

Flavour Seesaw and Phenomenological Consequences

Manfred Lindner



➔ **Sudip Jana, Sophie Klett, ML, PRD 105 (2022) 11, 115015 - arXiv: 2112.09155**

A banner for the FLASY 2022 workshop. It features a stylized particle detector on the left, a silhouette of a city skyline at the bottom, and text on the right. The text includes the workshop title, dates, location, and organizing institution.

FLASY 2022 | 9th Workshop on Flavour Symmetries and Consequences in Accelerators and Cosmology

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IST Congress Centre, Instituto Superior Técnico
Lisbon, Portugal

Organised by Centro de Física Teórica de Partículas (CFTP)

The Need for BSM Physics

Experimental facts:

- **Dark Matter & Dark Energy** exist!
- **Neutrino masses** have been detected!
- **Baryon asymmetry** of the universe
 $\leftrightarrow m_\nu > 0$
- **various 2-3 σ indications / hints**
g-2, LHCb, low E_R -excess @XENON, ...
 \leftrightarrow BSM-implications

Theoretical arguments:

SM **does not exist without** cutoff
Higgs-doublet = only **simplest extension**
Gauge hierarchy problem
Which: **particles (representations)**
Many parameters (9+? masses, 4+? mixings)
Charge quantization, unification: GUTs, ...,
Gravity: ...
...

3 generations + apparent regularities!
 \rightarrow does not require BSM, but is a hint!

How to solve these problems?

\rightarrow pick some direction for an extension:

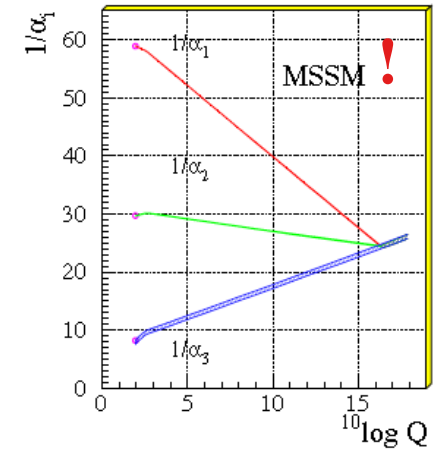
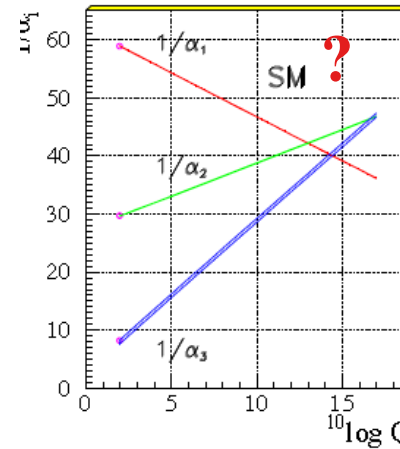
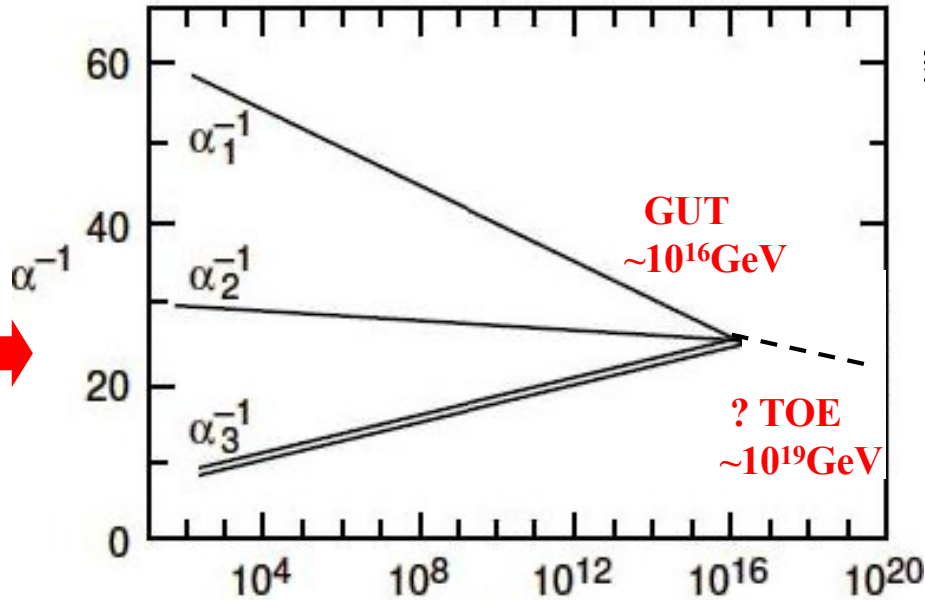
- **more representations (particles)**
- **enlarged symmetries & breakings**
- **new concepts beyond 4d QFT**
- **combinations thereof**
- **more and more complexity...**

\rightarrow Q: Are we on the right track or are we over-engineering problems?

\rightarrow Starting point: Some suggestive feature or interesting concept...

Indications pointing to SUSY + GUTs

gauge bosons



Higgs

gauge hierarchy problem:
 $\delta m_H^2 \sim \Lambda^2$

quarks
leptons

flavour problem: 3 generations
many parameters (m_i , mixings)
unification into GUTs

$$m_\nu = (m^D)^T M_R^{-1} m_D$$

- SM particles fit nicely into GUT representations
- charge quantization
- evidence for some unification
- SO(10): 16
- SU(5): $\bar{5}, 10$

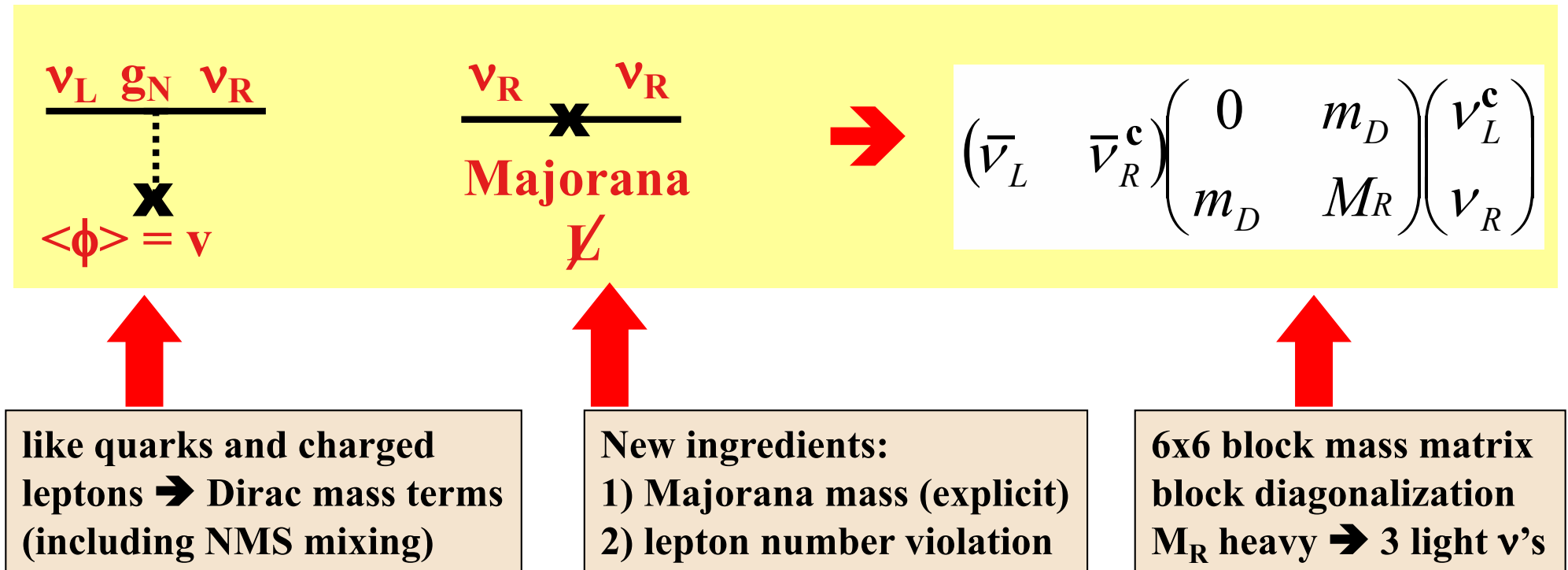
→ SUSY @ TeV ...?

$\Lambda_{\text{GUT}} \leftrightarrow$ seesaw scale?

supports „generations“

Flavour: Adding Neutrino Mass Terms

1) Simplest possibility: add 3 right handed neutrino fields



NEW ingredients:

\rightarrow SM+

- SM=1-scale \rightarrow multi-scale
- modified global symmetries (henn – egg)
- 9 parameters

Other Neutrino Mass Operators

2) new Higgs triplets Δ_L : 

→ left-handed Majorana mass term:

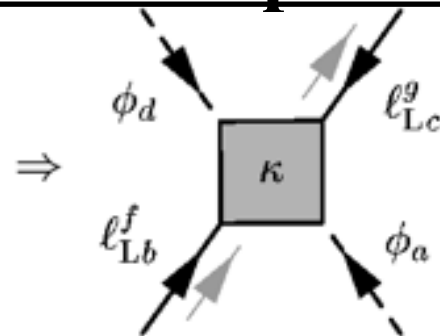
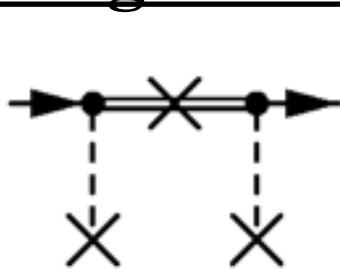
$$\rightarrow M_L \bar{L} L^c$$

3) Both ν_R and new Higgs triplets Δ_L :

→ see-saw type II

$$m_\nu = M_L - m_D M_R^{-1} m_D^T$$

4) Higher dimensional operators: $d=5, \dots$



⇔

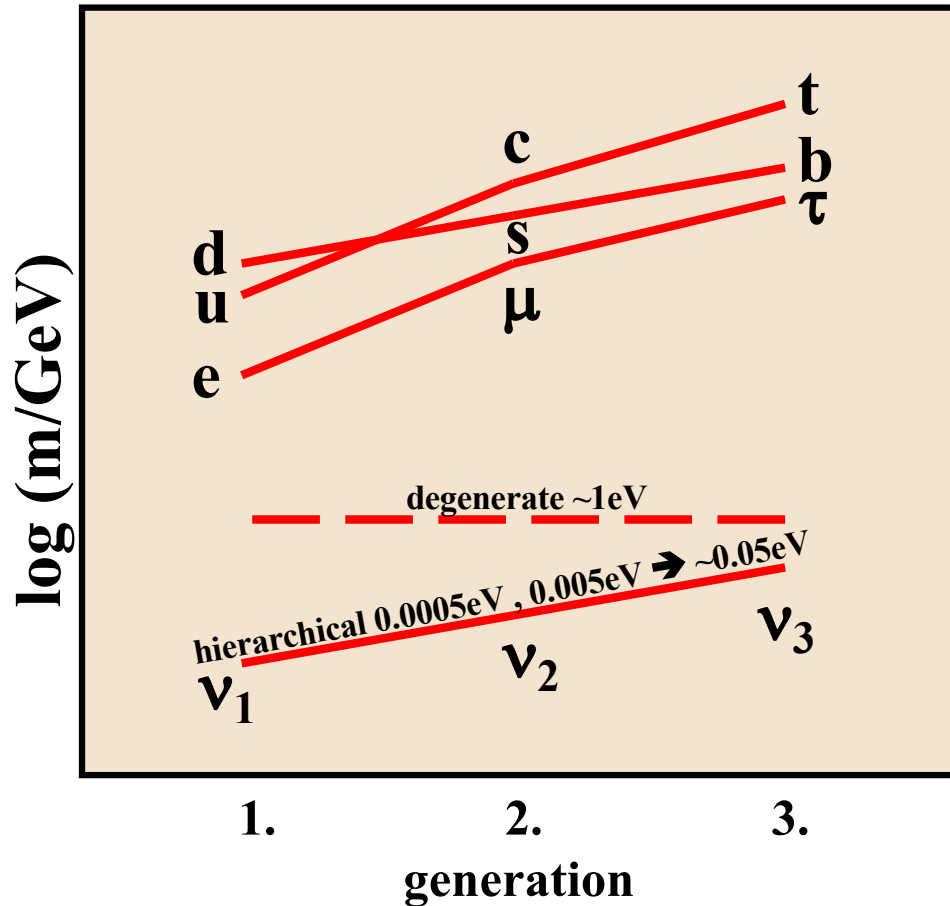
$$\mathcal{L}_{mass} = \kappa \cdot \bar{\nu}_L^C \nu_L \Phi^T \Phi$$

$$\rightarrow M_L \bar{L} L^c$$

5-N) ...

Empirical Regularities

Quarks & charged leptons → mass hierarchy pattern → neutrinos?



Quarks and charged leptons:

$$m_D \sim H^n ; n = 0, 1, 2 \rightarrow H \geq 20 \dots 200$$

Neutrinos:

$$m_\nu \sim H^n \rightarrow H \leq \sim 10$$

See-saw:

$$m_\nu = -m_D^T M_R^{-1} m_D$$

	↑	↑	↑	↑
H	~10	≥20	?	≥20

- less hierarchy in m_D or correlated hierarchy in M_R ?
→ theoretically connected?
- mixing patterns...

Obtaining apparent numerical Regularities

Quark and lepton masses as well as their mixings show apparent regularities

- moderate inter-generation scale hierarchies
- most mixings are either small or almost maximal
 - how can this be understood?

Toolbox: Mechanisms to naturally explain moderate scale hierarchies:

- scale ratios – e.g. two VEVs of two scalars (not too far apart... \leftrightarrow hierarchy...)
- small ratios from diagonalization in seesaw like structures
- loop suppression factors $\sim 1/16\pi^2$
- group theory factors from flavour symmetries
- ...

One mechanism \leftrightarrow more than one mechanism at work?

nicer? simpler?

one mechanism versus one new ingredient?

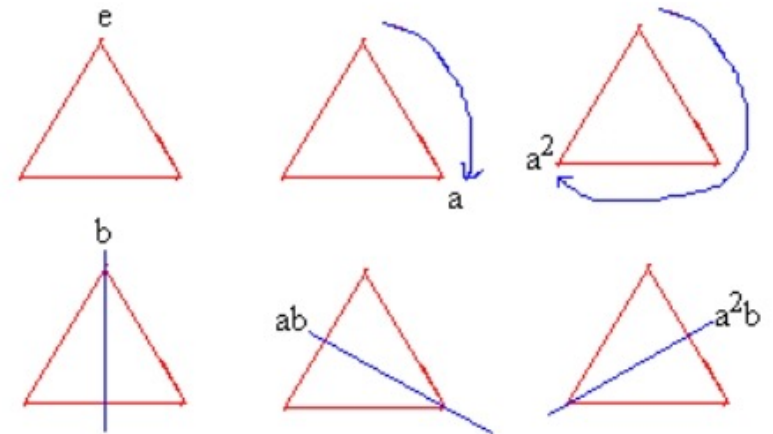
e.g. few fields, simpler groups, new flavour physics at moderate scales, ...

Flavour Symmetries

- **the basic idea:**
horizontal symmetry (discrete, continuous)
- **broken at high scales**
- **by flavons**
→ **explain observed masses and mixings**

e.g. dihedral groups D_n

geometric origin of D_3 :



Many groups and breaking scenarios have been discussed
a phenomenologically promising example: D_5 [Hagedorn, ML, Plentinger](#)

task: search for mass terms which are for suitable Higgs singlets under D_5

1) assign all fermions to representations $L = \{L_1, L_2, L_3\}$

2) write down any possible mass term using scalars \leftrightarrow singlet under symmetry

→ D₅ Allowed Mass Terms

Dirac mass terms:

$$\lambda_{ij} L_i^T (i\sigma_2) \phi L_j^c$$

Majorana mass terms:

$$\lambda_{ij} L_i^T \equiv \phi L_j$$

Scalars (flavons) → D5 symmetry induced mass matrices:

$$\begin{aligned} \Phi_1 &\sim \mathbf{1}_1 \\ \Phi_2 &\sim \mathbf{1}_2 \\ \Psi_1 &\sim \mathbf{2}_1 \end{aligned}$$

L	L^c	Mass Matrix
$(1_2, 1_1, 1_1)$	$(2_1, 1_1)$	$\begin{pmatrix} \kappa_1 \psi_2^1 & -\kappa_1 \psi_1^1 & \kappa_4 \phi^2 \\ \kappa_2 \psi_2^1 & \kappa_2 \psi_1^1 & \kappa_5 \phi^1 \\ \kappa_3 \psi_2^1 & \kappa_3 \psi_1^1 & \kappa_6 \phi^1 \end{pmatrix}$

→ check phenomenology → ... successful models + “predictions”

BUT: Flavour models get more and more complex, often require an intricate selection / interplay special of symmetries, carefully chosen representations, several flavons and specific breakings

→ all very interesting, but is it the right track or is it overengineering?

Flavor Seesaw Mechanism

S. Jana, S. Klett, ML, 2112:09155

Assumed fermion sector structure: up- or down- sector ($n=3$) + vector fermions

$|h\rangle$ - n -element vector including Yukawa couplings

M_P - explicit mass of vector fermion(s)

v_{EW} - the electro-weak VEV

v_S - VEV of a suitable new scalar

$$\begin{pmatrix} \mathbf{0} & v_{EW}|h\rangle \\ v_S\langle h| & M_P \end{pmatrix}$$

→ model details will follow

→ two non-zero and $n - 1$ zero mass-eigenvalues

• assume seesaw limit: $M_P \gg v_{EW}, v_S$

→ non-zero eigenvalues: $m_t \simeq - (v_{EW} v_S / M_P) \langle h|h \rangle$
 $m_p \simeq M_P$

Notes:

- “0” will be a consequence of representations
- vector masses M_P can have without problems arbitrary values
- for $n=3$ → only 3rd generation masses ; two vanishing eigenvalues
- accidental symmetry of non-interacting theory @ tree level
- want: minimal set of extra scalars (avoid many “flavons”)
- want: v_S as low as possible?

Step 1: ~universal seesaw Berezghiani, Chang and Mohapatra, Davidson and Wali
 ~ here not successful

- loop effects create δM
- and corrections $|h\rangle \rightarrow |\alpha\rangle$
- final quark mass matrix:

$$M_T = \left(\begin{array}{c|c} \delta M & v_{EW}|\alpha\rangle \\ \hline v_S\langle\alpha| & M_P \end{array} \right)$$

$$\left. \begin{array}{l} \delta M|x\rangle + v_{EW}|\alpha\rangle x_{n+1} = 0 \\ v_S\langle\alpha|x\rangle + M_P x_{n+1} = 0 \end{array} \right\} \begin{array}{l} \text{eliminate } x_{n+1} \\ (\delta M + a_0|\alpha\rangle\langle\alpha|)|x\rangle \equiv M|x\rangle = 0 \end{array}$$

→ $M = \delta M - (v_{EW}^* v_S / M_P) |\alpha\rangle\langle\alpha|$

Outcome: If M has rank r

→ $M_T M_T^\dagger$ possesses r + 1 nonzero eigenvalues

→ r generations become massive

n=3: m_t generated by see-saw

2nd generation @ 1-loop

1st generation...

} **interplay of different mechanisms**

Realizing the Flavor Seesaw

$$G \rightarrow G_{\text{SM}} \times U(1)_{\text{B-L}}$$

→ additional fields

$j = 1, 2, 3$ (will also cancel anomalies)

two generations of vector-like fermions:

$\mathbf{T}_k, \mathbf{B}_k, \mathbf{E}_k, \mathbf{N}_k$ with $k = 1, 2$

SM singlet - will break $U(1)_{\text{B-L}}$ →

Particle	SU(3) _C	SU(2) _L	U(1) _Y	U(1) _{B-L}
$Q_{jL} = \begin{pmatrix} u_j \\ d_j \end{pmatrix}_L$	3	2	1/3	1/3
u_{jR}	3	1	4/3	1/3
d_{jR}	3	1	-2/3	1/3
$\Psi_{jL} = \begin{pmatrix} \nu_j \\ e_j \end{pmatrix}_L$	1	2	-1	-1
ν_{jR}	1	1	0	-1
e_{jR}	1	1	-2	-1
T_{kL}, T_{kR}	3	1	4/3	2/3
B_{kL}, B_{kR}	3	1	-2/3	0
N_{kL}, N_{kR}	1	1	0	-2/3
E_{kL}, E_{kR}	1	1	-2	-4/3
$\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$	1	2	1	1/3
η	1	1	0	1/3

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}G_{\mu\nu}^a G^{\mu\nu a} - \frac{1}{4}W_{\mu\nu}^i W^{\mu\nu i} - \frac{1}{4}B_{\mu\nu} B^{\mu\nu} - \frac{1}{4}X_{\mu\nu} X^{\mu\nu}$$

↑
important!
→ "0"
→ Dirac v's

Scalar sector:

$$\mathcal{L}_{\text{scalar}} = (D_\mu \phi)^\dagger (D^\mu \phi) + (D_\mu \eta)^\dagger (D^\mu \eta) - V(\phi, \eta)$$

$$V(\phi, \eta) = -\mu_\phi^2 \phi^\dagger \phi + \frac{1}{2} \lambda_\phi (\phi^\dagger \phi)^2 - \mu_\eta^2 \eta^\dagger \eta + \frac{1}{2} \lambda_\eta (\eta^\dagger \eta)^2 + \lambda_{\phi\eta} (\phi^\dagger \phi) (\eta^\dagger \eta)$$

Symmetry breaking: $\langle \phi \rangle = \begin{pmatrix} 0 \\ v_{EW}/\sqrt{2} \end{pmatrix}$ $\langle \eta \rangle = \frac{v_S}{\sqrt{2}}$

assume: $v_S \gg v_{EW}$

$$SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L} \xrightarrow{\langle \eta \rangle} SU(3)_C \times SU(2)_L \times U(1)_Y \\ \xrightarrow{\langle \phi \rangle} SU(3)_C \times U(1)_{EM}$$

η is a weak singlet \rightarrow charged boson masses like in SM

$$M_{W^\pm}^2 = \frac{g^2 v_{EW}^2}{4}.$$

neutral gauge bosons \rightarrow (B,W₃,X) mass matrix (mixing by U(1)_{B-L} charge of Φ)

$$\mathcal{M}^2 = \frac{1}{4} \begin{pmatrix} g'^2 v_{EW}^2 & -gg' v_{EW}^2 & g' g_X q_\phi v_{EW}^2 \\ -gg' v_{EW}^2 & g^2 v_{EW}^2 & -g g_X q_\phi v_{EW}^2 \\ g' g_X q_\phi v_{EW}^2 & -g g_X q_\phi v_{EW}^2 & g_X^2 (q_\phi^2 v_{EW}^2 + q_\eta^2 v_S^2) \end{pmatrix}$$

$$s_w \equiv \sin \theta_w = g' / \sqrt{g^2 + g'^2}$$

$$\begin{pmatrix} A \\ Y \\ X \end{pmatrix} = \begin{pmatrix} c_w & s_w & 0 \\ -s_w & c_w & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} B \\ W^3 \\ X \end{pmatrix} \rightarrow M_{YY}^2 = \frac{v_{EW}^2 [g^2 c_w^2 (1 + 2s_w^2) + g'^2 s_w^2 (1 + 2c_w^2)]}{4}$$

$$M_{XX}^2 = \frac{g_X^2 (q_\phi^2 v_{EW}^2 + q_\eta^2 v_S^2)}{4}$$

$$\begin{pmatrix} Z \\ Z' \end{pmatrix} = \begin{pmatrix} c_\xi & s_\xi \\ -s_\xi & c_\xi \end{pmatrix} \begin{pmatrix} Y \\ X \end{pmatrix}$$

$$M_{YX}^2 = -\frac{g_X q_\phi v_{EW}^2 (g c_w + g' s_w)}{4}$$

$$\tan 2\xi = \frac{2M_{YX}^2}{M_{YY}^2 - M_{XX}^2}$$


$$M_{Z,Z'}^2 = \frac{1}{2} \left(M_{YY}^2 + M_{XX}^2 \mp (M_{YY}^2 - M_{XX}^2) \sqrt{1 + \tan^2 2\xi} \right)$$

Yukawa interactions: Only between quarks (leptons) and vector quarks (leptons)

$$\mathcal{L}_{\text{Yuk}} = -y_a^q \bar{Q}_{jL} \tilde{\phi} T_{kR} - y_b^q \bar{T}_{kL} \eta u_{jR} - y_c^q \bar{Q}_{jL} \phi B_{kR} - y_d^q \bar{B}_{kL} \eta^\dagger d_{jR} \\ - y_a^\ell \bar{\Psi}_{jL} \tilde{\phi} N_{kR} - y_b^\ell \bar{N}_{kL} \eta \nu_{jR} - y_c^\ell \bar{\Psi}_{jL} \phi E_{kR} - y_d^\ell \bar{E}_{kL} \eta^\dagger e_{jR} + h.c.$$

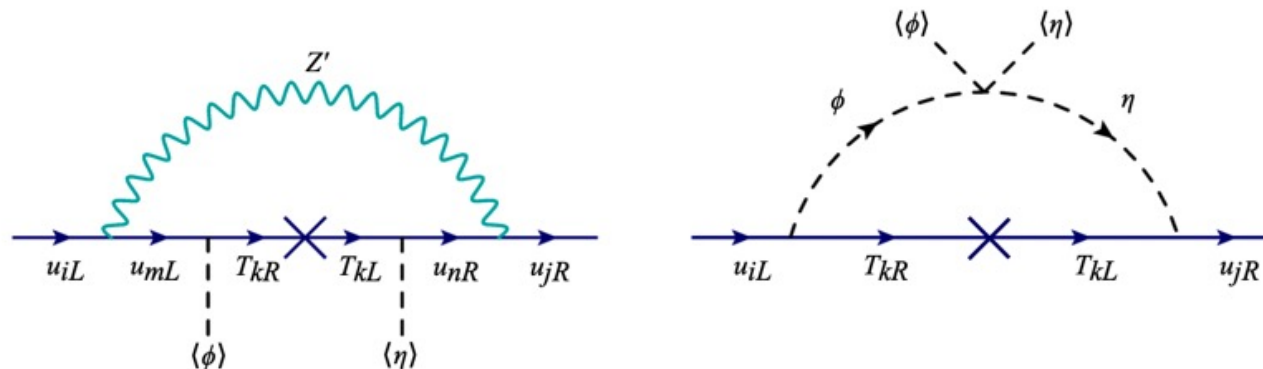
Explicit masses for vector fermions:

$$\mathcal{L}_{\text{explicit}} = -\mathcal{M}_T \bar{T}_{kL} T_{kR} - \mathcal{M}_B \bar{B}_{kL} B_{kR} - \mathcal{M}_N \bar{N}_{kL} N_{kR} - \mathcal{M}_E \bar{E}_{kL} E_{kR} + h.c.$$


 $\mathcal{M}_T = \begin{pmatrix} M_{T1} & 0 \\ 0 & M_{T2} \end{pmatrix}$
 diagonal

Note: Neutrinos must be Dirac particles \leftrightarrow U(1)_{B-L} charges

Radiative generation of further mass terms and mixings @ 1-loop:



Resulting up-type mass matrix:

$$\bar{\mathbf{u}}_L \mathcal{M}_u^{(1)} \mathbf{u}_R \equiv \begin{pmatrix} \bar{u}_{1L} & \bar{u}_{2L} & \bar{u}_{3L} & \bar{T}_{1L} & \bar{T}_{2L} \end{pmatrix} \left(\frac{\delta \mathcal{M}^u}{(y_b^q)^T \langle \eta \rangle} \middle| \frac{y_a^q \langle \phi \rangle}{\mathcal{M}_T} \right) \begin{pmatrix} u_{1R} \\ u_{2R} \\ u_{3R} \\ T_{1R} \\ T_{2R} \end{pmatrix}$$

@1-loop:

$$\begin{aligned} \delta M_{ij}^u &= \sum_{k=1}^2 \sum_{m,n=1}^3 i \frac{3g_X^2 q_Q q_u [y_a^q]_{mk} [y_b^q]_{kn} v_{EW} v_S}{8} \int \frac{d^4 k}{(2\pi)^4} \frac{\gamma_\mu (\not{k} + M_{Tk}) \gamma_\nu g^{\mu\nu}}{k^2 (k^2 - M_{Tk}^2) ((p-k)^2 - M_{Z'}^2)} \\ &= \sum_{k=1}^2 \sum_{m,n=1}^3 \frac{3g_X^2 q_Q q_u [y_a^q]_{mk} [y_b^q]_{kn} v_{EW} v_S}{32\pi^2} \frac{M_{Tk}}{(M_{Z'}^2 - M_{Tk}^2)} \log \frac{M_{Z'}^2}{M_{Tk}^2}. \end{aligned}$$

Analogous for down-type, charged and neutral lepton mass matrices

...

→ CKM and PMNS mixings including small unitarity violations from VF's

What did we win?

- moderate hierarchies from simple interplay of seesaw, two VEVs and loops
- no or less need to put hierarchies into SM Yukawas
- TeV-ish flavour physics?

Outcome: A mapping between SM Yukawas \leftrightarrow new Yukawas

It is highly non-trivial to obtain the known SM parameters!

\rightarrow two benchmark points:

$$\epsilon_1 = 10^{-2}, \quad \epsilon_2 = 10^{-1} * \epsilon_1^2$$

$$M_{Z'} = 300 \text{ TeV}$$

$$M_{T1} = 8 \text{ TeV} \quad ; \quad M_{T2} = 17.91 \text{ TeV}$$

$$M_{B1} = 40 \text{ TeV} \quad ; \quad M_{B2} = 65.69 \text{ TeV}$$

$$M_{E1} = 50 \text{ TeV} \quad ; \quad M_{E2} = 80 \text{ TeV}$$

$$M_{N1} = 1.15 \times 10^6 \text{ TeV} \quad ; \quad M_{N2} = 1.25 \times 10^6 \text{ TeV}$$

Mapped Yukawa couplings $O(10^{-2}) - O(1)$

\leftrightarrow hierarchies in SM Yukawas emerge mostly from seesaw, VEV ratio and loops

- Does not explain numbers
 \rightarrow parameter mapping
- But maybe easier to explain?!

Yukawa Couplings	Benchmark Points	
	BP1	BP2
y_a^q	$\begin{pmatrix} 0.625 & 0.513 \\ 0.186 \times e^{i0.05} & 0.159 \\ 0.401 & 0.327 \end{pmatrix}$	$\begin{pmatrix} 0.631 & 0.522 \\ 0.183 \times e^{i0.05} & 0.158 \\ 0.412 & 0.344 \end{pmatrix}$
y_b^q	$\begin{pmatrix} 0.332 & 0.229 \\ 0.340 & 0.224 \\ 0.294 & 0.232 \times e^{i0.05} \end{pmatrix}$	$\begin{pmatrix} 0.335 & 0.227 \\ 0.352 & 0.222 \\ 0.290 & 0.230 \times e^{i0.05} \end{pmatrix}$
y_c^q	$\begin{pmatrix} 1.539 & 1.287 \\ 0.195 & 0.538 \\ 1.060 & 0.754 \end{pmatrix} \epsilon_1$	$\begin{pmatrix} 1.501 & 1.341 \\ 0.205 & 0.600 \\ 1.061 & 0.795 \end{pmatrix} \epsilon_1$
y_d^q	$\begin{pmatrix} 1.110 & 0.332 \\ 0.850 & 1.506 \\ 13.969 & 12.405 \end{pmatrix} \epsilon_1$	$\begin{pmatrix} 1.129 & 0.331 \\ 0.811 & 1.624 \\ 14.076 & 13.236 \end{pmatrix} \epsilon_1$
y_a^ℓ	$\begin{pmatrix} 1.306 & 1.494 \\ 0.175 & 1.257 \\ 0.538 & 0.333 \end{pmatrix} \epsilon_1^2$	$\begin{pmatrix} 1.235 \times e^{i0.1} & 0.948 \\ 0.195 & 2.283 \\ 0.443 & 0.347 \end{pmatrix} \epsilon_2$
y_b^ℓ	$\begin{pmatrix} 0.319 & 1.048 \\ 0.285 & 0.668 \\ 0.967 & 0.328 \end{pmatrix} \epsilon_1^2$	$\begin{pmatrix} 0.101 & 1.210 \\ 2.871 & 0.947 \\ 0.243 & 1.589 \end{pmatrix} \epsilon_2$
y_c^ℓ	$\begin{pmatrix} 0.678 & 1.166 \\ 0.854 & 0.574 \\ 1.474 & 0.820 \end{pmatrix} \epsilon_1$	$\begin{pmatrix} 0.484 & 1.468 \\ 0.999 & 1.281 \\ 0.617 & 0.809 \end{pmatrix} \epsilon_1$
y_d^ℓ	$\begin{pmatrix} 1.398 & 0.960 \\ 0.740 & 0.780 \\ 0.747 & 1.445 \end{pmatrix} \epsilon_1$	$\begin{pmatrix} 1.555 & 0.479 \\ 1.355 & 1.381 \\ 0.858 & 0.982 \end{pmatrix} \epsilon_1$

Resulting SM observables for the two benchmark points

Quark Sector				Lepton Sector				
Observable (Masses in GeV)	3σ Exp. Range	Model Prediction		Observable (Masses in GeV)	3σ Exp. Range (NH)	3σ Exp. Range (IH)	Model Prediction (NH)	Model Prediction (IH)
		BP1	BP2				BP1	BP2
$m_u/10^{-3}$	1.38 → 3.63	2.12	3.07	$\frac{\Delta m_{21}^2}{10^{-5} \text{ eV}^2}$	6.82 → 8.04	6.82 → 8.04	7.583	7.898
m_c	1.21 → 1.33	1.29	1.25	$\frac{\Delta m_{3\ell}^2}{10^{-3} \text{ eV}^2}$	2.421 → 2.598	-2.583 → -2.412	2.567	-2.432
m_t	171.7 → 174.1	172.3	174.1	$m_e/10^{-3}$	0.485 → 0.537		0.511	0.527
$m_d/10^{-3}$	4.16 → 6.11	4.34	5.08	m_μ	0.100 → 0.111		0.109	0.109
m_s	0.078 → 0.126	0.122	0.109	m_τ	1.688 → 1.866		1.862	1.839
m_b	4.12 → 4.27	4.18	4.13	$\sin^2(\theta_{12})$	0.269 → 0.343	0.269 → 0.343	0.315	0.320
$ V_{ud} $	0.973 → 0.974	0.974	0.974	$\sin^2(\theta_{23})$	0.407 → 0.618	0.411 → 0.621	0.444	0.413
$ V_{us} $	0.222 → 0.227	0.227	0.226	$\sin^2(\theta_{13})$	0.02034 → 0.02430	0.02053 → 0.02436	0.02053	0.02300
$ V_{ub} /10^{-4}$	31.0 → 45.4	38.4	44.8	δ_{cp}/\circ	107 → 403	192 → 360	0	250
$ V_{cd} $	0.209 → 0.233	0.226	0.226					
$ V_{cs} $	0.954 → 1.020	0.973	0.973					
$ V_{cb} /10^{-3}$	36.8 → 45.2	42.3	41.9					
$ V_{td} /10^{-4}$	71.0 → 89.0	84.0	78.7					
$ V_{ts} /10^{-3}$	35.5 → 42.1	41.6	41.4					
$ V_{tb} $	0.923 → 1.103	0.999	0.999					
$\mathcal{J}/10^{-5}$	2.73 → 3.45	3.12	3.40					

Notes:

- The two benchmark points are just two solutions of a numerical search
→ there may exist solutions with even lower scales
- Many other model realizations should exist

BSM Phenomenology

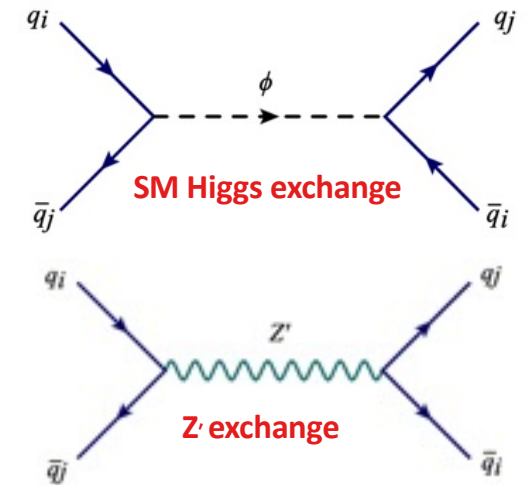
TeV-ish new physics...

- **FCNC's @ quark sector** → **neutral meson mixing**
estimate effects: $D^0 - \bar{D}^0$, $K^0 - \bar{K}^0$, $B_d^0 - \bar{B}_d^0$ and $B_s^0 - \bar{B}_s^0$
dominating contributions: SM Higgs and Z' exchange

...

→ new physics contributions

Observable (in GeV)	Model Prediction	
	BP1	BP2
$\Delta m_{B_d}^{\text{NP}}$	-1.402×10^{-13}	-1.495×10^{-14}
$\Delta m_{B_s}^{\text{NP}}$	2.663×10^{-14}	3.003×10^{-14}
Δm_D^{NP}	2.405×10^{-15}	2.036×10^{-15}
Δm_K^{NP}	0.504×10^{-15}	0.109×10^{-15}



the benchmark points are consistent with data

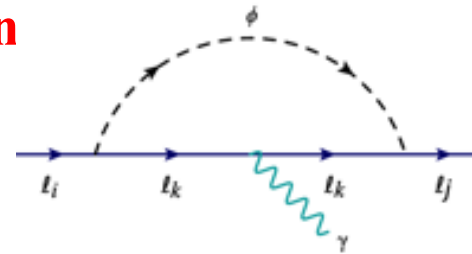
$$\Delta m_{B_d}^{\text{exp}} - \Delta m_{B_d}^{\text{SM}} = (-0.141 \pm 0.513) \times 10^{-13} \text{ GeV}$$

$$\Delta m_{B_s}^{\text{exp}} - \Delta m_{B_s}^{\text{SM}} = (-0.036 \pm 0.178) \times 10^{-11} \text{ GeV}$$

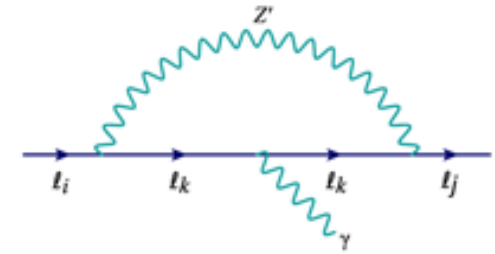
$$\Delta m_K^{\text{exp}} - \Delta m_K^{\text{SM}} = (0.410 \pm 0.922) \times 10^{-15} \text{ GeV}$$

- Charged Lepton Flavor Violation**

1-loop contributions to $l_i \rightarrow l_j \gamma$

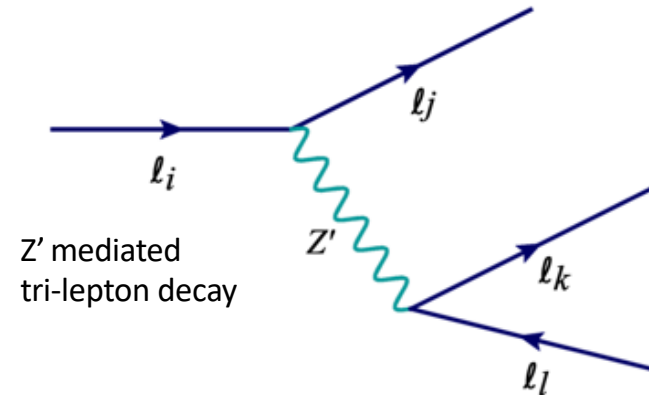


SM Higgs exchange



Z' exchange

tree level three lepton decay $l_i \rightarrow \bar{l}_j l_k l_l$



Z' mediated tri-lepton decay

Process	Experimental Limit	Model Prediction	
		BP1	BP2
$\text{BR}(\mu^- \rightarrow e^- \gamma)$	$< 4.2 \times 10^{-13}$	1.8×10^{-14}	6.3×10^{-15}
$\text{BR}(\tau^- \rightarrow e^- \gamma)$	$< 3.3 \times 10^{-8}$	2.2×10^{-14}	2.2×10^{-14}
$\text{BR}(\tau^- \rightarrow \mu^- \gamma)$	$< 4.4 \times 10^{-8}$	6.8×10^{-15}	3.0×10^{-15}
$\text{BR}(\mu^- \rightarrow e^- e^+ e^-)$	$< 1.0 \times 10^{-12}$	1.3×10^{-18}	3.9×10^{-19}
$\text{BR}(\tau^- \rightarrow e^- e^+ e^-)$	$< 2.7 \times 10^{-8}$	2.2×10^{-17}	1.8×10^{-17}
$\text{BR}(\tau^- \rightarrow \mu^- e^+ e^-)$	$< 1.8 \times 10^{-8}$	9.5×10^{-18}	4.1×10^{-18}

the benchmark points are consistent with data

- Other implications**

the presence of new TeV-ish physics can lead to various other effects not relevant for us – multi TeV-ish scales

lower scale solutions may exist \rightarrow check which BSM signals...

Conclusions

- **Tremendous success of renormalizable QFTs: QED \rightarrow QCD \rightarrow SM**
- **Conceptual: 4d QFT \oplus symmetries \oplus representations \oplus numbers**
 - \rightarrow type (q,l) and number (copies) of fermion representations is not understood
 - \rightarrow generations appear special due to mass values and anomaly cancellation
- **Ideas to “understand” apparent regularities in fermion masses and mixings**
 - \rightarrow flavour symmetries \oplus high scale breakings by flavons
 - \rightarrow many models, become rather complex \leftrightarrow observed structures
- **Other routes?**
 - \rightarrow flavour seesaw \leftrightarrow structure following from representations
 - \rightarrow moderate VEV ratio plus loops
 - \rightarrow discussed a simple model realization with extra $U(1)_{B-L}$
 - \rightarrow mapping of parameters to new patterns
 - \rightarrow allows multi-TeV-ish flavour physics
 - \rightarrow interesting phenomenological consequences: FCNC's , Dirac neutrinos, ...