Scotogenic $A_5 \to A_4$ Dirac Neutrinos

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Contents

Two Useful Ideas

(A) Neutrino masses exist because of dark matter, i.e. scotogenically. [Ma, PRD 73, 077301 (2006)]

For Majorana neutrinos, dark parity may be related to lepton parity: $\pi_D = (-1)^{L+2j}$, where j is the intrinsic spin of the particle. [Ma, PRL 115, 011801 (2015)]

For Dirac neutrinos, dark number may be related to lepton number: $D = L - (2j)_{[mod\ 2]}$.
[Ma, PLB 809, 135736 (2020)]

(B) Non-Abelian discrete flavor symmetries such as A_4 allow all charged-lepton masses to be different, and yet predict a pattern for neutrino masses and mixing.

[Ma/Rajasekaran, PRD 64, 113012 (2001)]

Up to 2012, the focus of model building was

Tribimaximal: $\theta_{13} = 0$, $\sin^2\theta_{23} = 1/2$, $\sin^2\theta_{12} = 1/3$.

[Harrison/Perkins/Scott, PLB 530, 167 (2002)]

At present, the data are closer to

Cobimaximal: $\theta_{13} \neq 0$, $\sin^2\theta_{23} = 1/2$, $\delta_{CP} = \pm\pi/2$.

[Babu/Ma/Valle, PLB 552, 207 (2003);

Grimus/Lavoura, PLB 579, 113 (2004)]

Tetrahedron and Pentatope

In 3 dimensions, the points $(1,0,0)$, $(0,1,0)$, $(0,0,1)$, $(1,1,1)$ form a tetrahedron with the symmetry A_4.
It has the irreducible representations $\underline{1}$, $\underline{1}'$, $\underline{1}''$, $\underline{3}$.
In the decomposition $\underline{3} \times \underline{3} = \underline{1} + \underline{1}' + \underline{1}'' + \underline{3} + \underline{3}$,

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

is obtained with $\omega = \exp(2\pi i/3) = -1/2 + i\sqrt{3}/2$.
[Cabibbo, PLB 72, 333 (1978); Wolfenstein, PRD 18, 958 (1978)]

In most applications, the charged-lepton mass matrix is diagonalized on the left by U_ω, then the PMNS matrix is obtained from multiplying $U_\omega^\dagger$ to the unitary matrix U_ν diagonalizing the neutrino mass matrix also on the left. If

$$U_\nu = \begin{pmatrix} 0 & 1 & 0 \\ 1/\sqrt{2} & 0 & -i/\sqrt{2} \\ 1/\sqrt{2} & 0 & i/\sqrt{2} \end{pmatrix},$$

then tribimaximal mixing occurs. [Ma, PRD 70, 031901R (2004)] If U_ν is an orthogonal matrix, then cobimaximal mixing occurs. [Fukuura/Miura/Takasugi/Yoshimura, PRD 61, 073002 (2000)]

In 4 dimensions, the points $(2,0,0,0)$, $(0,2,0,0)$, $(0,0,2,0)$, $(0,0,0,2)$, $(\varphi,\varphi,\varphi,\varphi)$, where $\varphi=(1+\sqrt{5})/2=1.618$ is the golden ratio, form a pentatope with the symmetry A_5.

[Everett/Stuart, PRD 79, 085005 (2009)].

It has the irreducible representations $\underline{1}$, $\underline{3}$, $\underline{3}'$, $\underline{4}$, $\underline{5}$.

Their multiplication rules are $3\times 3 = 1+3+5$, $3'\times 3' = 1+3'+5$, $3\times 3' = 4+5$, $3\times 4 = 3'+4+5$, $3'\times 4 = 3+4+5$, $4\times 4 = 1+3+3'+4+5$, etc.

The decompositions to A_4 representations are $1\sim 1$, $3\sim 3$, $3'\sim 3$, $4\sim 3+1$, $5\sim 3+1'+1''$.

Model Particle Content

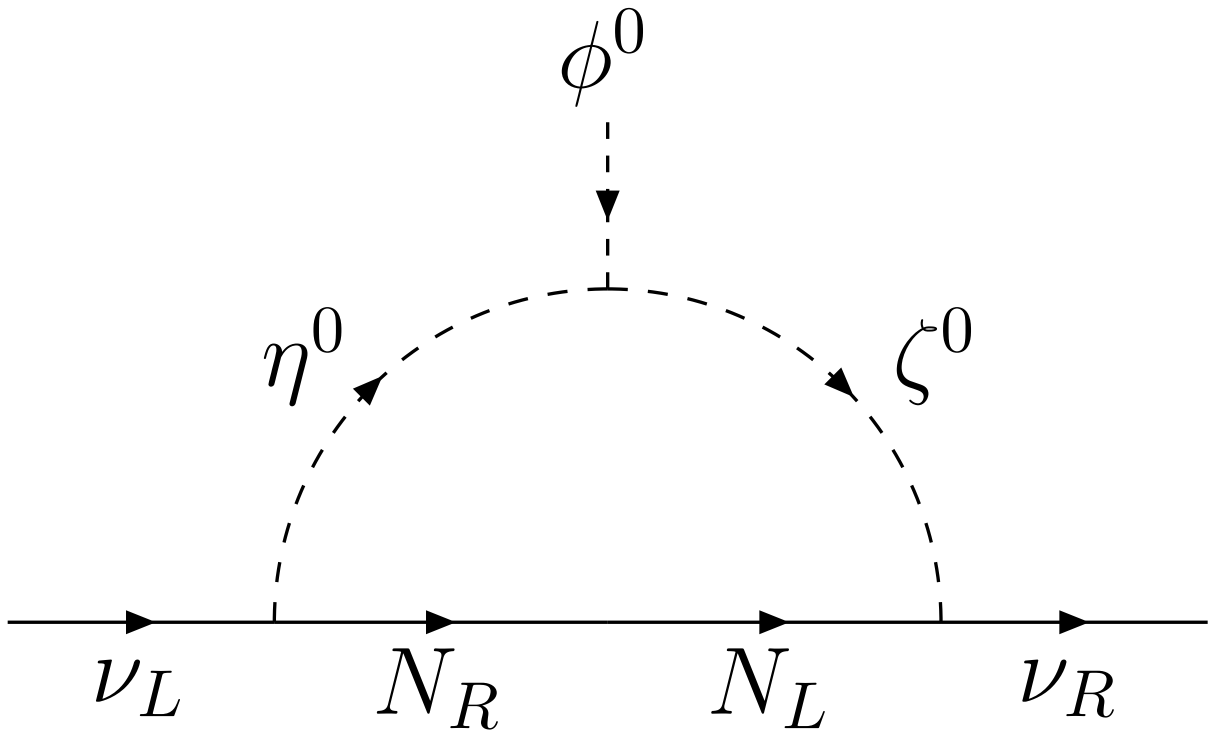
Consider the following particle content:

fermion/scalar	$SU(2)_L \times U(1)_Y$	A_5	A_4	L	$D = L - 2j$
$L_L = (\nu, l)_L$	$(2, -1/2)$	3	3	1	0
l_R	$(1, -1)$	3	3	1	0
ν_R	(1,0)	$3'$	3	1	0
$E_{L,R}$	$(1, -1)$	3	3	1	0
$\Phi = (\phi^+, \phi^0)$	(2,1/2)	1	1	0	0
$N_{L,R}$	(1,0)	4	3,1	0	-1
$\eta = (\eta^0, \eta^-)$	$(2, -1/2)$	$3'$	3	1	1
ζ^0	(1,0)	3	3	1	1
ζ^-	$(1, -1)$	$3'$	3	1	1

There is only one Higgs doublet Φ as in the Standard Model (SM). Only renormalizable terms are allowed in this model, i.e. it is ultraviolet complete. All dimension-4 terms must obey A_5, whereas A_4 and its breaking occur through soft terms.

Allowed: $\bar{l}_R \Phi^\dagger L_L$, $\bar{E}_R \Phi^\dagger L_L$ $(3 \times 1 \times 3)$;
$\bar{L}_L \eta N_R$ $(3 \times 3' \times 4)$; $\bar{\nu}_R \zeta^0 N_L$ $(3' \times 3 \times 4)$.
Forbidden: $\bar{\nu}_R \tilde{\Phi}^\dagger L_L$ $(3' \times 1 \times 3)$; $\bar{N}_R \tilde{\Phi}^\dagger L_L$ $(4 \times 1 \times 3)$.

The soft trilinear term $\Phi \eta \bar{\zeta}^0$ $(1 \times 3' \times 3)$ breaks A_5, but not A_4. Hence ν_L cannot couple to ν_R at tree level because of A_5, but is linked to it in one loop under A_4.


ϕ^0
η^0
ζ^0
ν_L
N_R
N_L
ν_R

Seesaw Charged Leptons

Since both l_R and E_R transform as $\underline{3}$ under A_5, the latter may be defined to be the one which couples to l_L through ϕ^0 and the former does not. Once chosen this way, both l_R and E_R will couple to E_L. Hence the 6×6 mass matrix linking $(l, E)_L$ to $(l, E)_R$ is

$$\mathcal{M} = \begin{pmatrix} 0 & M_{lE} \\ M_{El} & M_{EE} \end{pmatrix}.$$

Under A_5, all the 3×3 entries M_{lE}, M_{El}, M_{EE} are proportional to the identity, with M_{lE} coming from $\langle \phi^0 \rangle$.

It is now assumed that the soft term M_{El} breaks A_5 in a way compatible with A_4, using $E \sim 3$ and $l_R \sim 3$ and a gauge singlet flavon ~ 3. [Ma, MPLA 21, 2931 (2006)]

$$M_{El} = \begin{pmatrix} h_1 v' & h_2 v'' & h_3 v'' \\ h_3 v'' & h_1 v' & h_2 v'' \\ h_2 v'' & h_3 v'' & h_1 v' \end{pmatrix}$$

This magically decomposes to $U_\omega \; M_{diag} \; U_\omega{}^\dagger$, with eigenvalues $h_1 v' + (h_2 + h_3) v''$, $h_1 v' + (h_2 \omega + h_3 \omega^2) v''$, $h_1 v' + (h_2 \omega^2 + h_3 \omega) v''$. The seesaw 3×3 charged-lepton mass matrix is $M_{ll} = M_{lE} M_{EE}^{-1} M_{El}$, as desired.

Scotogenic Dirac Neutrino Mass

Let $\psi_1^0 = \eta^0 \cos\theta - \zeta^0 \sin\theta$, $\psi_2^0 = \eta^0 \sin\theta + \zeta^0 \cos\theta$, with masses $m_{1,2}$, then the radiative Dirac neutrino mass matrix $(\mathcal{M}_\nu)_{ij}$ is [using $F(x,y) = x \ln(x/y)/(x-y)$]

$$\frac{\sin\theta\cos\theta}{16\pi^2} \sum_{k,k',a} f^L_{ika} f^\phi_{k'k4} f^R_{k'ja} [F(m_2^2, M_a^2) - F(m_1^2, M_a^2)].$$

The f couplings come from $3 \times 3' \to 4$ of A_5. Let $(a_1, a_2, a_3) \sim 3$, $(b_1, b_2, b_3) \sim 3'$, then the four components of N are $3^{-1/2}[\varphi^{-1} a_3 b_2 - \varphi a_1 b_3, \varphi a_3 b_1 + \varphi^{-1} a_2 b_3, -\varphi^{-1} a_1 b_1 + \varphi a_2 b_2, a_2 b_1 - a_1 b_2 + a_3 b_3]$.

If $M_{1,2,3,4}$ are all equal, then all three Dirac neutrino masses are equal as well. Assume instead that N_4 is very heavy (breaking $A_5 \to A_4$), and all entries of the 3×3 mass matrix spanning $N_{1,2,3}$ are much smaller than $m_{1,2}$. In that case, $\mathcal{M}_\nu$ reduces to [Ma, PLB 717, 235 (2012)]

$$\frac{\sin\theta\cos\theta\ln(m_2^2/m_1^2)}{16\pi^2}\sum_{k,k',a,b} f^L_{ika} f^\phi_{k'k4} f^R_{k'jb} M_{ab}.$$

$$\propto \tilde{\mathcal{M}} = \begin{pmatrix} \varphi^2 M_{12} & M_{11}+M_{33} & \varphi^{-2}M_{32} \\ M_{22}+M_{33} & \varphi^{-2}M_{21} & \varphi^2 M_{31} \\ \varphi^{-2}M_{13} & \varphi^2 M_{23} & M_{11}+M_{22} \end{pmatrix}.$$

To obtain cobimaximal mixing, $\tilde{\mathcal{M}}$ is assumed to be diagonalized by an orthogonal matrix $\mathcal{O}$ on the left. This means that $\tilde{\mathcal{M}}\tilde{\mathcal{M}}^\dagger$ should be real. Using

$$U_{l\nu} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ (s_{12} - ic_{12}s_{13})/\sqrt{2} & (-c_{12} - is_{12}s_{13})/\sqrt{2} & ic_{13}/\sqrt{2} \\ (s_{12} + ic_{12}s_{13})/\sqrt{2} & (-c_{12} + is_{12}s_{13})/\sqrt{2} & -ic_{13}/\sqrt{2} \end{pmatrix},$$

and the central values $s_{13} = 0.148$, $c_{13} = 0.989$, $s_{12} = 0.554$, $c_{12} = 0.832$,

$$\mathcal{O} = U_\omega U_{l\nu} = \begin{pmatrix} 0.927 & -0.363 & 0.085 \\ 0.336 & 0.714 & -0.614 \\ 0.162 & 0.598 & 0.785 \end{pmatrix}.$$

As a numerical example, assume first a reduction of parameters: $\varphi^2(M_{12}, M_{31}, M_{23}) = \varphi^{-2}(M_{21}, M_{13}, M_{32})$, then $\mathcal{M}_\nu$ is of the form

$$\mathcal{M}_\nu = \begin{pmatrix} y_1 & x_1 & y_3 \\ x_2 & y_1 & y_2 \\ y_2 & y_3 & x_3 \end{pmatrix}, \quad \text{with } \mathcal{M}_\nu \mathcal{M}_\nu^\dagger = \mathcal{O} \begin{pmatrix} m_1^2 & 0 & 0 \\ 0 & m_2^2 & 0 \\ 0 & 0 & m_3^2 \end{pmatrix} \mathcal{O}^T,$$

resulting in

$$\begin{pmatrix} x_1^2 + y_1^2 + y_3^3 \\ x_2^2 + y_1^2 + y_2^2 \\ x_3^2 + y_2^2 + y_3^2 \end{pmatrix} = \begin{pmatrix} 0.859 & 0.113 & 0.026 \\ 0.132 & 0.510 & 0.358 \\ 0.007 & 0.377 & 0.616 \end{pmatrix} \begin{pmatrix} m_1^2 \\ m_2^2 \\ m_3^2 \end{pmatrix},$$

and

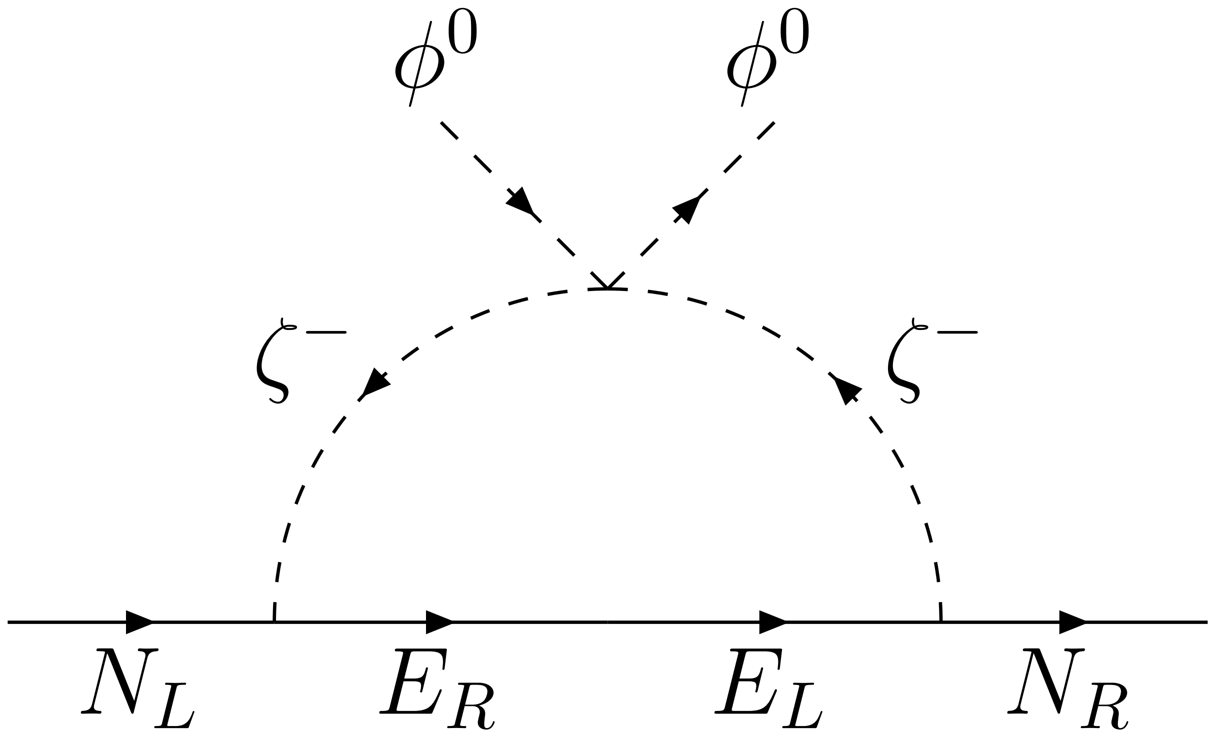
$$\begin{pmatrix} y_1(x_1+x_2)+y_2y_3 \\ y_3(x_1+x_3)+y_1y_2 \\ y_2(x_2+x_3)+y_1y_3 \end{pmatrix} = \begin{pmatrix} -0.337 & 0.240 & 0.097 \\ 0.079 & -0.206 & 0.127 \\ -0.031 & -0.438 & 0.469 \end{pmatrix} \begin{pmatrix} m_1^2 \\ m_2^2 \\ m_3^2 \end{pmatrix}.$$

Assuming normal ordering of neutrino masses with $\Delta m^2_{21} = 7.53 \times 10^{-5}$ eV^2 and $\Delta m^2_{32} = 2.453 \times 10^{-3}$ eV^2, the above 6 equations are solved for $y_1 = 0$, resulting in $x_1 = 0.00597$ eV, $x_2 = 0.01334$ eV, $x_3 = 0.02711$ eV, $y_2 = 0.02850$ eV, $y_3 = 0.00924$ eV, $m_1 = 0.00684$ eV. Hence $m_2 = 0.011$ eV, $m_3 = 0.051$ eV, with $\sum m_\nu = 0.07$ eV, less than 0.15 eV from cosmology.

Freeze-In Dark Matter

Defining $D = L - (2j)_{[mod\ 2]}$, the charged leptons and neutrinos have $D = 0$, whereas $D = 1$ for η, ζ and $D = -1$ for N.

The dark neutral fermions N are presumably light, say of order GeV in mass, and they are weakly coupled to the leptons. They are not suitable for freeze-out dark matter, but with the help of the charged scalar gauge singlet $\zeta^- \sim 3'$, they may be rare decay products of the SM Higgs boson, with a lifetime long enough to be freeze-in dark matter.


ϕ^0
ϕ^0
ζ^-
ζ^-
N_L
E_R
E_L
N_R

The effective Higgs Yukawa coupling f_h to $\bar{N}N$ is

$$f_h = \frac{\lambda_\zeta f_\zeta^L f_\zeta^R v m_E}{16\pi^2}\left[\frac{1}{m_\zeta^2 - m_E^2} - \frac{m_E^2 \ln(m_\zeta^2/m_E^2)}{(m_\zeta^2 - m_N^2)^2}\right].$$

The decay rate of h to $\bar{N}N$ is

$$\Gamma_h = \frac{f_h^2 m_h}{8\pi}\sqrt{1-4r^2}(1-2r^2), \quad r = m_N/m_h.$$

If the reheat temperature of the Universe is $T_R \sim 1$ to 10 TeV, and $f_h \sim 10^{-11}$, then N has the correct relic density, as a feebly interacting massive particle (FIMP).

Concluding Remarks

Flavor symmetries may relate Dirac neutrino masses and mixing to their dark matter counterparts through the scotogenic mechanism, using the symmetry A_5 breaking to A_4. A numerical example of cobimaximal neutrino mixing fitting present data with normal ordering of neutrino masses is obtained, implying a 3×3 dark fermion mass matrix of the form (in units of order GeV)

$$\mathcal{M}_N \sim \begin{pmatrix} 0.987 & 0 & 7.461 \\ 0 & 1.724 & 0.353 \\ 1.089 & 2.419 & -0.390 \end{pmatrix}.$$

Details of this presentation appear in PLB 829, 137104 (2022) [arXiv:2202.13031].

Further development:

The lepton number L is an input global symmetry assumed in this model. However, if the dark number D is gauged and is broken by 3 (or more) units, then D remains a global symmetry.

[Ma/Picek/Radovcic, PLB 726, 744 (2013)]

The linkage of N and η, ζ to the leptons through $A_5 \to A_4$ would again lead to $D = L - (2j)_{[mod\ 2]}$.

[Ma, arXiv:2203.12034]

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