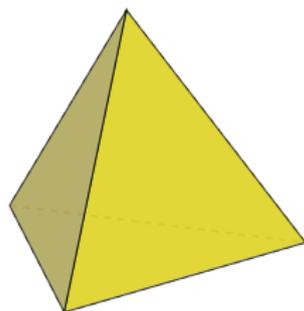
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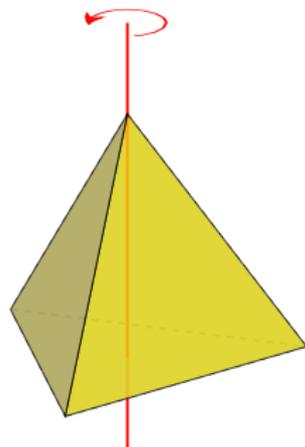
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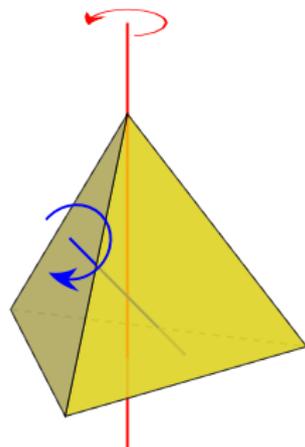
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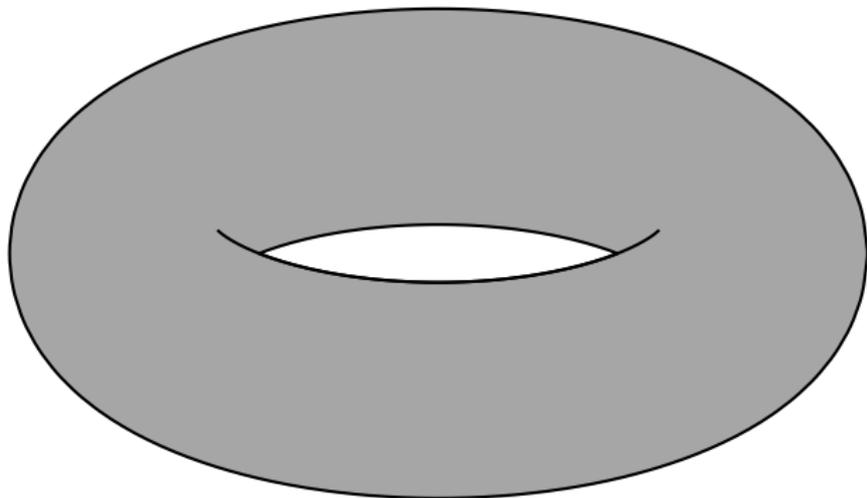
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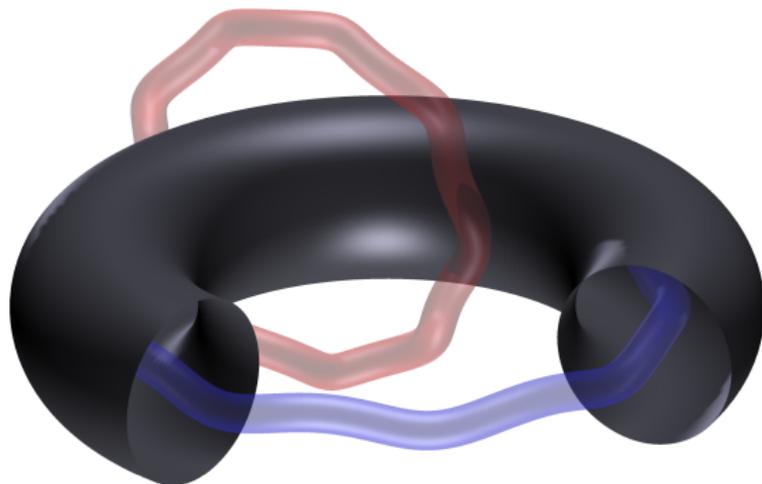
UV origin of modular flavor symmetries



This talk:

Derive modular flavor symmetries from explicit tori

UV origin of modular flavor symmetries



This talk:

Derive modular flavor symmetries from explicit tori ...
including those appearing in string theory

Metaplectic Flavor Symmetries

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from

Magnetized Tori

Magnetized Tori

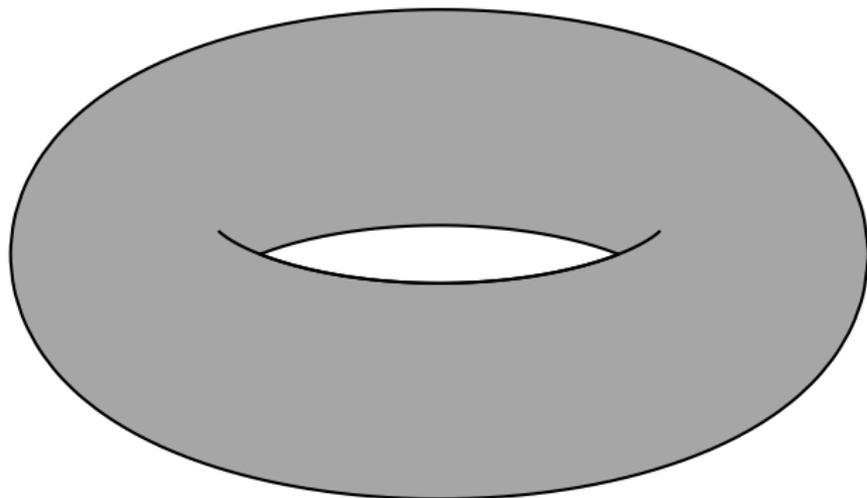
Magnetized tori

👉 6D $\mathcal{N} = 1$ SYM theory w/ $U(N)$ gauge symmetry

$$\mathcal{N} = 1 \text{ in 6D} \simeq \mathcal{N} = 2 \text{ in 4D}$$

Magnetized tori

- 6D $\mathcal{N} = 1$ SYM theory w/ $U(N)$ gauge symmetry
- Compactify two dimensions on a torus



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- 6D $\mathcal{N} = 1$ SYM theory w/ $U(N)$ gauge symmetry
- Compactify two dimensions on a torus
- Add magnetic flux:
 - breaks $\mathcal{N} = 2$ SUSY to $\mathcal{N} = 1$ SUSY in 4D

[Bachas \[1995\]](#)

Magnetized tori

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- Compactify two dimensions on a torus
- Add magnetic flux:
 - breaks $\mathcal{N} = 2$ SUSY to $\mathcal{N} = 1$ SUSY in 4D
 - gives rise to chiral zero-modes

[Bachas \[1995\]](#)

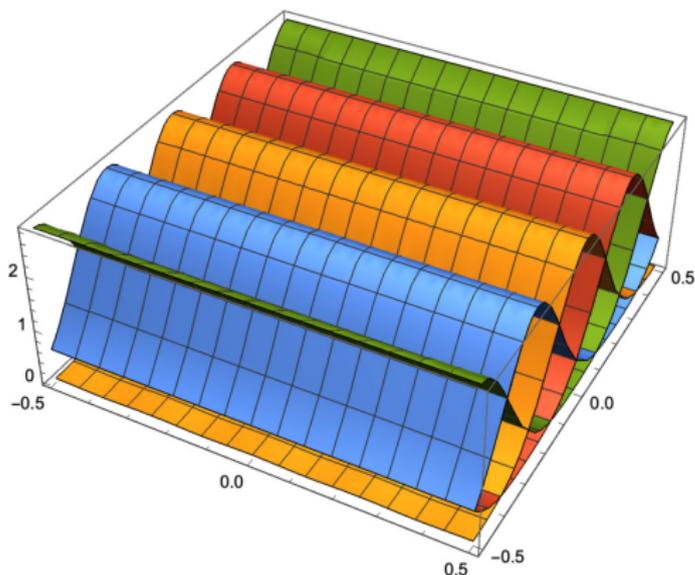
[Cremades, Ibáñez & Marchesano \[2004\]](#)

Chiral zero-modes on magnetized tori

[Cremades, Ibáñez & Marchesano \[2004\]](#)

☞ Torus with magnetic flux carries M chiral zero modes

$$\psi^{j,M}(z, \tau, \zeta) = \mathcal{N} e^{\pi i M (z + \zeta) \frac{\text{Im}(z + \zeta)}{\text{Im} \tau}} \vartheta \left[\begin{matrix} j \\ M \\ 0 \end{matrix} \right] (M(z + \zeta), M\tau)$$



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flux parameter \curvearrowright # of zero modes

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Wilson line

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Jacobi ϑ -function

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- ↳ Normalization \curvearrowright Kähler metrics and modular weights

$$\mathcal{N} = \left(\frac{2M \text{Im} \tau}{\mathcal{A}^2} \right)^{1/4} \quad \curvearrowright \quad k_\psi = -1/2$$

area of torus

$$\mathcal{A} = (2\pi R)^2 \text{Im} \tau$$

Flux

- Flux in $U(N)$ gauge theory w/ $N = N_a + N_b + N_c$

$$F_{z\bar{z}} = \frac{\pi i}{\text{Im } \tau} \begin{pmatrix} \frac{m_a}{N_a} \mathbb{1}_{N_a \times N_a} & 0 & 0 \\ 0 & \frac{m_b}{N_b} \mathbb{1}_{N_b \times N_b} & 0 \\ 0 & 0 & \frac{m_c}{N_c} \mathbb{1}_{N_c \times N_c} \end{pmatrix}$$

Assumption: $s_\alpha = \frac{m_\alpha}{N_\alpha} \in \mathbb{Z}$

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- $|\mathcal{I}_{\alpha\beta}|$ counts the number of $(\mathbf{N}_\alpha, \overline{\mathbf{N}}_\beta)$ or $(\overline{\mathbf{N}}_\alpha, \mathbf{N}_\beta)$ flavors

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- $|\mathcal{I}_{\alpha\beta}|$ counts the number of $(N_\alpha, \overline{N_\beta})$ or $(\overline{N_\alpha}, N_\beta)$ flavors
- “Sum rule”: $\mathcal{I}_{ab} + \mathcal{I}_{bc} + \mathcal{I}_{ca} = 0$

Yukawa couplings between chiral zero modes

Yukawa couplings are given by overlap integrals

[Cremades, Ibáñez & Marchesano \[2004\]](#)

$$Y_{ijk}(\tau) = g \sigma_{abc} \int_{\mathbb{T}^2} d^2z \psi^{i, \mathcal{I}_{ab}}(z, \tau, 0) \psi^{j, \mathcal{I}_{ca}}(z, \tau, 0) (\psi^{k, \mathcal{I}_{cb}}(z, \tau, 0))^*$$

gauge coupling

sign

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for simplicity we set the Wilson lines to zero

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- Yukawa couplings be expressed as sums of ϑ -functions

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$$\mathcal{N}_{abc} = g \sigma_{abc} \left(\frac{2 \operatorname{Im} \tau}{\mathcal{A}^2} \right)^{1/4} \left| \frac{\mathcal{I}_{ab} \mathcal{I}_{ca}}{\mathcal{I}_{bc}} \right|^{1/4}$$

Yukawa couplings between chiral zero modes

Yukawa couplings are given by overlap integrals

[Cremades, Ibáñez & Marchesano \[2004\]](#)

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- Obviously no sum for $\mathcal{I}_{bc} = 1$ (but there might still be a sum for $\gcd(\mathcal{I}_{ab}, \mathcal{I}_{ca}, \mathcal{I}_{bc}) = 1$)

Yukawa couplings for general flux parameters

[Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, MR & Shukla \[2021\]](#)

[▶ details](#)

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normalization \rightarrow later

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Euler ϕ -function

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- At most $\lambda = \text{lcm}(|\mathcal{I}_{ab}|, |\mathcal{I}_{ca}|, |\mathcal{I}_{bc}|)$ independent couplings, e.g. a model with $(\mathcal{I}_{ab}, \mathcal{I}_{ca}, \mathcal{I}_{bc}) = (1, 2, -3)$ has as many independent couplings as a model with $(\mathcal{I}_{ab}, \mathcal{I}_{ca}, \mathcal{I}_{bc}) = (3, 3, -6)$

Yukawa couplings for general flux parameters

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bottom-line:

Magnetized tori with $\lambda = \text{lcm}(\# \text{ of flavors})$ exhibit a $\tilde{\Gamma}_{2\lambda}$ modular (metaplectic) flavor symmetry

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- 😬 However, not true for odd flux parameters M

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- 👉 Alternatively one can use Scherk-Schwarz phases to define a consistent transformation law for odd M

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Connection to bottom–up model building

- ☞ Metaplectic flavor symmetries have been studied in bottom–up model building

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- ☞ Realistic fits of the neutrino masses have been achieved in the bottom–up approach. . .
- ... but seemingly at the expense of introducing representations and fixing their modular weights at will

Physical vs. holomorphic couplings

cf. [Kaplunovsky & Louis \[1993\]](#)

$$Y_{ijk}(\tau) = e^{\widehat{K}/2} \frac{\mathcal{Y}_{ijk}(\tau)}{(K_{i\bar{i}} K_{j\bar{j}} K_{k\bar{k}})^{1/2}}$$

holomorphic

physical

Kähler potential
of moduli

Kähler metrics
of matter fields

Physical vs. holomorphic couplings

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“area”

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label distinct Yukawas

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➤ Representation matrices:
$$\begin{cases} \rho_{\lambda}(\widetilde{S})_{\widehat{\alpha}\widehat{\beta}} = -\frac{e^{i\pi/4}}{\sqrt{\lambda}} \exp\left(\frac{2\pi i \widehat{\alpha} \widehat{\beta}}{\lambda}\right) \\ \rho_{\lambda}(\widetilde{T})_{\widehat{\alpha}\widehat{\beta}} = \exp\left(\frac{i\pi \widehat{\alpha}^2}{\lambda}\right) \delta_{\widehat{\alpha}\widehat{\beta}} \end{cases}$$

Modular weights

☞ Kähler metrics of zero modes

$$K_{i\bar{i}} \propto \frac{1}{(\text{Im } \tau)^{-1/2}}$$

e.g. from

$$\frac{e^{\widehat{K}/2}}{(K_{i\bar{i}} K_{j\bar{j}} K_{k\bar{k}})^{1/2}} \stackrel{!}{\propto} (\text{Im } \tau)^{1/4} \quad \text{where } e^{\widehat{K}/2} \propto (\text{Im } \tau)^{-1/2}$$

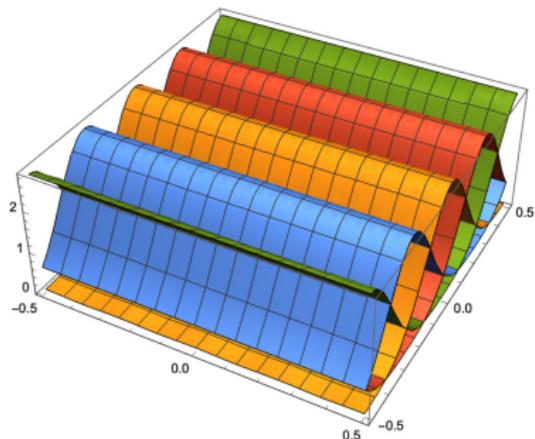
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Note: the modular weight of a brane field is 0 whereas the modular weight of a bulk field is -1 so the chiral zero modes are something in between



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- ➡ Modular weight of superpotential is -1 whereas in bottom-up models it is often set to 0

Kähler transformations

- ☞ Textbook Kähler transformations (Planck units!)

cf. [Wess & Bagger \[1992\]](#)

$$\mathcal{W}(\Phi) \mapsto e^{-\mathcal{F}(\Phi)} \mathcal{W}(\Phi)$$

$$K(\Phi, \bar{\Phi}) \mapsto K(\Phi, \bar{\Phi}) + \mathcal{F}(\Phi) + \overline{\mathcal{F}(\Phi)}$$

4D superfields

arbitrary holomorphic

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$$\widehat{K} \supset -\ln(\mathcal{U} + \bar{\mathcal{U}}) = -\ln(-i\tau + i\bar{\tau})$$

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- ☞ Under a modular transformation

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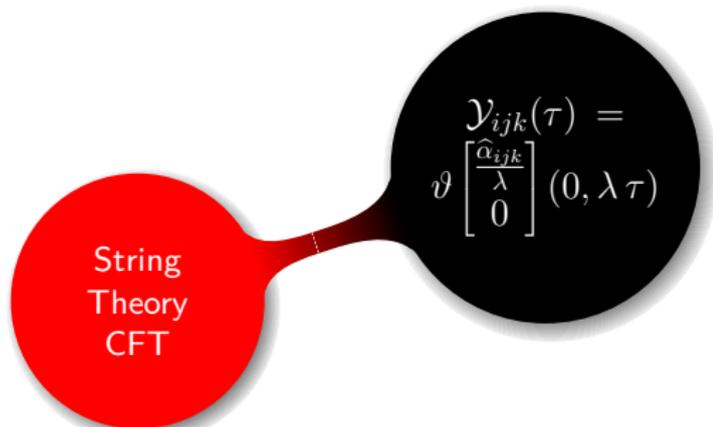
- Supergravity Kähler function is invariant

$$G(\Phi, \bar{\Phi}) = K(\Phi, \bar{\Phi}) + \ln |\mathcal{W}(\Phi)|^2$$

Three ways to compute Yukawa couplings

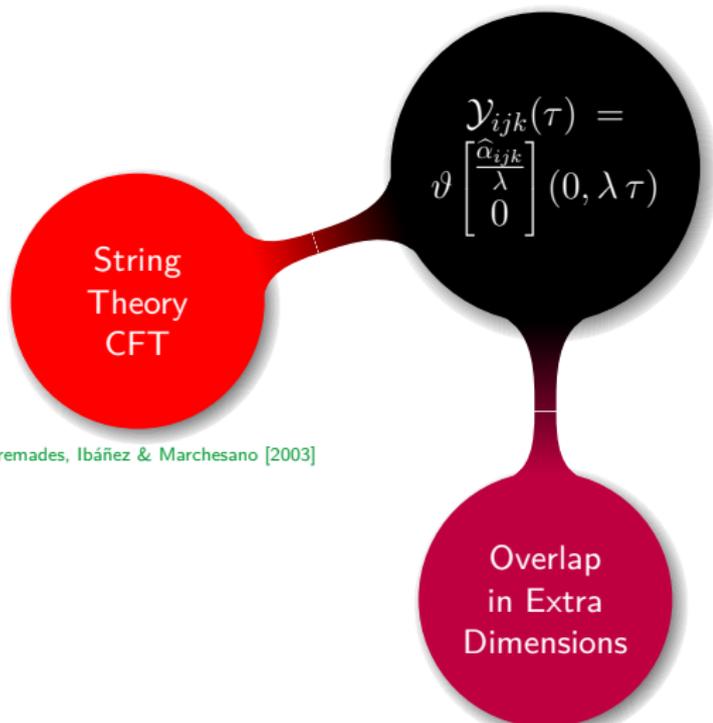
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[Cremades, Ibáñez & Marchesano \[2003\]](#)

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String
Theory
CFT

Bottom-up
Modular
Flavor
Symmetries

[Cremades, Ibáñez & Marchesano \[2003\]](#)

[Liu, Yao, Qu & Ding \[2020\]](#)
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Overlap
in Extra
Dimensions

[Cremades, Ibáñez & Marchesano \[2004\]](#)

Role of supersymmetry in modular flavor symmetries

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Role of supersymmetry in modular flavor symmetries

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- ☞ In a nonsupersymmetric model the scalar is expected to pick up a large mass and not to have a profile that resembles a zero mode, so at first sight we should not be allowed to use the above Yukawa couplings
- ☞ Obviously the SM Higgs is almost a zero mode, i.e. its mass is much smaller than the compactification scale. So maybe it is OK to use the modular form Yukawa couplings after all?
- ☞ Incidentally some scalars are immune to mass corrections in nonsupersymmetric flux compactifications [☞ Buchmüller, Dierigl, Dudas & Schweizer \[2017\]](#)
[☞ Ghilencea & Lee \[2017\]](#); [☞ Buchmüller, Dierigl & Dudas \[2018\]](#); [☞ Hirose & Maru \[2019\]](#)

Modular Flavor Symmetries

Modular Flavor Symmetries

from

Strings

Strings

Eclectic flavor symmetries in heterotic orbifolds

see talk by Andreas Trautner

[Baur, Nilles, Trautner & Vaudrevange \[2019\]](#)

[Nilles, Ramos-Sánchez & Vaudrevange \[2021\]](#); [Baur, Kade, Nilles, Ramos-Sanchez & Vaudrevange \[2021\]](#)

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- ☞ Discrete flavor symmetries are identified as the outer automorphisms of the Narain space group
- ☞ These symmetries include:
 - traditional flavor symmetries
 - modular flavor symmetries
 - R symmetries (including non-Abelian discrete R symmetries)
 - \mathcal{CP} symmetries and \mathcal{CP} -like transformations

Lessons from string models

(Yukawa) couplings *are* modular forms

e.g.  Quevedo [1996]

This is nothing but one of the $SL(2, \mathbf{Z})_{T,U}$ transformation for toroidal orbifold compactifications ($a = b = d = 1, c = 0$ in eq. (10)). Therefore the only conditions these symmetries impose on W is that it should transform as a modular form of a given weight ($W \rightarrow (cT + d)^{-3} W$ for the simplest toroidal orbifolds with T the overall size of the compactification space)[36]. In fact, explicit calculations for specific orbifold models show that

$$W_{tree}(T, Q^I) = \chi_{IJK}(T) Q^I Q^J Q^K + \dots \quad (19)$$

with $\chi(T)$ a particular modular form of $SL(2, \mathbf{Z})$ or any other duality group and the ellipsis represent higher powers of Q , exponentially suppressed. The identification of $\chi(T)$ with modular forms was a highly nontrivial check of the explicit orbifold calculations which were performed in refs. [37] without any relation (nor knowledge) of the underlying duality symmetry $SL(2, \mathbf{Z})$. This kind of symmetry puts also strong constraints to the higher order, nonrenormalizable, corrections to W , since each matter field Q transforms in a particular way under that symmetry ($Q \rightarrow (cT + d)^n Q$ with n the modular weight of Q). There are also other discrete symmetries, as those defined by the point group \mathcal{P} and space group \mathcal{S} of an orbifold which have to be respected by the superpotential W . These ‘selection rules’ are very important to find vanishing couplings and uncover flat directions which can be used to break the original gauge symmetries and construct quasi-realistic models.

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[Ferrara, Lüst, Shapere & Theisen \[1989\]](#); [Chun, Mas, Lauer & Nilles \[1989\]](#)

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- ☞ Observation: Yukawa couplings have integer modular weights in closed string models and can have half-integer modular weights in open string models
- ☞ Negative modular weights of the superpotential avoid the most offending corrections to the Kähler potential discussion by [Feruglio et al.](#)

“Magical” absence of Kähler corrections?

- ➡ Many (bottom–up) models assume canonical kinetic terms

“Magical” absence of Kähler corrections?

- Many (bottom–up) models assume canonical kinetic terms
- However, generically the kinetic terms can have higher order contributions

[Leurer, Nir & Seiberg \[1994\]](#); [Dudas, Pokorski & Savoy \[1995\]](#)

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 - the ability to forbid soft terms (“sequestering”) but this seems to be impossible
 - cf. ☞ Kachru, McAllister & Sundrum [2007]

Recall that the soft terms come from operators of the form

$$\int d^4\theta \frac{X^\dagger X}{\Lambda^2} \Phi^\dagger \Phi \quad \text{etc.}$$

messenger scale

$$F_X \neq 0$$

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 - the ability to forbid soft terms (“sequestering”) but this seems to be impossible
 - cf. ☞ Kachru, McAllister & Sundrum [2007]
- ☞ The higher order Kähler terms *can* be computed in string models
 - e.g. ☞ Antoniadis, Gava, Narain & Taylor [1994]

Summary

Summary

&

Outlook

Outlook

Summary (Lessons from the top-down)

- ➡ Modular flavor symmetries can be derived from explicit tori

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Three ways to compute Yukawa couplings

[Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, MR & Shukla \[2021\]](#)

$$y_{ijk}(\tau) = \vartheta \begin{bmatrix} \hat{\alpha}_{ijk} \\ \lambda \\ 0 \end{bmatrix} (0, \lambda \tau)$$

String
Theory
CFT

Bottom-up
Modular
Flavor
Symmetries

Overlap
in Extra
Dimensions

[Cremades, Ibáñez & Marchesano \[2003\]](#)

[Liu, Yao, Qu & Ding \[2020\]](#)
[Ding, Feruglio & Liu \[2021\]](#)

[Cremades, Ibáñez & Marchesano \[2004\]](#)

Summary (Lessons from the top–down)

- Modular flavor symmetries can be derived from explicit tori
- Metaplectic flavor symmetries from magnetized tori are an explicit example in which bottom–up and top–down analyses merge (and so far the only example of this kind)
- Supersymmetry arguably less essential than usually assumed

Outlook



- Detailed analysis of explicit string models
- Nonsupersymmetric modular flavor symmetries
- Other new ideas

Muito obrigado!

Backup slides

Backup slides

Metaplectic

Μεταπλεκτική

Flavor

Είδη

Symmetries

Συμμετρίες

(Details)

(Ρεφινιέρ)

Yukawa couplings between chiral zero modes

- Yukawa couplings are given by overlap integrals

$$Y_{ijk}(\tilde{\zeta}, \tau) = g \sigma_{abc} \int_{\mathbb{T}^2} d^2 z \psi^{i, \mathcal{I}_{ab}}(z, \tau, \zeta_{ab}) \psi^{j, \mathcal{I}_{ca}}(z, \tau, \zeta_{ca}) (\psi^{k, \mathcal{I}_{cb}}(z, \tau, \zeta_{cb}))^*$$

gauge coupling

sign

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Wilson lines

Yukawa couplings

[Cremades, Ibáñez & Marchesano \[2004\]](#)

Yukawa couplings be expressed as sums of ϑ -functions

$$Y_{ijk}(\tilde{\zeta}, \tau) = \mathcal{N}_{abc} e^{\frac{H(\tilde{\zeta}, \tau)}{2}} \sum_{m \in \mathbb{Z}_{\mathcal{I}_{bc}}} \delta_{k, i+j+\mathcal{I}_{ab} m} \cdot \vartheta \left[\begin{array}{c} \mathcal{I}_{ca} i - \mathcal{I}_{ab} j + \mathcal{I}_{ab} \mathcal{I}_{ca} m \\ -\mathcal{I}_{ab} \mathcal{I}_{bc} \mathcal{I}_{ca} \\ 0 \end{array} \right] (\tilde{\zeta}, \tau | \mathcal{I}_{ab} \mathcal{I}_{bc} \mathcal{I}_{ca} |)$$

$$\mathcal{N}_{abc} = g \sigma_{abc} \left(\frac{2 \operatorname{Im} \tau}{\mathcal{A}^2} \right)^{1/4} \left| \frac{\mathcal{I}_{ab} \mathcal{I}_{ca}}{\mathcal{I}_{bc}} \right|^{1/4}$$

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“collective” Wilson line

$$\tilde{\zeta} := -\mathcal{I}_{ab} \mathcal{I}_{ca} (\zeta_{ca} - \zeta_{ab}) = d^{\alpha\beta\gamma} s_{\alpha} \zeta_{\alpha} \mathcal{I}_{\beta\gamma} \\ w/ d^{\alpha\beta\gamma} = \begin{cases} 1 & \text{if } \{\alpha, \beta, \gamma\} \text{ is even perm. of } \{1, 2, 3\} \\ 0 & \text{otherwise} \end{cases}$$

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$$\frac{H(\tilde{\zeta}, \tau)}{2} := \frac{\pi i}{\text{Im } \tau} (\mathcal{I}_{ab} \zeta_{ab} \text{Im } \zeta_{ab} + \mathcal{I}_{bc} \zeta_{bc} \text{Im } \zeta_{bc} + \mathcal{I}_{ca} \zeta_{ca} \text{Im } \zeta_{ca}) \\ = \frac{\pi i}{\text{Im } \tau} |\mathcal{I}_{ab} \mathcal{I}_{bc} \mathcal{I}_{ca}|^{-1} \tilde{\zeta} \text{Im } \tilde{\zeta}$$

Yukawa couplings

[Cremades, Ibáñez & Marchesano \[2004\]](#)

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[Cremades, Ibáñez & Marchesano \[2003\]](#)

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- There might still be a sum for $\gcd(\mathcal{I}_{ab}, \mathcal{I}_{ca}, \mathcal{I}_{bc}) = 1$

[Alumin, Chen, Knapp-Pérez, Ramos-Sánchez, MR & Shukla \[2021\]](#)

Linear congruences and Euler's Theorem

☞ We need to find an integer such that

▶ back

$$\mathcal{I}_{ab} m + i + j = k \pmod{\mathcal{I}_{bc}}$$

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☞ Divide by d

$$|\mathcal{I}'_{ab}| m = m' \pmod{|\mathcal{I}'_{bc}|} \quad \text{where } m' = (k - i - j)/d \in \mathbb{Z}$$

$$\mathcal{I}'_{\alpha\beta} = \mathcal{I}_{\alpha\beta}/d$$

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$$|\mathcal{I}'_{ab}| m = m' \pmod{|\mathcal{I}'_{bc}|} \quad \text{where } m' = (k - i - j)/d \in \mathbb{Z}$$

☞ Euler's Theorem provides us with a closed-form solution

$$m_0 = (\mathcal{I}'_{ab})^{\phi(|\mathcal{I}'_{bc}|)-1} \frac{k - i - j}{d} \pmod{|\mathcal{I}'_{bc}|}$$

Euler ϕ -function

Yukawa couplings for general flux parameters

[Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, MR & Shukla \[2021\]](#)

[back](#)

- Using Euler's Theorem one can reduce the Yukawa coupling to a single ϑ -function

$$Y_{ijk}(\tilde{\zeta}, \tau) = \mathcal{N}_{abc} e^{\frac{H(\tilde{\zeta}, \tau)}{2}} \Delta_{i+j,k}^{(d)} \cdot \vartheta \left[\frac{\mathcal{I}'_{ca} i - \mathcal{I}'_{ab} j + \mathcal{I}'_{ca} (\mathcal{I}'_{ab}) \phi(|\mathcal{I}'_{bc}|) (k-i-j)}{\lambda} \middle| \begin{matrix} \tilde{\zeta} \\ d \end{matrix}, \lambda \tau \right]$$

Euler ϕ -function

$$\lambda = \text{lcm}(|\mathcal{I}_{ab}|, |\mathcal{I}_{ca}|, |\mathcal{I}_{bc}|)$$

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$\mathcal{I}'_{ij} = \mathcal{I}_{ij}/d$

Yukawa couplings for general flux parameters

[Almumin, Chen, Knapp-Pérez, Ramos-Sánchez, MR & Shukla \[2021\]](#)

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- At most $\text{lcm}(|\mathcal{I}_{ab}|, |\mathcal{I}_{ca}|, |\mathcal{I}_{bc}|)$ independent couplings, e.g. a model with $(\mathcal{I}_{ab}, \mathcal{I}_{ca}, \mathcal{I}_{bc}) = (1, 2, -3)$ has as many independent couplings as a model with $(\mathcal{I}_{ab}, \mathcal{I}_{ca}, \mathcal{I}_{bc}) = (3, 3, -6)$

Yukawa couplings for general flux parameters

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bottom-line:

Magnetized tori with $\lambda = \text{lcm}(\# \text{ of flavors})$ exhibit a $\tilde{\Gamma}_{2\lambda}$ modular flavor symmetry

Metaplectic transformations

cf. [Liu, Yao, Qu & Ding \[2020\]](#)

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👉 Double cover of $SL(2, \mathbb{Z})$: the so-called metaplectic group $\tilde{\Gamma} = Mp(2, \mathbb{Z})$

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Generators \tilde{S} and \tilde{T} of $\tilde{\Gamma}$ satisfy the presentation

$$\tilde{S}^8 = (\tilde{S}\tilde{T})^3 = \mathbb{1} \quad \text{and} \quad \tilde{S}^2\tilde{T} = \tilde{T}\tilde{S}^2$$

Our choice

$$\tilde{S} = (S, -\sqrt{-\tau}) \quad \text{and} \quad \tilde{T} = (T, +1), \quad S, T \in \Gamma$$

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Multiplication rule

$$(\gamma_1, \varphi(\gamma_1, \tau)) (\gamma_2, \varphi(\gamma_2, \tau)) = (\gamma_1\gamma_2, \varphi(\gamma_1, \gamma_2\tau)\varphi(\gamma_2, \tau))$$

Modular vs. metaplectic flavor symmetries

- ☞ The zero modes have halfinteger modular weights

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object	$\psi^{j,M}$	$\phi^{j,M}$	$\Omega^{j,M}$	Y_{ijk}	\mathcal{W}
modular weight k	$1/2$	$-1/2$	0	$1/2$	-1

internal

4D

$$\Omega^{j,M} = \phi^{j,M}(x^\mu) \otimes \psi^{j,M}(z, \tau)$$

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- ☞ One has to be careful with signs in modular transformations: metaplectic symmetries

Transformation laws for 4D superfields (for odd M)

$$\begin{aligned}
 \psi^{j,M}(z, \tau, 0) &\xrightarrow{S} \frac{e^{i\frac{\pi}{4}}}{\sqrt{M}} \left(-\frac{\tau}{|\tau|}\right)^{1/2} \sum_{k=0}^{M-1} e^{2\pi i j k / M} \psi^{k,M}(z, \tau, 0) \\
 &= - \left(-\frac{\tau}{|\tau|}\right)^{1/2} \left[\rho(S)_M^\psi\right]_{jk} \psi^{k,M}(z, \tau, 0) \\
 \psi^{j,M}(z, \tau, 0) &\xrightarrow{T} e^{i\pi M \frac{\text{Im } z}{2 \text{Im } \tau}} e^{i\pi j(j/M+1)} \psi^{j,M}(z - 1/2, \tau, 0) \\
 &= e^{i\pi M \frac{\text{Im } z}{2 \text{Im } \tau}} \left[\rho(T)_M^\psi\right]_{jk} \psi^{k,M}(z - 1/2, \tau, 0)
 \end{aligned}$$

Representation matrices of generators

$$\begin{aligned}
 \left[\rho(S)_M^\psi\right]_{jk} &= -\frac{e^{i\pi/4}}{\sqrt{M}} \exp\left(\frac{2\pi i j k}{M}\right) \\
 \left[\rho(T)_M^\psi\right]_{jk} &= \exp\left[i\pi j \left(\frac{j}{M} + 1\right)\right] \delta_{jk}
 \end{aligned}$$

Transformation laws for Yukawa couplings

$$\mathcal{Y}_{\hat{\alpha}}(\tau) \xrightarrow{\tilde{\gamma}} \mathcal{Y}_{\hat{\alpha}}(\tilde{\gamma}\tau) = \pm(c\tau + d)^{1/2} \rho_{\lambda}(\tilde{\gamma})_{\hat{\alpha}\hat{\beta}} \mathcal{Y}_{\hat{\beta}}(\tau)$$

☞ Representation matrices of generators

$$\rho_{\lambda}(\tilde{S})_{\hat{\alpha}\hat{\beta}} = -\frac{e^{i\pi/4}}{\sqrt{\lambda}} \exp\left(\frac{2\pi i \hat{\alpha} \hat{\beta}}{\lambda}\right)$$

$$\rho_{\lambda}(\tilde{T})_{\hat{\alpha}\hat{\beta}} = \exp\left(\frac{i\pi \hat{\alpha}^2}{\lambda}\right) \delta_{\hat{\alpha}\hat{\beta}}$$

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