Lepton flavor phenomena in modular symmetry

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T. Kobayashi, H. Otsuka, M, K. Yamamoto, PRD 105 (2022) no.5, arXiv:2112.00493,
arXiv:2204.12325

Outline of my talk

1 Introduction

2 Modular symmetry

3 Modular invariant flavor model

4 Dipole operators in modular symmetry

5 Flavor structure of Dipole operators

$(g-2)_{\mu, e}$ $\mu \to e\gamma$. .....

6 Summary

1 Introduction

There are a lot of works challenging
Flavor Problems of quarks and leptons
by using Modular Symmetries.

- Flavor mixing
- CP violation
- Mass hierarchy

Successful results are obtained for quark / lepton sector.

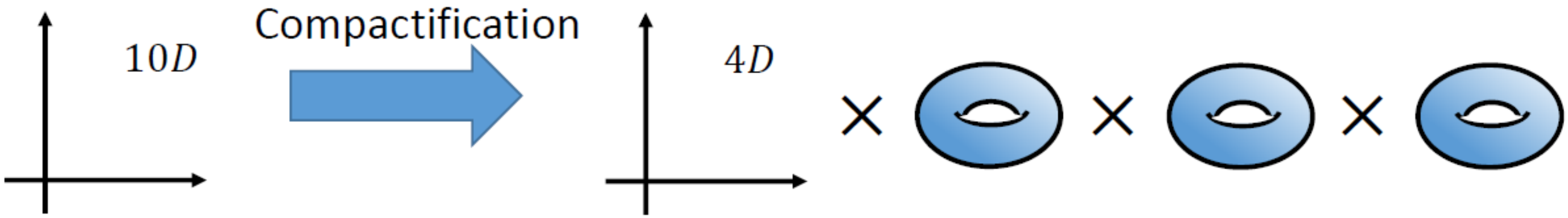
Challenge SMEFT

2 Modular symmetry

Superstring theory 10D
Our universe is 4D

→ The extra 6D should be compactified.

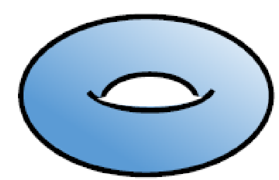
Torus compactification



We get 4D effective Lagrangian by integrating out over 6D.

$$S = \int d^4x d^6y \, \mathcal{L}_{10D} \rightarrow \int d^4x \, \mathcal{L}_{\mathrm{eff}}$$

→ $\mathcal{L}_{\mathrm{eff}}$ depends on the structure of

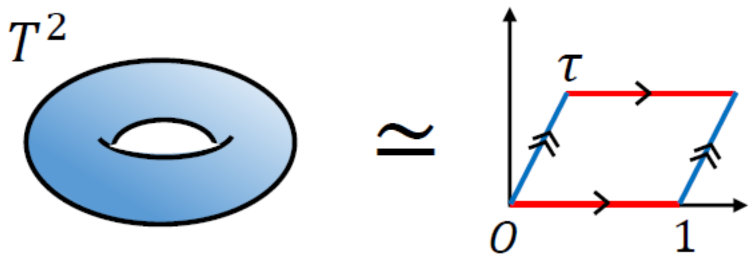


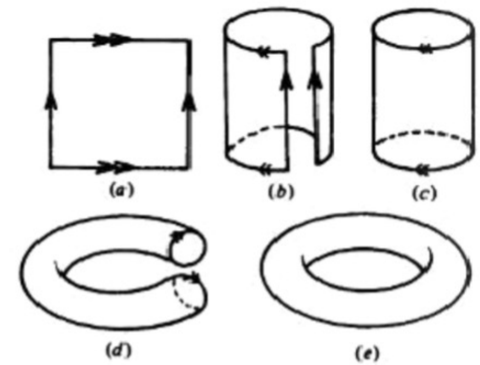
- $4D$ effective theory depends on internal space

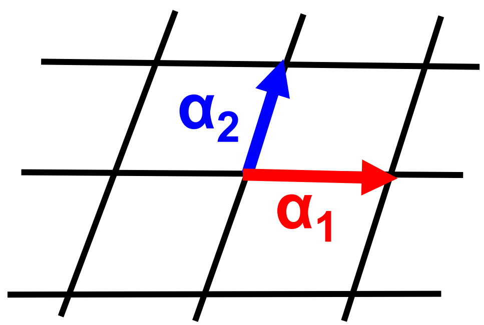
from T.H. Tatsuishi's slides

$2D$ torus (T^2) is equivalent to
parallelogram with identification of confronted sides.

by Feruglio







$(x,y) \sim (x,y) + n_1\alpha_1 + n_2\alpha_2$

Two-dimensional torus T^2 is obtained as

$$T^2 = \mathbb{R}^2 / \Lambda$$

Λ is two-dimensional lattice,
which is spanned by two lattice vectors

$\alpha_1 = 2\pi R$ and $\alpha_2 = 2\pi R\tau$

$\tau = \alpha_2 / \alpha_1$ is a modulus parameter (complex).

The same lattice is spanned by other bases under the transformation.

$$\begin{pmatrix} \alpha'_2 \\ \alpha'_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix}$$

ad-bc=1
a,b,c,d are integer SL(2,Z)

$$\begin{pmatrix} \alpha_2' \\ \alpha_1' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} \alpha_2 \\ \alpha_1 \end{pmatrix}$$

ad-bc=1
a,b,c,d are integer

$\tau = \alpha_2 / \alpha_1$

$$\tau \xrightarrow{\gamma} \tau' = \frac{a\tau + b}{c\tau + d}$$

Modular transformation

Modular transf. does not change the lattice (torus)

$4D$ effective theory (depends on τ)
must be invariant under modular transf.

e.g.) $\mathcal{L}_{\rm eff} \supset Y(\tau)_{ij}\phi\overline{\psi_i}\psi_j$

The modular transformation is generated by S and T .

$$\tau \xrightarrow{\gamma} \tau' = \frac{a\tau + b}{c\tau + d}$$

$$S : \tau \longrightarrow -\frac{1}{\tau}$$

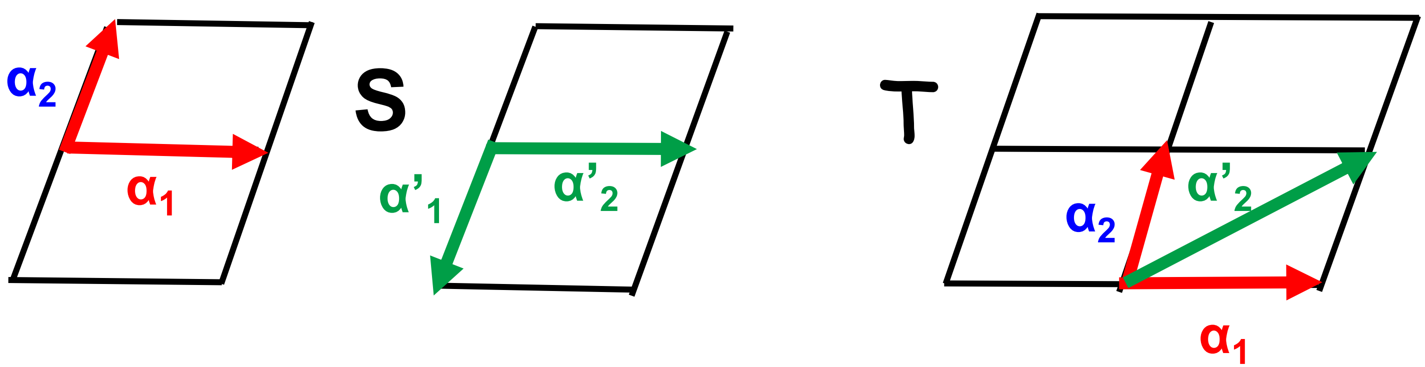
duality

$$T : \tau \longrightarrow \tau + 1$$

Dicrete shift symmetry

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$



$\tau = \alpha_2/\alpha_1$

$$S : \tau \longrightarrow -\frac{1}{\tau},$$

Duality

$$T : \tau \longrightarrow \tau + 1.$$

Dicrete shift symmetry

$$S^2 = 1, \qquad (ST)^3 = 1.$$

±1 is identified

generate infinite discrete group

Modular group

Modular group

$\Gamma \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I}\}$

Modular group has subgroups

Impose congruence condition $\Gamma(N) = \{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2, Z), \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \pmod{N} \}$

called principal congruence subgroups (normal subgroup)

$\Gamma_N \equiv \Gamma / \Gamma(N)$ quotient group finite group of level N

$\Gamma_N \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I}, T^N = \mathbb{I}\}$

$\Gamma_2 \simeq S_3$ $\Gamma_3 \simeq A_4$ $\Gamma_4 \simeq S_4$ $\Gamma_5 \simeq A_5$

isomorphic

We can consider effective theories with Γ_N symmetry.

$$\mathcal{L}_{\rm eff} \ni f(\tau)\phi^{(1)}\cdots\phi^{(n)}$$

modular form

$f(\tau), \phi^{(I)}$: non-trivial rep. of Γ_N

In cases of Γ_N (N=2,3,4,5) (S_3, A_4, S_4, A_5) explicit forms of f(τ) have been obtained.

γ= S, T

$$\tau \longrightarrow \tau' = \gamma\tau = \frac{a\tau + b}{c\tau + d}$$

Modular transformation

Automorphy factor

$$f_i(\tau) \longrightarrow f_i(\gamma\tau) = (c\tau + d)^k \rho(\gamma)_{ij} f_j(\tau)$$

modular forms of weight k

representation matrix

K is modular weight

Chiral superfields

$$(\phi^{(I)})_i(x) \longrightarrow (c\tau + d)^{-k_I} \rho(\gamma)_{ij} (\phi^{(I)})_j(x)$$

Modular forms are explicitly given if weight k is fixed.

On the other hand, chiral superfields are not modular forms and we have no restriction on the possible value of weight k_I, a priori.

Consider $f_i(\tau)\ \phi^{(I)}\phi^{(J)}H$

Automorphy factor $(c\tau+d)^k(c\tau+d)^{-k_I}(c\tau+d)^{-k_J} = (c\tau+d)^{k-k_I-k_J}$
vanishes if $k = k_I + k_J$

$\mathcal{L}_{\text{eff}}$ is modular invariant if sum of weights satisfy $\sum k_I = k$.

Modular invariant kinetic terms of matters

Simplest

Kähler potential $$K^{\text{matter}} = \frac{1}{[i(\bar{\tau}-\tau)]^{k_I}}|\phi^{(I)}|^2$$

A_4 Modular symmetry

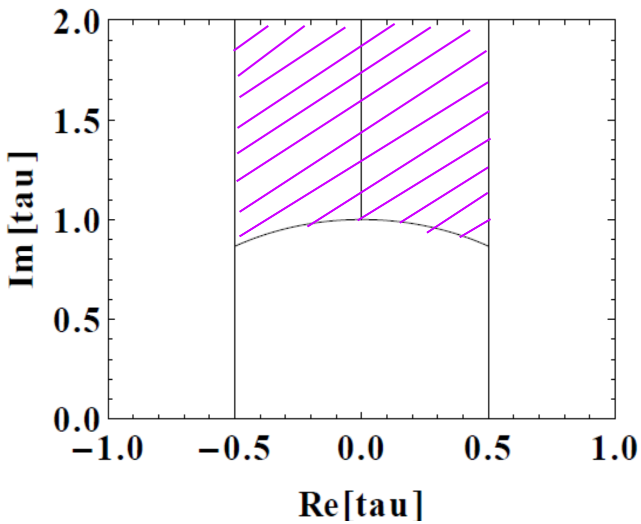
$$\Gamma_N \simeq \{S, T | S^2 = \mathbb{I}, (ST)^3 = \mathbb{I}, T^N = \mathbb{I}\}$$

Taking $T^3=1$, we get A_4 modular group (Γ_3). N=3 ～A_4

# of modular forms is k+1 (for N=3) k: weight

There are 3 linealy independent modular forms for weight 2 , which forms A_4 triplet.

Fundamental domain of τ on SL(2,Z)



$$S = \frac{1}{3}\begin{pmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{pmatrix}$$

$$T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}$$

A_4 triplet of modular forms with weight 2

$$f_i(\gamma\tau) = (c\tau + d)^k \rho(\gamma)_{ij} f_j(\tau)$$

F. Feruglio, arXiv:1706.08749

$$\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)} - \frac{27\eta'(3\tau)}{\eta(3\tau)}\right), \\
Y_2(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega^2\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right), \\
Y_3(\tau) &= \frac{-i}{\pi}\left(\frac{\eta'(\tau/3)}{\eta(\tau/3)} + \omega\frac{\eta'((\tau+1)/3)}{\eta((\tau+1)/3)} + \omega^2\frac{\eta'((\tau+2)/3)}{\eta((\tau+2)/3)}\right),
\end{aligned}$$

$$\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n)$$

Dedekind eta-function

$$Y_2^2 + 2Y_1Y_3 = 0$$

$$\eta(-1/\tau) = \sqrt{-i\tau}\,\eta(\tau), \qquad \eta(\tau+1) = e^{i\pi/12}\eta(\tau)$$

$$\begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} = \begin{pmatrix} 1 + 12q + 36q^2 + 12q^3 + \dots \\ -6q^{1/3}(1 + 7q + 8q^2 + \dots) \\ -18q^{2/3}(1 + 2q + 5q^2 + \dots) \end{pmatrix}$$

$$q = e^{2\pi i\tau}$$

Modular forms with higher weights k=4, 6 … are constructed by them.

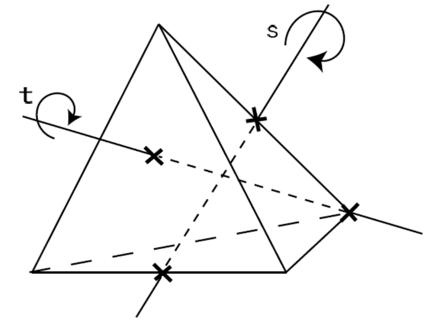
3 Modular invariant flavor model

We can construct quark / lepton mass matrices
in the framework of modular symmetry.

Non-Abelian Discrete Symmetry

A_4 group

Irreducible representations: 1, 1′, 1″, 3
The minimum group containing triplet



Symmetry of tetrahedron

It could be adjusted to Family Symmetry.

3: (e_L, μ_L, τ_L) , 1: e_R , 1″: μ_R , 1′: τ_R

Flavor symmetry should be broken !
We should know how to break the flavor symmetry.

Key : Modulus τ and Modular forms

We can construct a simple mass matrix by using weight 2 modular forms

A_4 assignments: left-handed doublet 3 right-handed singlets 1, 1", 1'

$$M_E = v_d \begin{pmatrix} \alpha_e & 0 & 0 \\ 0 & \beta_e & 0 \\ 0 & 0 & \gamma_e \end{pmatrix} \begin{pmatrix} Y_1 & Y_3 & Y_2 \\ Y_2 & Y_1 & Y_3 \\ Y_3 & Y_2 & Y_1 \end{pmatrix}_{RL}$$

Typical mass matrix of fermions by using weight 2 modular forms

Simple model of CP violation in Lepton sector

H.Okada, M.Tanimoto, JHEP 03(2021),010 [arXiv:2012.01688 [hep-ph]]

	L	(e^c,μ^c,τ^c)	H_u	H_d	$\mathbf{Y_r^{(2)}},\ \mathbf{Y_r^{(4)}}$
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{2}$	$\mathbf{1}$
A_4	$\mathbf{3}$	$(\mathbf{1},\ \mathbf{1}'',\ \mathbf{1}')$	$\mathbf{1}$	$\mathbf{1}$	$\mathbf{3},\ \{\mathbf{3},\mathbf{1},\mathbf{1}'\}$
$-k_I$	-2	$(0,\ 0,\ 0)$	0	0	$2,\quad 4$

$$M_E = v_d \begin{pmatrix} \alpha_e & 0 & 0 \\ 0 & \beta_e & 0 \\ 0 & 0 & \gamma_e \end{pmatrix} \begin{pmatrix} Y_1 & Y_3 & Y_2 \\ Y_2 & Y_1 & Y_3 \\ Y_3 & Y_2 & Y_1 \end{pmatrix}_{RL}$$

3, 1, 1'

$$w_\nu = -\frac{1}{\Lambda}(H_u H_u L L \mathbf{Y_r^{(k)}})_1$$

Weinberg operator by using weight 4 modular forms

5 modular forms

$$M_\nu = \frac{v_u^2}{\Lambda}\left[\begin{pmatrix} 2Y_1^{(4)} & -Y_3^{(4)} & -Y_2^{(4)} \\ -Y_3^{(4)} & 2Y_2^{(4)} & -Y_1^{(4)} \\ -Y_2^{(4)} & -Y_1^{(4)} & 2Y_3^{(4)} \end{pmatrix} + g_{\nu 1}\mathbf{Y_1^{(4)}}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + g_{\nu 2}\mathbf{Y_{1'}^{(4)}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}\right]_{LL}$$

of modular forms is k+1

weight 4 k=4

5 modular forms

$$Y_{\mathbf{1}}^{(4)}(\tau) = Y_1(\tau)^2 + 2Y_2(\tau)Y_3(\tau)\,, \qquad Y_{\mathbf{1'}}^{(4)}(\tau) = Y_3(\tau)^2 + 2Y_1(\tau)Y_2(\tau)\,,$$

$$Y_{\mathbf{1''}}^{(4)}(\tau) = Y_2(\tau)^2 + 2Y_1(\tau)Y_3(\tau) = 0\,, \qquad Y_{\mathbf{3}}^{(4)}(\tau) = \begin{pmatrix} Y_1^{(4)}(\tau) \\ Y_2^{(4)}(\tau) \\ Y_3^{(4)}(\tau) \end{pmatrix} = \begin{pmatrix} Y_1(\tau)^2 - Y_2(\tau)Y_3(\tau) \\ Y_3(\tau)^2 - Y_1(\tau)Y_2(\tau) \\ Y_2(\tau)^2 - Y_1(\tau)Y_3(\tau) \end{pmatrix}$$

CP symmetry

P. P. Novichkov, J. T. Penedo, S. T. Petcov and A. V. Titov, JHEP 07 (2019) 165

CP transformation

$$\tau \xrightarrow{\text{CP}} -\tau^* , \qquad \psi(x) \xrightarrow{\text{CP}} X_r \overline{\psi}(x_P) , \qquad \mathbf{Y}_{\mathbf{r}}^{(\mathrm{k})}(\tau) \xrightarrow{\text{CP}} \mathbf{Y}_{\mathbf{r}}^{(\mathrm{k})}(-\tau^*) = \mathbf{X}_{\mathbf{r}} \mathbf{Y}_{\mathbf{r}}^{(\mathrm{k})*}(\tau)$$

$$\mathbf{X}_{\mathbf{r}} \rho_{\mathbf{r}}^*(g) \mathbf{X}_{\mathbf{r}}^{-1} = \rho_{\mathbf{r}}(g') , \qquad g,\, g' \in G$$

$\mathbf{X}_{\mathbf{r}} = \mathbb{1}_{\mathbf{r}}$ can be taken in the base of symmetric S and T.

After fixing τ, real part of τ gives imaginary part of the mass matrices.

$$\begin{aligned} Y_1(\tau) &= 1 + 12q + 36q^2 + 12q^3 + \cdots , \\ Y_2(\tau) &= -6q^{1/3}(1 + 7q + 8q^2 + \cdots), \\ Y_3(\tau) &= -18q^{2/3}(1 + 2q + 5q^2 + \cdots). \end{aligned}$$

$$q = e^{2\pi i \tau}$$

$|q| \ll 1$

$$Y_2^2 + 2Y_1Y_3 = 0$$

$$M_E(\tau) \xrightarrow{CP} M_E(-\tau^*) = M_E^*(\tau) = v_d \begin{pmatrix} \alpha_e & 0 & 0 \\ 0 & \beta_e & 0 \\ 0 & 0 & \gamma_e \end{pmatrix} \begin{pmatrix} Y_1(\tau)^* & Y_3(\tau)^* & Y_2(\tau)^* \\ Y_2(\tau)^* & Y_1(\tau)^* & Y_3(\tau)^* \\ Y_3(\tau)^* & Y_2(\tau)^* & Y_1(\tau)^* \end{pmatrix}_{RL}$$

$$M_\nu(\tau) \xrightarrow{CP} M_\nu(-\tau^*) = M_\nu^*(\tau)$$

$$= \frac{v_u^2}{\Lambda} \left[\begin{pmatrix} 2Y_1^{(4)*}(\tau) & -Y_3^{(4)*}(\tau) & -Y_2^{(4)*}(\tau) \\ -Y_3^{(4)*}(\tau) & 2Y_2^{(4)*}(\tau) & -Y_1^{(4)*}(\tau) \\ -Y_2^{(4)*}(\tau) & -Y_1^{(4)*}(\tau) & 2Y_3^{(4)*}(\tau) \end{pmatrix} + g_1^{\nu *} \mathbf{Y}_{\mathbf{1}}^{(4)*}(\tau) \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} + g_2^{\nu *} \mathbf{Y}_{\mathbf{1'}}^{(4)*}(\tau) \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \right]$$

Impose CP symmetry Re τ=0 and boundary of fundamental region

$M_E(\tau) = M_E^*(\tau)\,,$ $M_\nu(\tau) = M_\nu^*(\tau)$ which leads to g_1^ν and g_2^ν being real.

6 parameters + τ = 8 parameters CP violation is realized by τ !

3 charged lepton masses+ 2 neutrino mass differences+ 3 mixing angles = 8

CP phase and mass absolute values can be predicted !

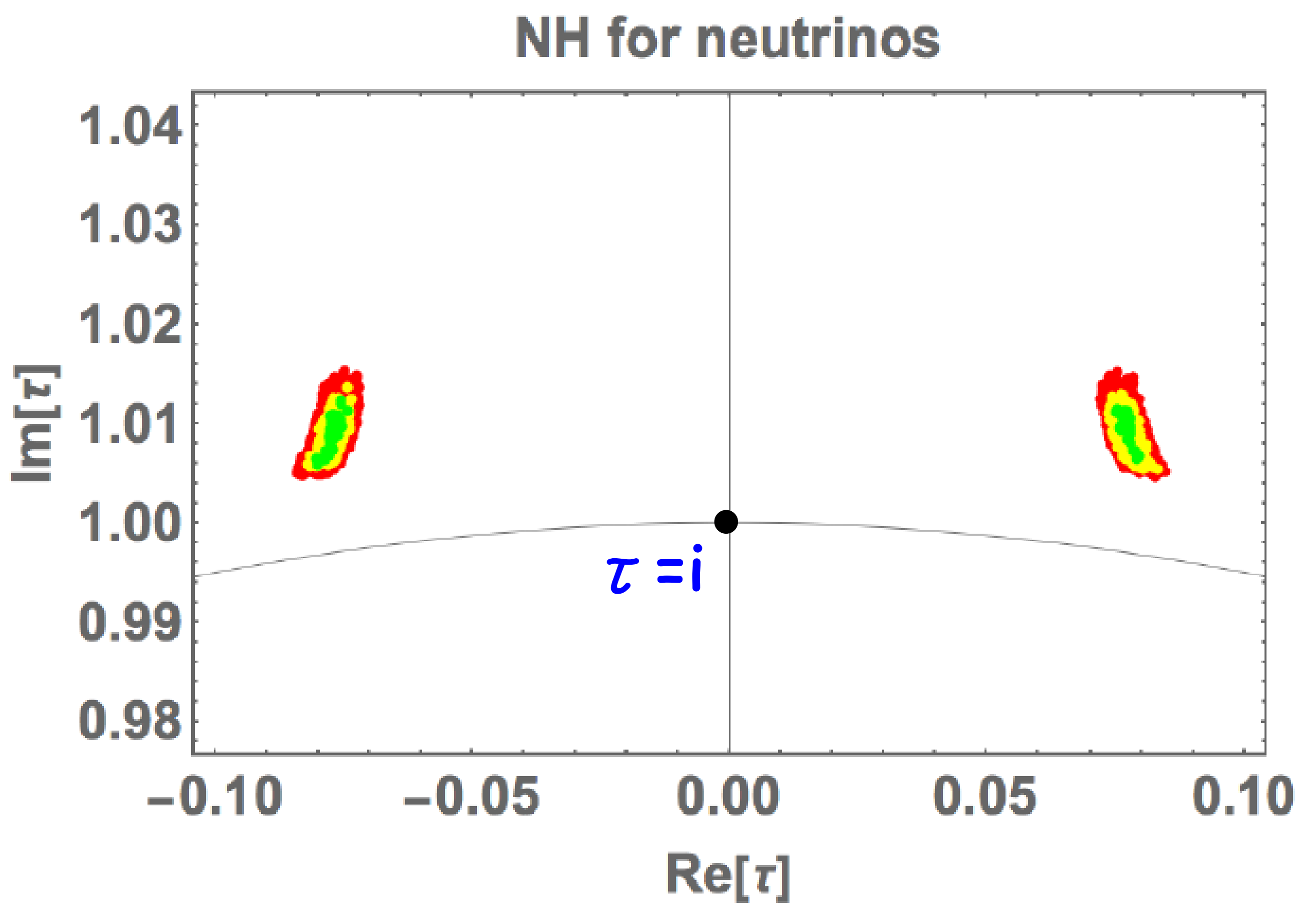
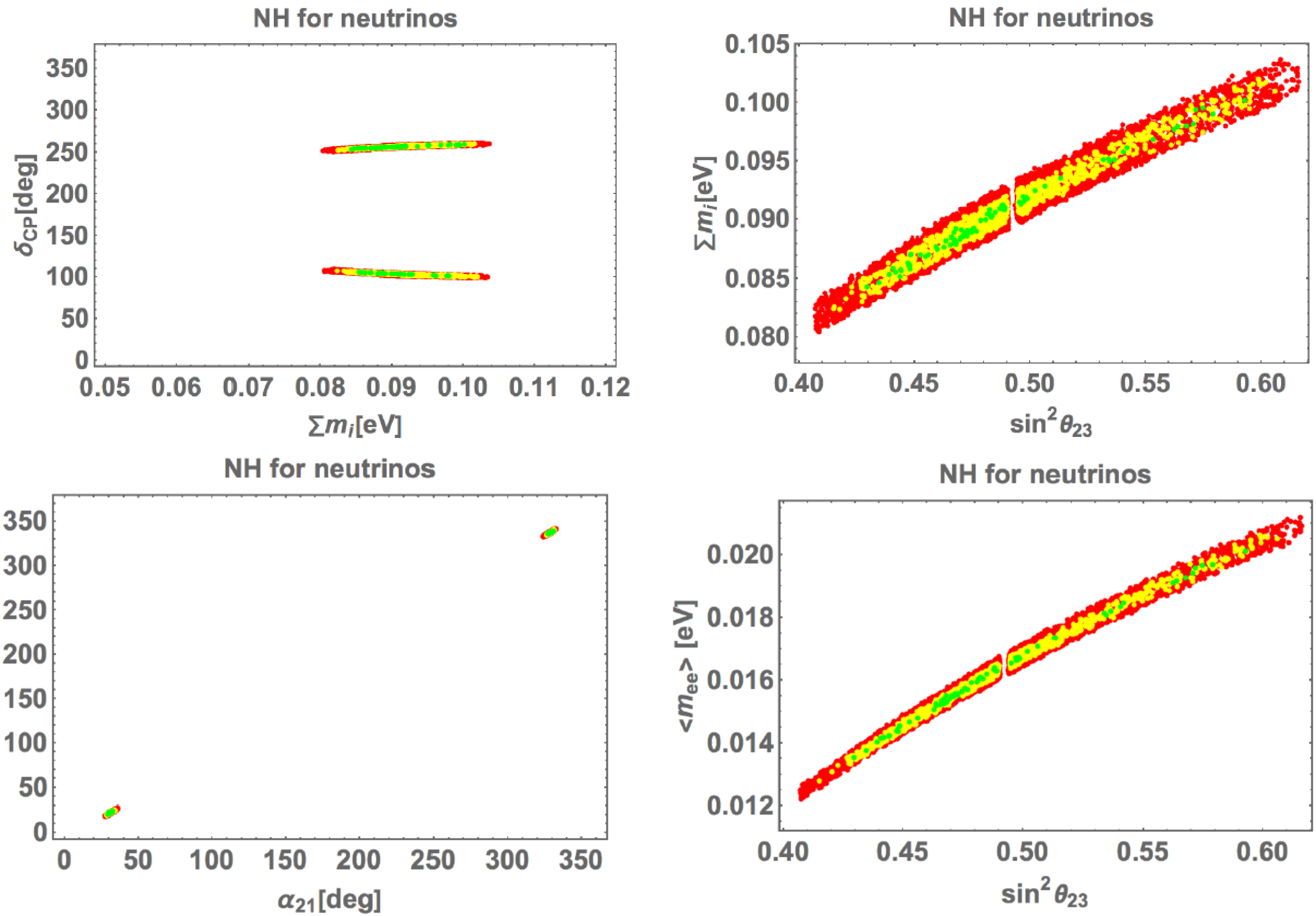


Figure 1: Allowed regions of τ for NH. Green, yellow and red correspond to 2σ, 3σ, 5σ confidence levels, respectively. The solid curve is the boundary of the fundamental domain, $|\tau| = 1$.

δ_{CP} is predicted clearly in $[98°, 110°]$ and $[250°, 262°]$ at $3\,\sigma$ confidence level.



Modular Symmetry meets

SM Effective Field Theory (SMEFT)

T. Kobayashi, H. Otsuka, Eur. Phys. J. C82 (2022) no.1, 25,
arXiv:2108.02700

"On stringy origin of Minimum Flavor Violation"

Kikuchi, Kobayashi, Nasu, Otsuka, Takada, Uchida, arXiv:2203.14667

"Modular symmetry of soft SUSY breaking terms"

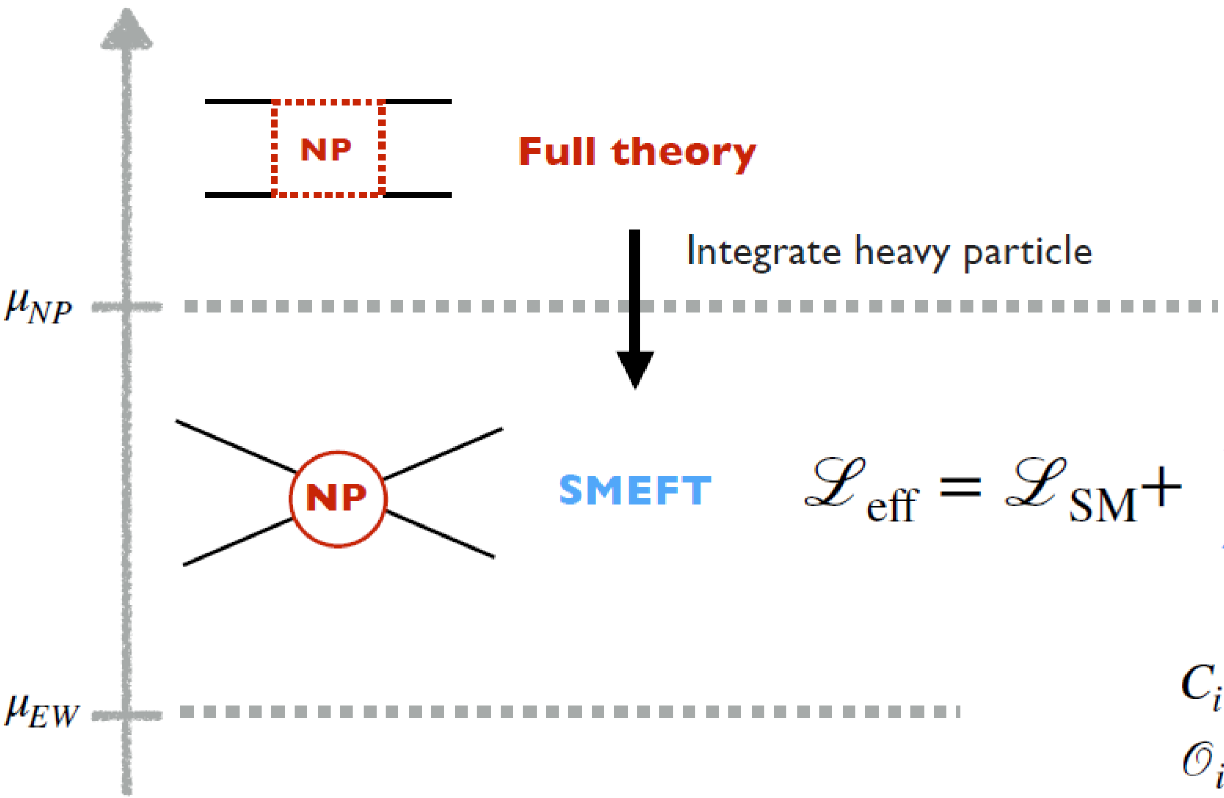
SUSY breaking terms are invariant (covariant) under modular transformation in moduli-mediated SUSY breaking scenario

We can consider modular invariant SMEFT
by supposing modular forms to be spurion !

SM Effective Field Theory (SMEFT)

B. Grzadkowski, M. Iskrzynski, M. Misiak and J. Rosiek [1008.4884]

- SMEFT is a effective theory based on $SU(3)_c \times SU(2)_L \times U(1)_Y$ at scale $\mu_{\rm EW} < \mu < \mu_{\rm NP}$



$$\mathscr{L}_{\rm eff} = \mathscr{L}_{\rm SM} + \sum_i \frac{C_i}{\Lambda^2} \mathscr{O}_i^{d=6}$$

C_i : Wilson coefficients

$\mathscr{O}_i$: Local operators

4 Dipole operators in modular symmetry

Stringy Ansatz

String compactifications leads to 4-dim low energy field theories with the specific structure:

T. Kobayashi, H. Otsuka, Eur. Phys. J. C82 (2022) no.1, 25, arXiv:2108.02700

4-point coupling comes from three point coupling in superstring theory *Kobayashi and Otsuka [2108.02700]*

m is virtual mode
H, ...

$$y^{(4)}_{ijk\ell} = \sum_m y^{(3)}_{ijm} y^{(3)}_{mk\ell}$$

→

$Q^{(1)}_{qq}$ $(\bar{q}_p \gamma_\mu q_r)(\bar{q}_s \gamma^\mu q_t)$

$Q^{(1)}_{\ell q}$ $(\bar{\ell}_p \gamma_\mu \ell_r)(\bar{q}_s \gamma^\mu q_t)$

Our model : A$_4$ modular symmetry

Assignments of weights

focus on charged-lepton sector (weight 2)

modular forms

	L_L	$(e_R^c,\ \mu_R^c,\ \tau_R^c)$	H_d	$Y(\tau_q),\ \ Y(\tau_e)$
$SU(2)$	$\mathbf{2}$	$\mathbf{1}$	$\mathbf{2}$	$\mathbf{1}$
A_4	$\mathbf{3}$	$(\mathbf{1},\ \mathbf{1}'',\ \mathbf{1}')$	$\mathbf{1}$	$\mathbf{3}$
k	2	(0, 0, 0)	0	2

k_i : modular weights

$\mathscr{L}_{\text{eff}} \supset Y(\tau)_{ij}\phi\bar{\psi}_i\psi_j$ modular invariant

Modular forms

The holomorphic and anti-holomorphic modular forms with weight 2 compose the A_4 triplet

$$Y_{\mathbf{3}}^{(2)}(\tau) = \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix}, \qquad \overline{Y_{\mathbf{3}}^{(2)}}(\tau) \equiv Y_{\mathbf{3}}^{(2)*}(\tau) = \begin{pmatrix} Y_1^*(\tau) \\ Y_3^*(\tau) \\ Y_2^*(\tau) \end{pmatrix}$$

$$[\bar{L}_R L_L] \longrightarrow [\bar{L}_R Y(\tau_q) L_L]$$

A_4 : $\{1,1'',1'\} \otimes 3$ $\quad$ $\{1,1'',1'\} \otimes 3 \otimes 3$

k_I : 0 2 $\quad$ 0 2 2

$(\bar{L}R)$ structure in the modular symmetry

$$[\bar{L}_R \otimes Y(\tau) \otimes L_L]_{\mathbf{1}} = \alpha_e\, \bar{e}_R \otimes (Y_1 e_L + Y_2 \tau_L + Y_3 \mu_L)_{\mathbf{1}} + \beta_e\, \bar{\mu}_R \otimes (Y_2 \mu_L + Y_1 \tau_L + Y_3 e_L)_{\mathbf{1}''}$$

$$+\gamma_e\, \bar{\tau}_R \otimes (Y_3 \tau_L + Y_1 \mu_L + Y_2 e_L)_{\mathbf{1}'}$$

$$= (\bar{e}_R, \bar{\mu}_R, \bar{\tau}_R) \begin{pmatrix} \alpha_e & 0 & 0 \\ 0 & \beta_e & 0 \\ 0 & 0 & \gamma_e \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix} \begin{pmatrix} e_L \\ \mu_L \\ \tau_L \end{pmatrix}$$

Same structure with mass matrix :

$$M_e = v_d \begin{pmatrix} \alpha_{e(m)} & 0 & 0 \\ 0 & \beta_{e(m)} & 0 \\ 0 & 0 & \gamma_{e(m)} \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix}_{RL}$$

Stringy Ansatz leads to

$$\alpha_e = \kappa \alpha_{e(m)}, \qquad \beta_e = \kappa \beta_{e(m)}, \qquad \gamma_e = \kappa \gamma_{e(m)}$$

if mode m is only Higgs.

Then, FC transition vanish in the mass basis. μ→eϒ never happen !

However, additional unknown modes (NP) causes flavor violations (for example, multi Higgs modes).

Suppose unknown mode contribution being small and couplings are Higgs-like.

$$\alpha_e - \alpha_{e(m)} \ll \alpha_e , \qquad \beta_e - \beta_{e(m)} \ll \beta_e , \qquad \gamma_e - \gamma_{e(m)} \ll \gamma_e .$$

$$\frac{\tilde{\beta}_e}{\tilde{\beta}_{e(m)}} = \frac{\tilde{\beta}_{e(m)} + c_\beta}{\tilde{\beta}_{e(m)}} = 1 + \frac{c_\beta}{\tilde{\beta}_{e(m)}} \equiv 1 + \delta_\beta ,$$
$$\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}} = \frac{\tilde{\alpha}_{e(m)} + c_\alpha}{\tilde{\alpha}_{e(m)}} = 1 + \frac{c_\alpha}{\tilde{\alpha}_{e(m)}} \equiv 1 + \delta_\alpha ,$$
$$\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}} = \frac{\tilde{\gamma}_{e(m)} + c_\gamma}{\tilde{\gamma}_{e(m)}} = 1 + \frac{c_\gamma}{\tilde{\gamma}_{e(m)}} \equiv 1 + \delta_\gamma ,$$

δ ’s are very small.

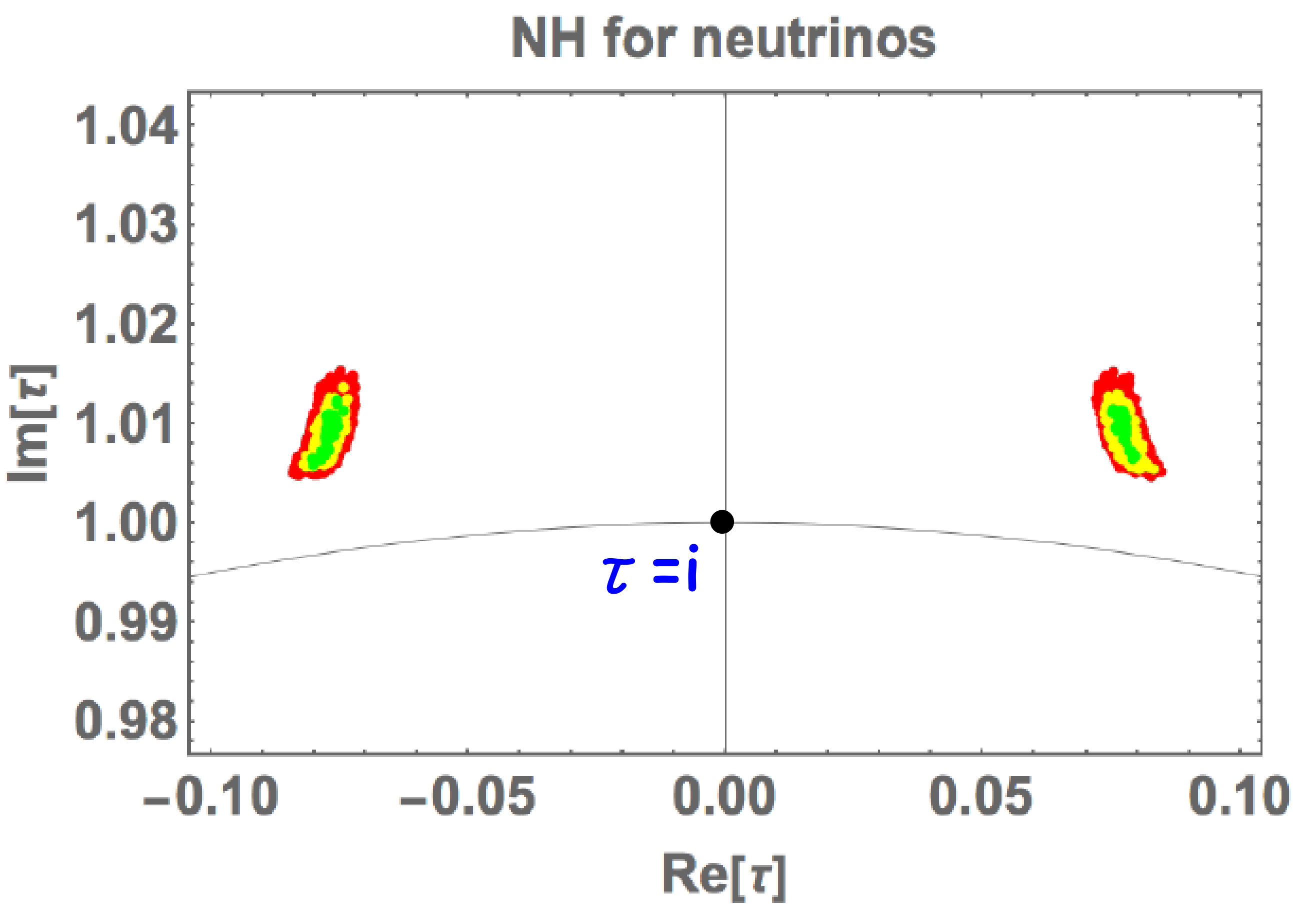


Figure 1: Allowed regions of τ for NH. Green, yellow and red correspond to 2σ, 3σ, 5σ confidence levels, respectively. The solid curve is the boundary of the fundamental domain, $|\tau| = 1$.

Mass matrix of charged leptons at nearby τ= i

$$M_e = v_d \begin{pmatrix} \alpha_{e(m)} & 0 & 0 \\ 0 & \beta_{e(m)} & 0 \\ 0 & 0 & \gamma_{e(m)} \end{pmatrix} \begin{pmatrix} Y_1(\tau) & Y_3(\tau) & Y_2(\tau) \\ Y_2(\tau) & Y_1(\tau) & Y_3(\tau) \\ Y_3(\tau) & Y_2(\tau) & Y_1(\tau) \end{pmatrix}_{RL}$$

$$Y(\tau_e = i) = Y_1(i) \begin{pmatrix} 1 \\ 1-\sqrt{3} \\ -2+\sqrt{3} \end{pmatrix}$$

$$\tau = i + \epsilon$$

$$\epsilon_1 = \frac{1}{2}\epsilon_2 = 2.05\, i\, \epsilon$$

$$\frac{Y_2(\tau)}{Y_1(\tau)} \simeq (1+\epsilon_1)\,(1-\sqrt{3})$$

$$\frac{Y_3(\tau)}{Y_1(\tau)} \simeq (1+\epsilon_2)\,(-2+\sqrt{3})$$

Move to mass eigenstate

$$E_L \to E_L^m \equiv U_{Lme}^\dagger U_S\, E_L\,, \qquad \bar{E}_L \to \bar{E}_L^m \equiv \bar{E}_L\, U_S^T U_{Lme}\,,$$

$$E_R \to E_R^m \equiv U_{Rme}^\dagger\, U_{12}^T\, E_R\,, \qquad \bar{E}_R \to \bar{E}_R^m \equiv \bar{E}_R\, U_{12}\, U_{Rme}\,,$$

$$U_S = \frac{1}{2\sqrt{3}} \begin{pmatrix} 2 & 2 & 2 \\ \sqrt{3}+1 & -2 & \sqrt{3}-1 \\ \sqrt{3}-1 & -2 & \sqrt{3}+1 \end{pmatrix}, \qquad U_{12} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$U_{Lme} \simeq P_e^* \begin{pmatrix} 1 & s_{L12}^e & s_{L13}^e \\ -s_{L12}^e & 1 & s_{L23}^e \\ s_{L12}^e s_{L23}^e - s_{L13}^e & -s_{L23}^e & 1 \end{pmatrix}, \quad U_{Rme} \simeq \begin{pmatrix} 1 & s_{R12}^e & s_{R13}^e \\ -s_{R12}^e & 1 & s_{R23}^e \\ s_{R12}^e s_{R23}^e - s_{R13}^e & -s_{R23}^e & 1 \end{pmatrix}$$

$$P_e = \begin{pmatrix} e^{i\eta_e} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}, \qquad \eta_e = \arg \epsilon_1\,.$$

$\bar{\mu}_R\Gamma\tau_L$ $\bar{\mu}_L\Gamma\tau_R$	$\bar{e}_R\Gamma\tau_L$ $\bar{e}_L\Gamma\tau_R$	$\bar{e}_R\Gamma\mu_L$ $\bar{e}_L\Gamma\mu_R$
$\frac{\sqrt{3}}{2}(\tilde{\alpha}_e + 2s^e_{R23}\tilde{\gamma}_e)$ $(\sqrt{3}s^e_{23L} + s^e_{12L}\lvert\epsilon_1^*\rvert)\tilde{\gamma}_e + \frac{3}{2}s^e_{R23}\tilde{\alpha}_e$	$\frac{\sqrt{3}}{2}(\tilde{\beta}_e - s^e_{12R}\tilde{\alpha}_e + 2(s^e_{R13} - s^e_{R12}s^e_{R23})\tilde{\gamma}_e)$ $(\sqrt{3}s^e_{13L} + \lvert\epsilon_1^*\rvert)\tilde{\gamma}_e$	$\frac{3}{2}(\tilde{\beta}_e + s^e_{12R}\tilde{\alpha}_e)$ $\frac{1}{2}(3s^e_{12L} - \sqrt{3}s^e_{13L} + 2\lvert\epsilon_1^*\rvert)\tilde{\alpha}_e$

$$\frac{\sqrt{3}}{2}(\tilde{\alpha}_e - \tilde{\alpha}_{e(m)})$$

A common overall factor $(1-\sqrt{3})$ is omitted

$$s^e_{L12} \simeq -\lvert\epsilon_1^*\rvert\,, \qquad s^e_{L23} \simeq -\frac{\sqrt{3}}{4}\frac{\tilde{\alpha}^2_{e(m)}}{\tilde{\gamma}^2_{e(m)}}\,, \qquad s^e_{L13} \simeq -\frac{\sqrt{3}}{3}\lvert\epsilon_1^*\rvert\,,$$

$$s^e_{R12} \simeq -\frac{\tilde{\beta}_{e(m)}}{\tilde{\alpha}_{e(m)}}\,, \qquad s^e_{R23} \simeq -\frac{1}{2}\frac{\tilde{\alpha}_{e(m)}}{\tilde{\gamma}_{e(m)}}\,, \qquad s^e_{R13} \simeq -\frac{1}{2}\frac{\tilde{\beta}_{e(m)}}{\tilde{\gamma}_{e(m)}}$$

$\tilde{\alpha}_{e(m)} = (6-3\sqrt{3})Y_1(i)\alpha_{e(m)}$, $\tilde{\beta}_{e(m)} = (6-3\sqrt{3})Y_1(i)\beta_{e(m)}$ and $\tilde{\gamma}_{e(m)} = (6-3\sqrt{3})Y_1(i)\gamma_{e(m)}$

$$\tau = -0.080 + 1.007\,i\,, \quad \lvert\epsilon_1\rvert = 0.165\,, \quad \frac{\tilde{\alpha}_{e(m)}}{\tilde{\gamma}_{e(m)}} \simeq \frac{\tilde{\alpha}_e}{\tilde{\gamma}_e} = 6.82\times 10^{-2}\,, \quad \frac{\tilde{\beta}_{e(m)}}{\tilde{\alpha}_{e(m)}} \simeq \frac{\tilde{\beta}_e}{\tilde{\alpha}_e} = 1.50\times 10^{-2}\,.$$

Best fit values of parameters in A_4 modular invariant model

Dipole operators in mass basis

$$\mathcal{O}_{\substack{e\gamma \\ LR}} = \frac{v}{\sqrt{2}} \overline{E}_L \sigma^{\mu\nu} E_R F_{\mu\nu} \,, \qquad \mathcal{C}'_{\substack{e\gamma \\ LR}} = \begin{pmatrix} \mathcal{C}'_{\substack{e\gamma \\ ee}} & \mathcal{C}'_{\substack{e\gamma \\ e\mu}} & \mathcal{C}'_{\substack{e\gamma \\ e\tau}} \\ \mathcal{C}'_{\substack{e\gamma \\ \mu e}} & \mathcal{C}'_{\substack{e\gamma \\ \mu\mu}} & \mathcal{C}'_{\substack{e\gamma \\ \mu\tau}} \\ \mathcal{C}'_{\substack{e\gamma \\ \tau e}} & \mathcal{C}'_{\substack{e\gamma \\ \tau\mu}} & \mathcal{C}'_{\substack{e\gamma \\ \tau\tau}} \end{pmatrix}$$

$$\mathcal{O}_{\substack{e\gamma \\ RL}} = \frac{v}{\sqrt{2}} \overline{E}_R \sigma^{\mu\nu} E_L F_{\mu\nu} \,, \qquad \mathcal{C}'_{\substack{e\gamma \\ RL}} = \mathcal{C}'^{\dagger}_{\substack{e\gamma \\ LR}} \,,$$

4-point couplings   LRHW(B)

$$\mathcal{L}_{\text{dipole}} = \frac{1}{\Lambda^2} \left(\mathcal{C}'_{\substack{e\gamma \\ LR}} \mathcal{O}_{\substack{e\gamma \\ LR}} + \mathcal{C}'_{\substack{e\gamma \\ RL}} \mathcal{O}_{\substack{e\gamma \\ RL}} \right)$$

$$\mathcal{C}'_{\underset{\tau\mu}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\alpha}_e \left(1 - \frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\frac{\tilde{\alpha}_{e(m)}}{\tilde{\alpha}_e}\right),$$

$$\mathcal{C}'_{\underset{\tau e}{e\gamma}} = \frac{\sqrt{3}}{2}(1-\sqrt{3})\tilde{\beta}_e \left(1 + \frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e} - 2\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}}\right),$$

$$\mathcal{C}'_{\underset{\mu e}{e\gamma}} = \frac{3}{2}(1-\sqrt{3})\tilde{\beta}_e \left(1 - \frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right),$$

Absolute values of Wilson coefficients is unknown unless NP is specified.

Diagonal elements are given as:

$$\mathcal{C}'_{\underset{ee}{e\gamma}} = 3\,(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|\,, \qquad \mathcal{C}'_{\underset{\mu\mu}{e\gamma}} = \frac{3}{2}\,(1-\sqrt{3})\tilde{\alpha}_e\,, \qquad \mathcal{C}'_{\underset{\tau\tau}{e\gamma}} = \sqrt{3}\,(1-\sqrt{3})\tilde{\gamma}_e$$

5 Flavor structure of Dipole operators

Experimental constraints of Dipole operators

G. Isidori, J. Pages and F. Wilsch, JHEP03(2022) , arXiv:2111.13724

muon (g-2)

$$\Delta a_\mu = a_\mu^{\rm Exp} - a_\mu^{\rm SM} = (251 \pm 59) \times 10^{-11}$$

$$\Delta a_\mu = \frac{4m_\mu}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\mathrm{Re}\,[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}] \quad \Rightarrow \quad \frac{1}{\Lambda^2}\mathrm{Re}\,[\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}] \approx 1.0 \times 10^{-5}\,\mathrm{TeV}^{-2}$$

μ→eϒ

$$\mathcal{B}(\mu^+ \to e^+\gamma) < 4.2 \times 10^{-13} \ (90\% \ \mathrm{C.L.})$$

$$\mathcal{B}(\ell_r \to \ell_s\gamma) = \frac{m_{\ell_r}^3 v^2}{8\pi\Gamma_{\ell_r}}\frac{1}{\Lambda^4}\left(|\mathcal{C}'_{\underset{rs}{e\gamma}}|^2 + |\mathcal{C}'_{\underset{sr}{e\gamma}}|^2\right) \quad \Rightarrow \quad \frac{1}{\Lambda^2}|\mathcal{C}'_{\underset{e\mu(\mu e)}{e\gamma}}| < 2.1 \times 10^{-10}\,\mathrm{TeV}^{-2}$$

$$\left|\frac{\mathcal{C}'_{\underset{e\mu(\mu e)}{e\gamma}}}{\mathcal{C}'_{\underset{\mu\mu}{e\gamma}}}\right| < 2.1 \times 10^{-5}$$

muon and electron (g-2)

$$\mathcal{C}'_{\substack{e\gamma\\ee}} = 3\,(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*|\,, \qquad \mathcal{C}'_{\substack{e\gamma\\\mu\mu}} = \frac{3}{2}\,(1-\sqrt{3})\tilde{\alpha}_e\,, \qquad \mathcal{C}'_{\substack{e\gamma\\\tau\tau}} = \sqrt{3}\,(1-\sqrt{3})\tilde{\gamma}_e$$

$$\frac{\mathcal{C}'_{\substack{e\gamma\\ee}}}{\mathcal{C}'_{\substack{e\gamma\\\mu\mu}}} = 2\frac{\tilde{\beta}_e}{\tilde{\alpha}_e}|\epsilon_1^*| \simeq 4.9\times 10^{-3}\,, \qquad \frac{\mathcal{C}'_{\substack{e\gamma\\\mu\mu}}}{\mathcal{C}'_{\substack{e\gamma\\\tau\tau}}} = \frac{\sqrt{3}}{2}\frac{\tilde{\alpha}_e}{\tilde{\gamma}_e} \simeq 5.9\times 10^{-2}\,,$$

$$\Delta a_e = \frac{4m_e}{e}\frac{v}{\sqrt{2}}\frac{1}{\Lambda^2}\mathrm{Re}\,[\mathcal{C}'_{\substack{e\gamma\\ee}}] \simeq 5.8\times 10^{-14}$$

This result is agreement with the naive mass scaling $\Delta a_\ell \propto m_\ell^2$

observations

$$\Delta a_e^{Cs} = a_e^{\mathrm{Exp}} - a_e^{\mathrm{SM,CS}} = (-8.8\pm 3.6)\times 10^{-13}\,,$$
$$\Delta a_e^{Rb} = a_e^{\mathrm{Exp}} - a_e^{\mathrm{SM,Rb}} = (4.8\pm 3.0)\times 10^{-13}\,.$$

Wait for future measurements !

muon (g-2) versus μ→eγ

G. Isidori et al.

$$\left|\frac{C'_{\substack{e\gamma\\e\mu}}}{C'_{\substack{e\gamma\\\mu\mu}}}\right| = \frac{\tilde{\beta}_e}{\tilde{\alpha}_e}\left|1 - \frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right|$$ **< 2.1 x 10^{-5}**

$$\left|1 - \frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}}\frac{\tilde{\beta}_{e(m)}}{\tilde{\beta}_e}\right| \simeq |\delta_\beta - \delta_\alpha| < 1.4 \times 10^{-3}$$

$$\frac{\tilde{\beta}_e}{\tilde{\beta}_{e(m)}} = \frac{\tilde{\beta}_{e(m)} + c_\beta}{\tilde{\beta}_{e(m)}} = 1 + \frac{c_\beta}{\tilde{\beta}_{e(m)}} \equiv 1 + \delta_\beta\,,$$

$$\frac{\tilde{\alpha}_e}{\tilde{\alpha}_{e(m)}} = \frac{\tilde{\alpha}_{e(m)} + c_\alpha}{\tilde{\alpha}_{e(m)}} = 1 + \frac{c_\alpha}{\tilde{\alpha}_{e(m)}} \equiv 1 + \delta_\alpha\,,$$

$$\frac{\tilde{\gamma}_e}{\tilde{\gamma}_{e(m)}} = \frac{\tilde{\gamma}_{e(m)} + c_\gamma}{\tilde{\gamma}_{e(m)}} = 1 + \frac{c_\gamma}{\tilde{\gamma}_{e(m)}} \equiv 1 + \delta_\gamma\,,$$

τ→μγ and τ→eγ

$$\frac{\mathcal{C}'_{\underset{\tau e}{e\gamma}}}{\mathcal{C}'_{\underset{\mu e}{e\gamma}}} = \frac{1}{\sqrt{3}} \times \mathcal{O}(1)\,, \qquad \frac{\mathcal{C}'_{\underset{\tau e}{e\gamma}}}{\mathcal{C}'_{\underset{\tau \mu}{e\gamma}}} = \frac{\tilde{\beta}_e}{\tilde{\alpha}_e} \times \mathcal{O}(1) \sim 10^{-2}$$

$$\mathcal{B}(\tau \to \mu\gamma) : \mathcal{B}(\tau \to e\gamma) : \mathcal{B}(\mu \to e\gamma) \sim \;\; 10^4 \; : \; 1 \; : \; 10.$$

Present experimental upper bounds

$\mathcal{B}(\tau \to e\gamma)$ and $\mathcal{B}(\tau \to \mu\gamma)$ are 3.3×10^{-8} and 4.4×10^{-8}

6 Summary

Modular flavor symmetry meets SMEFT.

We need more studies of SMEFT with modular symmetry.

We should check whether our results is model dependent?

other models with S_4, A_5 ...

Approach to other flavor phenomena
in the quark sector $b \to s \gamma$ ...

Back up slides

Predictions at $U(2)$ case

$$\mathrm{BR}(\tau \to \mu\gamma) \gg \mathrm{BR}(\mu \to e\gamma) \gg \mathrm{BR}(\tau \to e\gamma)$$

Different prediction

Predictions at Modular $\tau = i$ case

$$\mathrm{BR}(\tau \to \mu\gamma) \gg \mathrm{BR}(\mu \to \mathrm{e}\gamma) \sim \mathrm{BR}(\tau \to \mathrm{e}\gamma)$$

U(2) prediction of Lepton Flavor Violation

	$\mu \to e\gamma$	$\tau \to \mu\gamma$	$\tau \to e\gamma$
$\bar{R}L$	$(\rho_1 s_e \delta'_e)^* [\bar{e}_R \sigma^{\mu\nu} \mu_L]$	$(\sigma_1 \epsilon_\ell \delta_e)^* [\bar{\mu}_R \sigma^{\mu\nu} \tau_L]$	$(\sigma_1 \epsilon_\ell s_e \delta'_e)^* [\bar{e}_R \sigma^{\mu\nu} \tau_L]$
$\bar{L}R$	$-\rho_1 s_e \delta_e [\bar{e}_L \sigma^{\mu\nu} \mu_R]$	$\beta_1 \epsilon_\ell [\bar{\mu}_L \sigma^{\mu\nu} \tau_R]$	-

Spurion order count

$$1 > \epsilon_i > \delta_i > \delta'_i > 0$$

Predictions at $U(2)$ case

$$\mathrm{BR}(\tau \to \mu\gamma) \gg \mathrm{BR}(\mu \to e\gamma) \gg \mathrm{BR}(\tau \to e\gamma)$$

EDM of electron

$$|d_e/e| \lesssim 1.1 \times 10^{-29}\,\mathrm{cm} = 5.6 \times 10^{-13}\,\mathrm{TeV}^{-1}$$

$$\mathcal{O}_{\mathrm{edm}} = -\frac{i}{2}\, d_e(\mu)\, \overline{e}\sigma^{\mu\nu}\gamma_5 e F_{\mu\nu}\,,$$

$$\mathcal{L}_{\mathrm{EDM}} = \frac{1}{\Lambda^2}\mathcal{C}'_{\substack{e\gamma\\ ee}}\mathcal{O}_{\substack{e\gamma\\ LR}} = \frac{1}{\Lambda^2}\mathcal{C}'_{\substack{e\gamma\\ ee}}\frac{v}{\sqrt{2}}\overline{e}_L\sigma^{\mu\nu}e_R F_{\mu\nu}$$

$$d_e = -\sqrt{2}\,\frac{v}{\Lambda^2}\,\mathrm{Im}\,[\mathcal{C}'_{\substack{e\gamma\\ ee}}]$$

Experimental constraint from electron EDM

$$\frac{1}{\Lambda^2}\mathrm{Im}\,[\mathcal{C}'_{\substack{e\gamma\\ ee}}] < 1.6\times 10^{-12}\,\mathrm{TeV}^{-2}$$

(g-2) gives

$$\frac{\mathrm{Im}\,[\mathcal{C}'_{\substack{e\gamma\\ ee}}]}{\mathrm{Re}\,[\mathcal{C}'_{\substack{e\gamma\\ ee}}]} \simeq (\mathrm{Im}\,\delta_\beta) < \frac{1.6\times 10^{-12}}{4.9\times 10^{-8}} = 3.3\times 10^{-5}$$

Since $\mathcal{C}'_{\substack{e\gamma\\ ee}} = 3\,(1-\sqrt{3})\tilde{\beta}_e|\epsilon_1^*| = 3\,(1-\sqrt{3})\tilde{\beta}_{e(m)}(1+\delta_\beta)|\epsilon_1^*|$

We get $\mathrm{Im}\,[\mathcal{C}'_{\substack{e\gamma\\ ee}}] \simeq 3\,(1-\sqrt{3})\tilde{\beta}_{e(m)}(\mathrm{Im}\,\delta_\beta)|\epsilon_1^*|\,,\qquad \mathcal{C}'_{\substack{e\gamma\\ \mu e}} \simeq \frac{3}{2}(1-\sqrt{3})\tilde{\beta}_{e(m)}(\delta_\beta-\delta_\alpha)\,.$

Then, we have prediction $\mathcal{B}(\mu^+\to e^+\gamma) < 2.3\times 10^{-16}.$

ν_L ν_R

$$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}_{\mathbf{3}} \otimes \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}_{\mathbf{3}} = (a_1b_1 + a_2b_3 + a_3b_2)_{\mathbf{1}} \oplus (a_3b_3 + a_1b_2 + a_2b_1)_{\mathbf{1'}}$$

$$\oplus (a_2b_2 + a_1b_3 + a_3b_1)_{\mathbf{1''}}$$

$$\oplus \frac{1}{3}\begin{pmatrix} 2a_1b_1 - a_2b_3 - a_3b_2 \\ 2a_3b_3 - a_1b_2 - a_2b_1 \\ 2a_2b_2 - a_1b_3 - a_3b_1 \end{pmatrix}_{\mathbf{3}} \oplus \frac{1}{2}\begin{pmatrix} a_2b_3 - a_3b_2 \\ a_1b_2 - a_2b_1 \\ a_1b_3 - a_3b_1 \end{pmatrix}_{\mathbf{3}}.$$

symmetric × 3_Y anti-symmetric × 3_Y