

Revisiting puzzles in lifetimes of singly charmed hadrons



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Lifetimes of singly charmed hadrons, in coll. with J. Gratex and Ivan Nisandzic (RBI, Zagreb),
2204.11935 [hep-ph] to appear in JHEP



A BIT OF HISTORY

First flavour anomalies were connected with lifetimes :

□ 80' - $\tau(D^+)/\tau(D_0) \sim 2.1$



IN 80's and
IN THIS TALK

□ 85' - $\tau(D_s)/\tau(D_0) \sim 1.5$ (when D_s was called F 😊)



+/- IN 80's and
IN THIS TALK

□ 90' - $\tau(\Lambda_b)/\tau(B) \sim 0.7-0.8$



NICE EXAMPLE OF
AN „ANOMALY“

□ 2000 – WA large → influence on V_{ub} inclusive



/ nonperturbative?

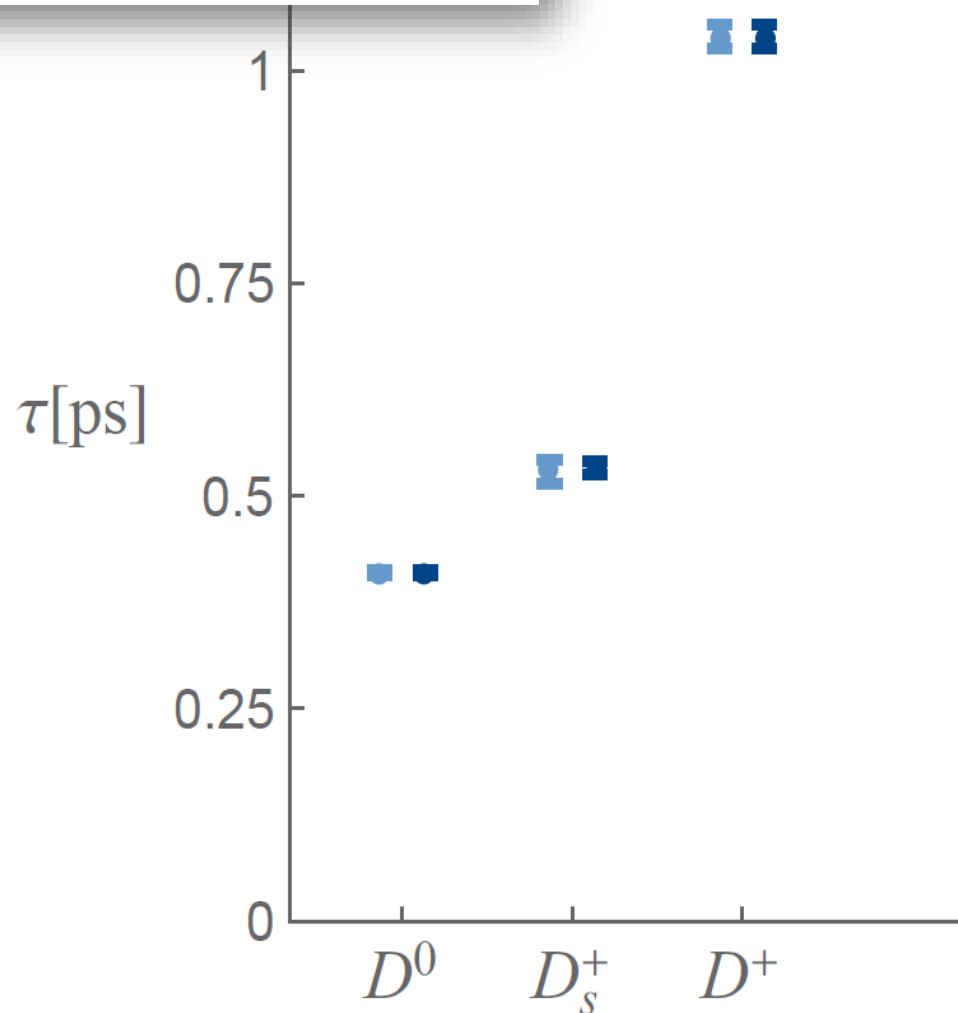
□ 2020 – $\tau(\Omega_c)$ – 3-4 times bigger then previously measured



IN THIS TALK

EXPERIMENTAL SITUATION

$D^0(c\bar{u})$, $D^+(c\bar{d})$, $D_s(c\bar{s})$



■ Mesons 2018

■ Mesons 2021

- practically unchanged lifetime pattern since 1980's

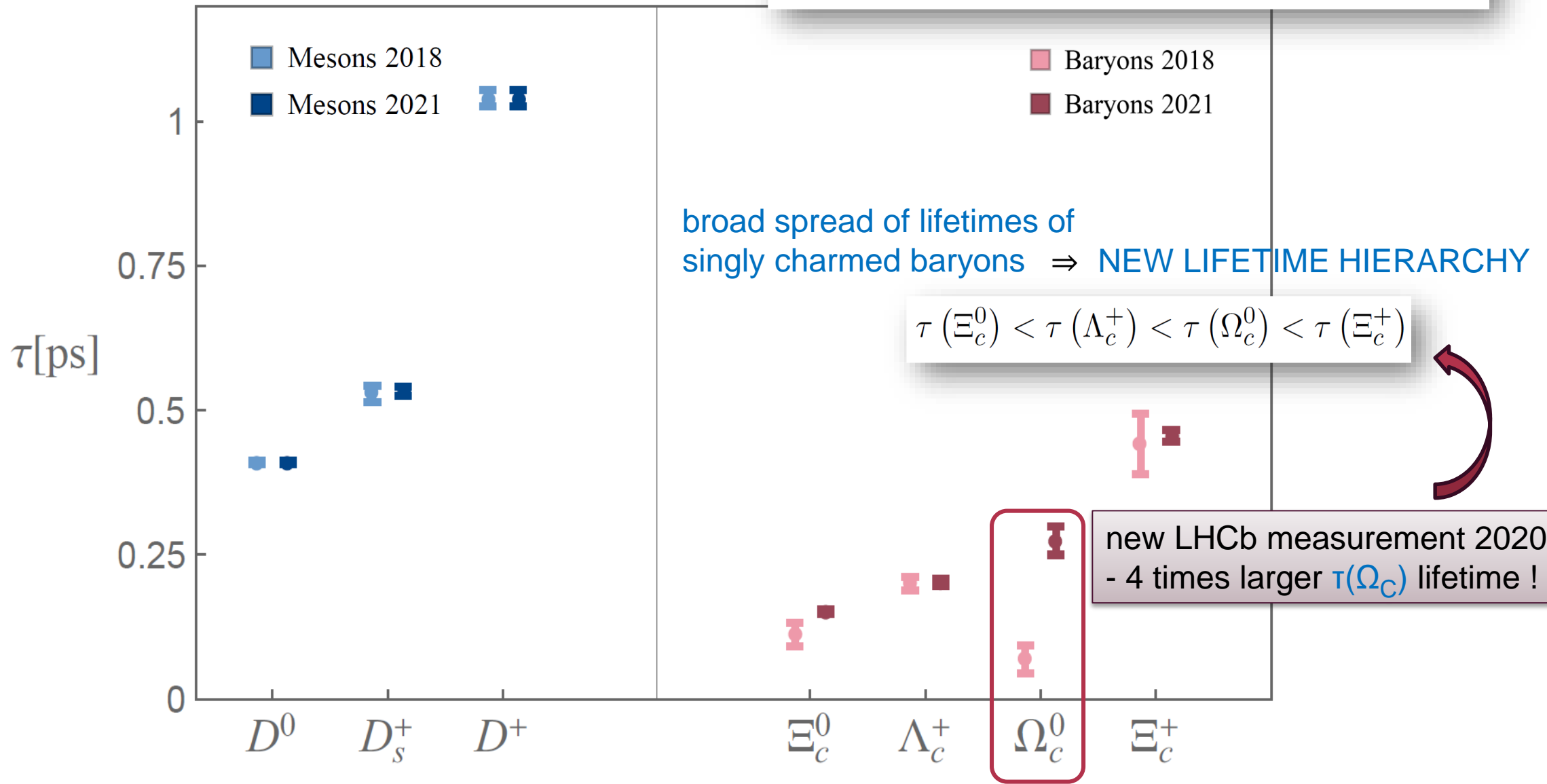
broad spread of lifetimes of singly charmed mesons

$$\frac{\tau(D^+)}{\tau(D^0)} = 2.54 \pm 0.02$$

$$\frac{\tau(D_s^+)}{\tau(D^0)} = 1.23 \pm 0.01$$

EXPERIMENTAL SITUATION

$\Lambda_c^+(cud)$, $\Xi_c^+(cus)$, $\Xi_c^0(cds)$, $\Omega_c^0(css)$



TOTAL DECAY WIDTH \rightarrow LIFETIMES

$$\frac{1}{\tau(H)} = \Gamma(H) = \frac{1}{2m_H} \langle H | \mathcal{T} | H \rangle$$

Shifman, Voloshin 85

$$\mathcal{T} = \text{Im } i \int d^4x T [\mathcal{H}_{eff}(x) \mathcal{H}_{eff}(0)]$$

forward-scattering amplitude

\mathcal{H}_{eff} = weak effective hamiltonian for a heavy Q decay

Buchalla, Buras, Lauterbach 96

$$\mathcal{H} = \frac{G_F}{\sqrt{2}} \left[\sum_{q,q'=d,s} V_{cq} V_{uq'}^* (C_1(\mu) Q_1^{(qq')} + C_2(\mu) Q_2^{(qq')}) - V_{ub} V_{cb}^* \sum_{k=3}^6 C_k(\mu) Q_k \right]$$

$$+ \sum_{\substack{q=d,s \\ \ell=e,u}} V_{cq} Q^{(q\ell)}$$

neglected for charm decays

non-leptonic(NL) and semileptonic (SL) decays included

WEAK HAMILTONIAN DIM6 and DIM7 OPERATORS

Dim 6 operators:

$$Q_1^{(qq')} = (\bar{c}^i \gamma_\mu (1 - \gamma_5) q^j) (\bar{q}'^j \gamma^\mu (1 - \gamma_5) u^i),$$

$$Q_2^{(qq')} = (\bar{c}^i \gamma_\mu (1 - \gamma_5) q^i) (\bar{q}'^j \gamma^\mu (1 - \gamma_5) u^j),$$

$$Q_{\text{SL}}^{(q\ell)} = (\bar{c} \gamma_\mu (1 - \gamma_5) q) (\bar{\ell} \gamma^\mu (1 - \gamma_5) \nu_\ell),$$

+ color-octet operators
+ μ -running and mixing

Dim 7 operators:

$$P_1^q = m_q (\bar{c}_i (1 - \gamma_5) q_i) (\bar{q}_j (1 - \gamma_5) c_j),$$

$$P_2^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho \gamma_\mu (1 - \gamma_5) D^\rho q_i) (\bar{q}_j \gamma^\mu (1 - \gamma_5) c_j),$$

$$P_3^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho (1 - \gamma_5) D^\rho q_i) (\bar{q}_j (1 + \gamma_5) c_j),$$

$$S_1^q = m_q (\bar{c}_i (1 - \gamma_5) t_{ij}^a q_j) (\bar{q}_k (1 - \gamma_5) t_{kl}^a c_l),$$

$$S_2^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho \gamma_\mu (1 - \gamma_5) t_{ij}^a D^\rho q_j) (\bar{q}_k \gamma^\mu (1 - \gamma_5) t_{kl}^a c_l),$$

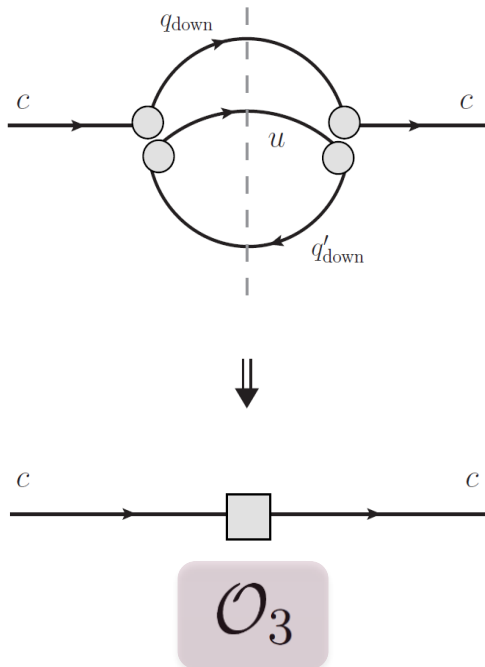
$$S_3^q = \frac{1}{m_Q} (\bar{c}_i \overleftarrow{D}_\rho (1 - \gamma_5) t_{ij}^a D^\rho q_j) (\bar{q}_k (1 + \gamma_5) t_{kl}^a c_l).$$

+ color-octet operators
+ non-local operators - *reabsorbed*
into dim6 matrix elements

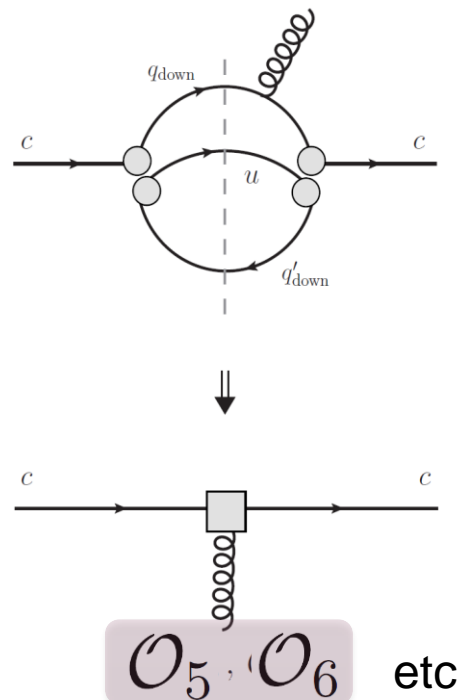
HEAVY QUARK EXPANSION (HQE) – systematic expansion in Λ_{QCD}/m_Q and α_s

$$\mathcal{T} = \left(c_3 \mathcal{O}_3 + \frac{c_5}{m_Q^2} \mathcal{O}_5 + \frac{c_6}{m_Q^3} \mathcal{O}_6 + \dots \right) + 16\pi^2 \left(\frac{\tilde{c}_6}{m_Q^3} \tilde{\mathcal{O}}_6 + \frac{\tilde{c}_7}{m_Q^4} \tilde{\mathcal{O}}_7 + \dots \right)$$

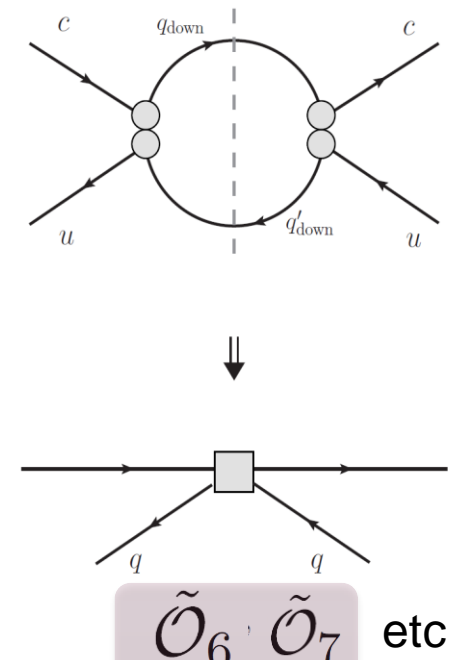
LEADING NON-SPECTATOR CONTRIBUTION



NON-LEADING NON-SPECTATOR CONTRIBUTION - in 90-ies HQE



FOUR-QUARK SPECTATOR CONTRIBUTIONS



$$\mathcal{T} = \left(c_3 \mathcal{O}_3 + \frac{c_5}{m_Q^2} \mathcal{O}_5 + \frac{c_6}{m_Q^3} \mathcal{O}_6 + \dots \right) + 16\pi^2 \left(\frac{\tilde{c}_6}{m_Q^3} \tilde{\mathcal{O}}_6 + \frac{\tilde{c}_7}{m_Q^4} \tilde{\mathcal{O}}_7 + \dots \right)$$

$$c_i = c_i^{(0)}(\mu, \mu_0) + c_i^{(1)}(\mu, \mu_0) \alpha_s(\mu) + c_i^{(2)}(\mu, \mu_0) \alpha_s(\mu)^2 + \dots,$$

MATRIX ELEMENTS OF DIFFERENT OPERATORS ARE NEEDED

universal leading contribution to all hadrons (up to mass corrections in c_3) $\sim m_Q^5$

$$\Gamma_0 = \frac{G_F^2 m_Q^5}{192\pi^3}$$

$$\Gamma(H) = \Gamma_0 \left[c_3 + \frac{c_\pi \mu_\pi^2 + c_G \mu_G^2}{m_Q^2} + \frac{c_\rho \rho_D^3}{m_Q^3} + \dots \right. \\ \left. + \frac{16\pi^2}{2m_H} \left(\sum_{i,q} \frac{c_{6,i}^q \langle H | O_i^q | H \rangle}{m_Q^3} + \sum_i \frac{c_{7,i}^q \langle H | P_i^q | H \rangle}{m_Q^4} + \dots \right) \right]$$



A BRIEF LOOK AT 1980s & 1990s “ANOMALIES”

“ANOMALIES” - 1st CASE

'80

$\tau(D^+)/\tau(D^0) \sim 2.1$

Guberina, Nussinov, Peccei, Ruckl 79

$$\mathcal{T} = \left(c_3 \mathcal{O}_3 + \frac{c_5}{m_Q^2} \mathcal{O}_5 + \frac{c_6}{m_Q^3} \mathcal{O}_6 + \dots \right) + 16\pi^2 \left(\frac{\tilde{c}_6}{m_Q^3} \tilde{\mathcal{O}}_6 + \frac{\tilde{c}_7}{m_Q^4} \tilde{\mathcal{O}}_7 + \dots \right)$$

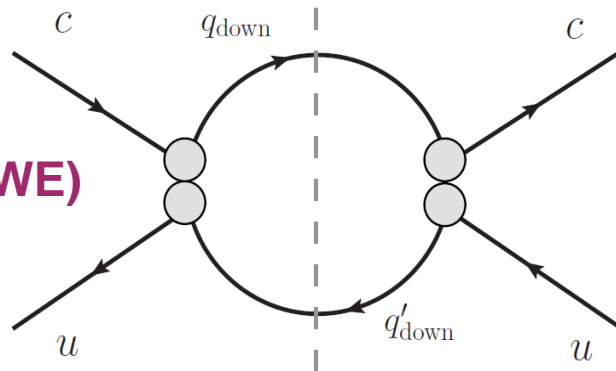
unknown pre-HQE 90'
unknown

$m_Q = m_c$ - slow convergence; spectator **contributions** $\sim 1/m_c^3$ **MIGHT BE IMPORTANT** - BUT WHY THERE WOULD BE SUCH DIFFERENCE IN D-MESON LIFETIMES?

$$\Gamma(H) = \Gamma_0 \left[c_3 + \frac{16\pi^2}{2m_H} \left(\sum_{i,q} \frac{c_{6,i}^q \langle H | O_i^q | H \rangle}{m_Q^3} \right) \right] \quad \Gamma_0 = \frac{G_F^2 m_Q^5}{192\pi^3}$$

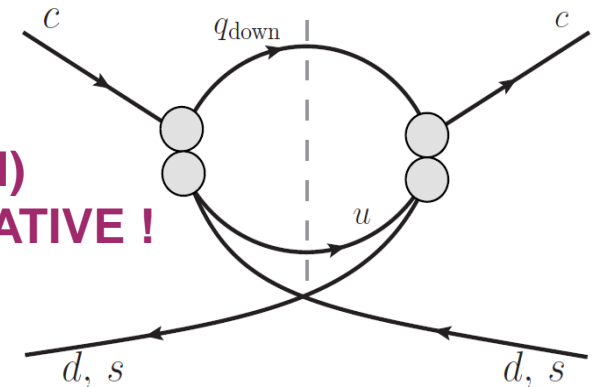
$D^0(c\bar{u})$

weak exchange (WE)



$D^+(c\bar{d})$

Pauli interference (PI)
- LARGE AND NEGATIVE !



“ANOMALIES” - 2nd CASE

'85

$\tau(D_s)/\tau(D_0) \sim 1.5$

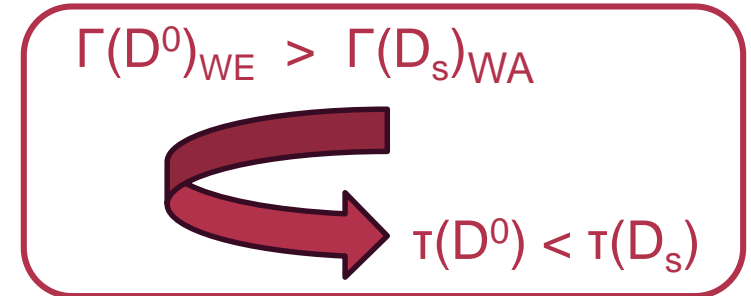
Shifman, Voloshin 86,87
see discussion in Bigi,Uratsev 9311243

$$\mathcal{T} = \left(c_3 \mathcal{O}_3 + \frac{c_5}{m_Q^2} \mathcal{O}_5 + \frac{c_6}{m_Q^3} \mathcal{O}_6 + \dots \right) + 16\pi^2 \left(\frac{\tilde{c}_6}{m_Q^3} \tilde{\mathcal{O}}_6 + \frac{\tilde{c}_7}{m_Q^4} \tilde{\mathcal{O}}_7 + \dots \right)$$

unknown pre-HQE 90'
unknown

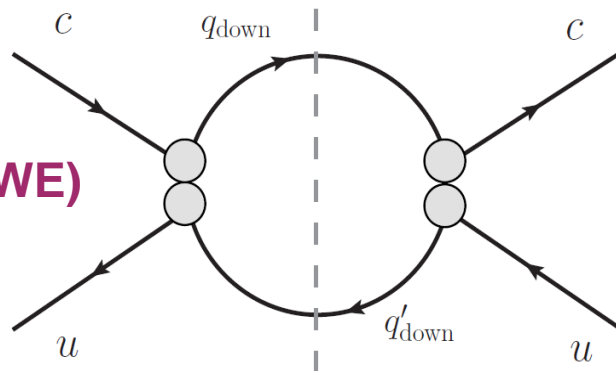
$m_Q = m_c$ - slow convergence; spectator **contributions** $\sim 1/m_c^3$ might BE IMPORTANT + SU(3) BREAKING

$$\Gamma(H) = \Gamma_0 \left[c_3 + \frac{16\pi^2}{2m_H} \left(\sum_{i,q} \frac{c_{6,i}^q \langle H | O_i^q | H \rangle}{m_Q^3} \right) \right]$$



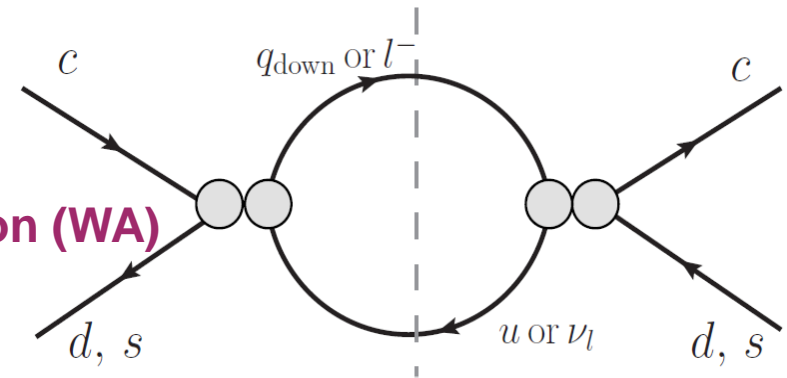
$D^0(c\bar{u})$

weak exchange (WE)



$D_s(c\bar{s})$

weak annihilation (WA)



“ANOMALIES” - 3st CASE

‘90-’00

$\tau(\Lambda_b)/\tau(B) \sim 0.7-0.8$

EXPERIMENT :

thanks to A. Lenz

| Year | Exp | Decay | $\tau(\Lambda_b)$ [ps] | $\tau(\Lambda_b)/\tau(B_d)$ |
|------|-------|---------------|------------------------|-----------------------------|
| 2003 | HFAG | average | 1.212 ± 0.052 | 0.798 ± 0.034 |
| 1998 | OPAL | $\Lambda_c l$ | 1.29 ± 0.25 | $0.85 \pm 0.16^*$ |
| 1998 | ALEPH | $\Lambda_c l$ | 1.21 ± 0.11 | $0.80 \pm 0.07^*$ |
| 1995 | ALEPH | $\Lambda_c l$ | 1.02 ± 0.24 | $0.67 \pm 0.16^*$ |
| 1992 | ALEPH | $\Lambda_c l$ | 1.12 ± 0.37 | $0.74 \pm 0.24^*$ |

THEORY :

$$\Gamma(H) = \Gamma_0 \left[c_3 + \frac{c_\pi \mu_\pi^2 + c_G \mu_G^2}{m_Q^2} + \frac{16\pi^2}{2m_H} \left(\sum_{i,q} \frac{c_{6,i}^q \langle H | O_i^q | H \rangle}{m_Q^3} \right) \right]$$

$16\pi^2/m_c^3 \sim 50$
 $16\pi^2/m_b^3 \sim 2$

$m_Q = m_b$ - fast convergence; spectator contributions $\sim 1/m_b^3$ highly suppressed

> CONCLUSION: $\tau(\Lambda_b)/\tau(B)$ CANNOT DEVIATE MUCH FROM 1

THEORY :

thanks to A. Lenz

| Year | Author | $\tau(\Lambda_b)/\tau(B_d)$ |
|------|---|-----------------------------|
| 2007 | Tarantino | 0.88 ± 0.05 |
| 2004 | Petrov et al. | 0.86 ± 0.05 |
| 2003 | Tarantino | 0.88 ± 0.05 |
| 2002 | Rome | 0.90 ± 0.05 |
| 2000 | Körner, Melic | $0.81 \dots 0.92$ |
| 1999 | Guberina, Melic, Stefanic | > 0.90 |
| 1999 | diPierro, Sachrajda, Michael | 0.92 ± 0.02 |
| 1999 | Huang, Liu, Zhu | 0.83 ± 0.04 |
| 1996 | Colangelo, deFazio | > 0.94 |
| 1996 | Neubert, Sachrajda | " > 0.90 " |
| 1992 | Bigi, Blok, Shifman, Uraltsev, Vainshtein | $> 0.85 \dots 0.90$ |
| x | only $1/m_b^2$ | 0.98 |

Colour coding:

- Wilson coefficient
- Matrix element of dimension 6 operator
- Numerical update

SOME OTHER THEORETICAL IDEAS: ALTARELLI et al

Many theory paper appeared

thanks to A. Lenz

Some claiming HQE fails

FAILURE OF LOCAL DUALITY IN INCLUSIVE NON-LEPTONIC HEAVY FLAVOUR DECAYS

G. Altarelli

Theoretical Physics Division, CERN, CH-1211 Geneva 23 and
Dipartimento di Fisica, Terza Università di Roma, Roma

G. Martinelli, S. Petrarca and F. Rapuano

Dip. di Fisica dell'Università *La Sapienza* and
INFN, Sez. di Roma I
P.le A. Moro 2, 00185 Roma, Italy

ABSTRACT

We argue that there is strong experimental evidence in the data of b - and c -decays that the pattern of power suppressed corrections predicted by the short distance expansion, the heavy quark effective theory and the assumption of local duality is not correct for the non-leptonic inclusive widths. The data indicate instead the presence of $1/m$ corrections that should be absent in the above theoretical framework. These corrections can be simply described by replacing the heavy quark mass by the mass of the decaying hadron in the m^5 factor in front of all the non-leptonic widths.

Nature (or experimentalists)
might be nasty

Experiment in 1996 shows

$$\Gamma^{\text{NL}} = \frac{G_F^2 m_{\text{Meson}}^5}{192\pi^3} \quad \text{vs.} \quad \Gamma^{\text{NL}} = \frac{G_F^2 m_b^5}{192\pi^3}$$



Works



Works
Not

APPEARED TO BE THEORETICALLY WRONG
- $1/m_Q$ TERMS ARE NOT EXISTING IN HQE

arXiv:hep-ph/9604202v3 5 Apr 1996

SOME OTHER THEORETICAL IDEAS: Grinstein et al

arXiv.org > hep-ph > arXiv:hep-ph/0304202v1

Search...

Help | Advance

thanks to A. Lenz

High Energy Physics - Phenomenology

Explicit Quark-Hadron Duality Violations in B-Meson Decays

Benjamin Grinstein, Michael Savrov

(Submitted on 22 Apr 2003 (this version), latest version 29 Apr 2003 (v2))

We consider the weak decay of heavy mesons in QCD. We compute the inclusive hadronic decay rate in leading order in the large N_c expansion, with masses chosen to insure the final state mesons recoil slowly (the SV limit). We find, by explicit computation, violations to quark-hadron duality at order $1/M$ in the heavy mass expansion. The violation to duality is linear in the slope of the form factor for the associated semileptonic decay. Differences in slopes of form factors may help understand the puzzle of lifetimes of b-hadrons.

hep-ph/0304202

Search...

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High Energy Physics - Phenomenology

Explicit Quark-Hadron Duality Violations in B-Meson Decays

Benjamin Grinstein, Michael Savrov

(Submitted on 22 Apr 2003 (v1), last revised 29 Apr 2003 (this version, v2))

Duality is not violated at order Δ/M once $j=3/2$ and $j=1/2+$ states are properly accounted for.

**WITHDRAWN BY AUTHORS
- NO DUALITY VIOLATION**

Comments: Paper withdrawn by authors, due to crucial omission of higher resonances

RESOLUTION OF THE „ANOMALY“ :

NEW EXPERIMENTAL MEASUREMENTS

thanks to A. Lenz

| Year | Exp | Decay | $\tau(\Lambda_b)$ [ps] | $\tau(\Lambda_b)/\tau(B_d)$ |
|------|------|---------------------|------------------------|-----------------------------|
| 2011 | HFAG | average | 1.425 ± 0.032 | 0.938 ± 0.022 |
| 2010 | CDF | $J/\psi\Lambda$ | 1.537 ± 0.047 | 1.020 ± 0.031 |
| 2009 | CDF | $\Lambda_c + \pi^-$ | 1.401 ± 0.058 | 0.922 ± 0.038 |
| 2007 | D0 | $\Lambda_c\mu\nu X$ | 1.290 ± 0.150 | $0.849 \pm 0.099^*$ |
| 2007 | D0 | $J/\psi\Lambda$ | 1.218 ± 0.137 | $0.802 \pm 0.090^*$ |
| 2006 | CDF | $J/\psi\Lambda$ | 1.593 ± 0.089 | 1.049 ± 0.059 |
| 2004 | D0 | $J/\psi\Lambda$ | 1.22 ± 0.22 | 0.87 ± 0.17 |

4.1 σ difference
to previous
measurements !

excellent agreement with the theory:

$$\frac{\tau(\Lambda_b)}{\tau(B_d)} = 0.935 \pm 0.054$$

A. Lenz, 1405.3601



GOING BACK TO THE PRESENT DEVELOPMENTS

$$\Gamma = \Gamma^{\text{NL}} + \Gamma^{\text{SL}}$$

$$\Gamma^{\text{NL}} = g_3^{(0)} + \alpha_s g_3^{(1)} + \frac{1}{m_c^2} \left(g_\pi^{(0)} + g_G^{(0)} \right) + \frac{1}{m_c^3} g_{\text{Darwin}}^{(0)} + \frac{16\pi^2}{m_c^3} \left(\tilde{g}_6^{(0)} + \alpha_s \tilde{g}_6^{(1)} + \frac{1}{m_c} \tilde{g}_7^{(0)} \right)$$

$$\Gamma^{\text{SL}} = g_3^{(0)} + \alpha_s g_3^{(1)} + \frac{1}{m_c^2} \left(g_\pi^{(0)} + \alpha_s g_\pi^{(1)} + g_G^{(0)} + \alpha_s g_G^{(1)} \right) + \frac{1}{m_c^3} g_{\text{Darwin}}^{(0)} + \frac{16\pi^2}{m_c^3} \left(\tilde{g}_6^{(0)} + \alpha_s \tilde{g}_6^{(1)} + \frac{1}{m_c} \tilde{g}_7^{(0)} \right)$$

| Semileptonic (SL) modes | |
|-------------------------|--|
| $\Gamma_3^{(3)}$ | Fael, Schönwald, Steinhauser '20 * ; Czakov, Czarnecki, Dowling '21 |
| $\Gamma_3^{(2)}$ | Czarnecki, Melnikov, v. Ritbergen, Pak, Dowling, Bonciani, Ferroglia, Biswas, Brucherseifer, Caola '97-'13 |
| $\Gamma_5^{(1)}$ | Alberti, Gambino, Nandi, Mannel, Pivovarov, Rosenthal '13-'15 |
| $\Gamma_6^{(1)}$ | Mannel, Pivovarov '19 |
| $\Gamma_7^{(0)}$ | Dassinger, Mannel, Turczyk '06 |
| $\Gamma_8^{(0)}$ | Mannel, Turczyk, Uraltsev '10 |

* see also talks by K. Schönwald and M. Fael

** Partial result

| Non-leptonic (NL) modes | |
|--------------------------|---|
| $\Gamma_3^{(2)}$ | Czarnecki, Slusarczyk, Tkachov '05 ** |
| $\Gamma_3^{(1)}$ | Ho-Kim, Pham, Altarelli, Petrarca, Voloshin, Bagan, Ball, Braun, Godzinsky, Fiol, Lenz, Nierste, Ostermaier, Krinner, Rauh '84-'13 |
| $\Gamma_5^{(0)}$ | Bigi, Uraltsev, Vainshtein, Blok, Shifman '92 |
| $\Gamma_6^{(0)}$ | Lenz, MLP, Rusov, Mannel, Moreno, Pivovarov '20-'21 |
| $\tilde{\Gamma}_6^{(1)}$ | Beneke, Buchalla, Greub, Lenz, Nierste, Franco, Lubicz, Mescia, Tarantino, Rauh '02-'13 |
| $\tilde{\Gamma}_7^{(0)}$ | Gabbiani, Onishchenko, Petrov '03-'04 |

CALCULATION OF MATRIX ELEMENTS

$$\mathcal{T} = \left(c_3 \mathcal{O}_3 + \frac{c_5}{m_Q^2} \mathcal{O}_5 + \frac{c_6}{m_Q^3} \mathcal{O}_6 + \dots \right) + 16\pi^2 \left(\frac{\tilde{c}_6}{m_Q^3} \tilde{\mathcal{O}}_6 + \frac{\tilde{c}_7}{m_Q^4} \tilde{\mathcal{O}}_7 + \dots \right)$$

$$\Gamma(H) = \frac{1}{2m_H} \langle H | \mathcal{T} | H \rangle$$

NON-SPECTATOR PART:

SPECTATOR PART:

$$\Gamma(H) = \Gamma_0 \left[c_3 + \frac{c_\pi \mu_\pi^2 + c_G \mu_G^2}{m_Q^2} + \frac{c_\rho \rho_D^3}{m_Q^3} + \dots \right] + \frac{16\pi^2}{2m_H} \left(\sum_{i,q} \frac{c_{6,i}^q \langle H | \mathcal{O}_i^q | H \rangle}{m_Q^3} + \sum_i \frac{c_{7,i}^q \langle H | \mathcal{P}_i^q | H \rangle}{m_Q^4} \right)$$

$$\mu_\pi^2(H) = \frac{-1}{2m_H} \langle H | \bar{c}_v (iD)^2 c_v | H \rangle, \text{ - kinetic parameter}$$

$$\mu_G^2(H) = \frac{1}{2m_H} \langle H | \bar{c}_v \frac{1}{2} \sigma \cdot (g_s G) c_v | H \rangle, \text{ - chromomagnetic parameter}$$

$$\rho_D^3(H) = \frac{1}{2m_H} \langle H | \bar{c}_v (iD_\mu) (i v \cdot D) (iD^\mu) c_v | H \rangle \text{ - Darwin term}$$

$$\langle H | \mathcal{O}_i^q | H \rangle$$

$$\langle H | \mathcal{P}_i^q | H \rangle$$

four-quark matrix elements

CALCULATION OF NON-SPECTATOR MATRIX ELEMENTS

NON-SPECTATOR PART:

- mainly universal – up to $SU(3)_f$ breaking and differences in spins of hadrons

μ_G^2

application of hadron mass formula:

$$m_H = m_c + \bar{\Lambda} + \frac{\mu_\pi^2(H)}{2m_c} - \frac{\mu_G^2(H)}{2m_c} + \mathcal{O}\left(\frac{1}{m_c^2}\right)$$

spin factor: $d_H = -2(S_H(S_H + 1) - S_h(S_h + 1) - S_l(S_l + 1))$

$$\mu_G^2(H) \equiv d_H \lambda_2 = d_H \frac{m_{H^*}^2 - m_H^2}{d_H - d_{H^*}}$$

| | | | | | |
|-------|-----|-------|---------------------------------|--------------|-----------------|
| H | D | D^* | $\Lambda_c^+, \Xi_c^+, \Xi_c^0$ | Ω_c^0 | Ω_c^{0*} |
| d_H | 3 | -1 | 0 | 4 | -2 |

μ_π^2

HQET SR

$$\mu_\pi^2 \geq \mu_G^2$$

ρ_D^3

applying EOM of $G_{\mu\nu}$ and relating to the dim6 operators

$$2m_H \rho_D^3 = g_s^2 \langle H | \left(-\frac{1}{8} O_1^q + \frac{1}{24} \tilde{O}_1^q + \frac{1}{4} O_2^q - \frac{1}{12} \tilde{O}_2^q \right) | H \rangle + \mathcal{O}(1/m_c)$$

$$\rho_D^3(D_q) = \frac{g_s^2}{18} f_{D_q}^2 m_{D_q} + \mathcal{O}(1/m_c)$$

CALCULATION OF NON-SPECTATOR MATRIX ELEMENTS

NON-SPECTATOR PART:

| | D^0 | D^+ | D_s^+ | Λ_c^+ | Ξ_c^+ | Ξ_c^0 | Ω_c^0 |
|----------------------------|-----------|-----------|-----------|---------------|-----------|-----------|--------------|
| μ_G^2 / GeV^2 | 0.41(12) | 0.41(12) | 0.44(13) | 0 | 0 | 0 | 0.26(8) |
| μ_π^2 / GeV^2 | 0.45(14) | 0.45(14) | 0.48(14) | 0.50(15) | 0.55(17) | 0.55(17) | 0.55(17) |
| ρ_D^3 / GeV^3 | 0.056(12) | 0.056(22) | 0.082(33) | 0.04(1) | 0.05(2) | 0.06(2) | 0.06(2) |

+ 30% uncertainties

ρ_D^3 much smaller parameter but with a surprisingly large Wilson coefficient C_ρ
- sizable contribution

CALCULATION OF SPECTATOR (FOUR-QUARK) MATRIX ELEMENTS

SPECTATOR PART FOR MESONS: - calculation of four-quark matrix elements

Dim 6 : $\langle D_q | \mathcal{O}_i^q | D_q \rangle = F_{D_q}(\mu)^2 m_{D_q} B_i^q,$

$$\langle D_q | \mathcal{O}_i^{q'} | D_q \rangle = F_{D_q}(\mu)^2 m_{D_q} \delta_i^{q'q}, \quad q \neq q'$$

$$F_{D_q}(\mu)^2 \rightarrow f_{D_q}^2 m_{D_q} \left(1 + \frac{4}{3} \frac{\alpha_s(m_c)}{\pi} \right)$$

HQET bag model parameters or lattice:

$$B_{1,2}^q \quad \epsilon_{1,2}^q \equiv B_{3,4}^q \quad \delta_i^{q'q}$$

Kirk, Lenz, Rauch, 1711.02100
King, Lenz, Rauch, 2112.03691
King et al, 2109.13219

Dim 7 : $\langle D_q | \mathcal{P}_1^q | D_q \rangle = -m_q F^2 m_{D_q} B_1^P,$

$$\langle D_q | \mathcal{P}_2^q | D_q \rangle = -\bar{\Lambda}_q F^2 m_{D_q} B_2^P,$$

$$\langle D_q | \mathcal{P}_2^q | D_q \rangle = -\bar{\Lambda}_q F^2 m_{D_q} B_2^P,$$

$$\langle D_q | \mathcal{R}_1^q | D_q \rangle = -F_{D_q}^2 m_{D_q} (\bar{\Lambda}_q - m_q) B_1^R,$$

$$\langle D_q | \mathcal{R}_2^q | D_q \rangle = F_{D_q}^2 m_{D_q} (\bar{\Lambda}_q - m_q) B_2^R,$$

Vacuum insertion approximation (VIA):

$$B_i^{P,R} = 1 \quad \epsilon_i^{P,R} = 0$$

for color-octet op

Decay constants in the $m_c \rightarrow \infty$ limit:

$$F_{D_q} \rightarrow f_{D_q} \sqrt{m_{D_q}}$$

CALCULATION OF SPECTATOR (FOUR-QUARK) MATRIX ELEMENTS

SPECTATOR PART FOR BARYONS : - calculation of four-quark matrix elements

NR CONSTITUENT QUARK MODEL

$$\frac{\langle \mathcal{B}_c | O_i^q | \mathcal{B}_c \rangle}{2m_{\mathcal{B}_c}} \sim |\psi_{cq}^{\mathcal{B}_c}(0)|^2 \quad \text{and} \quad |\Psi(0)|_{ij}^2 \sim \delta^3(0)$$

Rujula, Georgi, Glashow 1975

$$M_H = \sum_i m_i^H + \langle H_{\text{spin,H}} \rangle$$

$$H_{\text{spin, mesons}} = \frac{32\pi\alpha_s}{9} \frac{(\vec{s}_i \cdot \vec{s}_j)}{m_i^M m_j^M} \delta^3(\vec{r}_{ij})_{\text{M}}$$

$$H_{\text{spin, baryons}} = \sum_{i>j} \frac{16\pi\alpha_s}{9} \frac{(\vec{s}_i \cdot \vec{s}_j)}{m_i^{\mathcal{B}} m_j^{\mathcal{B}}} \delta^3(\vec{r}_{ij})_{\text{B}}$$

e.g.

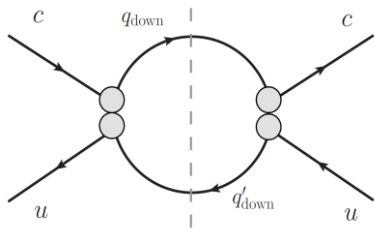
$$|\Psi_{cq}^{\Lambda_c^+}(0)|^2 = \frac{4}{3} \frac{M_{\Sigma_c^*} - M_{\Sigma_c}}{M_{D^*} - M_D} |\Psi_{cq}^{D^*}(0)|^2$$

Dim 7 operators are expressed similarly, in terms on dim 6 operators as above

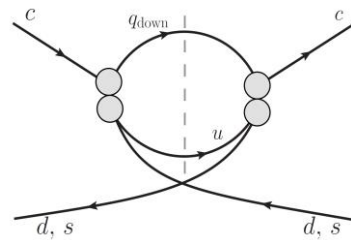
SPECTATOR (u,d,s) FOUR-QUARK CONTRIBUTIONS ARE IMPORTANT :

- one-loop i.e $16 \pi^2$ enhanced, although $1/m^3$ (dim6), $1/m^4$ (dim7) suppressed

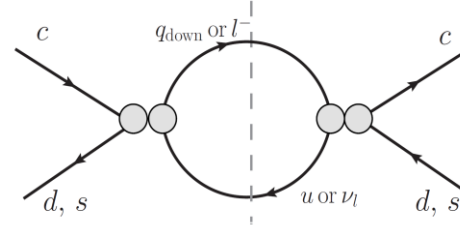
MESONS



WE

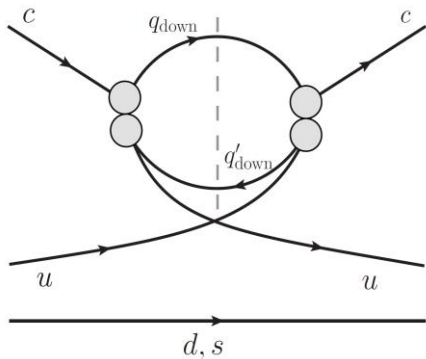


PI

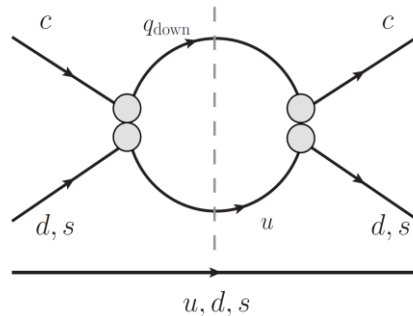


WA

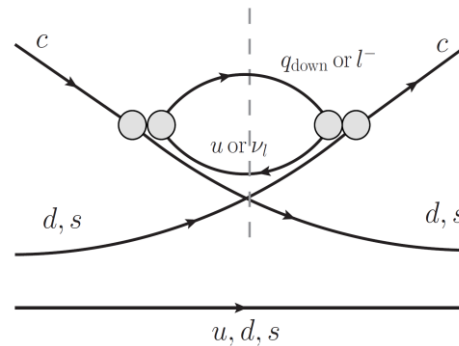
BARYONS



int-



exc



int+

| decay | CE NL | CE SL |
|------------------------|---|-------------------------------|
| H_c | $c \rightarrow s\bar{d}u$ | $c \rightarrow s\bar{l}\nu_l$ |
| $\bar{D}^0 (u\bar{c})$ | $\tilde{\Gamma}_{WE}$ | - |
| $D^- (d\bar{c})$ | $\tilde{\Gamma}_{PI}$ | - |
| $D_s^- (s\bar{c})$ | $\tilde{\Gamma}_{WA}$ | $\tilde{\Gamma}_{WA}^{SL}$ |
| $\Lambda_c^+ (udc)$ | $\tilde{\Gamma}_{exc} + \tilde{\Gamma}_{int-}$ | - |
| $\Xi_c^+ (usc)$ | $\tilde{\Gamma}_{int-} + \tilde{\Gamma}_{int+}$ | $\tilde{\Gamma}_{int+}^{SL}$ |
| $\Xi_c^0 (dsc)$ | $\tilde{\Gamma}_{exc} + \tilde{\Gamma}_{int+}$ | $\tilde{\Gamma}_{int+}^{SL}$ |
| $\Omega_c^0 (ssc)$ | $\tilde{\Gamma}_{int+}$ | $\tilde{\Gamma}_{int+}^{SL}$ |

CE = leading; Cabibbo enhanced

- effects are different in different mesons
- effects are different in different baryons
- there are effects in SL decays – different BR(SL) !

CHARM QUARK MASS

$$\Gamma_0 = \frac{G_F^2 m_Q^5}{192\pi^3}$$

POLE mass:

$$m_c^{\text{pole}} = \bar{m}_c(\bar{m}_c) \left[1 + \frac{4}{3} \frac{\alpha_s(\bar{m}_c)}{\pi} + 10.3 \left(\frac{\alpha_s(\bar{m}_c)}{\pi} \right)^2 + 116.5 \left(\frac{\alpha_s(\bar{m}_c)}{\pi} \right)^3 + \dots \right]$$

$$= \bar{m}_c(\bar{m}_c) (1 + 0.16 + 0.15 + \mathbf{0.21} + \dots)$$

IR renormalon –
divergent series starting from the third-loop...

renormalon-free mass definitions:

$$m_c^X(\mu_f) = m_c^{\text{pole}} - \delta m_c^X(\mu_f)$$

$$= \bar{m}_c(\bar{m}_c) + \bar{m}_c(\bar{m}_c) \sum_{n=1}^{\infty} \left[c_n(\mu, \bar{m}_c(\bar{m}_c)) - \frac{\mu_f}{\bar{m}_c(\bar{m}_c)} s_n^X(\mu/\mu_f) \right] \alpha_s^n(\mu)$$

- subtraction of IR renormalons
- rearrangement of α_s expansion - relevant for α_s -corrections in c_3 and c_6 terms

CHARM QUARK MASS

| $\overline{m}_c(\overline{m}_c) = 1.28 \text{ GeV}$ | 1-loop | 2-loop | 3-loop | 4-loop |
|---|--------|--------|--------|--------|
| m_c^{pole} | 1.49 | 1.68 | 1.95 | 2.43 |
| m_c^{kin} | 1.36 | 1.39 | 1.40 | - |
| m_c^{MSR} | 1.33 | 1.35 | 1.36 | 1.36 |

we provide results for all mass schemes... no large differences in the final results – rearrangements among $1/m_c$ and α_s -expansion !



RESULTS

RESULTS FOR BARYONS

Lifetime ratios for \mathcal{B}_c :

$$\frac{\tau(\mathcal{B}_c)}{\tau(\Lambda_c^+)} \equiv \frac{1}{1 + (\Gamma^{\text{th}}(\mathcal{B}_c) - \Gamma^{\text{th}}(\Lambda_c^+))\tau^{\text{exp}}(\Lambda_c^+)}$$

- some uncertainties cancel/subtract in the ratios

Inclusive SL branching ratios (e only) for \mathcal{B}_c :

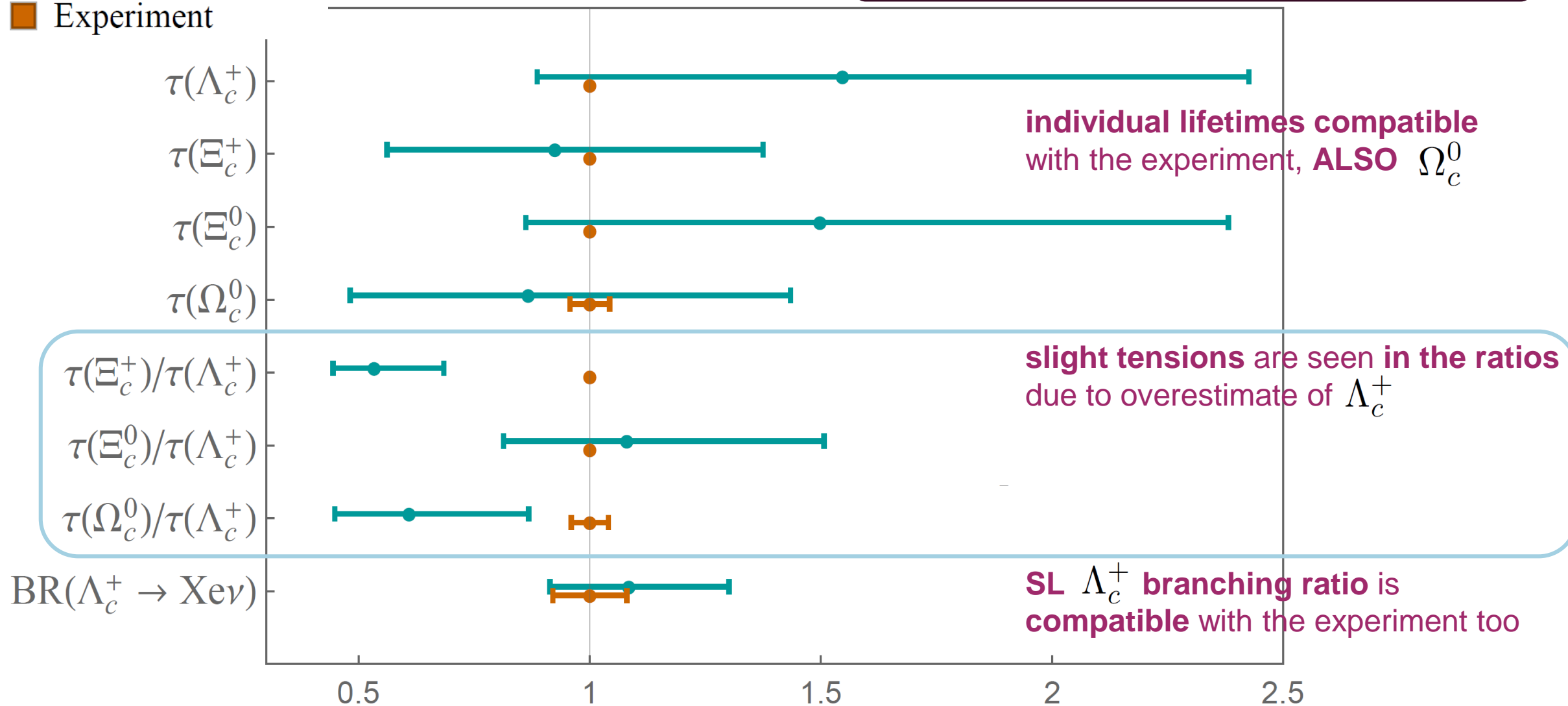
$$BR(\mathcal{B}_c \rightarrow X e \nu) \equiv \Gamma(\mathcal{B}_c \rightarrow X e \nu) \tau^{\text{exp}}(\mathcal{B}_c)$$

RESULTS FOR BARYONS

■ Our Results (MSR)

■ Experiment

$$\tau(\Xi_c^0) < \tau(\Lambda_c^+) < \tau(\Omega_c^0) < \tau(\Xi_c^+)$$



RESULTS FOR BARYONS - SL BRs

MSR mass scheme:

| | |
|--|-----------------------------------|
| $BR(\Lambda_c^+ \rightarrow X e \nu) / \%$ | $4.28^{+0.47+0.39}_{-0.37-0.30}$ |
| $BR(\Xi_c^+ \rightarrow X e \nu) / \%$ | $14.95^{+2.66+1.59}_{-2.45-1.50}$ |
| $BR(\Xi_c^0 \rightarrow X e \nu) / \%$ | $5.06^{+0.91+0.54}_{-0.84-0.51}$ |
| $BR(\Omega_c^0 \rightarrow X e \nu) / \%$ | $11.19^{+3.01+1.94}_{-2.89-2.09}$ |

SL decays are important to assess the validity of HQE in charm in baryons

- experimental measurements of $BR_{SL}(\Xi_c^+)$, $BR_{SL}(\Xi_c^0)$ and $BR_{SL}(\Omega_c^0)$ are needed

RESULTS FOR MESONS

Lifetime ratios :

$$\frac{\tau(D_{(s)}^+)}{\tau(D^0)} = 1 + \left(\Gamma^{\text{th}}(D^0) - \Gamma^{\text{th}}(D_{(s)}^+) \right) \tau^{\text{exp}}(D_{(s)}^+)$$

- some uncertainties cancel/subtract in the ratios

Inclusive SL branching ratios (e only) :

$$BR^{(e)}(D) = \Gamma^{(e)}(D) \tau^{\text{exp}}(D)$$

$$\frac{\Gamma^{(e)}(D_{(s)}^+)}{\Gamma^{(e)}(D^0)} = 1 + \left(\Gamma^{(e)\text{th}}(D_{(s)}^+) - \Gamma^{(e)\text{th}}(D^0) \right) \left(\frac{\tau(D^0)}{BR^{(e)}(D^0)} \right)^{\text{exp}}$$

RESULTS FOR MESONS

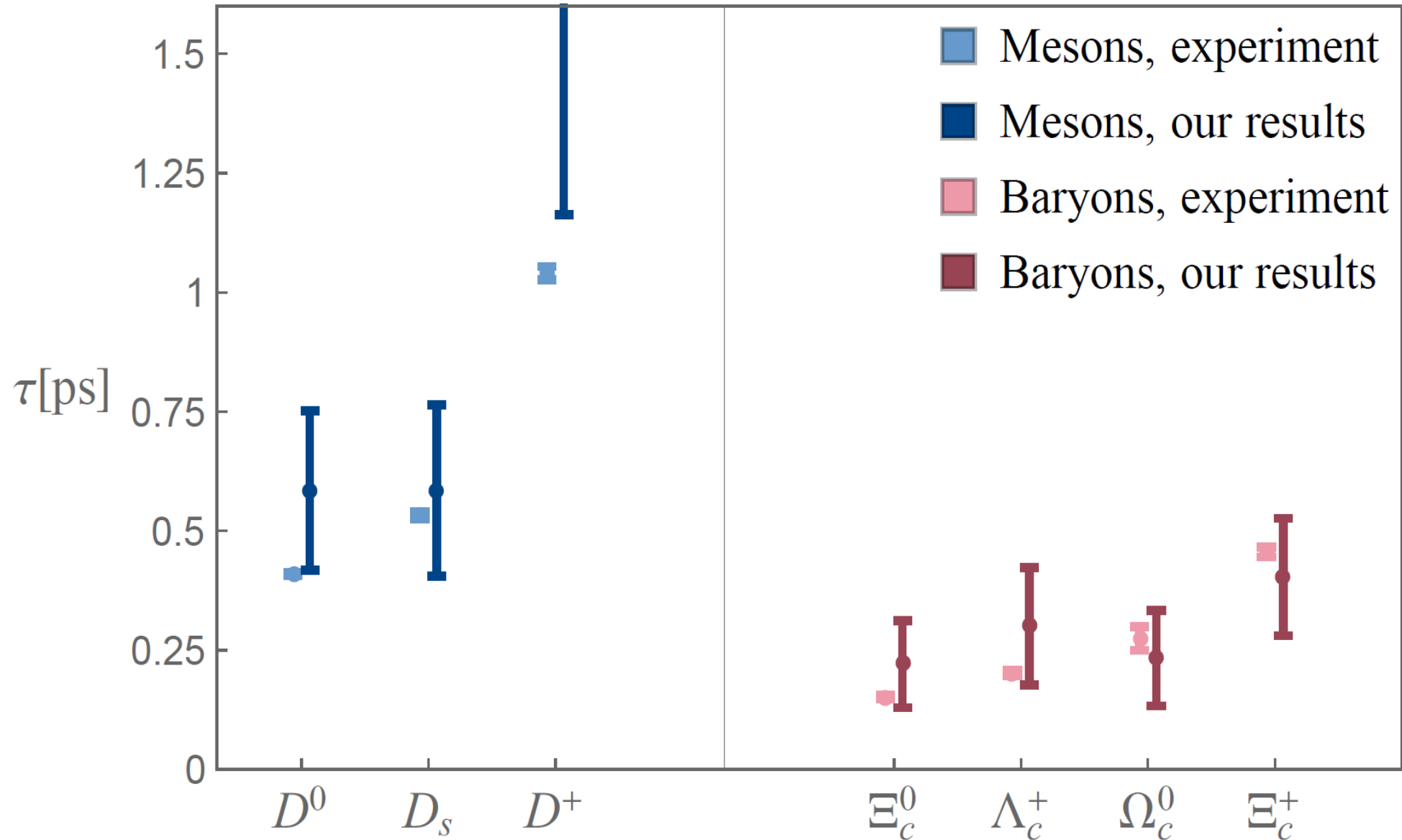
full agreement with King et al, 2109.13219

| Observable | MSR | Experiment |
|---------------------------------|-----------------------------------|-----------------|
| $\Gamma(D^0)$ | $1.68^{+0.38+0.53}_{-0.43-0.44}$ | 2.44 ± 0.01 |
| $\Gamma(D^+)$ | $-0.13^{+0.71+0.13}_{-0.64-0.11}$ | 0.96 ± 0.01 |
| $\tilde{\Gamma}(D_s^+)$ | $1.67^{+0.46+0.55}_{-0.56-0.46}$ | 1.88 ± 0.02 |
| $\tau(D^+)/\tau(D^0)$ | $2.89^{+0.66+0.42}_{-0.78-0.35}$ | 2.54 ± 0.02 |
| $\tilde{\tau}(D_s^+)/\tau(D^0)$ | $1.00^{+0.23+0.01}_{-0.21-0.01}$ | 1.30 ± 0.01 |

| Observable | MSR | Experiment |
|---|-----------------------------------|-------------------|
| $BR^{(e)}(D^0)$ [%] | $5.86^{+1.80+0.48}_{-2.07-0.41}$ | 6.49 ± 0.16 |
| $BR^{(e)}(D^+)$ [%] | $14.90^{+4.67+1.22}_{-5.37-1.06}$ | 16.07 ± 0.30 |
| $BR^{(e)}(D_s^+)$ [%] | $7.67^{+2.67+0.65}_{-3.10-0.56}$ | 6.30 ± 0.16 |
| $\Gamma^{(e)}(D^+)/\Gamma^{(e)}(D^0)$ | $1.00^{+0.02+0.00}_{-0.01-0.00}$ | 0.977 ± 0.031 |
| $\Gamma^{(e)}(D_s^+)/\Gamma^{(e)}(D^0)$ | $1.06^{+0.26+0.01}_{-0.29-0.01}$ | 0.790 ± 0.026 |

- ❖ results are largely compatible with the experiment
- ❖ difficulties with $\tau(D^+)$ – Pauli interference term can drive $\tau(D^+)$ large and even negative!
- ❖ slight tension with $\tau(D_s)/\tau(D^0)$ – theoretically closer to unity

CONCLUSIONS



CONCLUSIONS

- ❑ up-to-date results for lifetimes of weakly decaying hadrons with a single charm quark, with most complete set of contributions provided
- ❑ results compatible with experiment, albeit with large uncertainties, and favoring recent LHCb result for $\tau(\Omega_c^0)$ lifetime ($4\times$ old measurement)
- ❑ difficulty in predicting $\tau(D^+)$ – only marginally compatible - huge negative Pauli interference contribution
- ❑ predictions for unmeasured $BR_{SL}(H)$ are important for complete assessment
- ❑ conclusions above are largely independent of the charm mass scheme

OUTLOOK

- ❑ extending available contributions in $1/m_c$ and α_s series
- ❑ large uncertainties mean theory cannot compete with experiment – more control of hadronic parameters needed :
 - lattice determination of $\langle \tilde{\mathcal{O}}_6 \rangle$ planned (U Siegen)
 - higher α_s corrections planned (KIT) – NLO of 4q-dim7, NNLO of NL-dim3 etc..
 - exp. (BESIII, Belle II...) determination of kinetic, chromomagnetic and Darwin parameter from SL decays? too sensitive to four-quark oper. “leakage”?
- ❑ question of applicability of heavy quark approach to charm remains open
 $\Rightarrow \alpha_s(m_c) = 0.33, \Lambda_{\text{QCD}}/m_c = 0.30$ too large? (vs $\alpha_s(m_b) = 0.22, \Lambda_{\text{QCD}}/m_b = 0.10$)
- ❑ theoretical improvements: revisiting formulation of HQE in charm mass?
testing quark-hadron duality violation?

THANK YOU



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The banner features a blue background with a particle detector-like circular graphic on the left, a yellow beam of light, and a silhouette of a city skyline at the bottom. The text is in white and blue.