Joule heating of an emitter on the cathode surface by field electron emission current with account of the non-isolation of the vertex

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Theoretical background

- **Emitter geometry:** a cylindrical tip with radius in the range of $1nm \leqslant r \leqslant 10nm$.
- **One-dimensional heat conduction equation:**

$$c\frac{\partial T}{\partial t} = \lambda\frac{\partial^2 T}{\partial x^2} + q\left(T\right), \tag{1}$$

  where c is the volumetric heat capacity, ρ is the density of the emitter material, λ is the electronic thermal conductivity, q determines the power of heat sources.
- **Joule heating:**

$$q = \rho\left(T\right)j^2, \tag{2}$$

  where ρ is the resistivity.
- **Finite size effects** [1]**:**

$$\rho = \frac{a}{r}\frac{\rho_0}{T_0}T, \tag{3}$$

  where ρ_0 is the resistivity at some reference temperature T_0, $a = 70nm$.

[1] S. Parviainen et al. Computational Materials Science 50, (2011), pp.2075–2079.

Investigated cases

Three cases have been considered

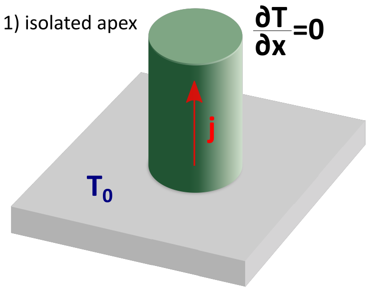

Fig. 1. Joule heating of an emitter with isolated apex

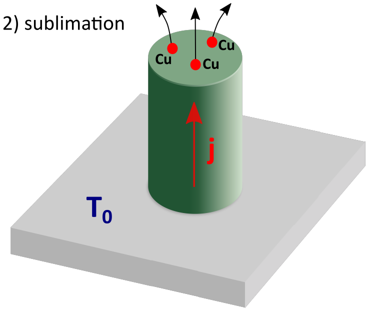

Fig. 2. Joule heating of an emitter with account of the evaporation

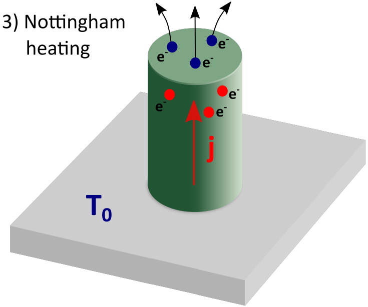

Fig. 3. Joule heating of an emitter with account of the Nottingham effect

Field electron emission current

- **Fowler-Nordheim equation for the Schottky–Nordheim barrier**

$$j = \frac{a}{t^2(y)} \frac{(\beta E)^2}{\varphi} \exp\left(-b\frac{\varphi^{3/2}}{\beta E}\vartheta(y)\right), \tag{4}$$

where

$$y = \frac{\sqrt{\beta E}}{\varphi}, a = \frac{e^3}{16\pi^2\hbar}, b = \frac{4\sqrt{2m}}{3\hbar e}. \tag{5}$$

- **Field enhancement factor**

$$\beta \approx \frac{h}{r}. \tag{6}$$

- **Approximate expressions of the special field emission elliptic functions**[2]

$$\vartheta(y) \approx 1 - y^2 + \frac{1}{3}y^2 lny, \tag{7}$$

$$t(y) \approx 1 + \frac{1}{9}\left(y^2 - y^2 lny\right). \tag{8}$$

[2] Richard G. Forbes. Applied Physics Letters 89, (2006), p.113122

Emitter with isolated apex

- **Analytical solution in the stationary case**

$$T(x) = T_0 \left\{ \cos(\sqrt{b}x) + tg(\sqrt{b}h) \sin(\sqrt{b}x) \right\}, \tag{9}$$

where $b = \frac{1}{L}\left(\frac{a\rho_0 j}{rT_0}\right)^2$, L is the Lorentz coefficient.

- **Temperature on the upper base of the tip**

$$T(h) = \frac{T_0}{\cos(\sqrt{b}h)}. \tag{10}$$

- **Critical value of the current density**

$$j_{cr} = \frac{\pi T_0 \sqrt{L}}{2a\rho_0 \beta}. \tag{11}$$

- **Melting current density**

$$j_m = \frac{T_0 \sqrt{L}}{a\rho_0 \beta} \arccos \frac{T_0}{T_m}. \tag{12}$$

Numerical calculation in the non-stationary case

- **Dimensionless heat conduction equation**

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} = \frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} + Q\tilde{T}, \tag{13}$$

where

$$\tilde{T} = \frac{T}{T_0}, \tilde{x} = \frac{x}{h}, \tilde{t} = \frac{t}{t_0}, t_0 = \frac{a\rho_0 ch^2}{rLT_0}, Q = \frac{1}{L}\left(\frac{a\rho_0}{T_0}\beta j\right). \tag{14}$$

- **Implicit finite difference schema**

$$\frac{\tilde{T}_i^{n+1} - \tilde{T}_i^n}{\delta\tilde{t}} = \frac{\tilde{T}_{i+1}^{n+1} - 2\tilde{T}_i^{n+1} + \tilde{T}_{i-1}^{n+1}}{\delta\tilde{x}^2} + Q\tilde{T}_i^{n+1}, \tag{15}$$

where $\delta\tilde{x}, \delta\tilde{t}$ are the steps in coordinate and time, n corresponds to the number of the time step, and i – to the coordinate.

Results for the isolated apex

Initial parameters: Cu, $r = 10$ nm, $\frac{h}{r} = 47.3$, $E = 170$ MV/m.

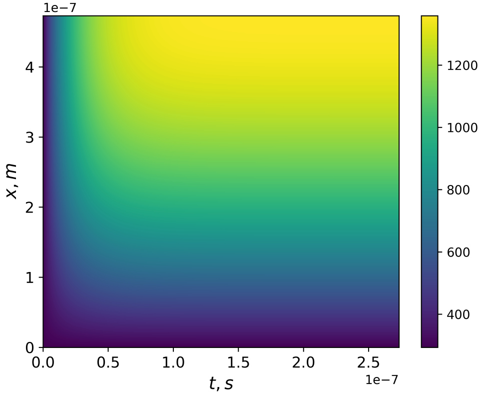

Fig. 4. Color map of the dependence of the emitter temperature on the coordinate and time.

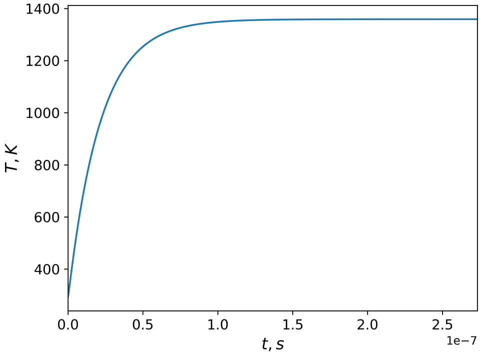

Fig. 5. Dependence of the apex temperature on the time

Results for the isolated apex

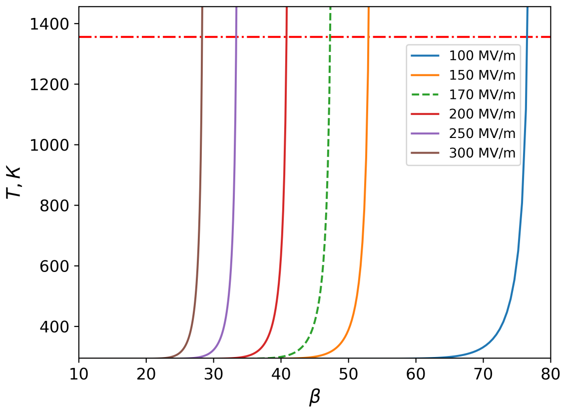

Fig. 6. Dependencies of the apex temperature on the emitter aspect ratio for different values of electric field

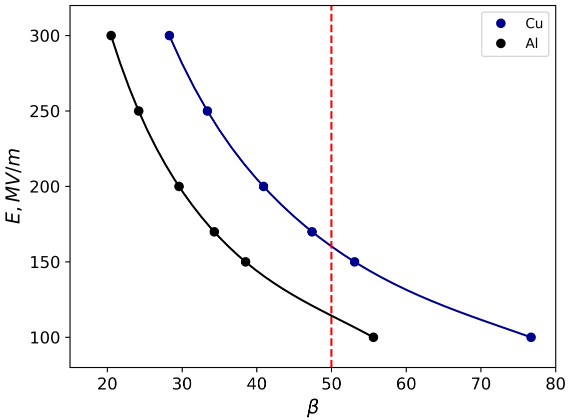

Fig. 7. Dependencies of the electric field on the emitter aspect ratio when the melting of the apex starts

- **Dimensionless heat conduction equation**

$$\frac{\partial \tilde{T}}{\partial \tilde{t}} = \frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} + Q\tilde{T}. \tag{16}$$

- **Boundary conditions**

$$\tilde{T}\Big|_{\tilde{x}=0} = 1, \left.\frac{\partial \tilde{T}}{\partial \tilde{x}}\right|_{\tilde{x}=1} = -\frac{w_{evap}E_{evap}}{\lambda T_0}h, \tag{17}$$

where w_{evap} is the sublimation rate, E_{evap} is the sublimation energy.
According to the Hertz–Knudsen kinetic theory

$$w_{evap} = \sqrt{\frac{M}{2\pi RT}}p, \tag{18}$$

where p is the saturated vapor pressure of a substance, which is a function of absolute temperature, and the empirical expression has the following form

$$\tilde{p} = 10^{A+\frac{B}{T}+C\lg T+D\cdot T\cdot 10^{-3}}. \tag{19}$$

Results with account of the sublimation

Initial parameters: Cu, $r = 10$ nm, $\frac{h}{r} = 47.3$, $E = 170$ MV/m.

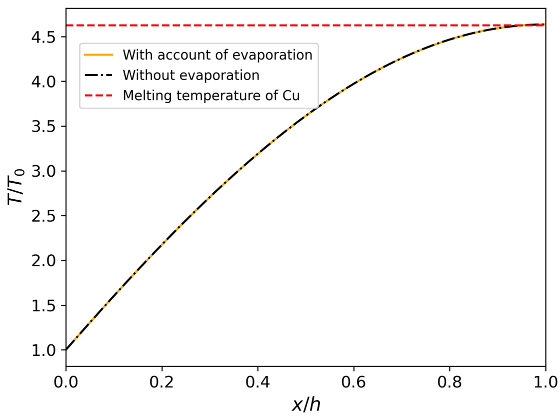

Fig. 8. Dependence of the emitter temperature on the coordinate

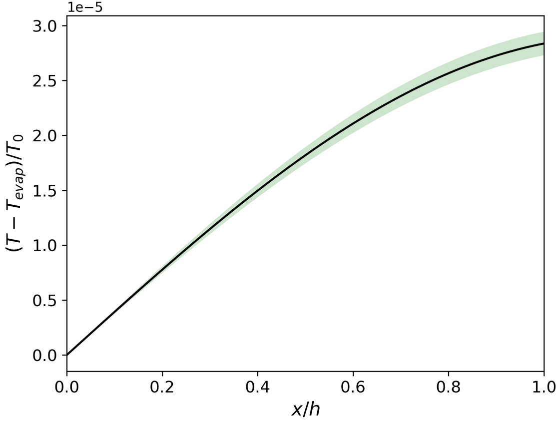

Fig. 9. Influence of the sublimation process on the dependence of the tip temperature on the coordinate.

Nottingham effect

- **Boundary conditions**

$$\left.\frac{\partial \tilde{T}}{\partial \tilde{x}}\right|_{\tilde{x}=1} = -\frac{j}{e}\frac{h}{\lambda T_0}\Delta\varepsilon(\tilde{T}), \tag{20}$$

where $\Delta\varepsilon$ is the difference between the average energy of the electrons that come from the depth of the material and the average energy of the electrons that leave the surface.

- **Energy difference in the case $T < T_{inver}$**[3]

$$\Delta\varepsilon(\tilde{T}) = \frac{\pi^2}{2}\left(\frac{kT_0}{\varepsilon_F}\tilde{T}\right)^2\varepsilon_F + 2\pi k T_0 \tilde{T} ctg\left(\frac{\pi T_0}{2T_{inver}}\tilde{T}\right), \tag{21}$$

where T_{inver} is the inversion temperature.

[3] E. A. Litvinov, G. A. Mesyats, D. I. Proskourovsky. Physics-Uspekhi. 139, (1983), p.265

Results

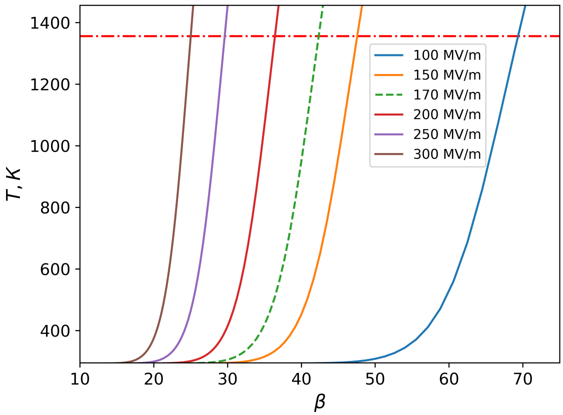

Fig. 10. Dependencies of the apex temperature on the emitter aspect ratio β for different values of electric field taking into account the Nottingham effect

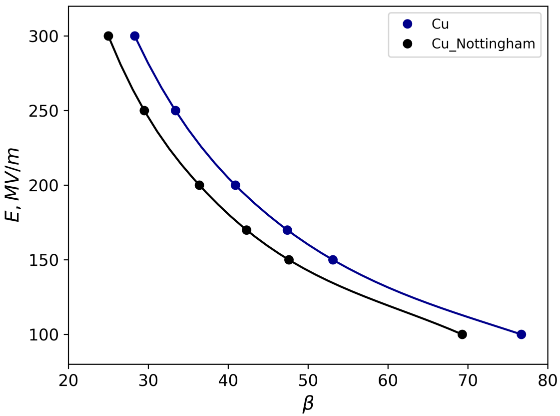

Fig. 11. Dependencies of the breakdown electric field on β (assuming that the breakdown occurs when the apex of the emitter begins to melt)

Conclusions

1. The non-stationary problem of Joule heating of a cylindrical nano-emitter on the cathode surface by field electron emission current has been considered.
2. The dependencies of the equilibrium temperature of the apex of the tip on the emitter aspect ratio have been obtained numerically for different values of the electric field. Assuming that the vacuum breakdown begins when the apex of the emitter melts, it is shown that for a copper emitter at an electric field strength of 100 MV/m, the emitter aspect ratio is 76.7, and for 170 MV/m it is 47.4, which is well consistent with the experimental results. If $\beta = 50$, it is found that the vacuum breakdown field will be equal to 160.3 MV/m for Cu, and 114.4 MV/m for Al.
3. The process of sublimation in the first stage of heating the emitter is taken into account and it is shown that this process is insignificant at this stage.
4. The Nottingham heating has been considered and it has been found that the breakdown field for the constant value of β is reduced. For example, for $\beta = 50$ the reduction is approximately 10%.

Thank you for your attention