Beamstrahlung in CAIN

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ICFA Higgs Factories
Focus Meeting: Beamstrahlung
https://indico.cern.ch/event/1100734/

CAIN Versions

- Started in 1984, named ABEL, for beam-beam interaction in JLC
  - ✓ Pinch effect
  - ✓ Beamstrahlung
- Later, renamed to CAIN when beam-laser interaction was included for gamma-gamma colliders
- Latest version: not clear.
  - ✓ Perhaps, CAIN2.44b, but a beta version
  - ✓ CAIN2.43 is better
- Code
  - ✓ Written in FORTRAN90 since recursive calls were included
  - ✓ Windows version and UNIX version
  - ✓ I have almost stopped revising the code.
  - ✓ Ask Tauchi san for UNIX version and compilation
- Manual available.
- Possible bugs
  - ✓ Sometimes a strange phenomena when too many macroparticles are used
  - ✓ There can be lots of bugs in the routines which are used only rarely

Structure of the Code

- All the particles (electron, positron, photon) are in one big array, containing the info of the space-time, energy-momentum, polarization, etc.
- Fields
  - ✓Beam-beam field
  - ✓External field (constant field, quadrupole field, laser field)

Commands

- Input data is a sequence of “commands”

- Math expression
  - ✓ SET, ARRAY
- Control :
  - ✓ DO, CYCLE, EXIT, ENDDO,
  - ✓ IF, ELSEIF, ELSE, ENDIF
- Output
  - ✓ WRITE, PRINT
  - ✓ PLOT (use very old software TOPDRAWER)
- Beam definition
  - ✓ BEAM
  - ✓ LASER
- Interaction control
  - ✓ BBFIELD beam-beam field
  - ✓ LASERQED
  - ✓ CFQED
    - beamstrahlung
    - coherent pair creation,
  - ✓ LUMINOSITY
  - ✓ PPINT (incoherent pair)
- Beamline definition
  - ✓ MAGNET, BEAMLINE, BLOPTICS, MATCHING
- Execution commands
  - ✓ PUSH, ENDPUSH
  - ✓ DRIFT
  - ✓ LORENTZ
  - ✓ TRANSPORT, ENDTRANSPORT

Polarization

- Spin of electron/positron and Stokes parameters of photons
- Treated as density matrix
- But not for all the interactions

		initial $e^{\pm}$	laser	final $e^{\pm}$	final γ
Beamstrahlung	$e^{\pm} \to e^{\pm} + \gamma$	LT	–	LT	LT
Linear laser-Compton	$e^{\pm} + \text{laser} \to e^{\pm} + \gamma$	LT	LT	LT	LT
Nonlinear laser-Compton	$e^{\pm} + n{\cdot}\text{laser} \to e^{\pm} + \gamma$	L	L*	L	L
	or	N	T*	N	T
		initial γ	laser	final $e^{\pm}$	
Coherent pair	$\gamma \to e^{+} + e^{-}$	LT	–	LT	
Linear laser-Breit-Wheeler	$\gamma + \text{laser} \to e^{+} + e^{-}$	LT	LT	LT	
Nonlinear laser-Breit-Wheeler	$\gamma + n{\cdot}\text{laser} \to e^{+} + e^{-}$	L	L*	L	
		initial	final pair		
Incoherent Breit-Wheeler	$\gamma + \gamma \to e^{+} + e^{-}$	L	N		
Incoherent Bethe-Heitler	$\gamma + e \to e + e^{+} + e^{-}$	N	N		
Incoherent Landau-Lifshitz	$e + e \to e + e + e^{+} + e^{-}$	N	N		
		initial	final		
Bremsstrahlung	$e + e \to e + e + \gamma$	N	N		

- L : Longitudinal spin of electron/positron (or circular polarization of photon).
- T : Transverse spin of electron/positron (or linear polarization of photon).
- * : 100% polarization only
- N : Not computed. (No change for existing particles, zero for created particles)
- - : Irrelevant

Beam Definition

- Courant-Snyder parameters: Following parameters can be used

$$N \text{ (bunch-population)},\ E,\ (t,x,y,s) \text{ (beam-center)},$$
$$\beta_{x,y},\ \alpha_{x,y},\ \eta_{x,y},\ \eta'_{x,y},\ \ \sigma_t,\ \sigma_\epsilon,$$
$$\psi_{x,y} \text{ (crab-angle)},\ \theta_{x,y} \text{ (crossing-angle)},\ \phi_{x,y}\ (x\text{–}y \text{ role}),$$
$$(\zeta_x,\zeta_y,\zeta_s) \text{ (spin)},\ \text{etc.}$$

- Beam data can also be read from files
  - ✓ CAIN standard format, MATHEMATICA format, FORTRAN NAMELIST)
  - ✓ Or, user-defined format (see Sec.3.5.2 of the manual)

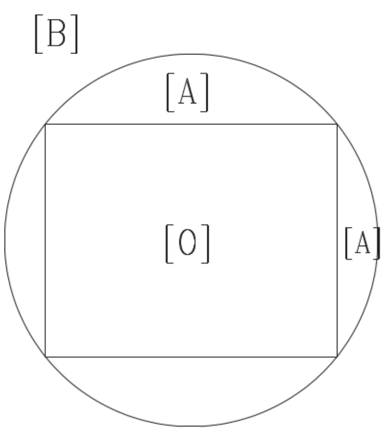
Beam-Beam Field

- Longitudinal slices
  - ✓No interaction between different slices (Lorentz contraction)
  - ✓Longitudinal mesh size must be defined by the user
- Interaction within each slice (2D)
  - ✓Main part
    - Equal-space, rectangular mesh
    - Kernel potential averaged over each mesh
    - Fast computation by FFT
    - Size of the “main part” and the mesh size are decided by the input data, not automatically

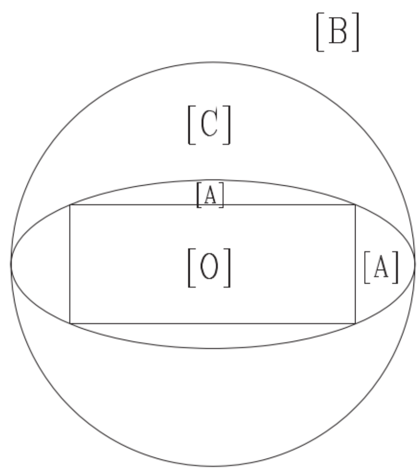
Beam-Beam Field (continued)

✓Outside region

- Ignore the contribution of the particles outside the mesh region
- But the force from the main part to outside particles is included by
  - either direct Coulomb force
  - or by harmonic expansion



Square (more or less) case



Very flat case

[O] mesh region
[A] Direct Coulomb field
[B] Harmonic expansion in polar coordinate
[C] Harmonic expansion in elliptic coordinate

Beamstrahlung

- Use the formula in constant (within the slice) magnetic field

$$dW = \frac{\alpha m}{\sqrt{3}\pi\gamma}\left[Ki_{5/3}(z') + \frac{x^2}{1-x}K_{2/3}(z)\right]dx,$$

$$x \equiv \omega_\gamma/E_0, \qquad z \equiv \frac{2}{3\Upsilon}\frac{x}{1-x}, \qquad \Upsilon \equiv \gamma\frac{B}{B_{sch}}$$

$$B_{sch} = m^2/e = 4.4 \times 10^9 \text{ Tesla}$$

$$Ki_\nu(z) = \int_z^\infty K_\nu(z')dz'$$

- Special functions such as $Ki_{5/3}$ and $K_{2/3}$ are approximated by appropriated polynomials
- Polarization included as much as possible (see below)
- See the manual for the algorithm of event generation
  - ✓ Cain244manual.pdf
  - ✓ uploaded in the indico site

Beamstrahlung (continued)

- Emission angle is not taken into account
  - Effects of the angle is not large during the collision
  - Emission angle causes 1-2 mm displacement at the beam dump window
  - Vertical angle should at least be included
  - Horizontal angle cannot be consistently included in the "Constant Field Approximation"
  - I do not have a complete formula including the angle and polarization effect

Polarization Effects

- Electron/positron polarization can be tracked during beam-beam interaction, including the precession and the spin-flip beamstrahlung
- Initial and final electron polarization (longitudinal and transverse) and the Stokes parameter of the final photon are included
- Included terms
  ζ: electron polarization vector,
  ξ: photon Stokes parameter
  suffix i/f : initial/final
  Terms involving 3 polarization operators are ignored

$$dW = \frac{\alpha dE_\gamma}{4\sqrt{3}\pi\gamma^2} \times \Bigg\{ F_{00}(1+\boldsymbol{\zeta}_i\cdot\overline{\boldsymbol{\zeta}}_f) - xK_{1/3}(\boldsymbol{e}_2\cdot\boldsymbol{\zeta}_i) - \frac{x}{1-x}K_{1/3}(\boldsymbol{e}_2\cdot\overline{\boldsymbol{\zeta}}_f) \\ -\frac{x^2}{1-x}\left[(\boldsymbol{e}_v\cdot\boldsymbol{\zeta}_i)(\boldsymbol{e}_v\cdot\overline{\boldsymbol{\zeta}}_f)Ki_{1/3} + (\boldsymbol{\zeta}_i\cdot\overline{\boldsymbol{\zeta}}_f - \boldsymbol{e}_v\cdot\boldsymbol{\zeta}_i\boldsymbol{e}_v\cdot\overline{\boldsymbol{\zeta}}_f)K_{2/3}\right] \\ +\frac{x}{1-x}K_{1/3}(\boldsymbol{e}_1\cdot\boldsymbol{\zeta}_i)\overline{\xi}_1 - \frac{x(2-x)}{2(1-x)}K_{2/3}(\boldsymbol{e}_v\cdot\boldsymbol{\zeta}_i)\overline{\xi_2} + \left[K_{2/3} - \frac{x}{1-x}K_{1/3}(\boldsymbol{e}_2\cdot\boldsymbol{\zeta}_i)\right]\overline{\xi_3}\Bigg\} \tag{5.152}$$

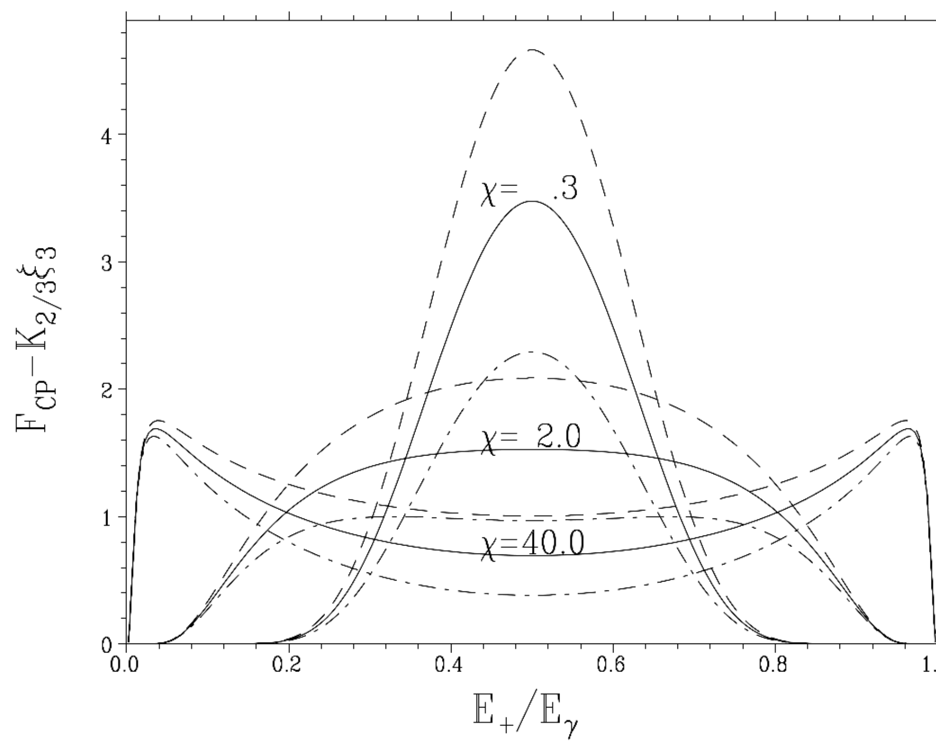
Coherent Pair Creation

➢Constant field formula

$$dW = \frac{\alpha m^2 dE_+}{4\sqrt{3}\pi\omega_\gamma^2} \times \left[Ki_{1/3}(z) + \left(\frac{E_-}{E_+} + \frac{E_+}{E_-}\right)\right] K_{2/3}(z)$$

$$E_- = \omega_\gamma - E_+, \quad z = \frac{2}{3\chi}\frac{\omega_\gamma^2}{E_+ E_-}, \quad \chi \equiv \frac{\omega_\gamma}{mc^2}\frac{B}{B_{sch}}$$

- Polarization of (e^+, e^-, γ) is included
  - ✓ The formula is much more complex than above
- Creation angle ignored
- The algorithm is inefficient for large χ (>~ 1000)



Application to ILC

- Characteristics of the beam-beam interaction in ILC
  - ✓Very large disruption parameter
    - Dy = 25 – 35
    - Accurate simulation is hard
    - Luminosity fluctuation a few % with 400k particles
  - ✓Beamstrahlung is moderate (compared with CLIC)
    - $n_\gamma \sim 2.0$, $\delta_E = 2.5$ (Z-pole, 500GeV) ~ 10 (1TeV)%

Parameters used here

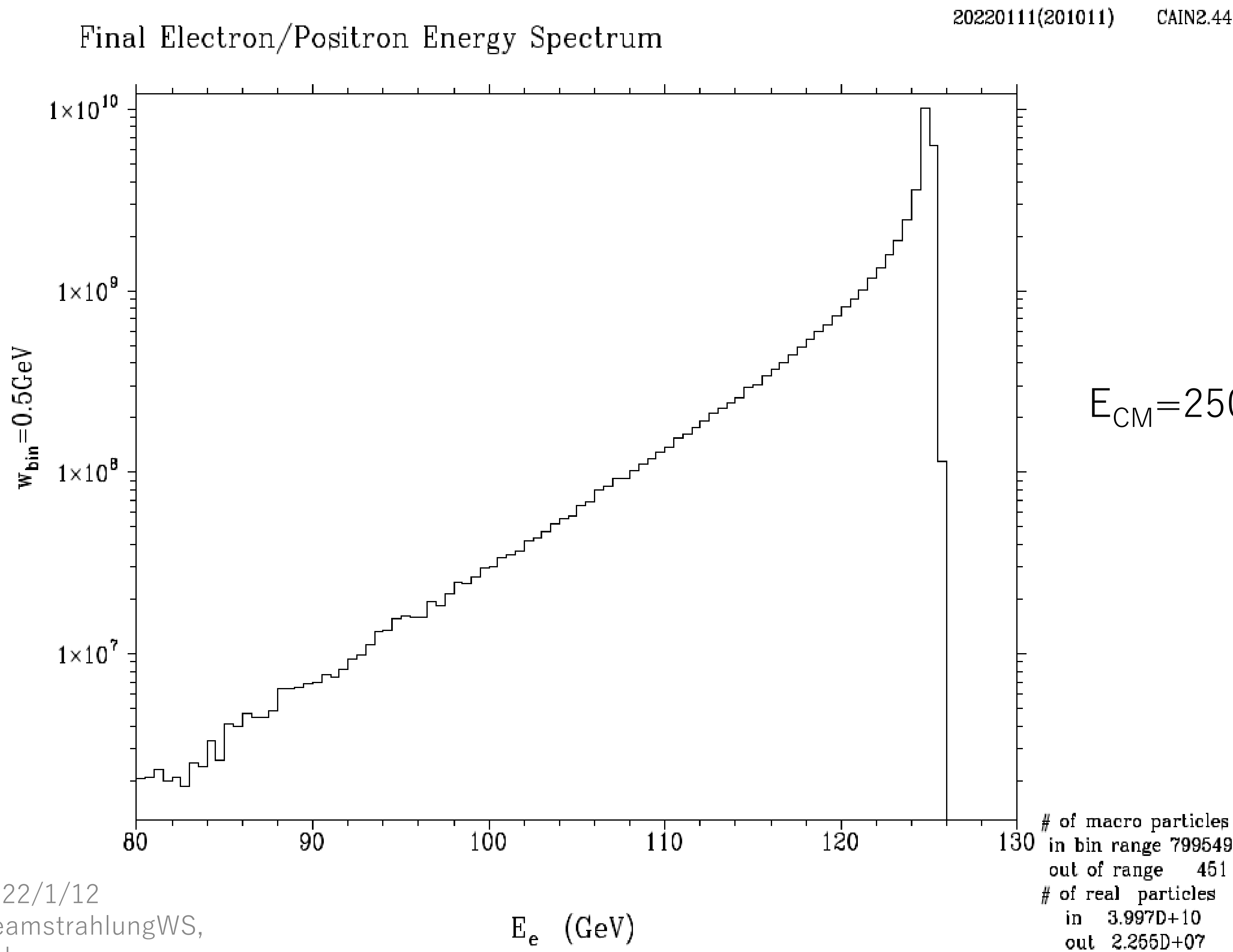
			ILC	ILC(TDR)	ILC(TDR,B)	CLIC(*)
Basic parameters						
	Ecm	GeV	250	500	1000	3000
	Bunch population	e10	2	2	1.737	0.372
	Number of bunches		1312	1312	2450	312
	Repetition rate	Hz	5	5	4	50
	RMS bunch length	mm	0.3	0.3	0.225	0.044
IP parameters						
	Electron energy spread	e-3	1.88	1.24	0.85	3.4
	Positron energy spread	e-3	1.5	0.7	0.47	3.4
	Horizontal emittance	e-6	5	5	5	1.1
	Vertical emittance	e-9	35	35	30	54
	Beta-x	mm	13	22	22	6.9
	Beta-y	mm	0.41	0.48	0.23	0.068
	sigma-x	nm	515	474	335	50.8
	sigma-y	nm	7.659	5.860	2.656	1.118
	Disruption param (x)		0.51	0.30	0.20	0.12
	Disruption param (y)		34.5	24.6	25.1	5.41
	Upsilon average		0.0283	0.0616	0.203	4.33

(*) CLIC parameters here are based on CDR but not exactly the same because the beam deformation in the BDS is included in the CDR

			ILC	ILC(TDR)	ILC(TDR,B)	CLIC(*)
	Ecm	GeV	250	500	1000	3000
Luminosity						
	Lum e+e- (geometric)	e34	0.529	0.751	2.643	3.021
	Lum e+e-	e34	1.453	1.823	5.272	5.845
	Lum e-g	e34	0.728	0.981	3.270	4.393
	Lum at top 1%	%	70.1	62.7	46.3	33.3
Beamstrahlung						
	n-gamma		1.928	1.856	2.077	1.977
	delta_E	%	2.62	4.24	10.01	28.1
Depolarization						
	e- (total)	%	1.412	1.522	2.675	9.436
	e+ (total)	%	1.406	1.511	2.664	9.423
	e- (precession)	%	1.293	1.219	1.310	3.798
	e+ (precession)	%	1.289	1.213	1.305	3.813
	e- (spin flip)	%	0.119	0.303	1.365	5.638
	e+ (spin flip)	%	0.117	0.298	1.359	5.610

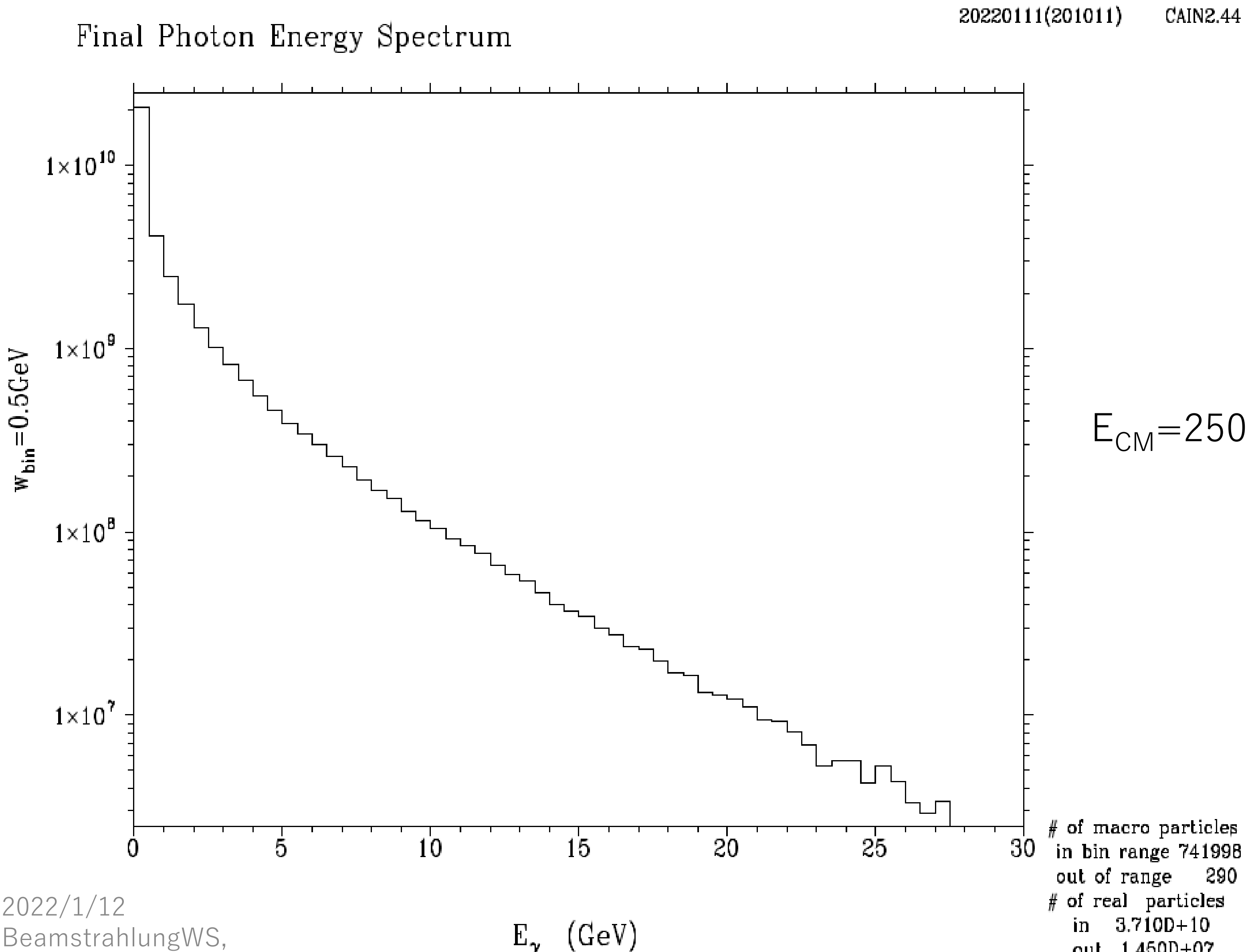
(Only one random number set was used for each. For demonstration only)

Final Electron/Positron Energy Spectrum



E_{CM}=250GeV

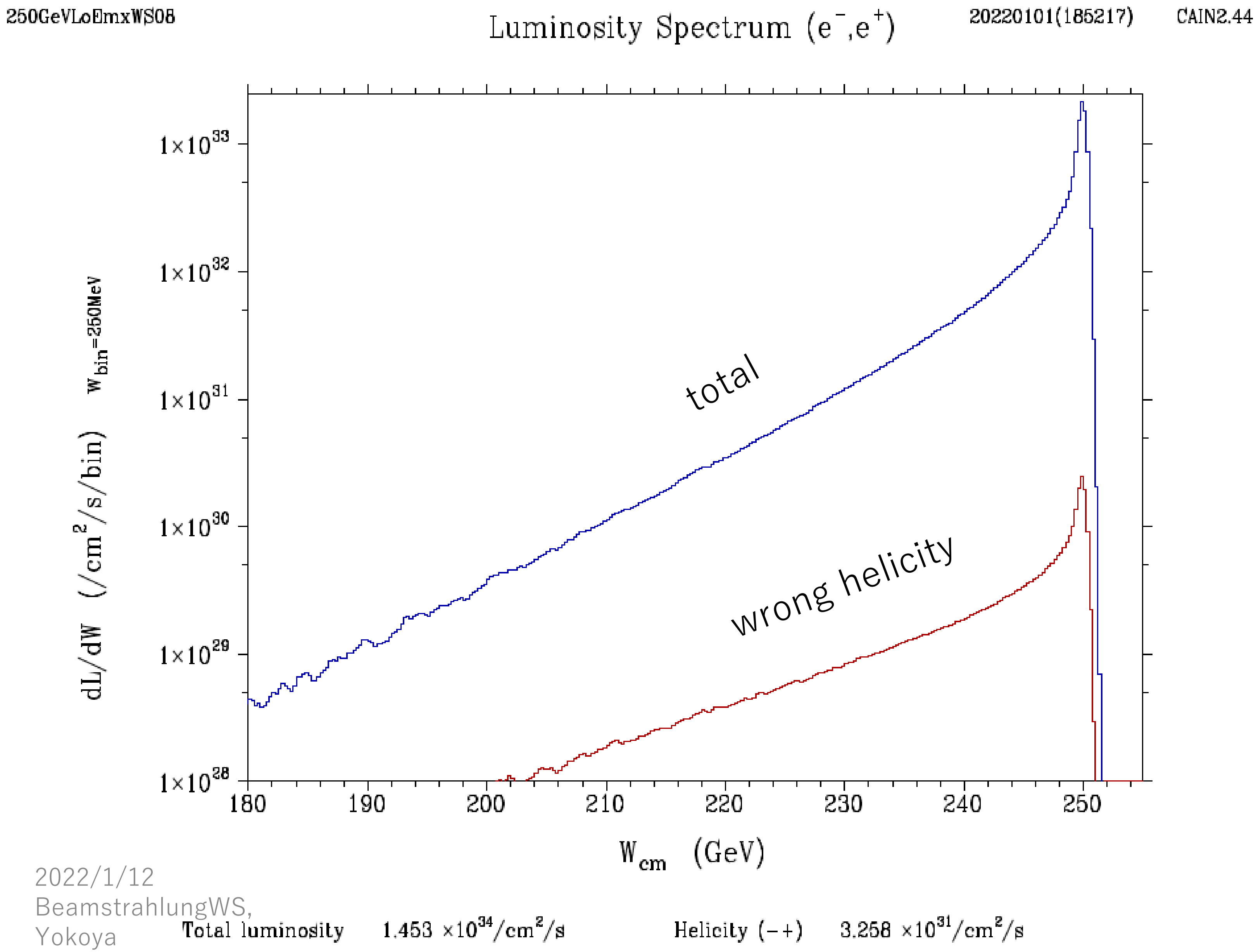
Photon Energy Spectrum



E_{CM}=250GeV

Luminosity Spectrum

E_{CM}=250GeV



Summary

- CAIN has been used since mid 1980's for beam-beam interaction in linear colliders
- Beamstrahlung has been included since the first version
- Angle of photon emission is not included
- Polarization effects due to both the precession and the spin-flip beamstrahlung are included