

# Beamstrahlung in CAIN

Kaoru Yokoya  
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ICFA Higgs Factories  
Focus Meeting: Beamstrahlung  
<https://indico.cern.ch/event/1100734/>

# CAIN Versions

- Started in 1984, named ABEL, for beam-beam interaction in JLC
  - ✓ Pinch effect
  - ✓ Beamstrahlung
- Later, renamed to CAIN when beam-laser interaction was included for gamma-gamma colliders
- Latest version: not clear.
  - ✓ Perhaps, CAIN2.44b, but a beta version
  - ✓ CAIN2.43 is better
- Code
  - ✓ Written in FORTRAN90 since recursive calls were included
  - ✓ Windows version and UNIX version
  - ✓ I have almost stopped revising the code.
  - ✓ Ask Tauchi san for UNIX version and compilation
- Manual available.
- Possible bugs
  - ✓ Sometimes a strange phenomena when too many macroparticles are used
  - ✓ There can be lots of bugs in the routines which are used only rarely

# Structure of the Code

- All the particles (electron, positron, photon) are in one big array, containing the info of the space-time, energy-momentum, polarization, etc.
- Fields
  - ✓ Beam-beam field
  - ✓ External field (constant field, quadrupole field, laser field)

# Commands

➤ Input data is a sequence of “commands”

➤ Math expression

- ✓ SET, ARRAY

➤ Control :

- ✓ DO, CYCLE, EXIT, ENDDO,
- ✓ IF, ELSEIF, ELSE, ENDIF

➤ Output

- ✓ WRITE, PRINT
- ✓ PLOT (use very old software TOPDRAWER)

➤ Beam definition

- ✓ BEAM
- ✓ LASER

➤ Interaction control

- ✓ BBFIELD beam-beam field
- ✓ LASERQED
- ✓ CFQED
  - beamstrahlung
  - coherent pair creation,
- ✓ LUMINOSITY
- ✓ PPINT (incoherent pair)

➤ Beamline definition

- ✓ MAGNET, BEAMLIN, BLOPTICS, MATCHING

➤ Execution commands

- ✓ PUSH, ENDPUSH
- ✓ DRIFT
- ✓ LORENTZ
- ✓ TRANSPORT, ENDTRANSPORT

# Polarization

- Spin of electron/positron and Stokes parameters of photons
- Treated as density matrix
- But not for all the interactions

		initial $e^\pm$	laser	final $e^\pm$	final $\gamma$
Beamstrahlung	$e^\pm \rightarrow e^\pm + \gamma$	LT	—	LT	LT
Linear laser-Compton	$e^\pm + \text{laser} \rightarrow e^\pm + \gamma$	LT	LT	LT	LT
Nonlinear laser-Compton	$e^\pm + n \cdot \text{laser} \rightarrow e^\pm + \gamma$	L	L*	L	L
		or N	T*	N	T
		initial $\gamma$	laser	final $e^\pm$	
Coherent pair	$\gamma \rightarrow e^+ + e^-$	LT	—	LT	
Linear laser-Breit-Wheeler	$\gamma + \text{laser} \rightarrow e^+ + e^-$	LT	LT	LT	
Nonlinear laser-Breit-Wheeler	$\gamma + n \cdot \text{laser} \rightarrow e^+ + e^-$	L	L*	L	
		initial	final pair		
Incoherent Breit-Wheeler	$\gamma + \gamma \rightarrow e^+ + e^-$	L	N		
Incoherent Bethe-Heitler	$\gamma + e \rightarrow e + e^+ + e^-$	N	N		
Incoherent Landau-Lifshitz	$e + e \rightarrow e + e + e^+ + e^-$	N	N		
		initial	final		
Bremsstrahlung	$e + e \rightarrow e + e + \gamma$	N	N		

- L : Longitudinal spin of electron/positron (or circular polarization of photon).
- T : Transverse spin of electron/positron (or linear polarization of photon).
- \* : 100% polarization only
- N : Not computed. (No change for existing particles, zero for created particles)

# Beam Definition

- Courant-Snyder parameters: Following parameters can be used

$N$  (bunch-population),  $E$ ,  $(t, x, y, s)$  (beam-center),  
 $\beta_{x,y}$ ,  $\alpha_{x,y}$ ,  $\eta_{x,y}$ ,  $\eta'_{x,y}$ ,  $\sigma_t$ ,  $\sigma_\epsilon$ ,  
 $\psi_{x,y}$  (crab-angle),  $\theta_{x,y}$  (crossing-angle),  $\phi_{x,y}$  ( $x$ – $y$  role),  
 $(\zeta_x, \zeta_y, \zeta_s)$  (spin), etc.

- Beam data can also be read from files
  - ✓ CAIN standard format, MATHEMATICA format, FORTRAN NAMELIST)
  - ✓ Or, user-defined format (see Sec.3.5.2 of the manual)

# Beam-Beam Field

## ➤ Longitudinal slices

- ✓ No interaction between different slices (Lorentz contraction)
- ✓ Longitudinal mesh size must be defined by the user

## ➤ Interaction within each slice (2D)

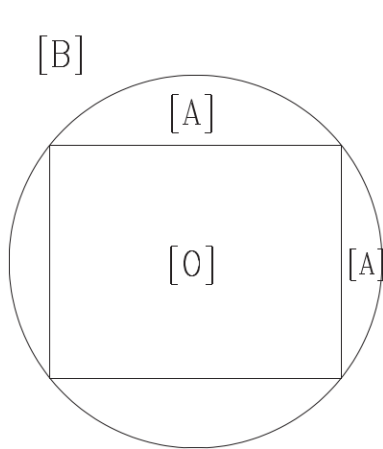
### ✓ Main part

- Equal-space, rectangular mesh
- Kernel potential averaged over each mesh
- Fast computation by FFT
- Size of the “main part” and the mesh size are decided by the input data, not automatically

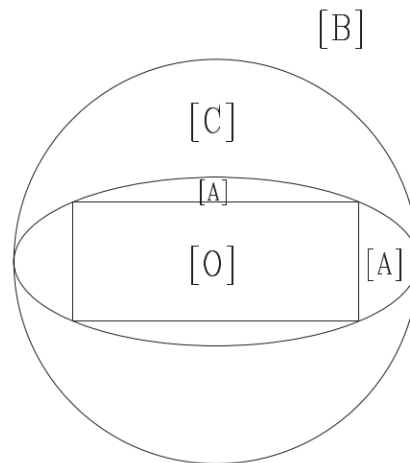
# Beam-Beam Field (continued)

## ✓ Outside region

- Ignore the contribution of the particles outside the mesh region
- But the force from the main part to outside particles is included by
  - either direct Coulomb force
  - or by harmonic expansion



Square (more or less) case



Very flat case

[O] mesh region  
[A] Direct Coulomb field  
[B] Harmonic expansion in polar coordinate  
[C] Harmonic expansion in elliptic coordinate



# Beamstrahlung

- Use the formula in constant (within the slice) magnetic field

$$dW = \frac{\alpha m}{\sqrt{3}\pi\gamma} \left[ Ki_{5/3}(z') + \frac{x^2}{1-x} K_{2/3}(z) \right] dx,$$

$$x \equiv \omega_\gamma/E_0, \quad z \equiv \frac{2}{3\Upsilon} \frac{x}{1-x}, \quad \Upsilon \equiv \gamma \frac{B}{B_{sch}}$$

$$B_{sch} = m^2/e = 4.4 \times 10^9 \text{ Tesla}$$

$$Ki_\nu(z) = \int_z^\infty K_\nu(z') dz'$$

- Special functions such as  $Ki_{5/3}$  and  $K_{2/3}$  are approximated by appropriated polynomials
- Polarization included as much as possible (see below)
- See the manual for the algorithm of event generation
  - ✓ Cain244manual.pdf
  - ✓ uploaded in the indico site

# Beamstrahlung (continued)

- Emission angle is not taken into account
  - ✓ Effects of the angle is not large during the collision
  - ✓ Emission angle causes 1-2 mm displacement at the beam dump window
  - ✓ Vertical angle should at least be included
  - ✓ Horizontal angle cannot be consistently included in the “Constant Field Approximation”
  - ✓ I do not have a complete formula including the angle and polarization effect

# Polarization Effects

- Electron/positron polarization can be tracked during beam-beam interaction, including the precession and the spin-flip beamstrahlung
- Initial and final electron polarization (longitudinal and transverse) and the Stokes parameter of the final photon are included
- Included terms
  - ζ: electron polarization vector,
  - ξ: photon Stokes parameter
  - suffix i/f : initial/final
  - Terms involving 3 polarization operators are ignored

$$\begin{aligned}
 dW = \frac{\alpha dE_\gamma}{4\sqrt{3}\pi\gamma^2} \times & \left\{ F_{00}(1 + \zeta_i \cdot \bar{\zeta}_f) - x K_{1/3}(\mathbf{e}_2 \cdot \zeta_i) - \frac{x}{1-x} K_{1/3}(\mathbf{e}_2 \cdot \bar{\zeta}_f) \right. \\
 & - \frac{x^2}{1-x} \left[ (\mathbf{e}_v \cdot \zeta_i)(\mathbf{e}_v \cdot \bar{\zeta}_f) K_{i1/3} + (\zeta_i \cdot \bar{\zeta}_f - \mathbf{e}_v \cdot \zeta_i \mathbf{e}_v \cdot \bar{\zeta}_f) K_{2/3} \right] \\
 & \left. + \frac{x}{1-x} K_{1/3}(\mathbf{e}_1 \cdot \zeta_i) \bar{\xi}_1 - \frac{x(2-x)}{2(1-x)} K_{2/3}(\mathbf{e}_v \cdot \zeta_i) \bar{\xi}_2 + \left[ K_{2/3} - \frac{x}{1-x} K_{1/3}(\mathbf{e}_2 \cdot \zeta_i) \right] \bar{\xi}_3 \right\} \quad (5.152)
 \end{aligned}$$

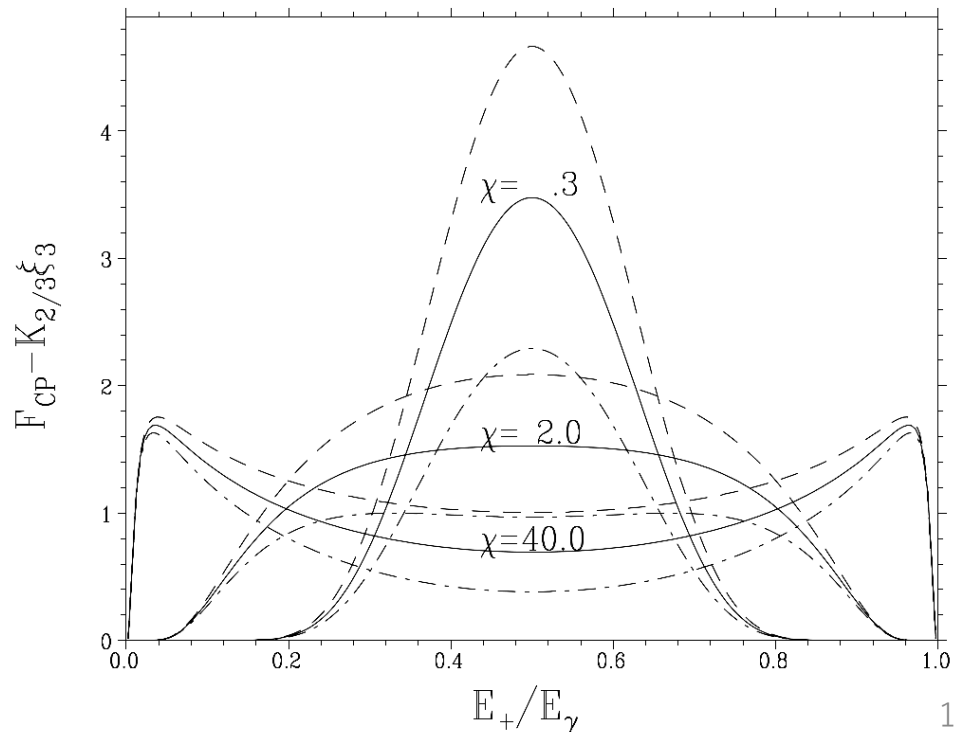
# Coherent Pair Creation

## ➤ Constant field formula

$$dW = \frac{\alpha m^2 dE_+}{4\sqrt{3}\pi\omega_\gamma^2} \times \left[ Ki_{1/3}(z) + \left( \frac{E_-}{E_+} + \frac{E_+}{E_-} \right) \right] K_{2/3}(z)$$

$$E_- = \omega_\gamma - E_+, \quad z = \frac{2}{3\chi} \frac{\omega_\gamma^2}{E_+ E_-}, \quad \chi \equiv \frac{\omega_\gamma}{mc^2} \frac{B}{B_{sch}}$$

- Polarization of (e<sup>+</sup>, e<sup>-</sup>, γ) is included
  - ✓ The formula is much more complex than above
- Creation angle ignored
- The algorithm is inefficient for large χ (>~ 1000)



# Application to ILC

## ➤ Characteristics of the beam-beam interaction in ILC

### ✓ Very large disruption parameter

- $D_y = 25 - 35$
- Accurate simulation is hard
- Luminosity fluctuation a few % with 400k particles

### ✓ Beamstrahlung is moderate (compared with CLIC)

- $n_\gamma \sim 2.0$ ,  $\delta_E = 2.5$  (Z-pole, 500GeV)  $\sim 10$  (1TeV)%

# Parameters used here

			ILC	ILC(TDR)	ILC(TDR,B)	CLIC <sup>(*)</sup>
Basic parameters						
	Ecm	GeV	250	500	1000	3000
	Bunch population	e10	2	2	1.737	0.372
	Number of bunches		1312	1312	2450	312
	Repetition rate	Hz	5	5	4	50
	RMS bunch length	mm	0.3	0.3	0.225	0.044
IP parameters						
	Electron energy spread	e-3	1.88	1.24	0.85	3.4
	Positron energy spread	e-3	1.5	0.7	0.47	3.4
	Horizontal emittance	e-6	5	5	5	1.1
	Vertical emittance	e-9	35	35	30	54
	Beta-x	mm	13	22	22	6.9
	Beta-y	mm	0.41	0.48	0.23	0.068
	sigma-x	nm	515	474	335	50.8
	sigma-y	nm	7.659	5.860	2.656	1.118
	Disruption param (x)		0.51	0.30	0.20	0.12
	Disruption param (y)		34.5	24.6	25.1	5.41
	Upsilon average		0.0283	0.0616	0.203	4.33

(\*) CLIC parameters here are based on CDR but not exactly the same because the beam deformation in the BDS is included in the CDR

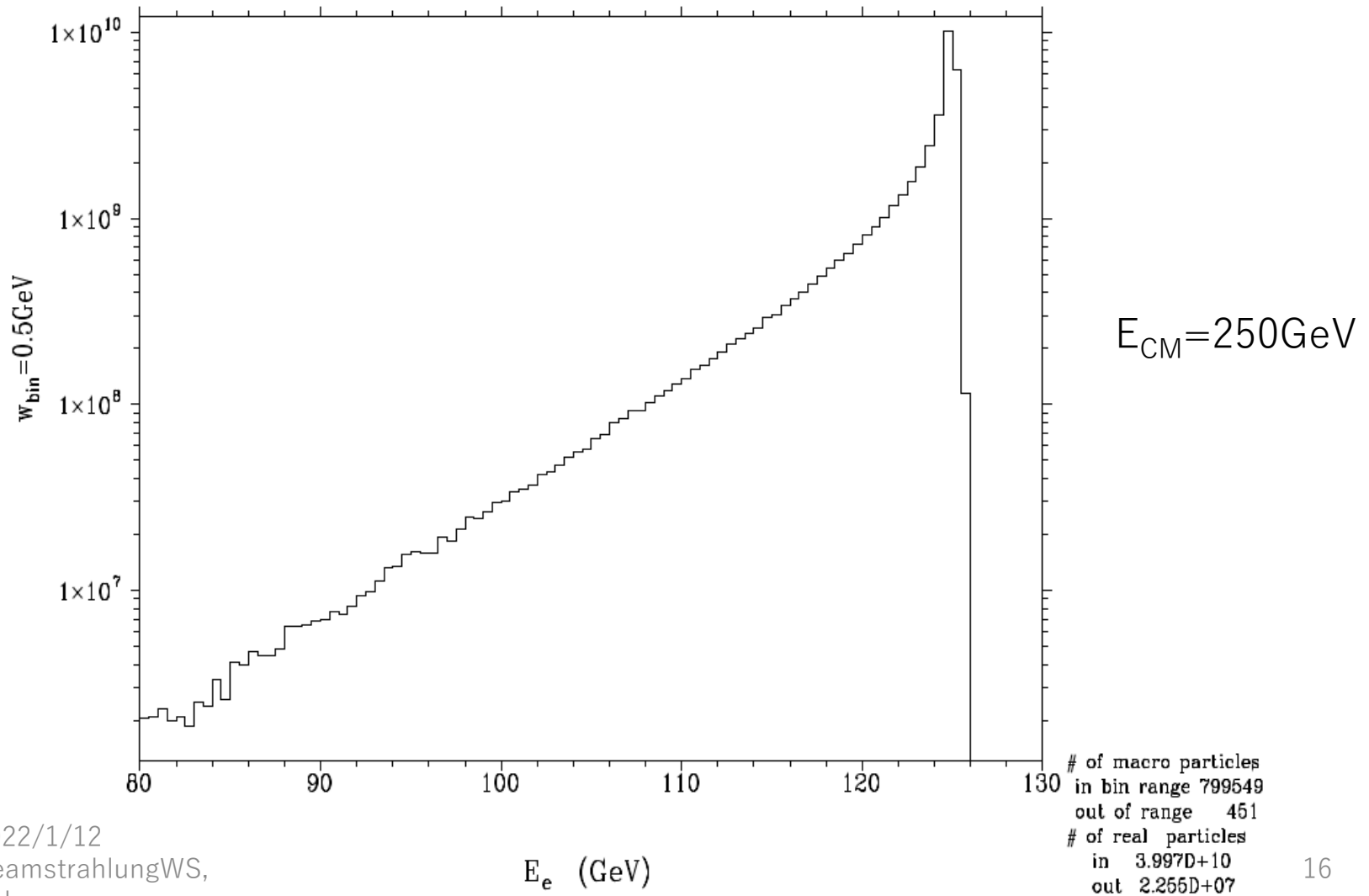
			ILC	ILC(TDR)	ILC(TDR,B)	CLIC <sup>(*)</sup>
	Ecm	GeV	250	500	1000	3000
Luminosity						
	Lum e+e- (geometric)	e34	0.529	0.751	2.643	3.021
	Lum e+e-	e34	1.453	1.823	5.272	5.845
	Lum e-g	e34	0.728	0.981	3.270	4.393
	Lum at top 1%	%	70.1	62.7	46.3	33.3
Beamstrahlung						
	n-gamma		1.928	1.856	2.077	1.977
	delta_E	%	2.62	4.24	10.01	28.1
Depolarization						
	e- (total)	%	1.412	1.522	2.675	9.436
	e+ (total)	%	1.406	1.511	2.664	9.423
	e- (precession)	%	1.293	1.219	1.310	3.798
	e+ (precession)	%	1.289	1.213	1.305	3.813
	e- (spin flip)	%	0.119	0.303	1.365	5.638
	e+ (spin flip)	%	0.117	0.298	1.359	5.610

(Only one random number set was used for each. For demonstration only)

# Final Electron/Positron Energy Spectrum

20220111(201011) CAIN2.44

Final Electron/Positron Energy Spectrum



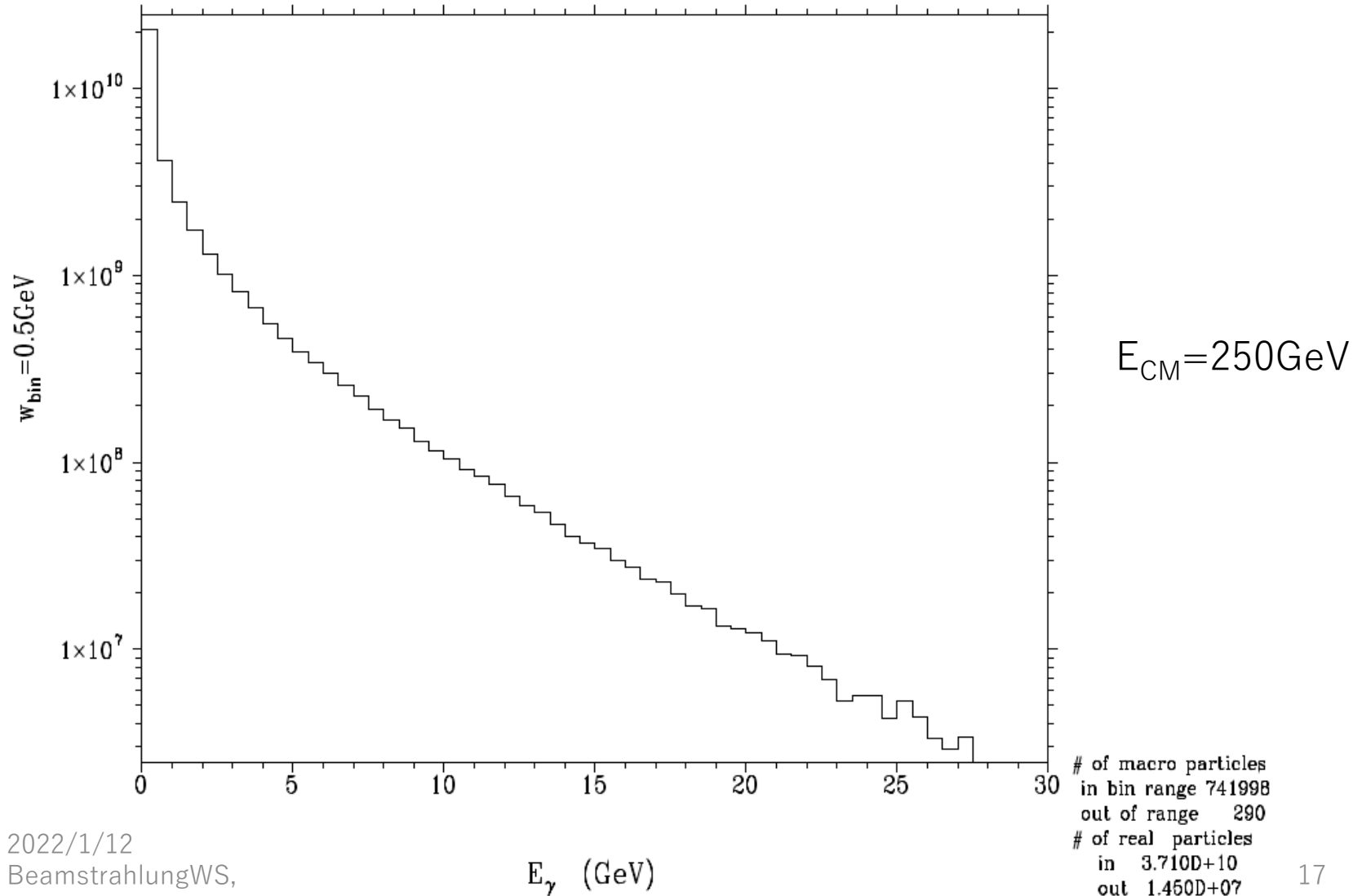


# Photon Energy Spectrum

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CAIN2.44

Final Photon Energy Spectrum



# Luminosity Spectrum

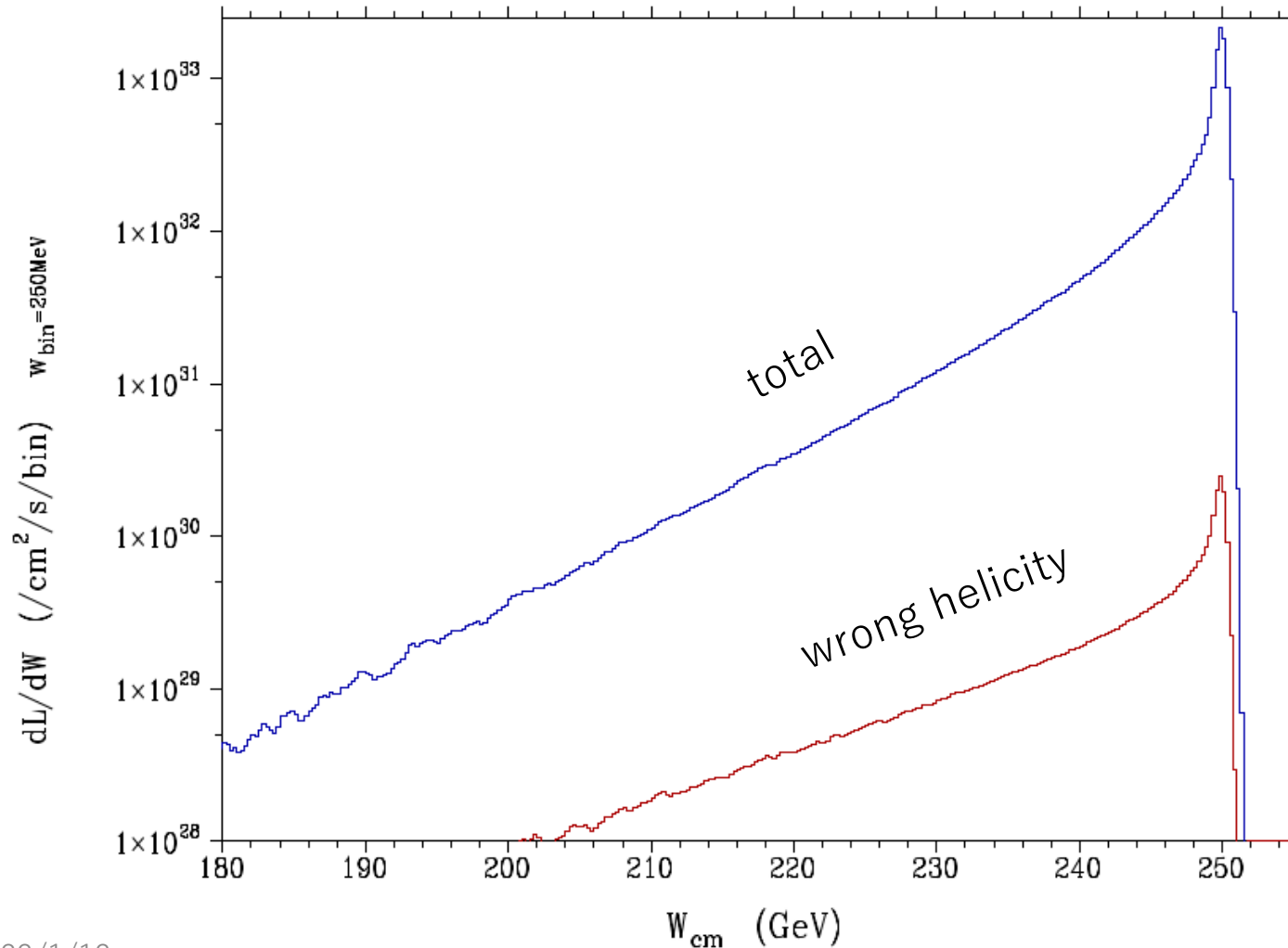
$E_{CM}=250\text{GeV}$

250GeVLoEmxWS08

Luminosity Spectrum ( $e^-e^+$ )

20220101(185217)

CAIN2.44



2022/1/12

BeamstrahlungWS,

Yokoya

Total luminosity  $1.453 \times 10^{34}/\text{cm}^2/\text{s}$

Helicity (-+)  $3.258 \times 10^{31}/\text{cm}^2/\text{s}$

# Summary

- CAIN has been used since mid 1980's for beam-beam interaction in linear colliders
- Beamstrahlung has been included since the first version
- Angle of photon emission is not included
- Polarization effects due to both the precession and the spin-flip beamstrahlung are included