Local Holography:
A new paradigm for quantum gravity
Based on symmetry

ENS–Lyon 2022

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A. Raclariu, S. Speziale, A. Speranza, N. Teh.

QG Questions

- What are the fundamental QG degrees of freedom?
- What is the geometrical entropy counting?
- What are the fundamental observables?
- Can we provide a model of quantum gravity that respect the presence of a Planckian cutoff and the principle of general covariance

- Local Holography: A new perspective which comes from

- Asking new fundamental questions :

- How do we decompose a gravitational sytems into subsystems?
- What is the nature of entanglement across subregions ?
- What are the symmetries of gravity ?

- Developing new tools: Covariant phase space, Coadjoint orbits
  Representation theory of sphere loop groups

Local Holography

- Local Holography: Fits the logic of LQG

- It takes seriously the **classical structure of gravity**
- It builds a quantization of GR where **symmetries are at the center** Diff, SU(2)
- Start with quasi-local data and build spacetime from **gluing and coarse graining**

- It provides new opportunity for LQG

- It reconciles **LQG** (bulk based) with **Holography** (boundary based)
- It provide a **semi-classical basis** for some of the LQG results (Qtion of area)
- It provides new quantization tools for LQG
- It provides a canonical generator for diffeomorphism and tools to resolve ambiguities
- It allows to derive **discretization** of geometry (loops/graph) **from quantization** of symmetries
- It allows to connect quantum gravity with S-matrix analysis and classical strong gravity

Local Holography: People

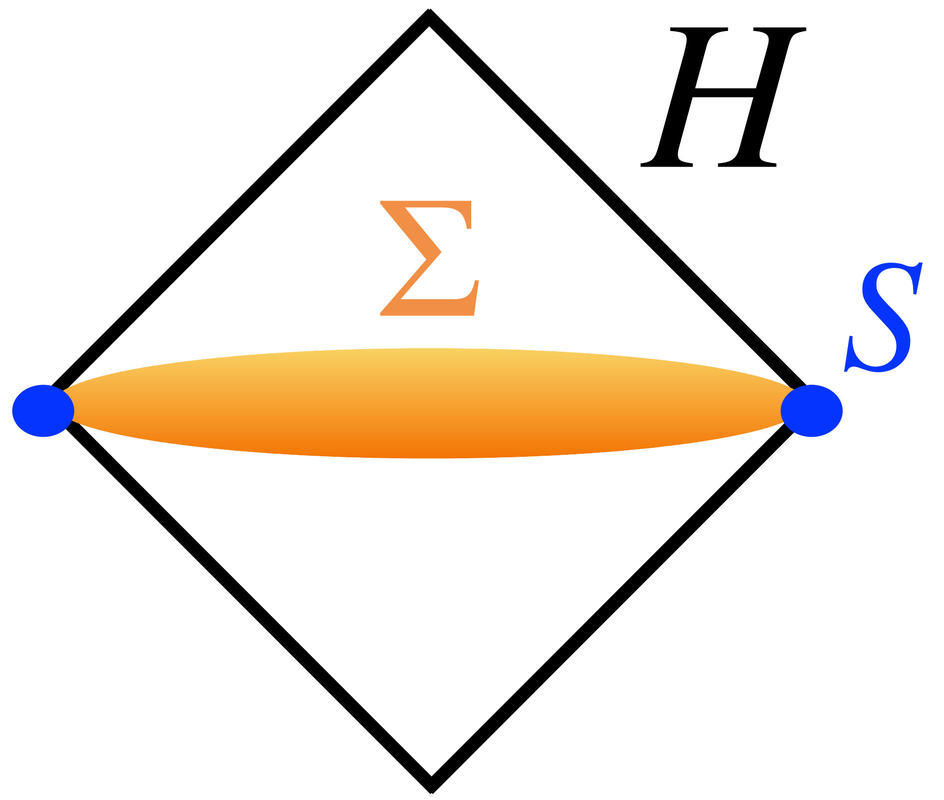
- Some of the ideas were already present in usual LQG: Black Hole entropy calculation, New vacua, Twisted geometry, spin foam with cosmological constant

Ashtekar, Baez, Bodendorfer, Bianchi, Dittrich, Geiller, Haggard, Han, Husain, Krasnov, Perez, Pranzetti, Riello, Speziale, Varadarajan, Wieland

- New Community:

Barnich, Ciambelli, Compere, Chandrasekaran, Donnay, Donnelly, Flanagan, Geiller, Grumiller, Marteau, Oliveri, Petropoulos, Pranzetti, Ruzziconi, Sheikh-Jabbari, Riello, Raclariu, Ruzziconi, Speranza, Speziale, Troesseart, Zwikel, Wieland,…

Why boundaries?



- Important physics happens @ boundaries: Codimension 1
  @ corners: Codimension 2

- In Gauge theory corners support local symmetry charges and symmetry algebra
- Boundaries support flux/balance laws of charges and radiation

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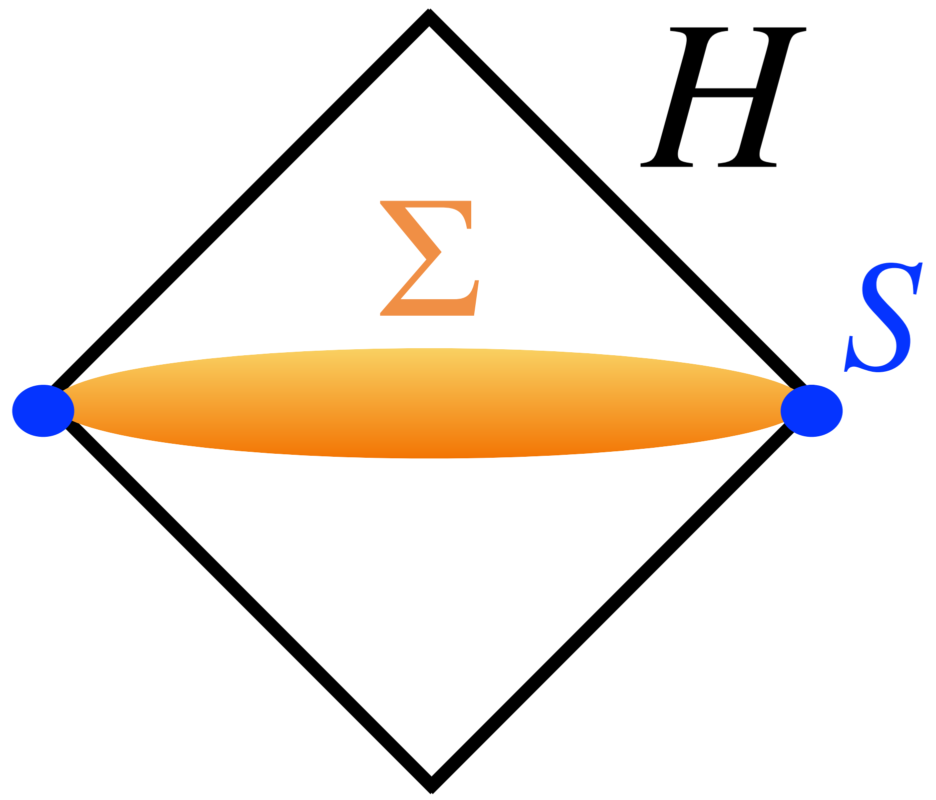

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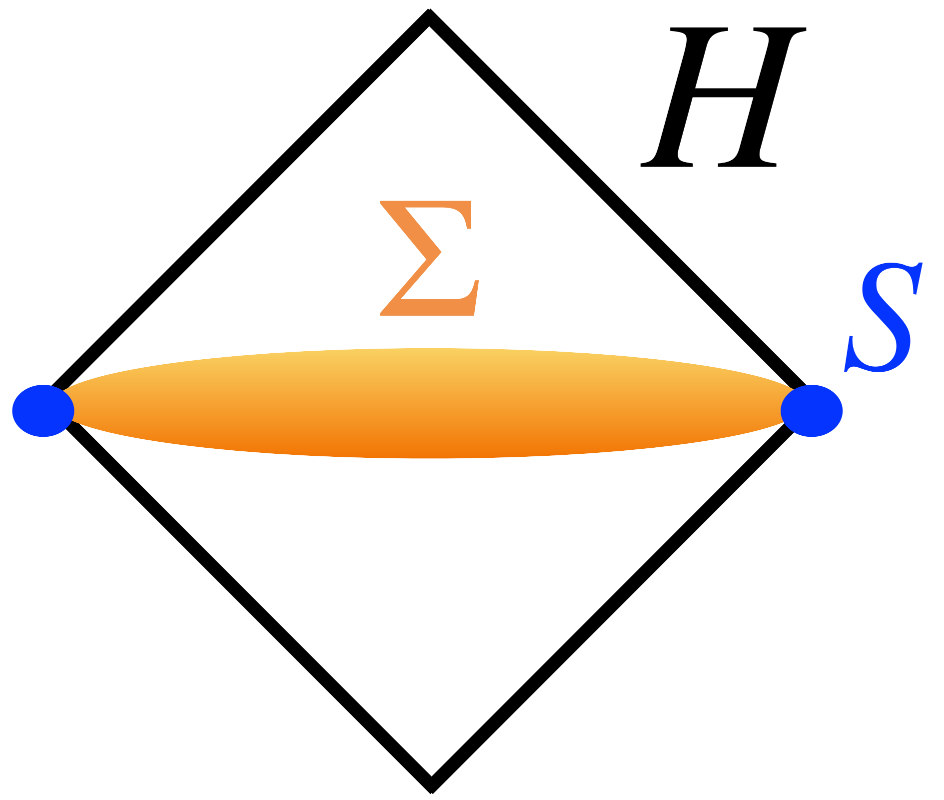
- Corners do not require boundary conditions for an Hamiltonian treatment while Boundaries do.
- Boundaries/Corners may be at ∞ (eg $\mathscr{I}^+/\, \iota_0$), Finite distance (BH Horizon/ Bifurcation surface), arbitrary (entangling surface).
- Boundaries/corners in Gravity appears in the NP study of aymptotic dynamics, the S-matrix (soft theorems), the **membrane paradigm**, the **fluid/gravity** correspondence, in the study of **isolated/dynamical horizons**.
- Boundaries/ corners support essential physical observables, **LQG Flux**, **ADM mass** and angular momenta, **memory observables**, **multipole moments**, BH and entanglement **entropy**. All related to corner symmetries !

Why Symmetries ?



- Classical Symmetries allows to resolve quantization ambiguities and gives us a precise control on the quantisation of algebra observables

$\longrightarrow$ This gives us a way to control the quantization of **geometry recast as symmetry observable**

- The **coadjoint orbit method** is a classical tools that allows us to understand quantum representation as classical orbit on the dual of the Lie algebra. Works in a variety of cases for finite and infinite dimensional groups!

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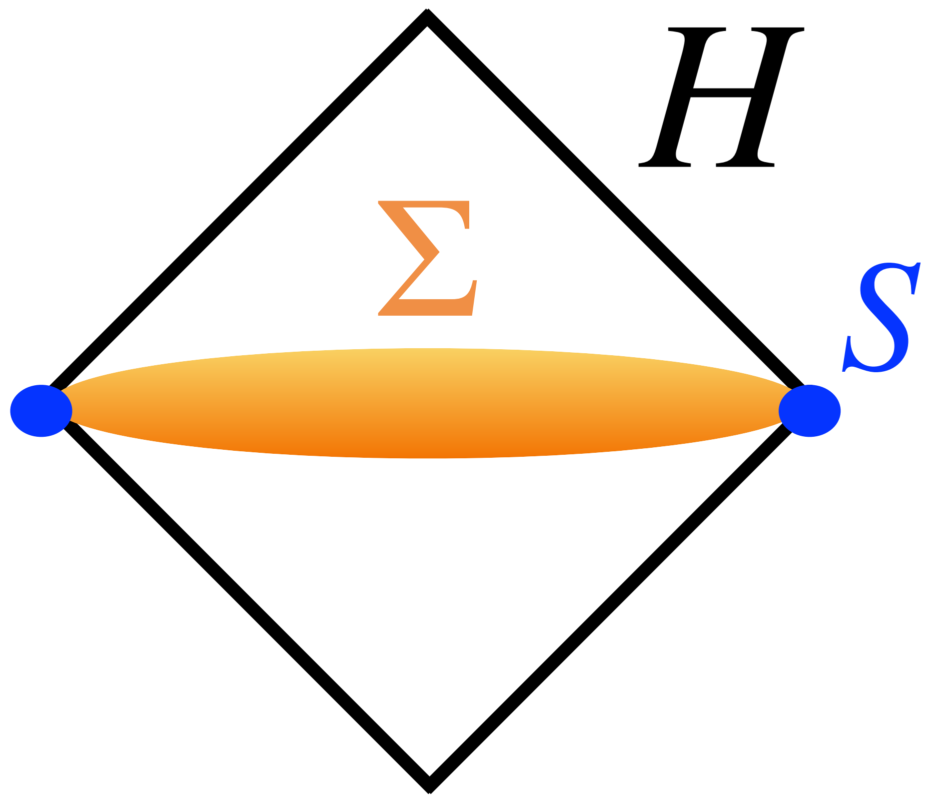

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- Several options: Once a symmetry is obtained at the classical level: look for a quantum theory that represents exactly the symmetry, look for a deformation.

- At the quantum level it is essential to understand the distinction between **gauge redundancy and symmetry**. ``Gauge fixing a symmetry" could be inoccuous at the classical evel but very dangerous at the quantum level. Eg $BMS \subset GBMS$ which accounts for the soft theorems

  → Miguel & Daniele's talks

- Corner symmetry = local symmetry $\neq$ global symmetry
- = Phase space symmetry $\neq$ geometrical symmetry (preserving a given geometrical structure)
- Conserved in the abscence of radiation, satisfy flux/balance laws in the presence of radiation
- One should look for the maximal symmetry group. **One should not gauge fix a symmetry!**

Noether theorems for local symmetry

- Global symmetries are associated with currents integrated on codimension one slices Σ

$$Q_\xi = \int_\Sigma J_\xi$$

- For **local symmetries** the Noether current is trivially conserved: this means that there is a Constraints such that $d(J_\xi - C_\xi) = 0$

- This implies that the charges of local symmetries such as diffeomorphisms supported inside a region Σ are entirely supported by its boundary $S = \partial\Sigma$

$$Q_\xi = \underbrace{\int_\Sigma C_\xi}_{\text{Constraints}} + \underbrace{\int_S q_\xi}_{\text{Charge aspect}}$$

$$\boxed{C_\xi \hat{=} 0}$$

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$$Q_\xi \hat{=} \int_S q_\xi \quad \longleftarrow \quad \text{Charge aspect}$$

- What distinguishes gauge from symmetry is the **non zero value** of the charge.

Extended Corner symmetry group GS

Charge and Flux

- Dynamical symmetries carry **Flux**

$$I_\xi \Omega = \delta Q_\xi + \mathscr{F}_\xi$$

Canonical variation = Noether + Flux

- Local conservation Law: Flux balance

$$\delta_\ell Q_\xi = Q_{[\xi,\ell]} + I_\xi \mathscr{F}_\ell$$

Evolution = Rotation + dissipation

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- The charges splits into kinematical charges $\mathscr{F}_\xi = 0$ and dynamical charges.
- The kinematical charges are canonical bracket that form a quantizable algebra:
  Corner symmetry group H_S

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Ashtekar Streubel '81
Wald, Zoupas' 00
Barnich Troessaert '10
Pasterski, Strominger, Zhib '18
Ladha, Campiglia '18
Pranzetti, Oliveri, Speiale,LF '21
Ciambelli, Leigh '21
Wieland'22

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- The Dynamical charges are also quantizable in a **extended phase space** that includes the radiative modes or the edge mode fields. **Extended Corner symmetry group** G_S

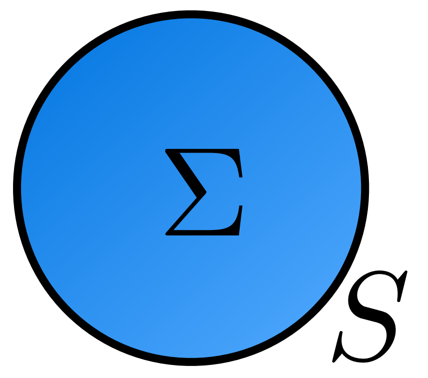
$\longrightarrow$ Non-commutativity of the corner metric components

- At the quantum level physical observables form representations of $H_S \subset G_S$

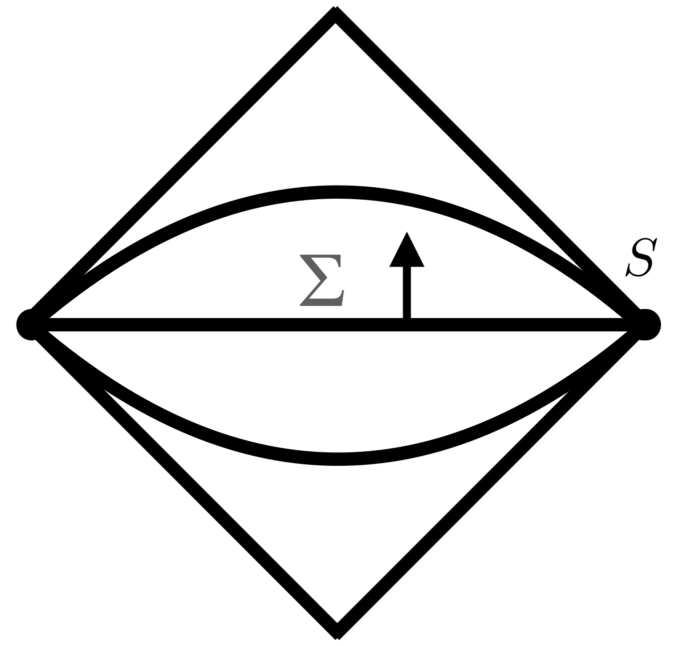
$\longrightarrow$ Quantising H_S, G_S = Quantizing geometry.

Summary

- Given a region R with slice Σ the symmetry charges are supported on codimension 2 corners = entangling sphere



- Local symmetries allows us to encapsulate the geometry of spacetime regions into corner symmetry charge



- The dynamics is encoded into flux-balance laws for these charges
- At the quantum level the project is that

$\longrightarrow$ Representing the Corner charge algebra = Quantization of geometry

$\longrightarrow$ Representing the Extended charge algebra = Quantization of dynamics

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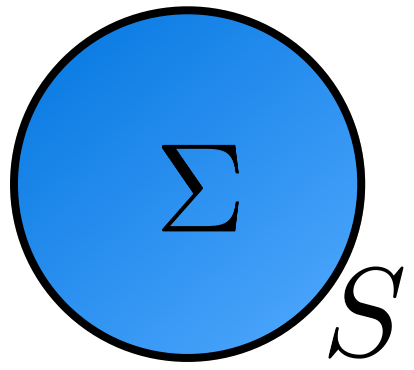

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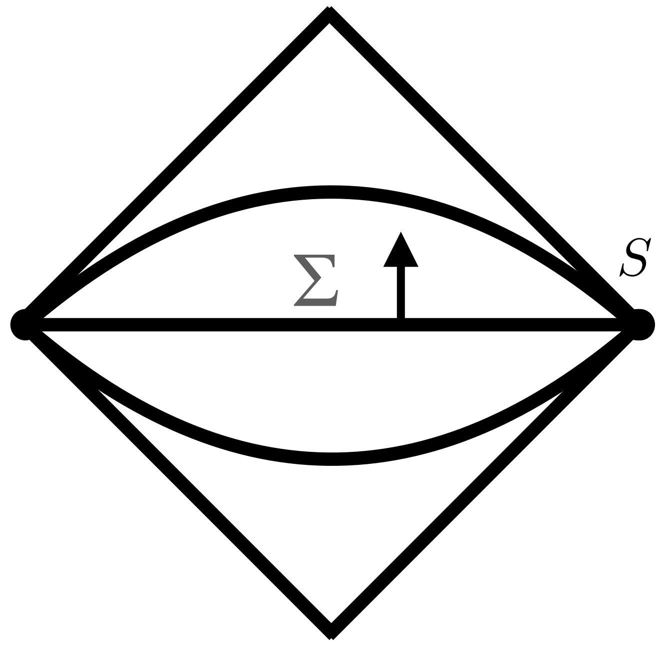

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- For Einstein-Hilbert gravity we have

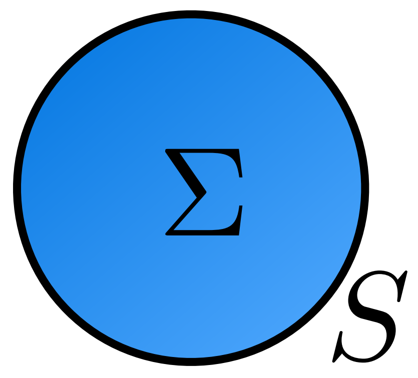
$$C_\xi = \xi^\mu \left(\frac{1}{8\pi G} G_\mu{}^\nu - T_\mu{}^\nu \right) \epsilon_\nu ,$$

$\longleftarrow$ codim1,2 volume forms

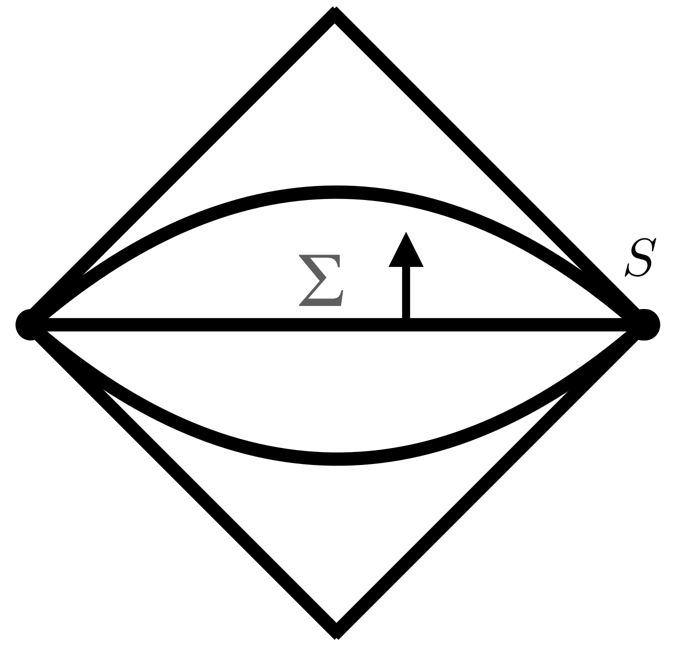
$$q_\xi = \frac{1}{8\pi G} \nabla^\mu \xi^\nu \epsilon_{\mu\nu}$$

$\longleftarrow$ Universal quasi-local formula encompassing the energy of matter coupled to gravity

Symmetries and Gravity



- Given a region R with slice Σ the symmetry charges are supported on codimension 2 corners S= entangling sphere



- The extended corner symmetry group G_S is the subgroup of Diff(M) which and possesses non zero Noether charges in the presence of S, its with kinematical subgroup $H_S \subset G_S$ preserves the region R.

- In metric gravity

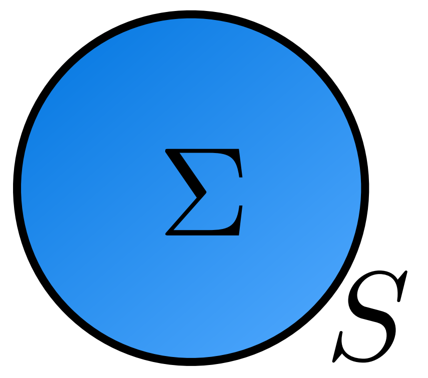
$$\boxed{G_S = (\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S) \ltimes \mathbb{R}^{2\bar{S}}}$$

Group = Kinematical + dynamical

W. Donnelly, L.F 2016
L.F, Leigh, Ciambelli' 21

Wald, Speranza'17

Symmetries and Gravity



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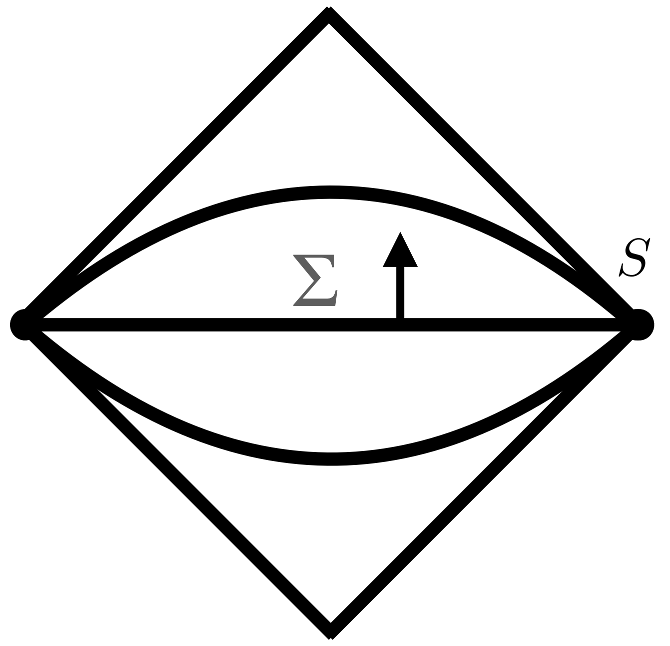

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W. Donnelly, L.F 2016
L.F, Leigh, Ciambelli' 21

- Double Universality of G_S for metric gravity!

  -Same group for infinitesimal diamond or very large ones

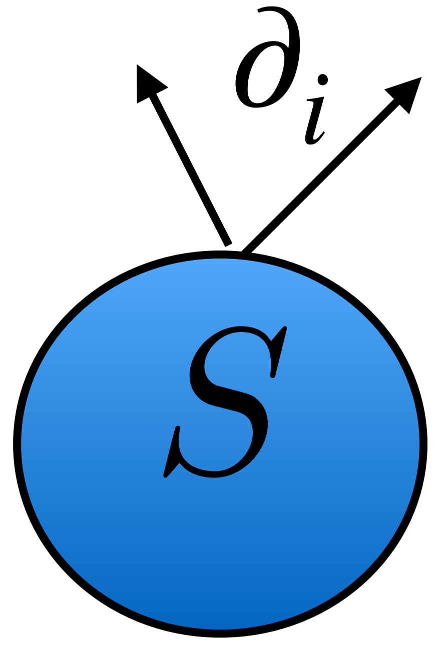
  -Same group for Einstein gravity or any other higher derivative formulation of gravity no matter how many extra derivative

Wald, Speranza'17

- What changes is either the choice of representation or the canonical representation of the symmetry generators

Extended corner symmetry group

- In metric gravity the group is $G_S = \left(\mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S\right) \ltimes \mathbb{R}^{2S}$
- Given a surface S embedded in space time the tangent bundle can be decomposed in tangential components with coordinates σ^A and normal component with coordinates $x^i = (t, r)$: $S = \{x^i = 0\}$



- The extended corner symmetry algebra is generated by vector fields

$$\xi = \underbrace{T^i\partial_i}_{\text{super-translation}} + \underbrace{Y^A\partial_A}_{\substack{\text{super-Lorentz}\\ \text{diff(S)}}} + \underbrace{W_i{}^j(x^i\partial_j)}_{\substack{\text{super-boost}\\ \text{Weyl}}}$$

- Metric

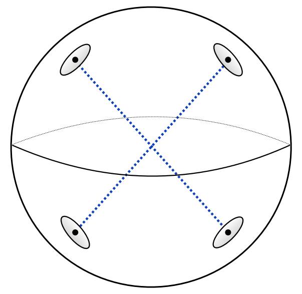
$$\mathrm{d}s^2 = \underbrace{h_{ij}}_{\text{normal metric}}\mathrm{d}x^i\mathrm{d}x^j + \underbrace{q_{AB}}_{\text{tangential metric}}(\mathrm{d}\sigma^A - V_i^A\mathrm{d}x^i)(\mathrm{d}\sigma^B - \underbrace{V_j^B}_{\text{Normal lapse}}\mathrm{d}x^j)$$

- Canonical diffeo aspect

$$P_A = q_{AB}(\partial_0 V_1^B - \partial_1 V_0^B + [V_0, V_1]^B_{\mathrm{Lie}})$$

Twist1-form

Extended corner symmetry group



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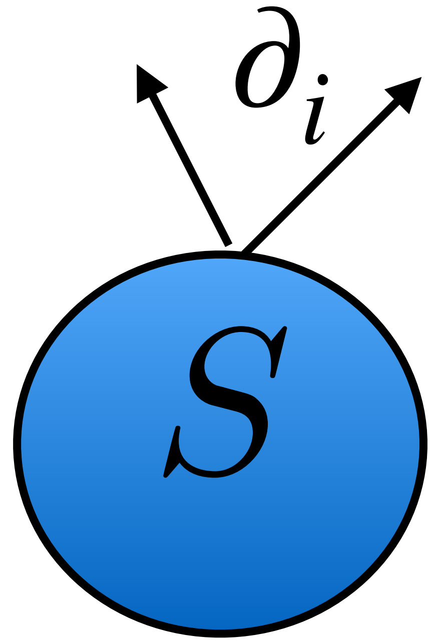

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- Metric

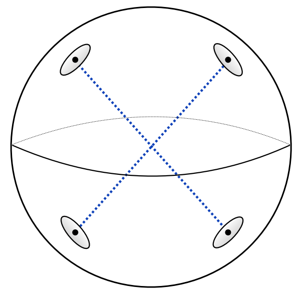
$$ds^2 = \underbrace{h_{ij}}_{\text{normal metric}} dx^i dx^j + \underbrace{q_{AB}}_{\text{tangential metric}}(d\sigma^A - V_i^A dx^i)(d\sigma^B - \underbrace{V_j^B}_{\text{Normal lapse}} dx^j)$$

- Canonical Boost aspect

$$N_j{}^i = \epsilon_{jk} h^{ki}$$

normal metric

Extended corner symmetry group



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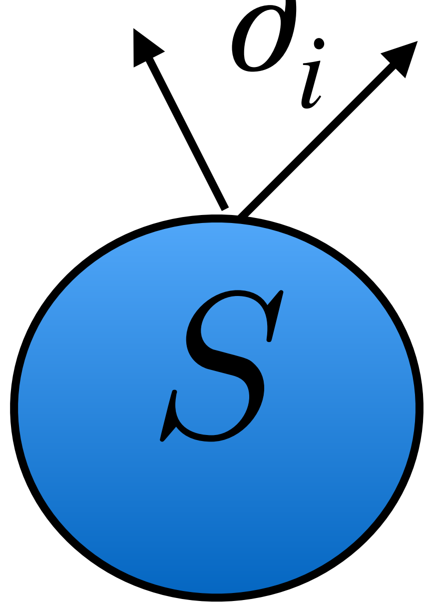

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- Canonical energy/momenta

$$M_j = D_i N_j^i$$

Expansions

Diffferent Corner symmetry

- For a different formulation of gravity we have

LF, Geiller, Pranzetti '20

$$\Omega_F = \Omega_{EH} + \mathrm{d}\Omega_{F/EH}$$

- Different formulation have different symmetry groups $\longrightarrow$ **Inequivalent quantization**

$$\boxed{G_S = (\mathrm{Diff}(S) \ltimes K_S) \ltimes \mathbb{R}^{2\bar{S}}}$$

Perez, Engle, Noui '10
Bodendorfer '13
Perez, LF '15

ADM	$K_S = 0$
Einstein-Hilbert	$K_S = \mathrm{SL}(2,\mathbb{R})^S_\perp$
Einstein-Cartan-Holst	$K_S = \mathrm{SL}(2,\mathbb{C})^S_\parallel \times \mathrm{SL}(2,\mathbb{R})^S_\parallel$ $\longleftarrow$ Tangential metric q_{AB}

$$\{e^i_A(\sigma), e^j_B(\sigma')\} = \gamma \epsilon_{AB} \delta^{ij} \delta^{(2)}(\sigma, \sigma')$$

Electric Flux

- In all cases we have that $\sqrt{\mathrm{Casimir}_2(K_S)} \propto \sqrt{q}$ Area form!

Loop gravity input

- $\mathrm{SU}(2)^S$ is a subgroup of Hs

Quantum Corner symmetry

$$H_S = \mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{R})^S$$

Donnelly, Moosavian, Speranza, LF

- What are the reps? what are the Casimirs?
- The little group is the group that preserves

$$C_{\mathrm{SL}(2,\mathbb{R})_\perp} = \det(q) > 0$$

- Representations are classified by representations of the area preserving Diffeomorphism subgroup.

- The **vorticity** generator Ω form a representation of the area preserving diffeomorphisms algebra

$$\Omega = \epsilon^{AB}\left[\partial_A P_B - \frac{1}{2}\epsilon_{abc}\partial_A N^a \partial_B N^b N^c\right]$$

- The Casimirs are then given by

$$\boxed{C_n = \int_S \sqrt{q}\,\Omega^n}$$

$$C_0 = \mathrm{Area}$$
$$C_1 = \mathrm{NUT\ charge}$$
$$C_2 = \mathrm{Fluid\ enstrophy}$$

Quantum fluid

W. Donnelly, A. Speranza, F.M Moosavian, L.F 2020

Arnold; Marsden, Ratiu,

H_S is isomorphic to the symmetry groups of 2d hydrodynamics

- Analogy: the area density $\sqrt{q}$ plays the role of the fluid density ρ

  The outer curvature plays the role of the fluid vorticity w

- The quantum representations are classified by a choice of area and vorticity densities (ρ, w) on S.
- Classical fluids correspond to a choice of density density measure $\rho > 0$ which is absolutely continuous wrt the Lebesgue measure
- Quantum fluids correspond to a choice of discrete measure where ρ is absolutely continuous wrt to the counting measure

- Both label quantum representation of the `fluid group' H_S

- Quantum fluid corresponds to a choice of quantization which contains constituents.

Quantum fluid

W. Donnelly, A. Speranza, F.M Moosavian, L.F 2020

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- This provides a **constituent picture** where

M. Geiller, D. Pranzetti, L.F 2021

Fluid molecularization = Area constituent
Vortex quantization = momenta quantization

$$\rho = \sum_i \rho_i \delta^{(2)}(\sigma, \sigma_i)$$

L.F 2022

Wieland '19

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M. Geiller, D. Pranzetti, L.F 2021

  Fluid molecularization = Area constituent
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$$\rho = \sum_i \rho_i \delta^{(2)}(\sigma, \sigma_i)$$

- Each constituent carries a density, weight and spin (ρ_i, Δ_i, s_i)

$$P_A = \sum_i \delta^{(2)}(\sigma, \sigma_i) D_A + (\Delta_i \delta_A^B + s_i \epsilon_A{}^B)\partial_B \delta^{(2)}(\sigma, \sigma_i)$$

L.F 2022

- Area constituent in the continuum from quantization!

- Einstein Cartan gravity with an Immirzi parameter implies that $\rho_i = \sqrt{j_i(j_i+1)}$. Area gap in the continuum!

Wieland '19

Quantum Radiation

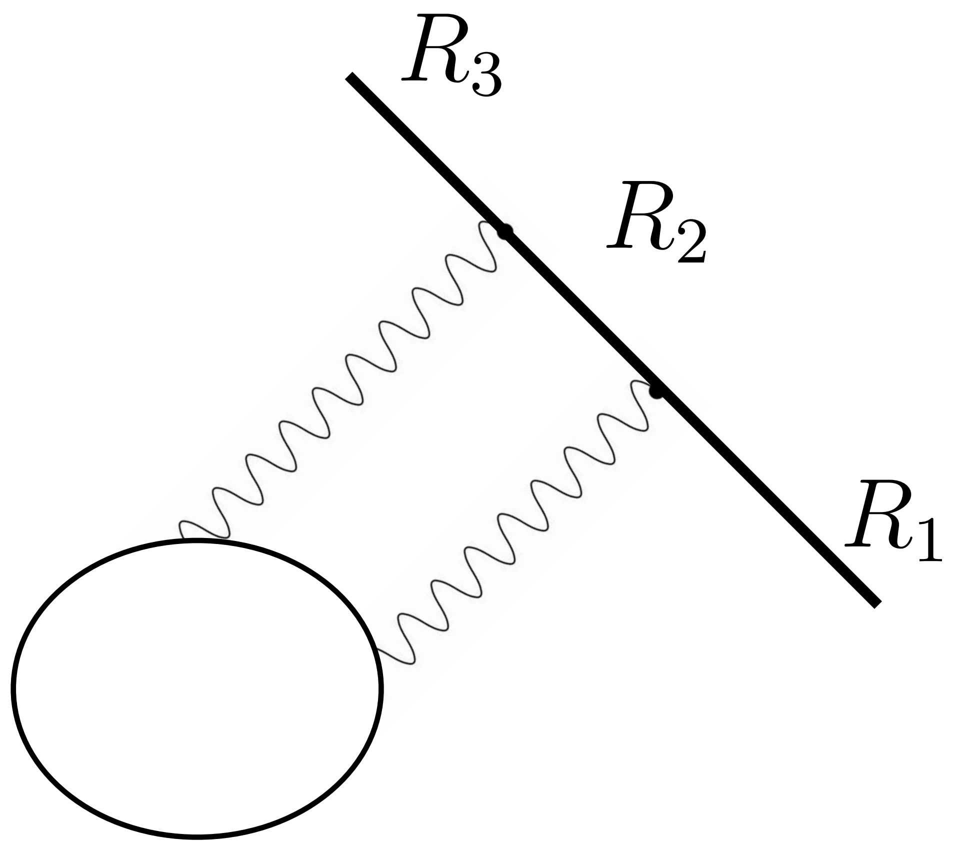
Wieland
Barrett

- Quantum Radiation can be described as elementary transitions between representation of H_S
- These transitions can be represented classically in terms of spacetimes carrying Impulsive waves

LF Pranzetti 21

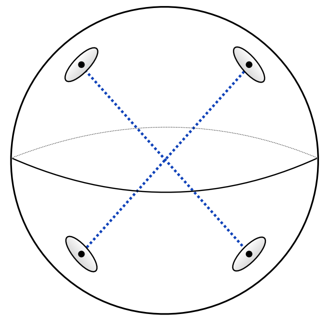
- Impulsive waves are solution of EE which carries non trivial radiations but no energy-momentum tensor along null sheets

$$\mathcal{N}^{AB} = \Delta n^{AB} \delta(u - u_0)$$



- At the quantum level we expect these transitions are intertwinners of the extended group G_s: A new picture of quantum dynamics.

Link with LQG and Spin foam



- The **quantum fluid constituents** corresponds to the intersection of the LQG links with a sphere surrounding the vertex loop gravity states contains information about the area but misses quantum numbers such as the momenta operator
  - The momenta quantum number is necessary in order to reconstruct the frame field and the Diffeo generators

E. Livine , D. Pranzetti, L.F' 19

  - State invariant under diffeomorphisms can be obtained using these additional quantum numbers and expressed as superposition of spin networks

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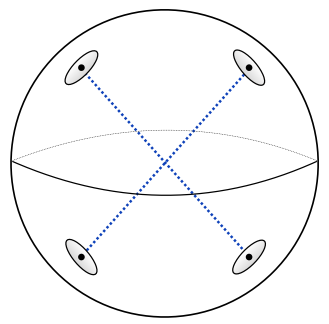

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E. Livine , D. Pranzetti, L.F' 19

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It also opens **future directions**:

- Providing consistency check on the Hamiltonian constraint consruction: The usual contruction of the constraint operator is such that $[C(1), C(2)] \hat{=} 0$ because states are assumed to be diff invariant.
- The way to check consistency with symmetry is to decompose $C = C_L + C_R$ and check That C_L satisfy the corner algebra $[C_L(1), C_L(2)] = P_A(V_{12})$
- For spin foam model the spin foam amplitude should be constraint by the action of the symmetry generators: The amplitude needs to satisfy ward identities

$$[Q, Z] = 0$$

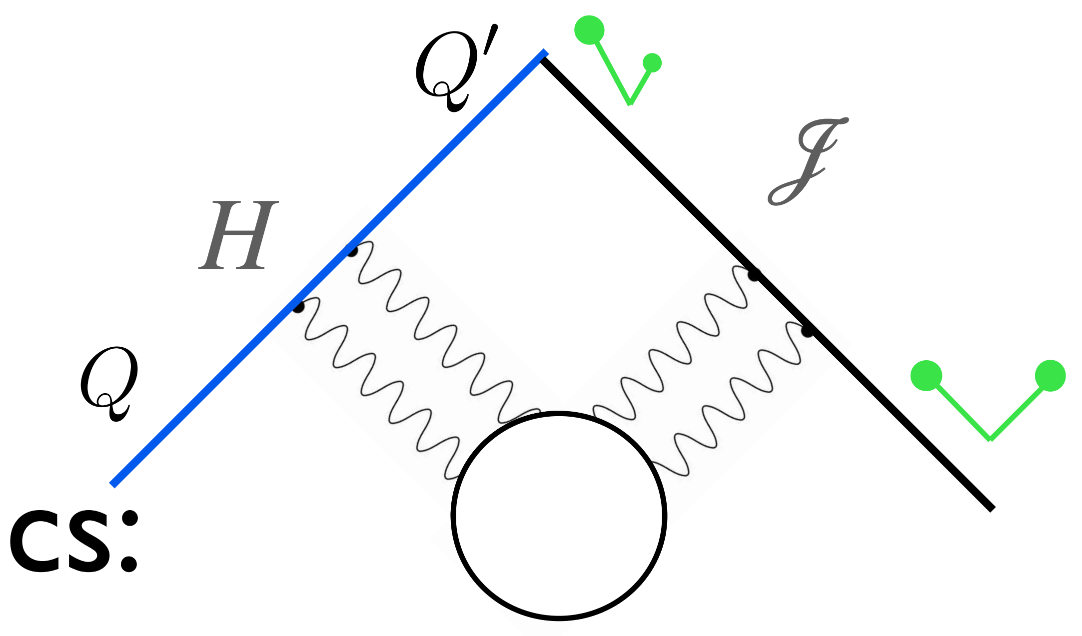
Symmetry Charge

Spin Foam amplitude

Dynamics along null surfaces

- In recent years there has been a tremendous effort and renewed understanding using symmetries of gravitational dynamics and Flux along null horizons including null infinity

Ashtekar, Adami, Barnich, Ciambelli, Compere, Chandrasekaran, Donnay, Flanagan, Freidel, Grumiller, Hopfmueller, Gobadazar, Marteau, Oliveri, Petropoulos, Perry, Ruzziconi, Sheikh-Jabbari, Speranza, Speziale, Troesseart, Zwikel, Wieland,…



→ Two main results for finite and asymptotic null surfaces outside caustics:

- The Einstein dynamics along null surfaces is encoded in the evolution Equation of a Carrolllian fluid: $c \to 0$, ultra-local limit of a relativistic fluid.
- This dynamics can be understood as the conservation of charge for a universal symmetry group called BMSW naturally derived from Gs

- The Gravitational dynamics projected on $\mathscr{N}$ can be recast as a set of Null conservation Laws $D_i T^i{}_j = 0$

Carrollian connection

Carrollian energy-momentum tensor

Donnay, Marteau '19
LF, Hopfmueller,'19; Sheikh-Jabbari'20
Speranza, Flanagan, Chandrasekaran 21

Lesson from ∞

- Asymptotic infinity also carry the structure of an asympotic fluid: Two generator of symmetry the mass aspect and angular momenta aspect.

$$GBMS = \mathrm{Diff}(S) \ltimes \mathbb{R}^S_{-1}$$

- Asymptotic infinity also carry the structure of an asympotic fluid: Two generators of symmetry the mass aspect and angular momenta aspect.

- Diff(S) rep are labelled by dimension and spin $(\Delta, \pm 2)$ with **angular momenta aspect**

$$\boxed{P_A = \sum_i \delta^{(2)}(\sigma, \sigma_i) D_A + (\Delta_i \delta_A^B \pm 2\epsilon_A{}^B)\partial_B \delta^{(2)}(\sigma, \sigma_i)}$$

- The **mass aspect** plays the role of a density operator that shifts dimension

$$M = \sum_i \hat{M}_i \delta^{(2)}(\sigma, \sigma_i) \qquad M_i G_\Delta = G_{\Delta+1}$$

- The (Δ, s) representations = insertions of the conformal gravitons = constituents at ∞

$$G_{\Delta,\pm 2}(\sigma) = \int_{-\infty}^{+\infty} \mathrm{d}\omega \omega^{\Delta-1} a_\pm(\omega q(\sigma)) \qquad q^2 = 0$$

Summary:

- The profound consequences of Noether theorem for gravitational theories leads to a new picture of quantum geometry as a state of representation of the corner symmetry group which capture the essence of subregions entanglement.

- It encodes the non-commutativity of geometrical observables associated with subregions representing the quantization of geometry.
- It leads discretization of space from the representation of continuous non-commutative infinite dimensional algebras represented as quantum fluid.
- This discretization is two-folds: It allows the possibility of corner constituents through molecularisation and the usual area gap from the presence of the immirzi parameter

- Dynamics along null surfaces is encoded into Carrolian conservation laws for the symmetry charges and activated at the quantum level by the representation of the dynamical charges

- These concepts can be extended to asymptotic Dynamics which connects to S-matrix calculations and reveals a new tower of higher spin symmetry responsible for all known soft theorems

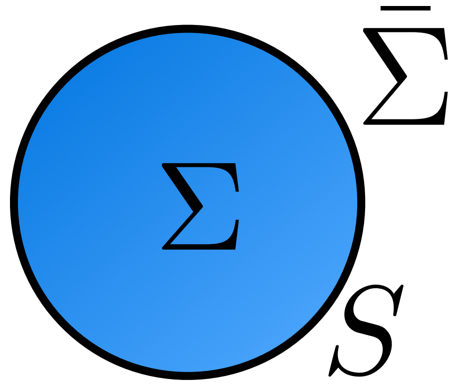
$\longrightarrow$ Miguel & Daniele's talks

Subregion entanglement

- The extended corner symmetry group $G_S = H_S \ltimes \mathbb{R}^{2S}$ is the subgroup of Diff(M) with non zero non zero Noether charges on S

- The Kinematical subgroup H_S controls the kinematical entanglement of subregions

- If we divide our slice into a subregion Σ and its complement $\bar{\Sigma}$ with interface S. The Hilbert space decomposes as a **Fusion product**



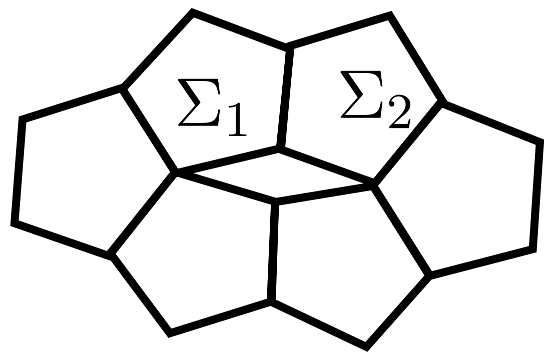
$$\mathcal{H} = \mathcal{H}^{\text{ext}}_{\Sigma} \boxtimes \quad \mathcal{H}^{\text{ext}}_{\bar{\Sigma}}$$

W. Donnelly, L.F 2016

Singlet under Hs

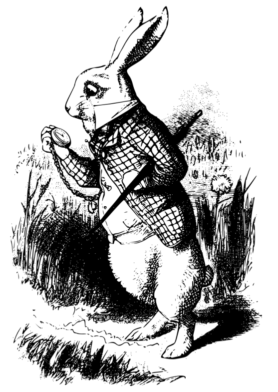
- More generally if we decompose a slice of spacetime into a collection of regions the gluing of Hilbert spaces associated with subregions requires the identifications of the corner symmetry representations.

$$\Sigma = \cup_i \Sigma_i$$



- Related to the dressing of physical observables $\longrightarrow$ P. Hoen talk

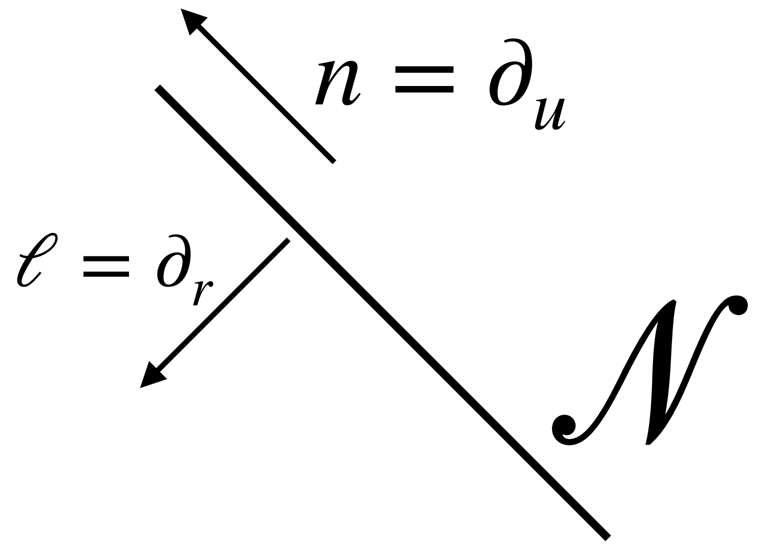
Carrollian Fluid



Levy-Leblond, Perry

A Carrollian Fluid is defined as the ultra-local limit $c \to 0$ of a relativistic fluid.
In a Carrollian universe spacelike separated observers are causally disconnected

Donnay, Marteau,



The Carrollian structure (n^a, q_{ab}) on a null surface $\mathcal{N}$ is connected to the presence of a preferred Null generator n, which is in the kernel of the Bundle map $\pi : \mathcal{N} \to S$

Mars, Senovilla

The embedding of $\mathcal{N}$ into spacetime defines a rigged structure on represented by a transverse one form ℓ_a with $\ell \cdot n = 1$, and allows the construction of a Carrollian connection.

- The Gravitational dynamics projected on $\mathcal{N}$ can be recast as a set of Null conservation Laws $D_i T^i{}_j = 0$

LF, Hopfmueller, 18, Sheikh-Jabbari,20
Speranza Flanagan Chandrasekaran 21

- These laws are the conservation of BMSW charges along $\mathcal{N}$

Split property

- In gauge theories the total Hilbert space does not decompose into tensor product instead we have

$$\mathcal{H} = \mathcal{H}^{\mathrm{ext}}_{\Sigma} \boxtimes_{G_S} \mathcal{H}^{\mathrm{ext}}_{\bar{\Sigma}}$$

W. Donnelly, L.F 2016

Singlet under Gs

- The corner symmetry group and its representation is an absolutely necessary ingredient to construct quantum Spacetime
- Physical Operations on different space components do not commute. They need to share the same representation of Gs on their shared corner.
- States of geometry are states of representation of Gs which encodes the gravitational entanglement

$\longrightarrow$ Goal: to insure that GS charges captures all geometric information around S

Split property and paradoxes

- In gauge theories the total Hilbert space does not decompose into tensor product

Due to the presence of infinitely many vacuum

- Instead $$\mathcal{H} = \mathcal{H}^{\text{ext}}_{\Sigma} \boxtimes_{G_S} \mathcal{H}^{\overleftarrow{\text{ext}}}_{\bar{\Sigma}}$$

- This means that the assumption made repeatedly in the recent litterature that the state of a system, in a standard theory of quantum gravity, can be specified independently on a bounded region and on its complement is incorrect.

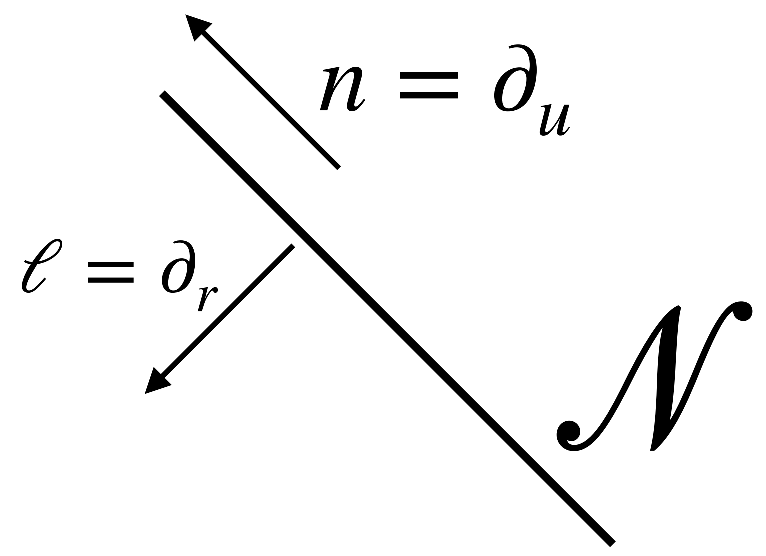
S. Raju 2021

- This incorrect, albeit appealing, assumption is the source of many information paradoxes such as the original Hawing paradox, the fuzzball proposal (monogamy of entanglement), the firewall paradox, and the existence of a Page curve. They all assume a split property of the Hilbert space.

- What the Noether theorem implies is the fact that any information localized inside the region Σ can be measured at the quantum level by operators (Charges) defined purely on the boundary (or exterior) of that region $\longrightarrow$ Information delocalization.

Asymptotic symmetry & Celestial Holography

- In the presence of a null surface the corner symmetry group reduces to the BMSW symmetry sub-group which is the group of symmetry preserving the asymptotic null generator $n = \partial_u$ and the surface gravity



- This subgroup is

$$\mathrm{BMSW} = (\mathrm{Diff}(S) \ltimes \mathrm{Weyl}) \ltimes \mathbb{R}^S$$

$$\boxed{\xi = T(\sigma)\partial_u + Y^A(\sigma)\partial_A + W(\sigma)(u\partial_u - r\partial_r)}$$

Chandrasekar, Flanagan, Prabhu'18
LF, Oliveri, Pranzetti Speziale '21

Barnich Troesseart '11
Campiglia, Ladha '16
Compere, Fiorucci, Ruzziconi'18

- At infinity, same group of symmetry !
- The gravitational dynamics is encoded in conservation laws for the corresponding symmetry charges.

Radiation

Moosavian,Pranzetti,LF, 22

Banich, Ruzziconi, 21

- The covariant mass M and the covariant momenta P_A transforms as moment maps for the group GBMS $= G_{-1}$ where

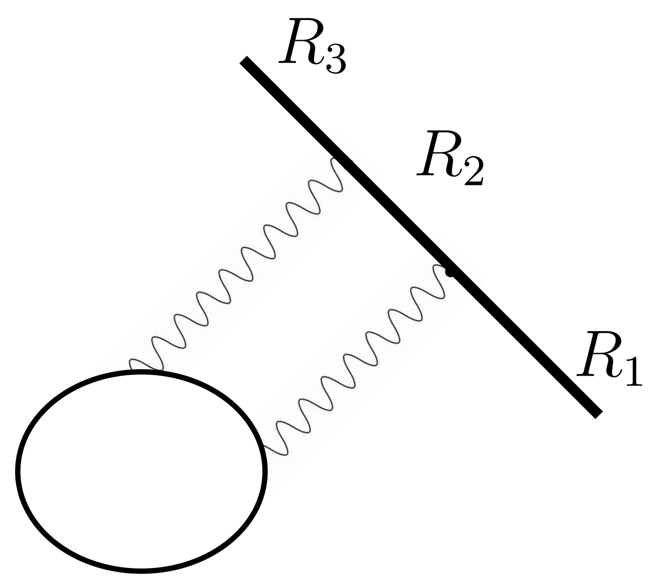
$(M, P_A) \in g_1^*$

$$G_\Delta = \mathrm{Diff}(S) \ltimes \mathbb{R}^S_\Delta$$

- Non radiative spacetime can therefore be understood as coadjoint orbits of the symmetry group GBMS $\subset$ HS

- Radiation can be described as elementary transitions between Vacua
- These transitions can be represented in terms of spacetimes carrying Impulsive waves

- Impulsive waves are solution of vaccum Einstein Equations which carries non trivial radiations but no energy-momentum tensor along null sheets



$$\dot{N}_{AB}(u) = \Delta n_{AB} \delta(u - u_0)$$

At the quantum level states are labelled by Unitary representation of G_Δ

Quantum Fluid Hologram

Moosavian,Pranzetti,LF, 22

Banich, Ruzziconi, 21

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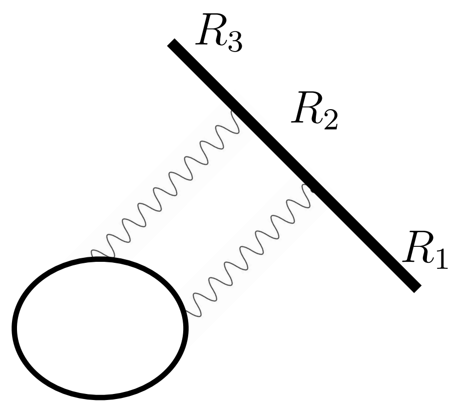


- G_0 is the symmetry group of perfect barotropic fluid with density ρ and momenta p_A

For strictly positive mass aspect we have that $G_0 \sim G_\Delta$ with $m = \rho^{\frac{3}{2}}$

The quantum representations (Casimirs) are labeled by the fluid vorticity

$$w = \epsilon^{AB}\left(m\partial_A p_B + \frac{2}{3} p_A \partial_B m\right) \longleftarrow$$

Super Pauli-Lubanski spin vector
Invariant under super-translation

The graviton appears as the fluid constituent $C_{\Delta,\pm 2}$

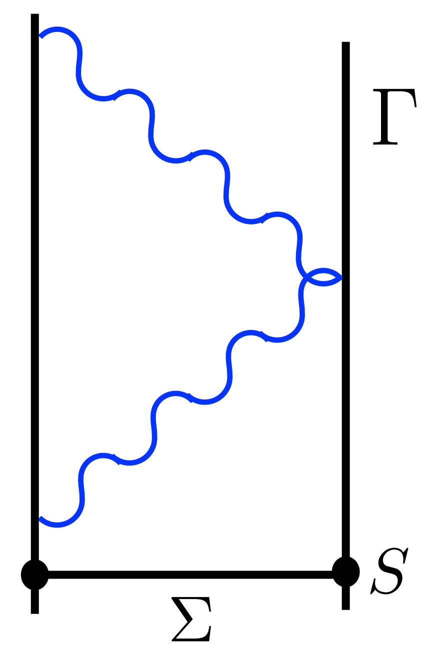
$$\rho(z) = \sum_i \rho_i \delta^{(2)}(z, z_i)$$

$$P(z) = \sum_i \delta(z - z_i)D + (\Delta_i \pm s_i)D\delta(z - z_i)$$

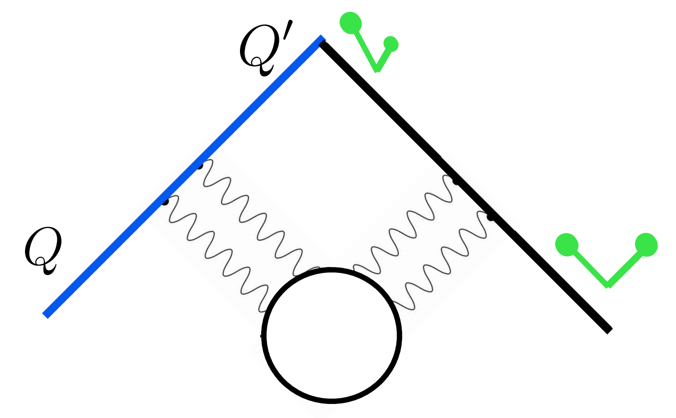
Holographies

There are several notions of Holographies

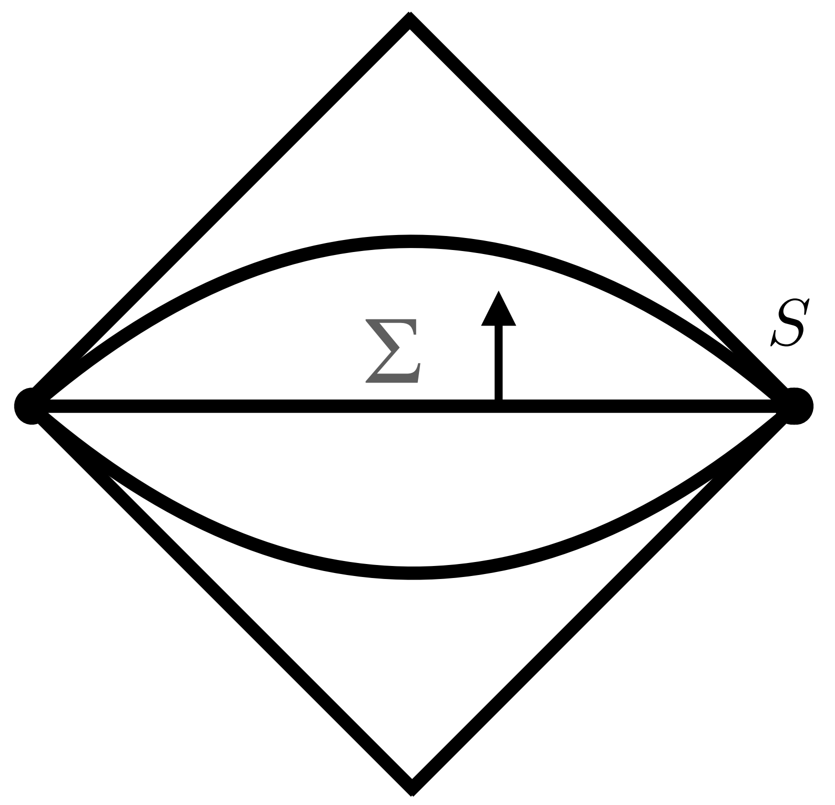
- AdS/CFT Holography: In this holography the boundary is asymptotic and timelike. It is rigidly defined by a boundary condition. It does not allow outgoing radiation= The system is closed and unitary by design.



- Celestial Holography: The boundary is asymptotic and null. It is less rigidly defined by a fall-off conditions. It does allow radiation= The system is open and allows energy loss.
  It connects to gravitational wave detection and S-matrix observables



- Local Holography: Holography associated with local causal diamonds. It requires the choice of a Cauchy domain attach to a finite corner. Does not require any boundary condition.



- Each Holography is associated with a symmetry group: Conformal, Generalized-BMS symmetry, or <u>Corner symmetry</u>

Thank You !

Quantum Algebra

- The Corner algebra contains a local factor

$$G_S^{\mathsf{ECH}} = \mathrm{Diff}(S) \ltimes (\mathrm{SL}(2,\mathbb{C})^S \times \mathrm{SL}(2,\mathbb{R})_{\|}^S)$$

- Internal normal $\quad n^I$
- Flux $\quad E_I := \frac{1}{2}(e^K \wedge e^L) n^J \epsilon_{IJKL}$
- Tangential metric $\quad q_{AB} = e_A^I e_B^J \eta_{IJ}$

- Lorentz Charge aspect

$$E_{IJ} = \underbrace{E_{[I} n_{J]}}_{\text{Boost}} + \underbrace{\frac{1}{\gamma} \epsilon_{IJKL} E^K n^L}_{\text{Spin}}$$

Quantum Algebra

- The flux form an ultralocal SU(2) algebra

$$[E_I(\sigma), E_J(\sigma')] = \gamma \epsilon_{IJKL} E^K(\sigma) n^L \, \delta^{(2)}(\sigma, \sigma')$$

- The tangential metric form an $\mathfrak{sl}(2,\mathbb{R})$ algebra

$$[q_{AB}(\sigma), q_{CD}(\sigma')] = i\gamma \left(\epsilon_{AC} q_{BD} + \epsilon_{BD} q_{AC} - \epsilon_{AD} q_{BC} - \epsilon_{BD} q_{AD}\right) \delta^{(2)}(\sigma, \sigma')$$

- Geometrical balance relation

$$C_{\mathrm{SL}(2,\mathbb{R})_{\|}} = -\left(\frac{\sqrt{q}}{\gamma}\right)^2 = C_{\mathrm{SU}(2)}$$

- Discrete area spectra from discrete series:

$$\boxed{\sqrt{q}(\sigma) = \gamma \sum_i \sqrt{j_i(j_i+1)} \, \delta^{(2)}(\sigma - \sigma_i)}$$

- Local Lorentz invariance is finally built in!

Simplicity constraint

- The corner algebra contains an ultralocal factor

$$G_{\mathrm{ECH}}(S) = \mathrm{Diff}(S) \ltimes \mathrm{SL}(2,\mathbb{C})^S \times \mathrm{SL}(2,\mathbb{R})_{\|}^S$$

- All the Casimir of the ultra local factors are proportional to the area element!

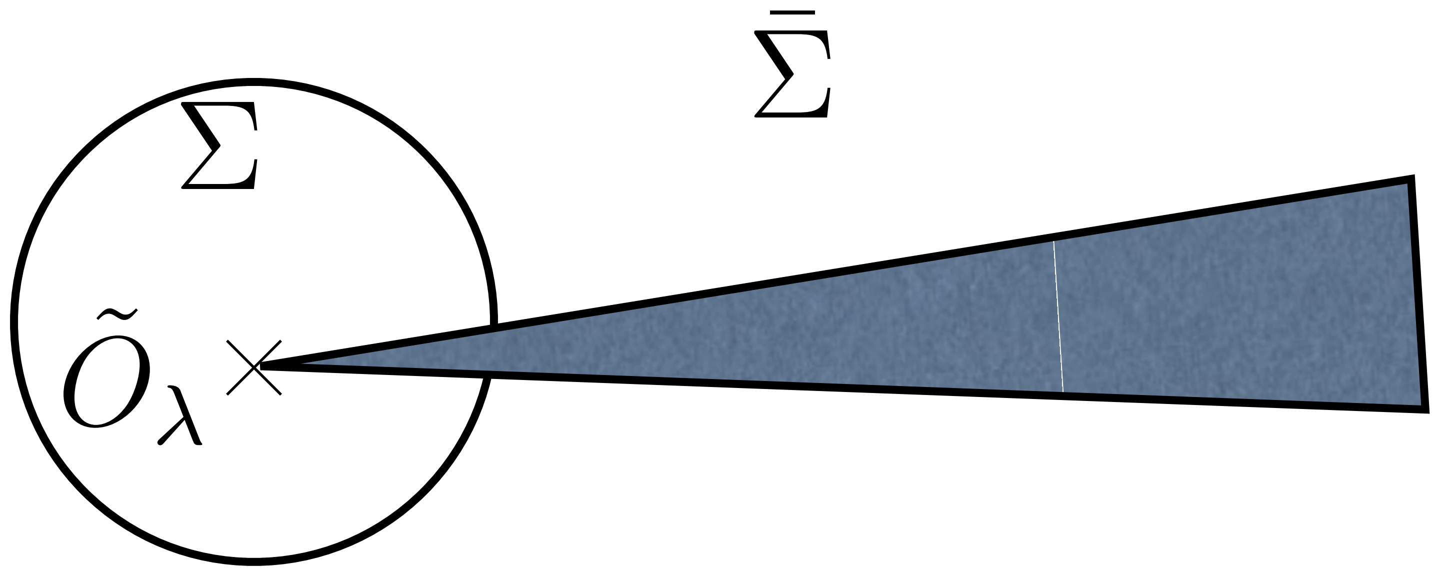
$$\boxed{C_{\mathfrak{sl}(2,\mathbb{R})_{\|}} = q, \quad C_{\mathfrak{su}(2,\mathbb{R})} = \beta^2 q, \quad C^1_{\mathfrak{sl}(2,\mathbb{C})} = (\beta^2 - 1) q, \quad C^2_{\mathfrak{sl}(2,\mathbb{C})} = 2\beta q.}$$

	Corner symmetries $\mathfrak{g}^S$				
Formulation of 4d gravity	diff(S)	$\mathfrak{sl}(2,\mathbb{R})_\perp$	$\mathfrak{sl}(2,\mathbb{R})_{\|}$	$\mathfrak{su}(2)$	boosts
Canonical general relativity (GR)	✓				
Einstein–Hilbert (EH)	✓	✓			
Einstein–Cartan (EC)	✓				✓
Einstein–Cartan–Holst (ECH)	✓		✓	✓	✓

- A part of the symmetry group is universal. Another depends on the form

Split property and Dressing

- In gravity, a localized matter excitation $O_\lambda(x) = e^{i\lambda\phi(x)}$
- localized inside a region violates the constraints
- Instead the operators needs to be gravitationally dressed. Since the insertion carry energy and energy is positive it cannot be screened and the dressing needs to extend to infinity



- The energy is locally defined on the boundary of and doesnt commute with the dressed localized operator

$$8\pi G E = \int_S (r^2 \partial_r g_{tt}) \qquad [E, \tilde{O}_\lambda(x)] = \partial_t \tilde{O}_\lambda \neq 0$$

- Charge operators generating Gs localized outside a region still capture information about operator insertion inside this region