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Why using the Nordheim parameter y in the mathematics of the special mathematical function "v" used in field emission theory should now be regarded as mathematically perverse

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This presentation forms part of my efforts to modernise the theories of field ion and electron emission and to encourage more consistent use of basic theory as between different groups of users. It primarily concerns the correction factor "v_F"that appears in the Murphy-Good (MG) theory of field electron emission (FE), which is based on the planar-image-rounded "Schottky-Nordheim"(SN) barrier. In MG FE theory, one can write the following expression for the emission current I in terms of the emitter local work function w (for simplicity, assumed uniform) and a characteristic local field-magnitude F_C (usually interpreted as the field magnitude at the apex of an emitting protrusion):

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I  =  A (a/w)(F\_C)\^{}2 exp[-(v\_F) b w\^{}(3/2)/F\_C]
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Here a and b are the Fowler-Nordheim constants, A is an area-like parameter, and (v_F) is an appropriate particular value of a special mathematical function (SMF) often just called "v".

It has been known since 2008 that the SMF "v"is a very special solution of the Gauss Hypergeometric Differential Equation (HDE); consequently, it is now considered best mathematical practice to take "v"as a function of the independent variable in the Gauss HDE, which I denote by "x"and call the Gauss variable. Hence, I now write v(x). This convention replaces the older convention of expressing "v"as a function of the Nordheim parameter y, which is given by y= +√x.

In recent years I have been arguing that we should make a separation between the mathematics of "v"and the use of this SMF in modelling FE and related phenomena. This separation would be analogous to the separation made between (a) the mathematics of the "abstract"SMF "sinx"and (b) its use in trigonometry, by setting x equal to an angle \theta.

I have also been arguing that continued use of the Nordheim parameter y in the mathematics of "v"should be considered mathematically perverse. The mathematics of "v"should now use the variable "x". Several examples will be presented to underline what I mean by mathematically perverse, and to show that "x"is a "better mathematical variable"than "y".

The SMF v(x) finds uses, not only in MG FE theory, but also in other scientific contexts, for example field ionization of inert gas atoms, as in gas field ion sources and field ion microscopy.

When applying the SMF v(x) to modelling FE via the MG FE equation, there are two possible conventions: (a) the "legacy convention"of setting x=yˆ2; and the modern "21st Century convention"of setting x equal to the "scaled-field f "defined by

f = F/(F_R).

where F_R is the reference field needed to pull the top of the SN barrier down to the Fermi level.

It will be argued that, for experimentalists and technologists, the "21st Century convention"of using f has several significant advantages (which will be detailed) over the legacy convention of using y. The question of a suitable name for "v"will be discussed.

Much of the FE theory used in vacuum breakdown contexts is in fact based on an approximation for "v"de-

veloped by Charbonnier and Martin in 1962. In modern mathematical form this is written:
v(x) \approx (v_CM)(x) = 0.956 −1.062 x .
This appears to have originally been derived (as a function of y) by fitting to a table of values generated by Burgess et al. in 1953 (Phys. Rev. 90, 515). A modern method of deriving this formula, based on a form of linear expansion, will be presented.
A more complete account of a modern theory of "v"can be found in [1]. A tabulation of values of v(x) and related functions, in terms of x (or equivalently f) can be found in [2].
[1] R.G. Forbes, Chap. 9 in: Modern Developments in Vacuum Electron Sources (Springer 2020).
[2] R.G. Forbes and J.H.B. Deane, J. Vac. Sci. Technol. B 28, C2A33 (2010).

Topic

Field Emission

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