Event Horizons are tunable factories of quantum entanglement

Ivan Agullo

Louisiana State University

Loops 22, Lyon, July 18 2022

Work in collaboration with A. Brady, D. Kranas and A. Delhom

Reference:

Quantum Aspects of Stimulated Hawking Radiation in an Optical Analog White-Black Hole Pair

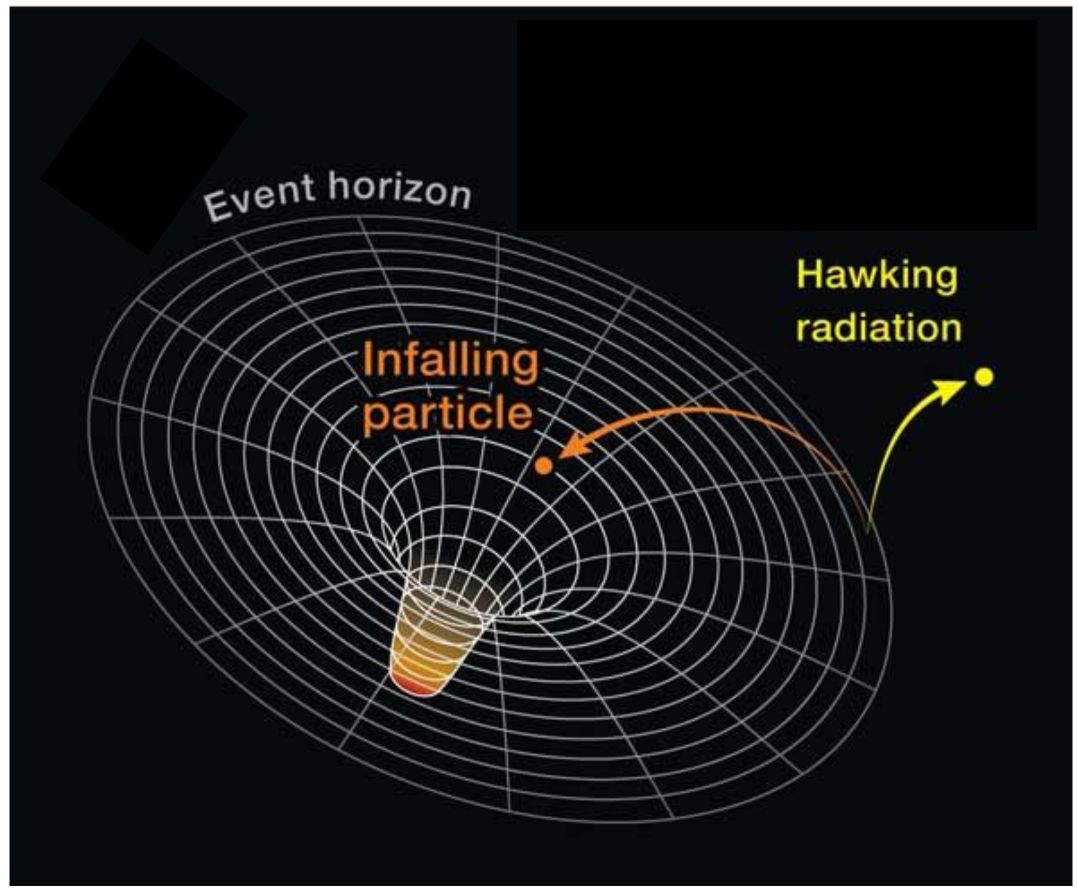
Ivan Agullo, Anthony J. Brady, and Dimitrios Kranas
Phys. Rev. Lett. **128**, 091301 – Published 28 February 2022

See Dimitris' talk on Tuesday, for further details

Motivation

S. Hawking '74

Black holes aren't black: they radiate as hot bodies:

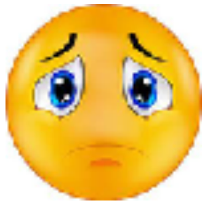


$$T_{\mathrm{H}} = \frac{\hbar c^3}{8\pi G k_{\mathrm{B}} M}$$

Hawking temperature

Observability?

$$T_H \approx 10^{-7}\, K\, \frac{M_\odot}{M}$$



Hawking radiation is over-shined by the Cosmic Microwave Background

Observability?

$$T_H \approx 10^{-7}\, K\, \frac{M_\odot}{M}$$

Hawking radiation is over-shined by the Cosmic Microwave Background

Unruh'81:

The Hawking effect is generic in presence of causal barriers (horizons)

↓

Hawking radiation in Analog Gravity systems

(fluids, optical systems, BEC's, etc)

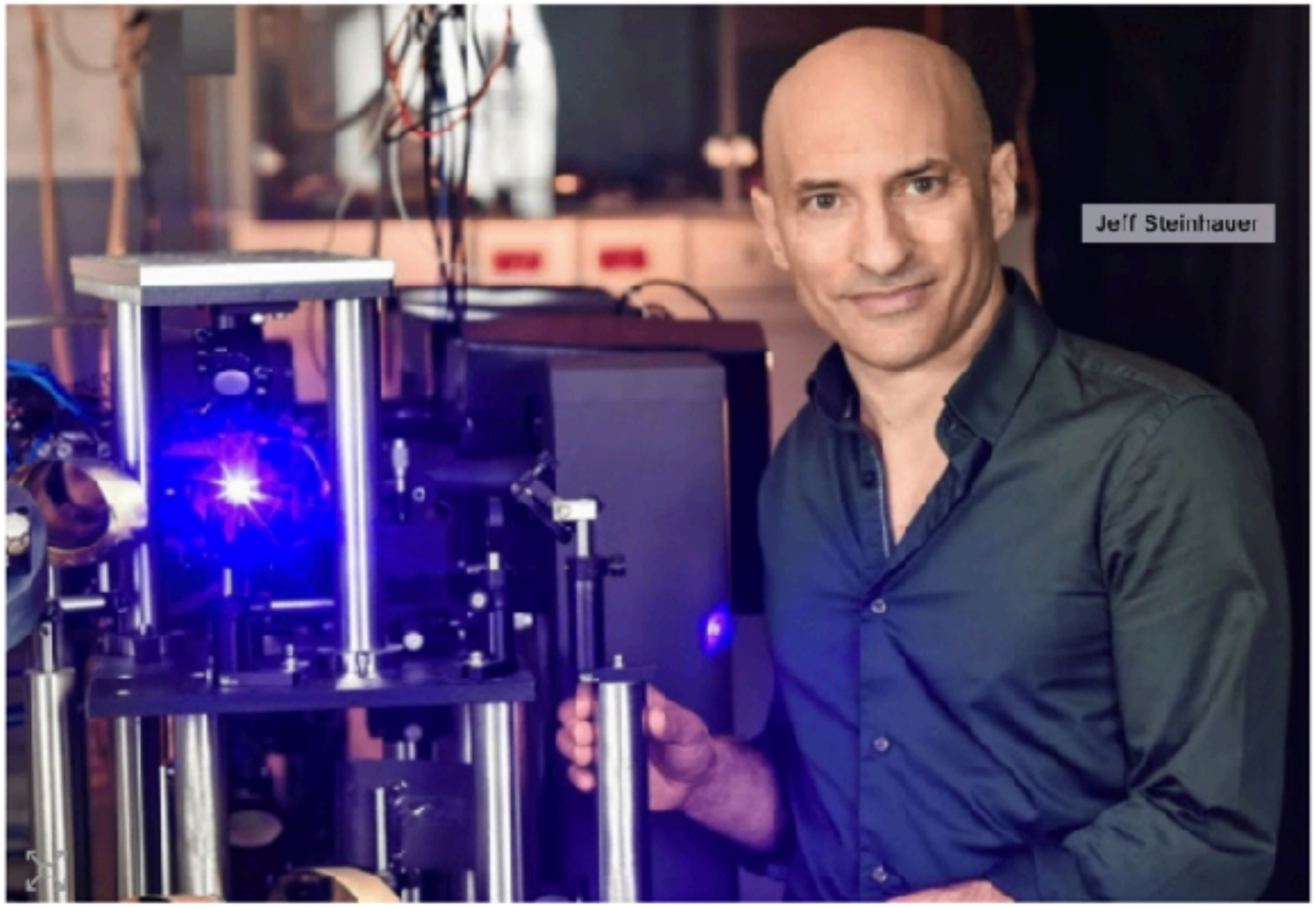
Spontaneous Hawking radiation is extremely weak. Difficult to observe.

See however:

GRAVITY | RESEARCH UPDATE

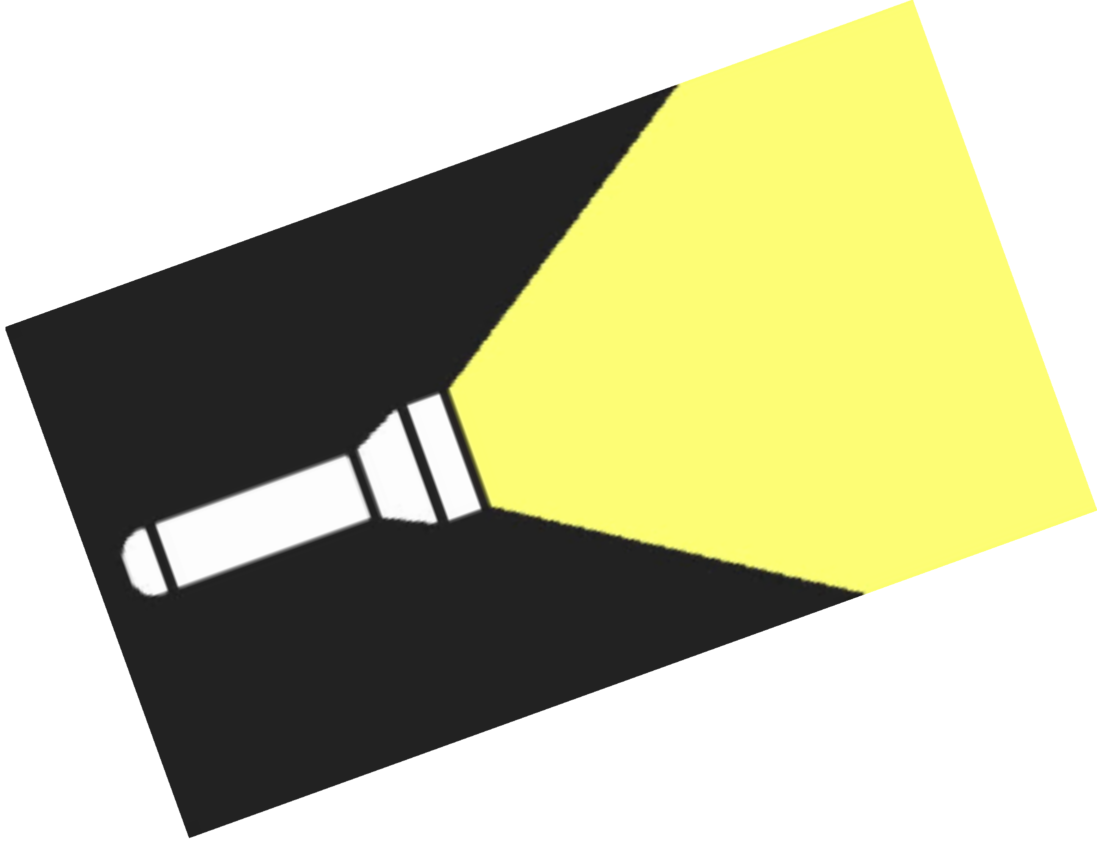
Thermal spectrum of analogue black hole puts Hawking radiation in a new light

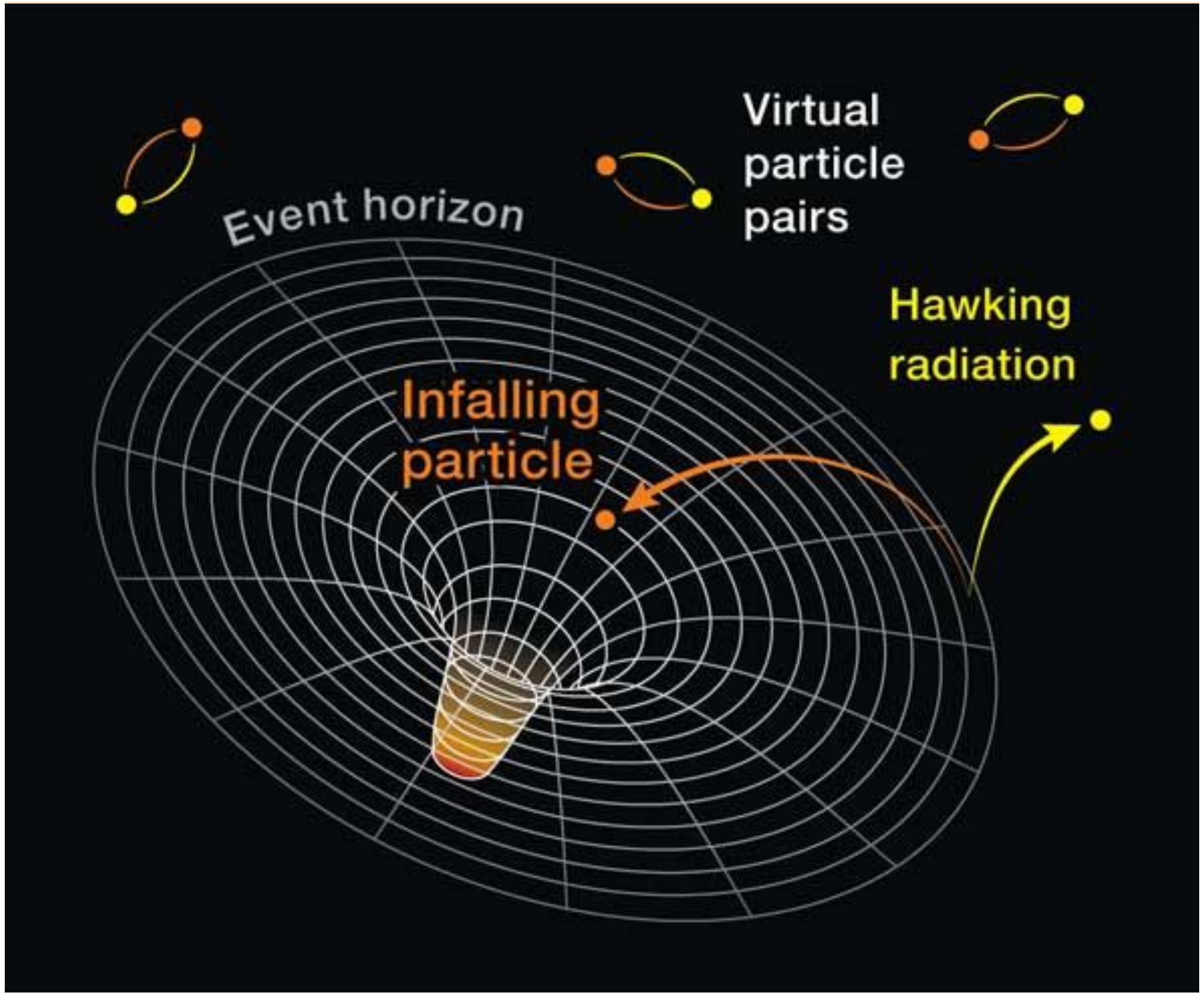
29 May 2019



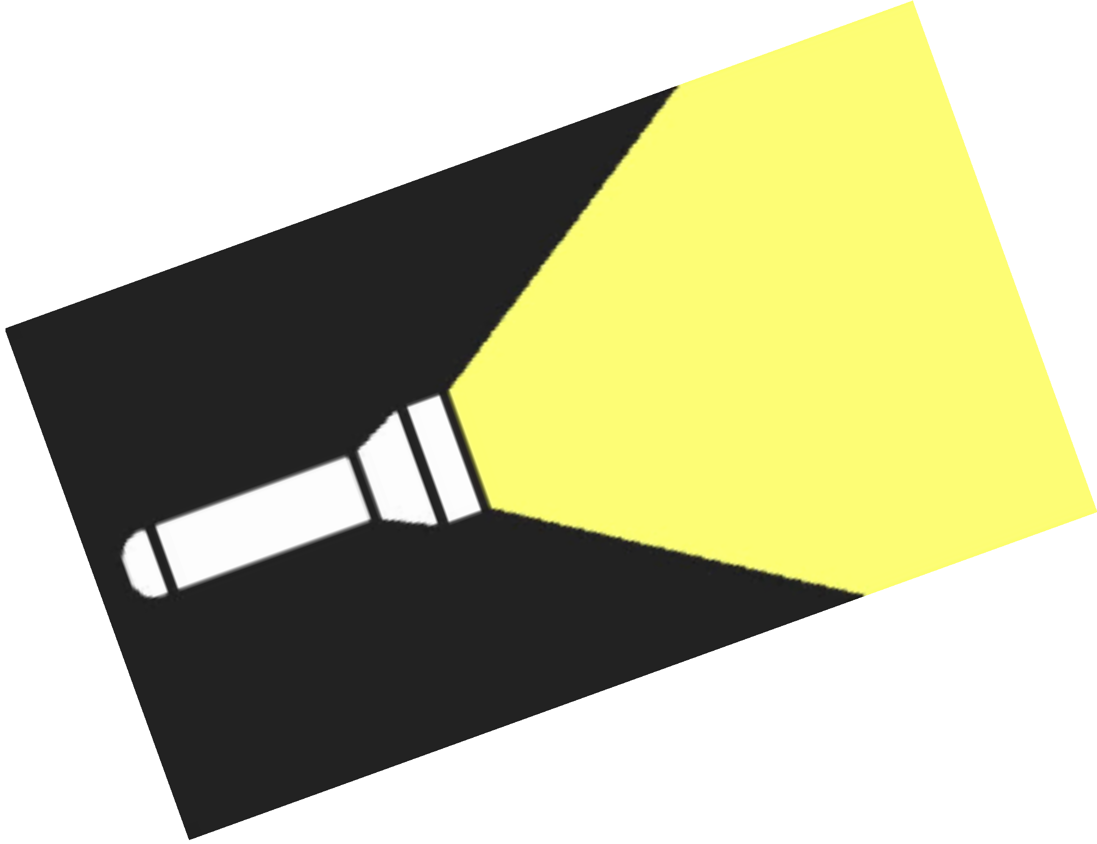
Quantum analogue: Jeff Steinhauer and colleagues have measured the temperature of an analogue black hole. (Courtesy: Technion)

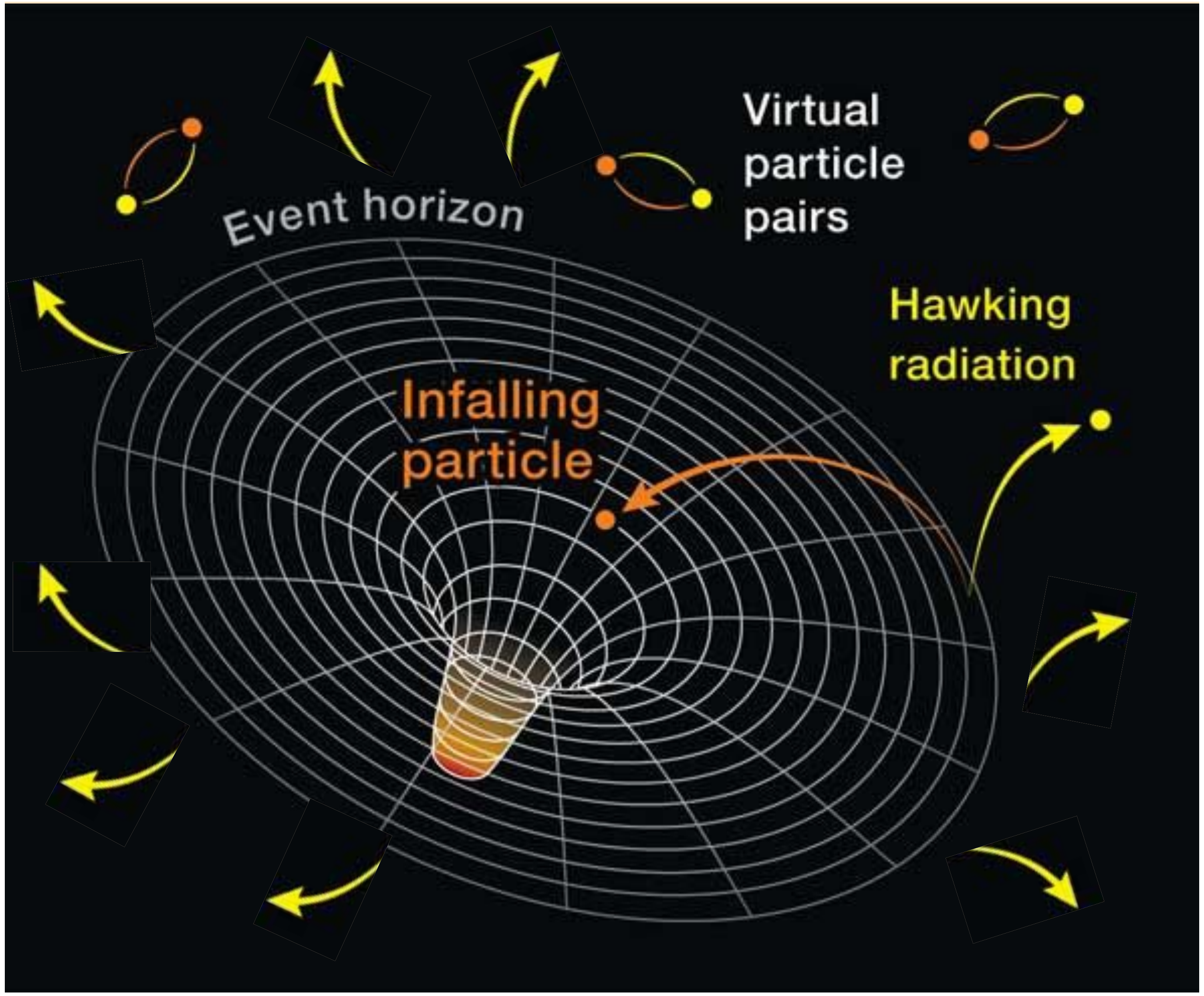
Stimulated Hawking radiation:



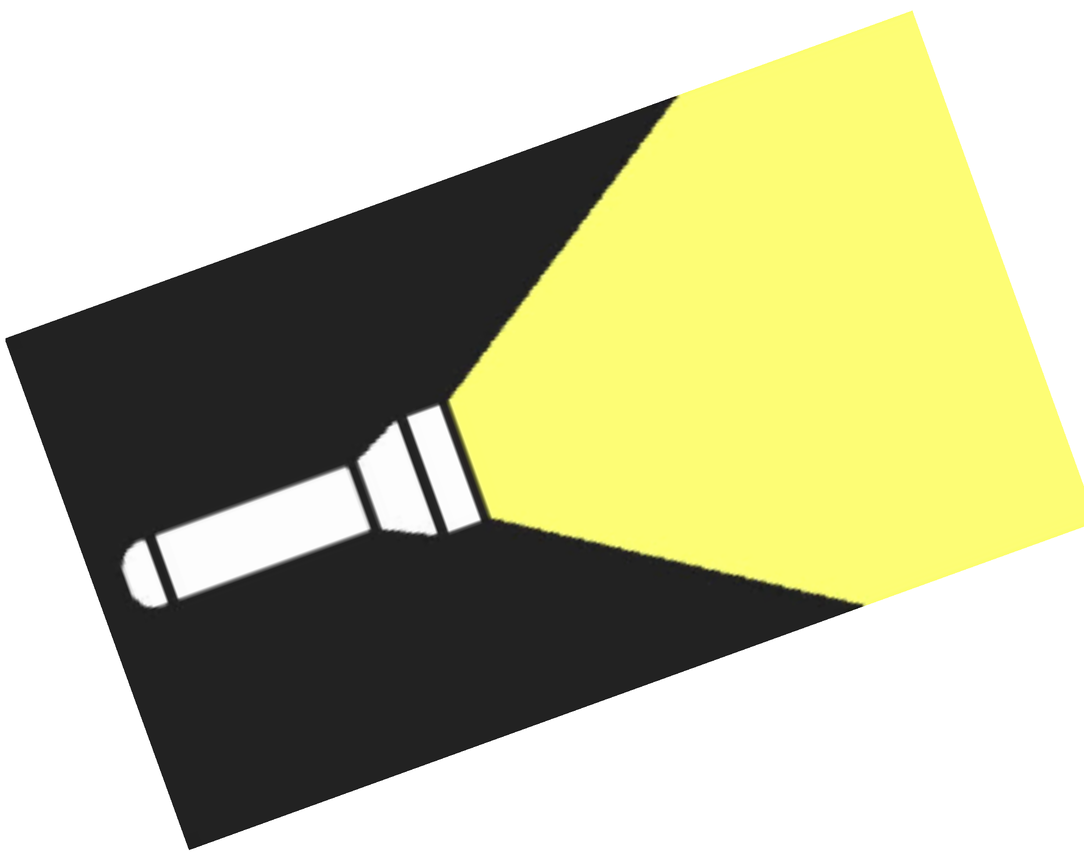


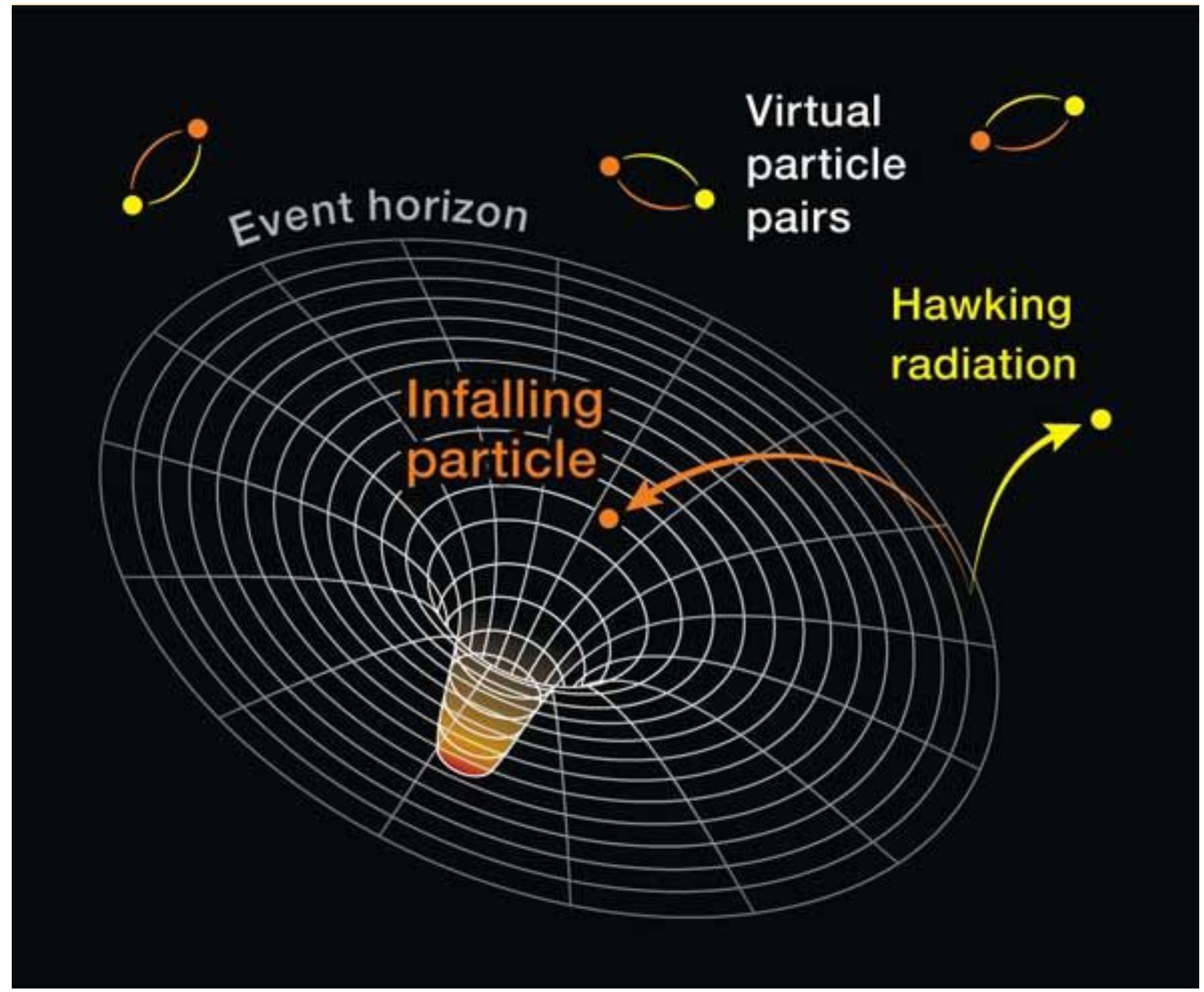
Stimulated Hawking radiation:





Stimulated Hawking radiation:





Measurement of stimulated Hawking emission in an analogue system

Silke Weinfurtner*, Edmund W. Tedford**, Matthew C. J. Penrice*, William G. Unruh*, and Gregory A. Lawrence**
*Department of Physics and Astronomy,
University of British Columbia,
Vancouver, Canada V6T 1Z1

Observation of Stimulated Hawking Radiation in an Optical Analogue

Jonathan Drori[1], Yuval Rosenberg[1], David Bermudez[2], Yaron Silberberg[1], and Ulf Leonhardt[1]
[1] Weizmann Institute of Science, Rehovot 7610001, Israel
[2] Departamento de Física, Cinvestav, A.P. 14-740, 07000 Ciudad de México, Mexico
(Dated: January 15, 2019)

But… there is nothing quantum in these experiments (agreed by the authors)

The stimulated Hawking effect is regarded as a classical phenomenon

Questions

(1) What is quantum and what is not in the stimulated Hawking effect?

(2) Can quantum effects be amplified?

Questions

(1) What is quantum and what is not in the stimulated Hawking effect?

Entanglement

(2) Can quantum effects be amplified?

Actually, yes!

"Event horizon are tunable factories of quantum entanglement"

→ Observational opportunity

Task: quantify the entanglement generated in the Hawking process

The Hawking Process for astrophysical BH's

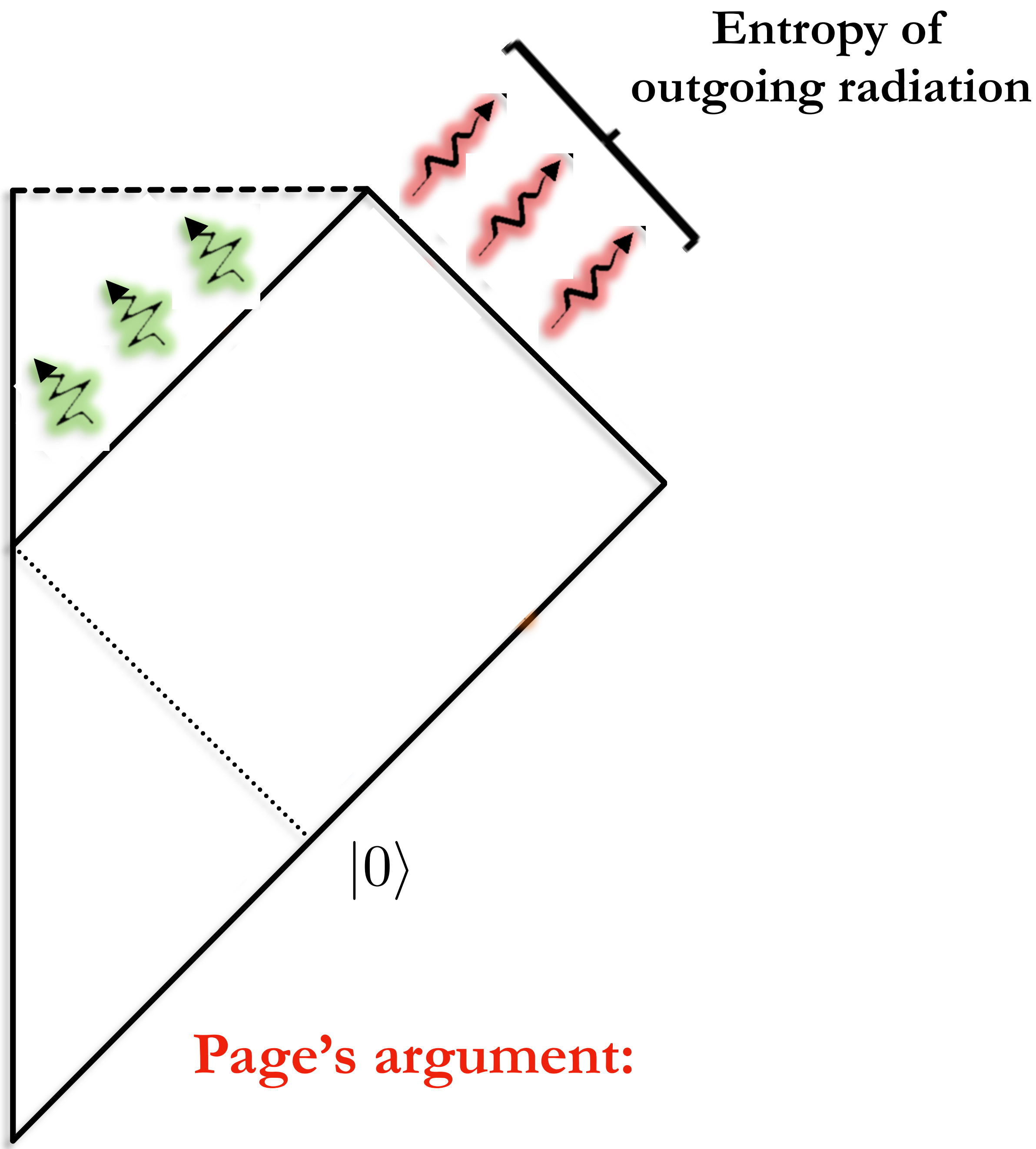
Previous work

D. Page'2013:

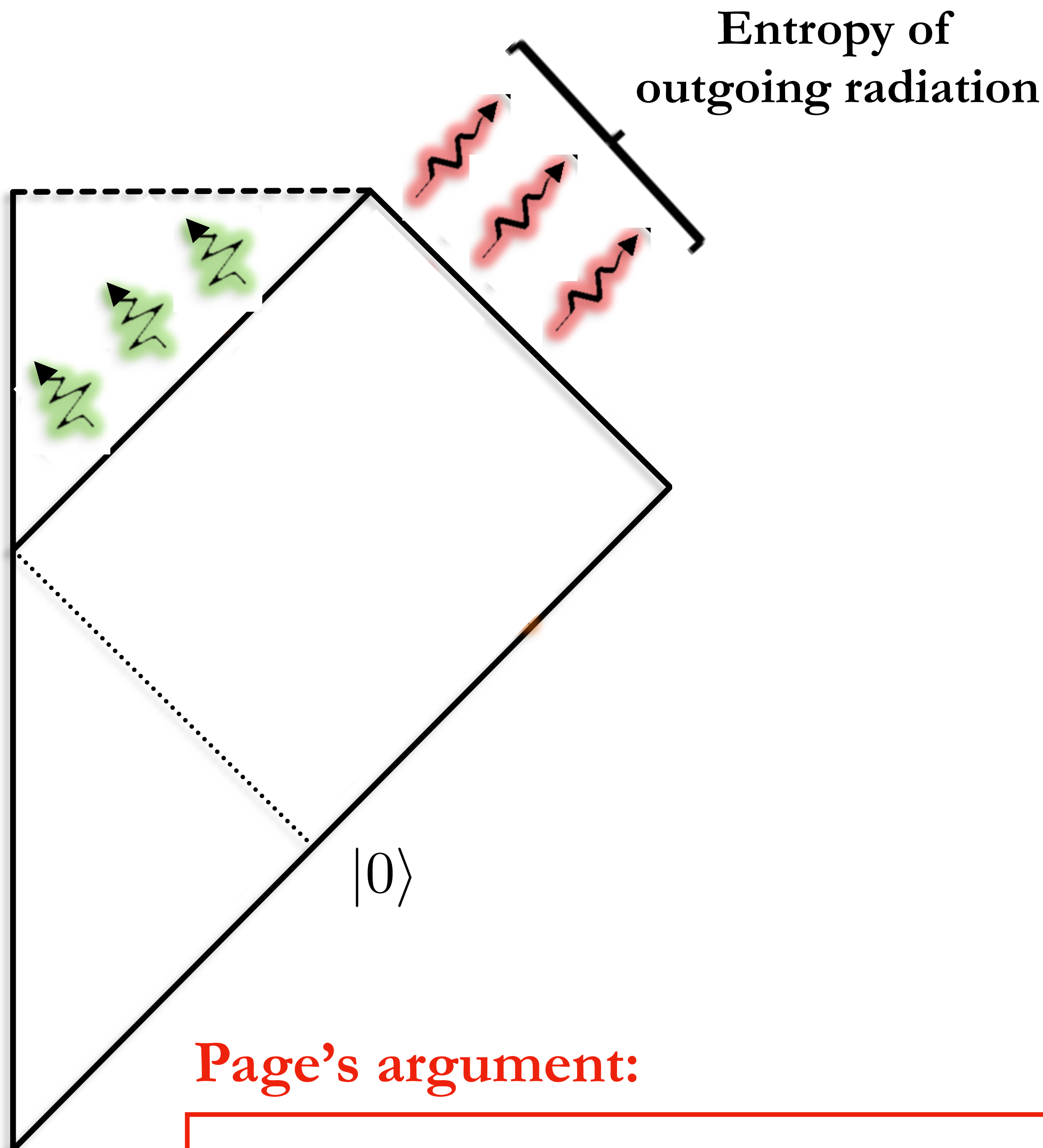
Journal of Cosmology and
Astroparticle Physics

Time dependence of Hawking radiation entropy

To cite this article: Don N. Page JCAP09(2013)028



Page's argument:



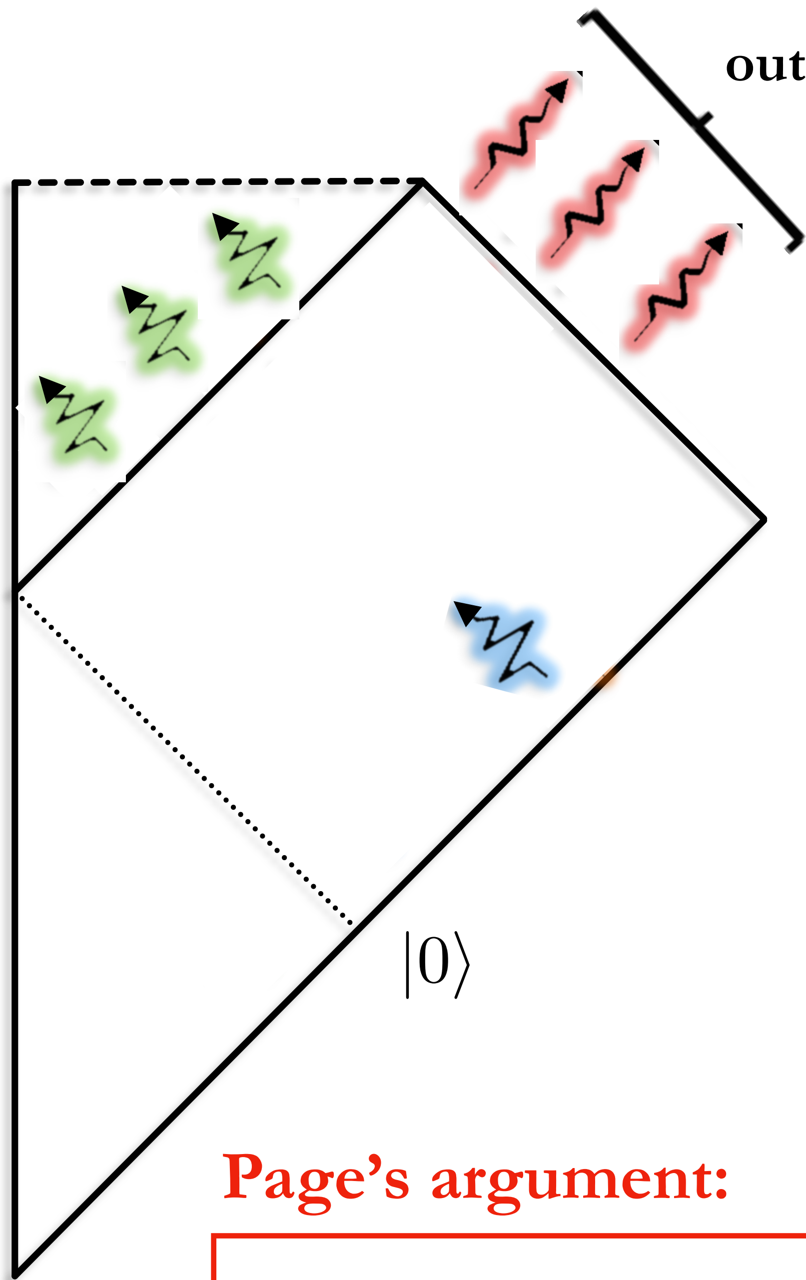
Page's argument:

Entropy of the radiation reaching infinity = entanglement entropy

⟶ Quantifier of Hawking-generated entanglement

Nice! But limited, because:

(1) Only true if "in" state is pure, and
(2) BH is in isolation (not satisfied for any BH's we know in nature)



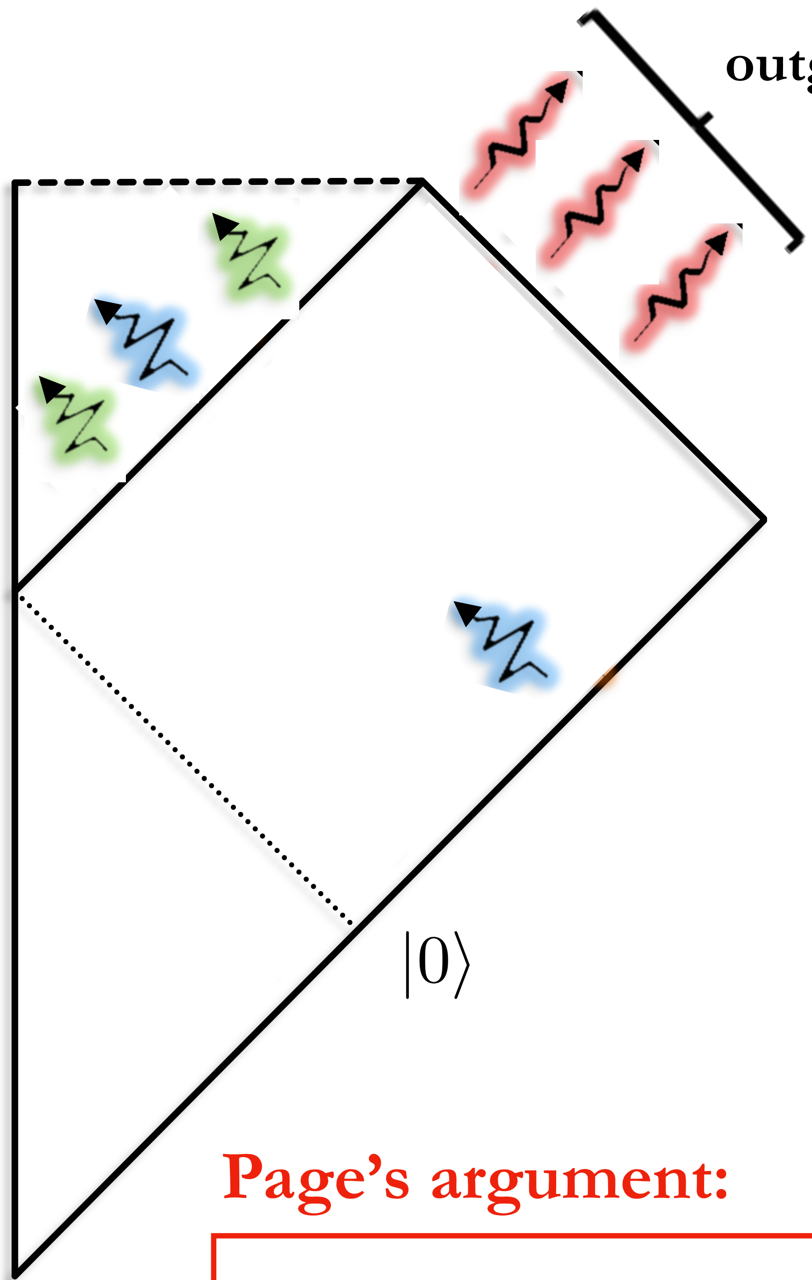
Page's argument:

Entropy of the radiation reaching infinity = entanglement entropy

⟶ Quantifier of Hawking-generated entanglement

Nice! But limited, because:

(1) Only true if "in" state is pure, and
(2) BH is in isolation (not satisfied for any BH's we know in nature)

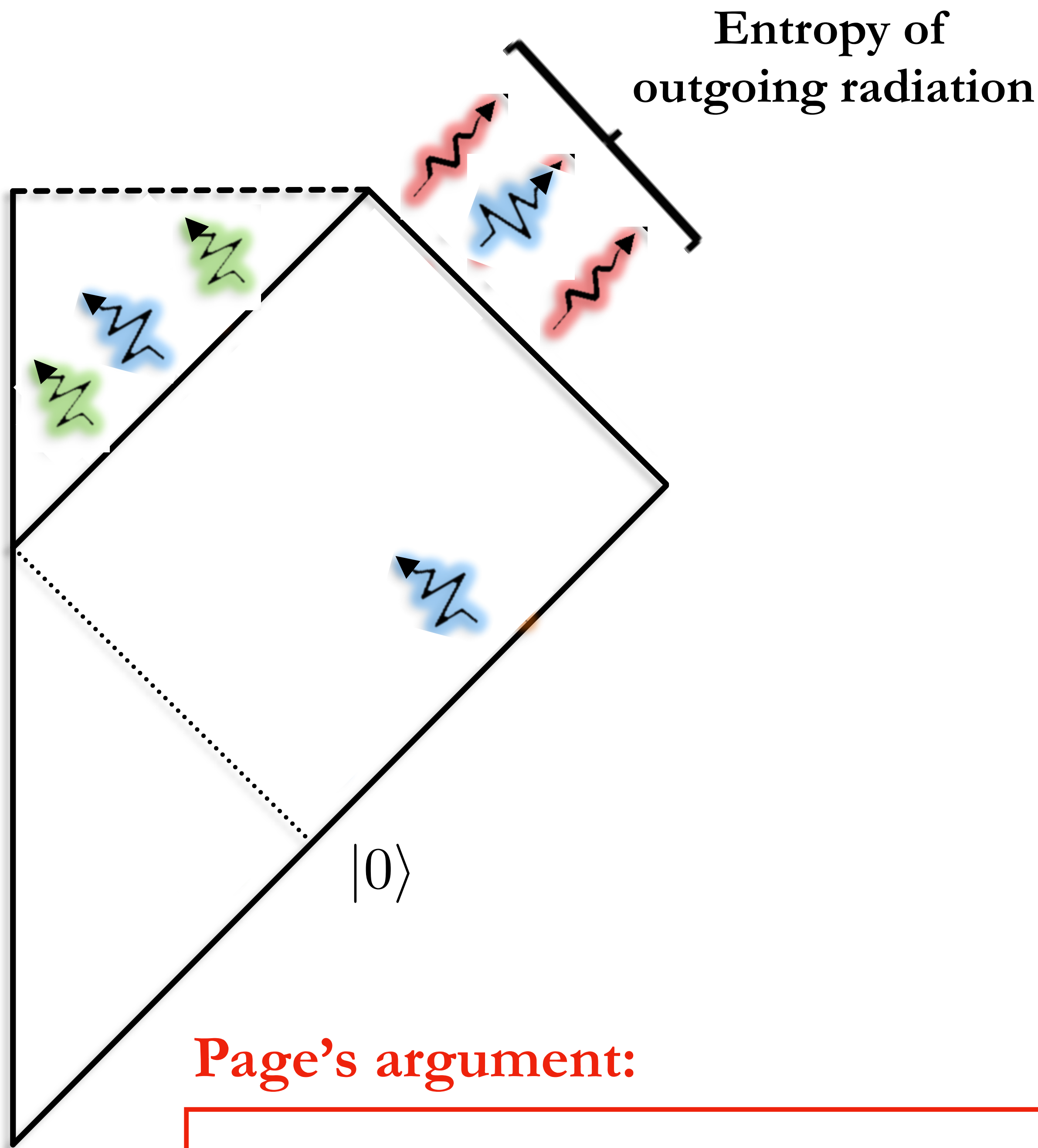


Page's argument:

Entropy of the radiation reaching infinity = entanglement entropy
⟶ Quantifier of Hawking-generated entanglement

Nice! But limited, because:

(1) Only true if "in" state is pure, and
(2) BH is in isolation (not satisfied for any BH's we know in nature)



Page's argument:

Entropy of the radiation reaching infinity = entanglement entropy
⟶ Quantifier of Hawking-generated entanglement

Nice! But limited, because:

(1) Only true if "in" state is pure, and
(2) BH is in isolation (not satisfied for any BH's we know in nature)

Goal:

Extend Page's analysis to quantify the entanglement generated in the Hawking process under different inputs

The tools
(pedagogical excursion)

Brief Review of Gaussian states for finite-dimensional bosonic quantum systems

Good reference:

[1] Alessio Serafini, *Quantum continuous variables: a primer of theoretical methods* (CRC press, 2017).

- **N-dimensional quantum (bosonic) system:** $\hat{x}_1, \hat{p}_1; \hat{x}_2, \hat{p}_2; \cdots \hat{x}_N, \hat{p}_N \equiv \hat{r}^i$

C.C.R's: $[\hat{r}^i, \hat{r}^j] = i\,\hbar\,\Omega^{ij}$

$$\Omega^{ij} = \oplus_N \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

- **N-dimensional quantum (bosonic) system:** $\hat{x}_1, \hat{p}_1; \hat{x}_2, \hat{p}_2; \cdots \hat{x}_N, \hat{p}_N \equiv \hat{r}^i$

C.C.R's: $[\hat{r}^i, \hat{r}^j] = i\,\hbar\,\Omega^{ij}$ $\qquad$ $\Omega^{ij} = \oplus_N \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

- **Gaussian state** $\hat{\rho}$ **: Completely and uniquely determined by its first and second moments**

$$\mu^i \equiv \mathrm{Tr}[\hat{\rho}\,\hat{r}^i]$$

$\mathrm{Tr}[\rho\,\hat{r}^i\,\hat{r}^j] \longrightarrow \sigma^{ij} = \mathrm{Tr}[\hat{\rho}\,\{(\hat{r}^i - \mu^i), (\hat{r}^j - \mu^j)\}]$ **covariance matrix**

- **N-dimensional quantum (bosonic) system:** $\hat{x}_1, \hat{p}_1; \hat{x}_2, \hat{p}_2; \cdots \hat{x}_N, \hat{p}_N \equiv \hat{r}^i$

  C.C.R's: $[\hat{r}^i, \hat{r}^j] = i\,\hbar\,\Omega^{ij}$ $\qquad$ $\Omega^{ij} = \oplus_N \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

- **Gaussian state** $\hat{\rho}$ **: Completely and uniquely determined by its first and second moments**

$$\mu^i \equiv \mathrm{Tr}[\hat{\rho}\,\hat{r}^i]$$

$$\mathrm{Tr}[\rho\,\hat{r}^i\,\hat{r}^j] \longrightarrow \sigma^{ij} = \mathrm{Tr}[\hat{\rho}\,\{(\hat{r}^i - \mu^i), (\hat{r}^j - \mu^j)\}]$$ **covariance matrix**

- **Restriction to a subsystem produces another Gaussian state:** $(\mu^i_{\mathrm{red}}, \sigma^{ij}_{\mathrm{red}})$

  Example: $\vec{\mu} = (\vec{\mu}^{\mathrm{red}}_A, \vec{\mu}^{\mathrm{red}}_B)$ $\qquad$ $\sigma = \begin{pmatrix} \sigma^{\mathrm{red}}_A & \sigma_{AB} \\ \sigma^{\top}_{AB} & \sigma^{\mathrm{red}}_B \end{pmatrix}$

- **N-dimensional quantum (bosonic) system:** $\hat{x}_1, \hat{p}_1; \hat{x}_2, \hat{p}_2; \cdots \hat{x}_N, \hat{p}_N \equiv \hat{r}^i$

C.C.R's: $[\hat{r}^i, \hat{r}^j] = i\,\hbar\,\Omega^{ij}$ $\qquad$ $\Omega^{ij} = \oplus_N \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

- **Gaussian state $\hat{\rho}$: Completely and uniquely determined by its first and second moments**

$$\mu^i \equiv \mathrm{Tr}[\hat{\rho}\,\hat{r}^i]$$

$\mathrm{Tr}[\rho\,\hat{r}^i\,\hat{r}^j] \longrightarrow \sigma^{ij} = \mathrm{Tr}[\hat{\rho}\,\{(\hat{r}^i - \mu^i), (\hat{r}^j - \mu^j)\}]$ **covariance matrix**

- **Restriction to a subsystem produces another Gaussian state:** $(\mu^i_{\mathrm{red}}, \sigma^{ij}_{\mathrm{red}})$

Example: $\vec{\mu} = (\vec{\mu}^{\,\mathrm{red}}_A, \vec{\mu}^{\,\mathrm{red}}_B)$ $\qquad$ $\sigma = \begin{pmatrix} \sigma^{\mathrm{red}}_A & \sigma_{AB} \\ \sigma^{T}_{AB} & \sigma^{\mathrm{red}}_B \end{pmatrix}$

- **N-dimensional quantum (bosonic) system:** $\hat{x}_1, \hat{p}_1; \hat{x}_2, \hat{p}_2; \cdots \hat{x}_N, \hat{p}_N \equiv \hat{r}^i$

  C.C.R's: $[\hat{r}^i, \hat{r}^j] = i\,\hbar\,\Omega^{ij}$ $\qquad \Omega^{ij} = \oplus_N \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

- **Gaussian state $\hat{\rho}$: Completely and uniquely determined by its first and second moments**

  $$\mu^i \equiv \mathrm{Tr}[\hat{\rho}\,\hat{r}^i]$$

  $$\mathrm{Tr}[\rho\,\hat{r}^i\,\hat{r}^j] \longrightarrow \sigma^{ij} = \mathrm{Tr}[\hat{\rho}\,\{(\hat{r}^i - \mu^i), (\hat{r}^j - \mu^j)\}]$$ **covariance matrix**

- **Restriction to a subsystem produces another Gaussian state:** $(\mu^i_{\mathrm{red}}, \sigma^{ij}_{\mathrm{red}})$

  Example: $\vec{\mu} = (\vec{\mu}_A^{\mathrm{red}}, \vec{\mu}_B^{\mathrm{red}})$ $\qquad \sigma = \begin{pmatrix} \sigma_A^{\mathrm{red}} & \sigma_{AB} \\ \sigma_{AB}^{\top} & \sigma_B^{\mathrm{red}} \end{pmatrix}$

- **Mean number of quanta in subsystem A:** $\langle \hat{n}_A \rangle = \frac{1}{4}\,\mathrm{Tr}[\sigma_A^{\mathrm{red}}] + \vec{\mu}_A^{\mathrm{red}\,\top} \cdot \vec{\mu}_A^{\mathrm{red}} - \frac{1}{2}\,N_A$

- **Elementary examples:**

Vacuum:	$\mu^i = 0$	$\sigma^{ij} = \mathbb{I}_{2N}$	}	**Pure**
Coherent state:	$\mu^i \neq 0$	$\sigma^{ij} = \mathbb{I}_{2N}$		
Squeezed:	$\mu^i = 0$	$\sigma^{ij} \neq \mathbb{I}_{2N}$		
Thermal:	$\mu^i = 0$	$\sigma^{ij} = \oplus_i^N (2\, n_i + 1)\, \mathbb{I}_2$	}	**Mixed**

- **A Gaussian state is pure iff the eigenvalues of** $\sigma^{ik}\Omega_{kj}$ **are** $\pm i$

  (beautiful connection with Kähler geometries)

Evolution:

If Hamiltonian is quadratic (= linear system), Gaussian states evolve to Gaussian states

$$(\mu^i_{\text{in}}, \sigma^{ij}_{\text{in}}) \longrightarrow (\mu^i_{\text{out}}, \sigma^{ij}_{\text{out}})$$

$S^i_{\ j}$ **= evolution matrix (2Nx2N)**

$$\vec{\mu}_{\text{out}} = S \cdot \vec{\mu}_{\text{in}}$$
$$\sigma_{\text{out}} = S \cdot \sigma_{\text{in}} \cdot S^{\top}$$

Because of linearity, $S^i_{\ j}$ turns out to be exactly the same matrix as the matrix implementing Hamiltonian evolution in the classical theory.

- Entanglement

  - Entanglement entropy is only a entanglement-quantifier for pure states

  - Logarithmic Negavity (based in the PPT criterion) is a convenient quantifier:

    - For Gaussian state and if either of the two subsystem is made of a single mode, LogNeg is a faithful quantifier.

    - Has an operational meaning: entanglement cost https://arxiv.org/abs/1809.09592

    - Units of "e-bits" (1 e-bit = the entanglement in a Bell pair)

    - Entanglement btw any bi-partition is determined entirely from σ^{ij} (μ^i is not involved)

Example 1: Two-mode squeezing of two h.o.'s

Evolution:

$$\hat{a}_1^{\text{in}} \to \hat{a}_1^{\text{out}} = \hat{a}_1^{\text{in}} \cosh r + \hat{a}_2^{\text{in}\,\dagger} \sinh r$$

$$\hat{a}_2^{\text{in}} \to \hat{a}_2^{\text{out}} = \hat{a}_1^{\text{in}\,\dagger} \sinh r + \hat{a}_2^{\text{in}} \cosh r$$

$$a_I = \frac{1}{\sqrt{2}}\,(x_I - i\,p_I)$$

$$\hat{r}^i_{\text{out}} = S^i{}_j\,\hat{r}^j_{\text{in}}$$

where $$S^i{}_j = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\cosh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}$$

Example 1: Two-mode squeezing of two h.o.'s

Evolution:

$$\hat{a}_1^{\rm in} \to \hat{a}_1^{\rm out} = \hat{a}_1^{\rm in}\, \cosh r + \hat{a}_2^{\rm in\,\dagger}\, \sinh r$$

$$\hat{a}_2^{\rm in} \to \hat{a}_2^{\rm out} = \hat{a}_1^{\rm in\,\dagger}\, \sinh r + \hat{a}_2^{\rm in}\, \cosh r$$

$$a_I = \frac{1}{\sqrt{2}}\,(x_I - i\,p_I)$$

$$\longrightarrow \quad \hat{r}^i_{\rm out} = S^i{}_j\, \hat{r}^j_{\rm in}$$

where $$S^i{}_j = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\cosh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}$$

- **Acting on vacuum:**

$$\vec{\mu}_{\rm in} = \vec{0} \longrightarrow \vec{\mu}_{\rm out} = S\cdot\vec{\mu}_{\rm in} = \vec{0}$$

$$\sigma_{\rm in} = \mathbb{I}_4 \longrightarrow \sigma_{\rm out} = S\cdot\sigma_{\rm in}\cdot S^\top = \begin{pmatrix} \cosh 2r & 0 & \sinh 2r & 0 \\ 0 & \cosh 2r & 0 & -2\,\cosh r\,\sinh r \\ \sinh 2r & 0 & \cosh 2r & 0 \\ 0 & -2\,\cosh r\,\sinh r & 0 & \cosh 2r \end{pmatrix}$$

Example 1: Two-mode squeezing of two h.o.'s

Evolution:

$$\hat{a}_1^{\text{in}} \to \hat{a}_1^{\text{out}} = \hat{a}_1^{\text{in}} \cosh r + \hat{a}_2^{\text{in}\,\dagger} \sinh r$$

$$\hat{a}_2^{\text{in}} \to \hat{a}_2^{\text{out}} = \hat{a}_1^{\text{in}\,\dagger} \sinh r + \hat{a}_2^{\text{in}} \cosh r$$

$$a_I = \frac{1}{\sqrt{2}}(x_I - i\,p_I)$$

$$\longrightarrow \quad \hat{r}^i_{\text{out}} = S^i{}_j\, \hat{r}^j_{\text{in}}$$

where
$$S^i{}_j = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\cosh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}$$

- **Acting on vacuum:**

$$\vec{\mu}_{\text{in}} = \vec{0} \longrightarrow \vec{\mu}_{\text{out}} = S \cdot \vec{\mu}_{\text{in}} = \vec{0}$$

Mixed

$$\sigma_{\text{in}} = \mathbb{I}_4 \longrightarrow \sigma_{\text{out}} = S \cdot \sigma_{\text{in}} \cdot S^\top = \begin{pmatrix} \cosh 2r & 0 & \sinh 2r & 0 \\ 0 & \cosh 2r & 0 & -2\cosh r\,\sinh r \\ \sinh 2r & 0 & \cosh 2r & 0 \\ 0 & -2\cosh r\,\sinh r & 0 & \cosh 2r \end{pmatrix}$$

Example 1: Two-mode squeezing of two h.o.'s

$$a_I = \frac{1}{\sqrt{2}}(x_I - i\,p_I)$$

Evolution:

$$\hat{a}_1^{\rm in} \to \hat{a}_1^{\rm out} = \hat{a}_1^{\rm in}\,\cosh r + \hat{a}_2^{\rm in\,\dagger}\,\sinh r$$

$$\hat{a}_2^{\rm in} \to \hat{a}_2^{\rm out} = \hat{a}_1^{\rm in\,\dagger}\,\sinh r + \hat{a}_2^{\rm in}\,\cosh r$$

$$\longrightarrow \quad \hat{r}^i_{\rm out} = S^i{}_j\,\hat{r}^j_{\rm in}$$

where
$$S^i{}_j = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\cosh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}$$

- **Acting on vacuum:**

$$\vec{\mu}_{\rm in} = \vec{0} \longrightarrow \vec{\mu}_{\rm out} = S\cdot\vec{\mu}_{\rm in} = \vec{0}$$

$$\sigma_{\rm in} = \mathbb{I}_4 \longrightarrow \sigma_{\rm out} = S\cdot\sigma_{\rm in}\cdot S^\top = \begin{pmatrix} \cosh 2r & 0 & \sinh 2r & 0 \\ 0 & \cosh 2r & 0 & -2\,\cosh r\,\sinh r \\ \sinh 2r & 0 & \cosh 2r & 0 \\ 0 & -2\,\cosh r\,\sinh r & 0 & \cosh 2r \end{pmatrix}$$

Mixed (diagonal blocks)

correlations (off-diagonal block)

Example 1: Two-mode squeezing of two h.o.'s

$$a_I = \frac{1}{\sqrt{2}}(x_I - i\,p_I)$$

Evolution:

$$\hat{a}_1^{\text{in}} \to \hat{a}_1^{\text{out}} = \hat{a}_1^{\text{in}}\,\cosh r + \hat{a}_2^{\text{in}\,\dagger}\,\sinh r$$

$$\hat{a}_2^{\text{in}} \to \hat{a}_2^{\text{out}} = \hat{a}_1^{\text{in}\,\dagger}\,\sinh r + \hat{a}_2^{\text{in}}\,\cosh r$$

$$\longrightarrow \quad \hat{r}^i_{\text{out}} = S^i{}_j\,\hat{r}^j_{\text{in}}$$

where $$S^i{}_j = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\cosh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}$$

- **Acting on vacuum:**

$$\vec{\mu}_{\text{in}} = \vec{0} \longrightarrow \vec{\mu}_{\text{out}} = S\cdot\vec{\mu}_{\text{in}} = \vec{0}$$

$$\sigma_{\text{in}} = \mathbb{I}_4 \longrightarrow \sigma_{\text{out}} = S\cdot\sigma_{\text{in}}\cdot S^{\top} = \begin{pmatrix} \cosh 2r & 0 & \sinh 2r & 0 \\ 0 & \cosh 2r & 0 & -2\,\cosh r\,\sinh r \\ \sinh 2r & 0 & \cosh 2r & 0 \\ 0 & -2\,\cosh r\,\sinh r & 0 & \cosh 2r \end{pmatrix}$$

Mixed

correlations

- **Entanglement:**

$$\text{LogNeg} = \ln_2 e^{2\,r} \quad \text{(e-bits)}$$

Conclusion: each oscillator is individually in a thermal state, and they are entangled: Two-mode squeezed vacuum

Example 1: Two-mode squeezing of two h.o.'s

Evolution:

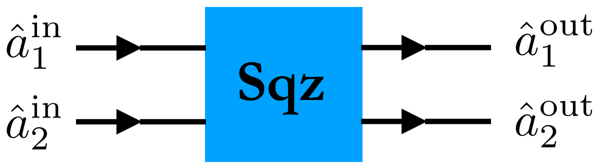
$$\hat{a}_1^{\rm in} \to \hat{a}_1^{\rm out} = \hat{a}_1^{\rm in}\,\cosh r + \hat{a}_2^{\rm in\,\dagger}\,\sinh r$$

$$\hat{a}_2^{\rm in} \to \hat{a}_2^{\rm out} = \hat{a}_1^{\rm in\,\dagger}\,\sinh r + \hat{a}_2^{\rm in}\,\cosh r$$

$$a_I = \frac{1}{\sqrt{2}}\,(x_I - i\,p_I)$$

$$\longrightarrow \quad \hat{r}^i_{\rm out} = S^i{}_j\,\hat{r}^j_{\rm in}$$

where $$S^i{}_j = \begin{pmatrix} \cosh r & 0 & \sinh r & 0 \\ 0 & \cosh r & 0 & -\cosh r \\ \sinh r & 0 & \cosh r & 0 \\ 0 & -\sinh r & 0 & \cosh r \end{pmatrix}$$



- **Acting on vacuum:**

$$\vec{\mu}_{\rm in} = \vec{0} \longrightarrow \vec{\mu}_{\rm out} = S\cdot\vec{\mu}_{\rm in} = \vec{0}$$

Mixed

correlations

$$\sigma_{\rm in} = \mathbb{I}_4 \longrightarrow \sigma_{\rm out} = S\cdot\sigma_{\rm in}\cdot S^\top = \begin{pmatrix} \cosh 2r & 0 & \sinh 2r & 0 \\ 0 & \cosh 2r & 0 & -2\,\cosh r\,\sinh r \\ \sinh 2r & 0 & \cosh 2r & 0 \\ 0 & -2\,\cosh r\,\sinh r & 0 & \cosh 2r \end{pmatrix}$$

- **Entanglement:**

$$\text{LogNeg} = \ln_2 e^{2\,r}\quad \text{(e-bits)}$$

Conclusion: each oscillator is individually in a thermal state, and they are entangled: Two-mode squeezed vacuum

Example 2: **Beam splitter for two h.o.'s**

Evolution:

$$\hat{a}_1^{\rm in} \to \hat{a}_1^{\rm out} = \hat{a}_1^{\rm in}\,\cos\theta + \hat{a}_2^{\rm in}\,\sin\theta$$

$$\hat{a}_2^{\rm in} \to \hat{a}_2^{\rm out} = -\hat{a}_1^{\rm in}\,\sin\theta + \hat{a}_2^{\rm in}\,\cos\theta$$

$$\longrightarrow \qquad r^i_{\rm out} = S^i{}_j\, r^j_{\rm in}$$

where $$S^i{}_j = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos r \end{pmatrix}$$

Example 2: Beam splitter for two h.o.'s

Evolution:

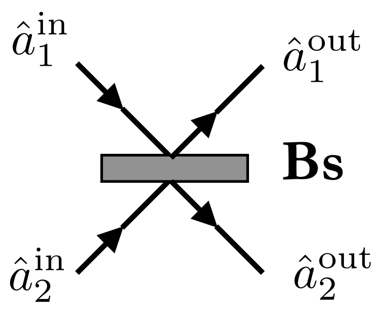
$$\hat{a}_1^{\rm in} \to \hat{a}_1^{\rm out} = \hat{a}_1^{\rm in}\ \cos\theta + \hat{a}_2^{\rm in}\ \sin\theta$$

$$\hat{a}_2^{\rm in} \to \hat{a}_2^{\rm out} = -\hat{a}_1^{\rm in}\ \sin\theta + \hat{a}_2^{\rm in}\ \cos\theta$$

$\longrightarrow$ $r^i_{\rm out} = S^i{}_j\, r^j_{\rm in}$

where $$S^i{}_j = \begin{pmatrix} \cos\theta & 0 & \sin\theta & 0 \\ 0 & \cos\theta & 0 & \sin\theta \\ -\sin\theta & 0 & \cos\theta & 0 \\ 0 & -\sin\theta & 0 & \cos r \end{pmatrix}$$

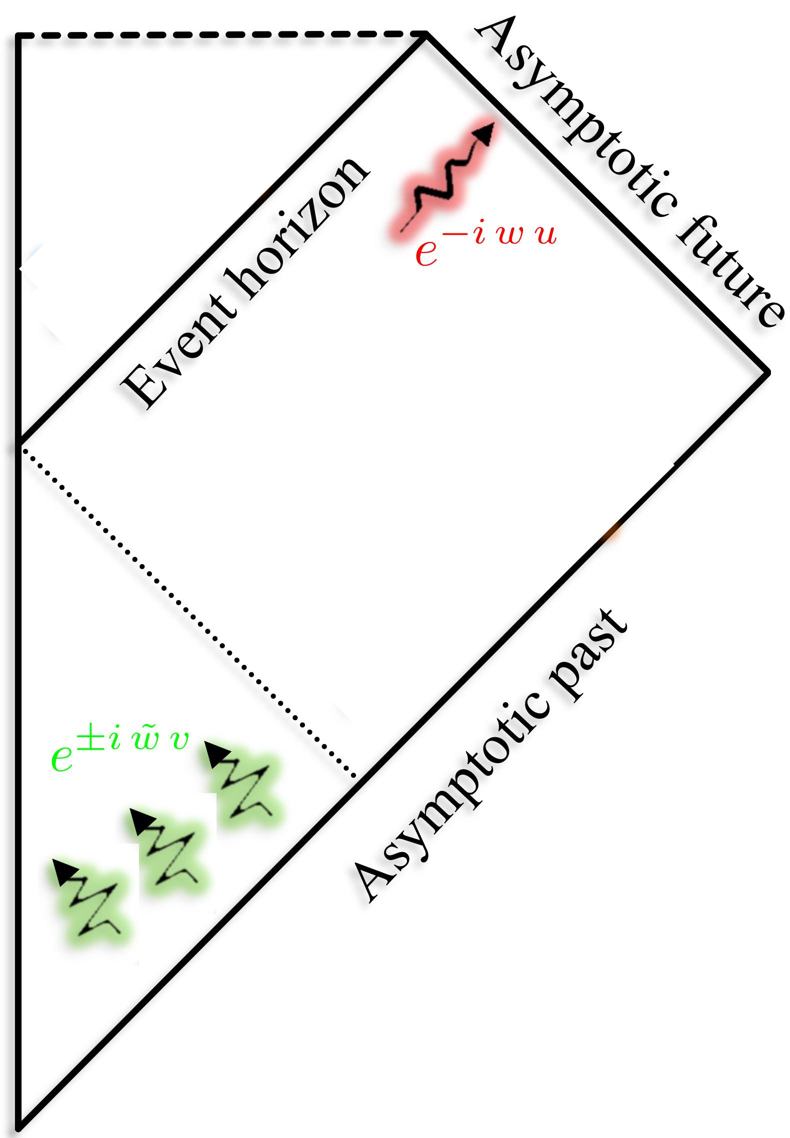
Beam Splitters divide amplitudes and entanglement



The Hawking Process for astrophysical BH's (Cont.)

Can we apply these techniques to quantify the generation of entanglement?

Difficulty: Infinitely many degrees of freedom involved



Evolution mixes infinitely many "in" modes to produce one "out" mode

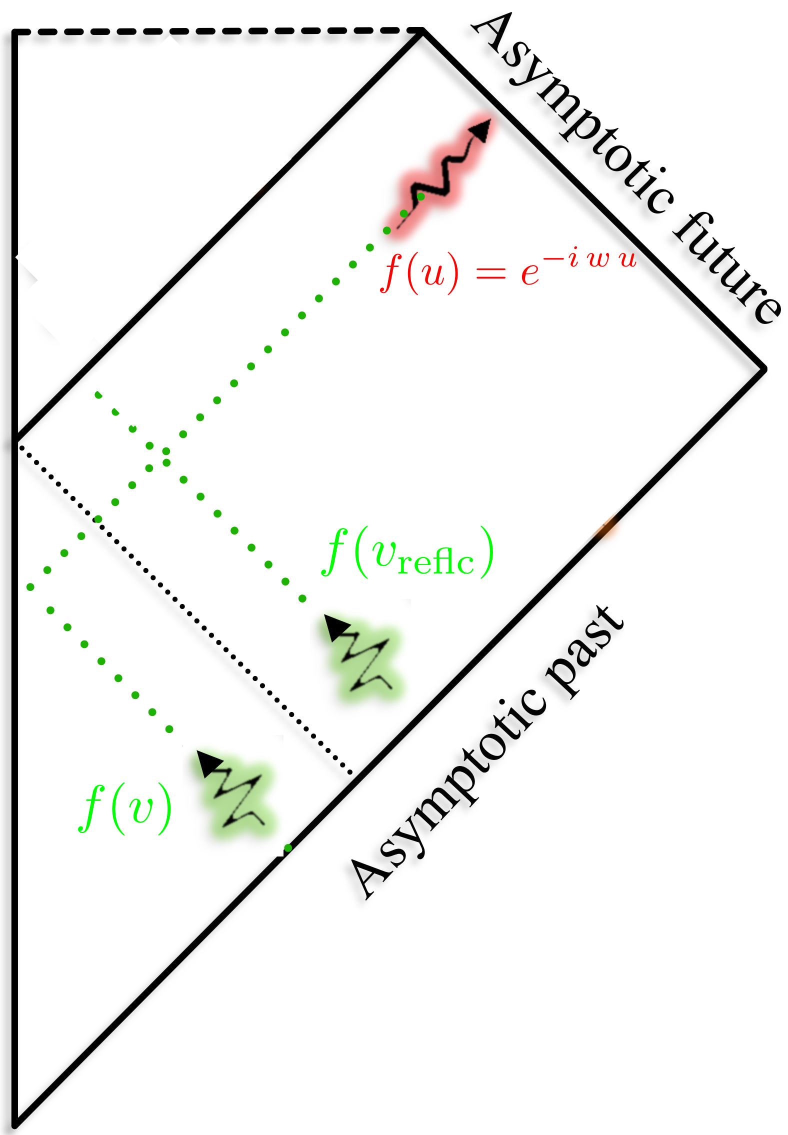
$$e^{-i\,w\,u} \to \int_0^\infty d\tilde{w}\,\left(\alpha_{w\tilde{w}}\, e^{-i\,\tilde{w}\,v} + \beta_{w\tilde{w}}\, e^{i\,\tilde{w}\,v}\right)$$

$e^{-i\,w\,u} \downarrow a_w^{\text{out}}$, $\quad e^{-i\,\tilde{w}\,v} \downarrow a_{\tilde{w}}^{\text{in}}$, $\quad e^{i\,\tilde{w}\,v} \downarrow a_{\tilde{w}}^{\text{in}\,\dagger}$

Is it possible to "diagonalize" this evolution to find the progenitors of the Hawking quanta?

Yes! Wald'75

Progenitors of the out modes: $F_I(w)\,,\,F_{II}(w)$
They do not have well-defined "in" frequency, but they are made of positive-frequency "in" modes, hence define the same "in" vacuum



Wald's Basis:

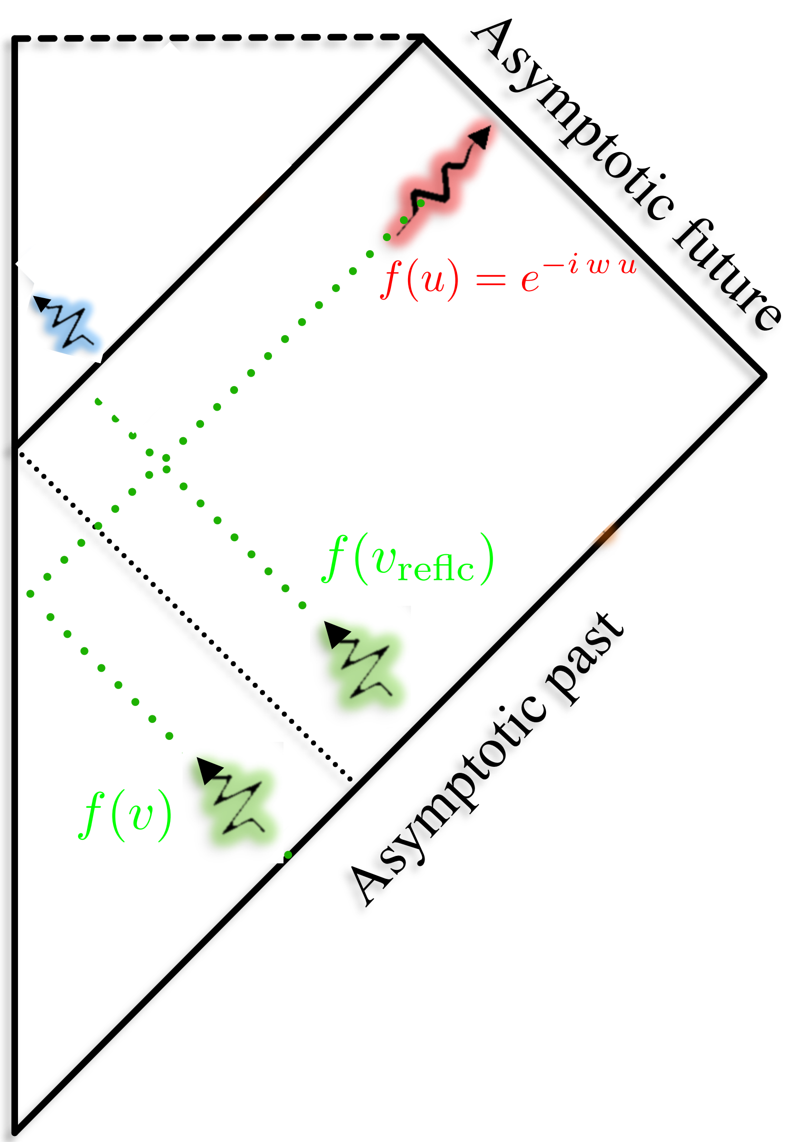
$$F_I(w) = N\left(f(v) + e^{-\pi w/\kappa}\, f(v_{reflc})\right)$$

$$F_{II}(w) = N\left(f^*(v_{reflc}) + e^{-\pi w/\kappa}\, f^*(v)\right)$$

Yes! **Wald'75**

Progenitors of the out modes: $F_I(w)\,,\,F_{II}(w)$

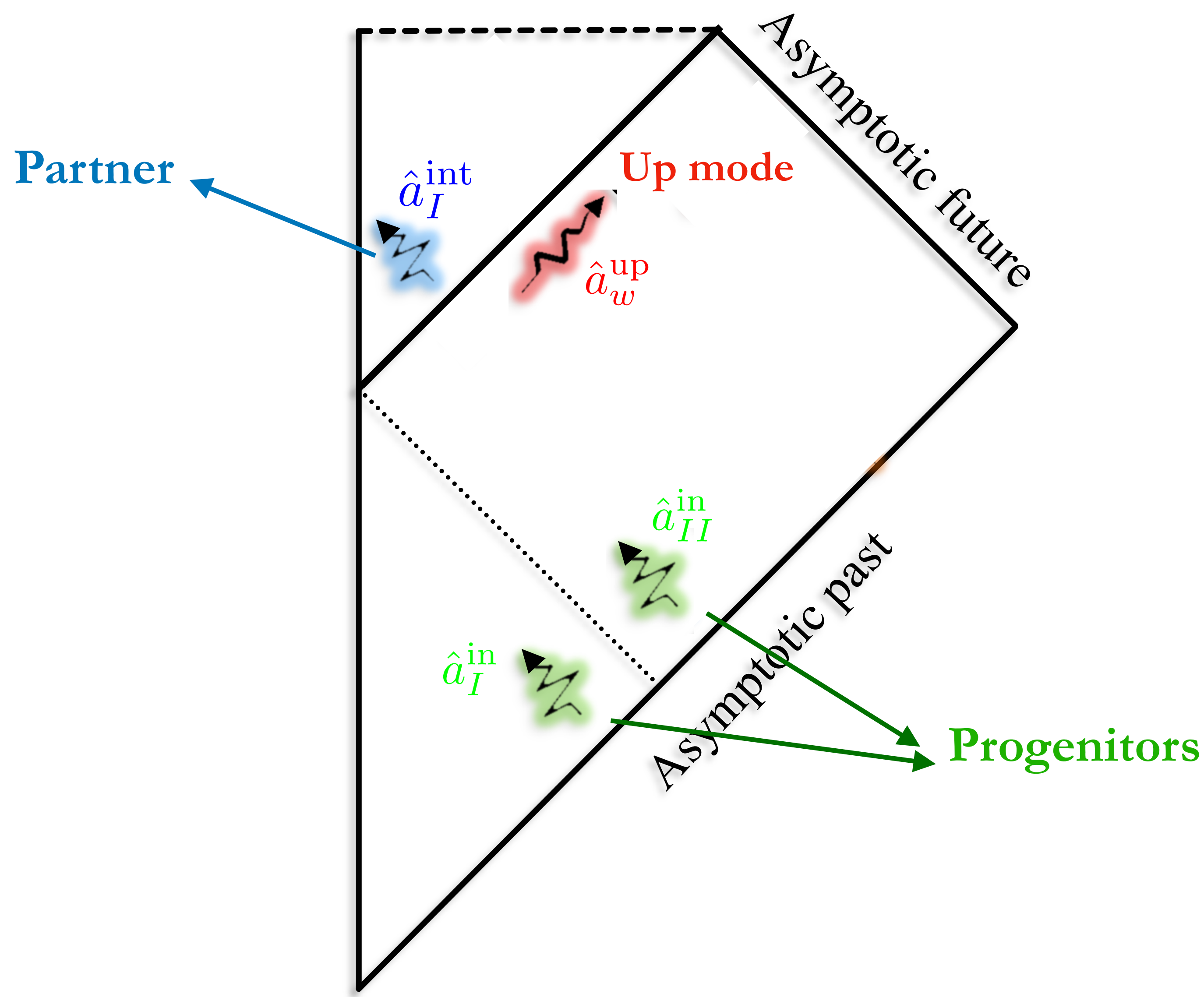
They do not have well-defined "in" frequency, but they are made of positive-frequency "in" modes, hence define the same "in" vacuum



Wald's Basis:

$$F_I(w) = N\left(f(v) + e^{-\pi w/\kappa}\, f(v_{reflc})\right)$$

$$F_{II}(w) = N\left(f^*(v_{reflc}) + e^{-\pi w/\kappa}\, f^*(v)\right)$$



Evolution (Hawking'74):

$$\hat{a}_I^{\rm in} \to \hat{a}_w^{\rm up} = \hat{a}_I^{\rm in} \cosh r_H - \hat{a}_{II}^{\rm in\,\dagger} \sinh r_H$$

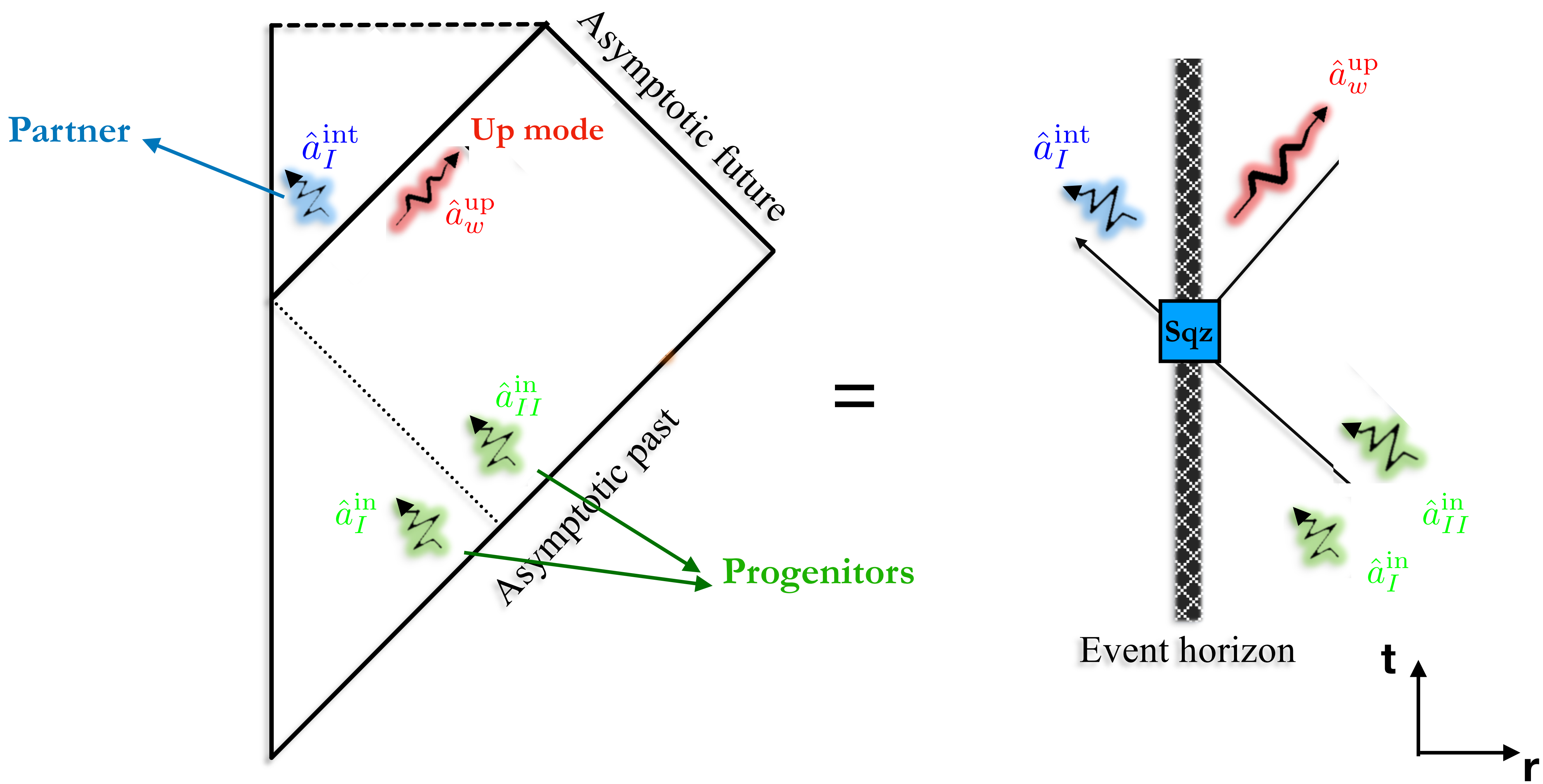
$$\hat{a}_{II}^{\rm in} \to \hat{a}_w^{\rm int} = -\hat{a}_I^{\rm in\,\dagger} \sinh r_H + \hat{a}_{II}^{\rm in} \cosh r_H$$

TWO-MODE SQUEEZER!

where

$$r_H(w) = \tanh^{-1} e^{-\frac{w}{2\,T_H}}$$

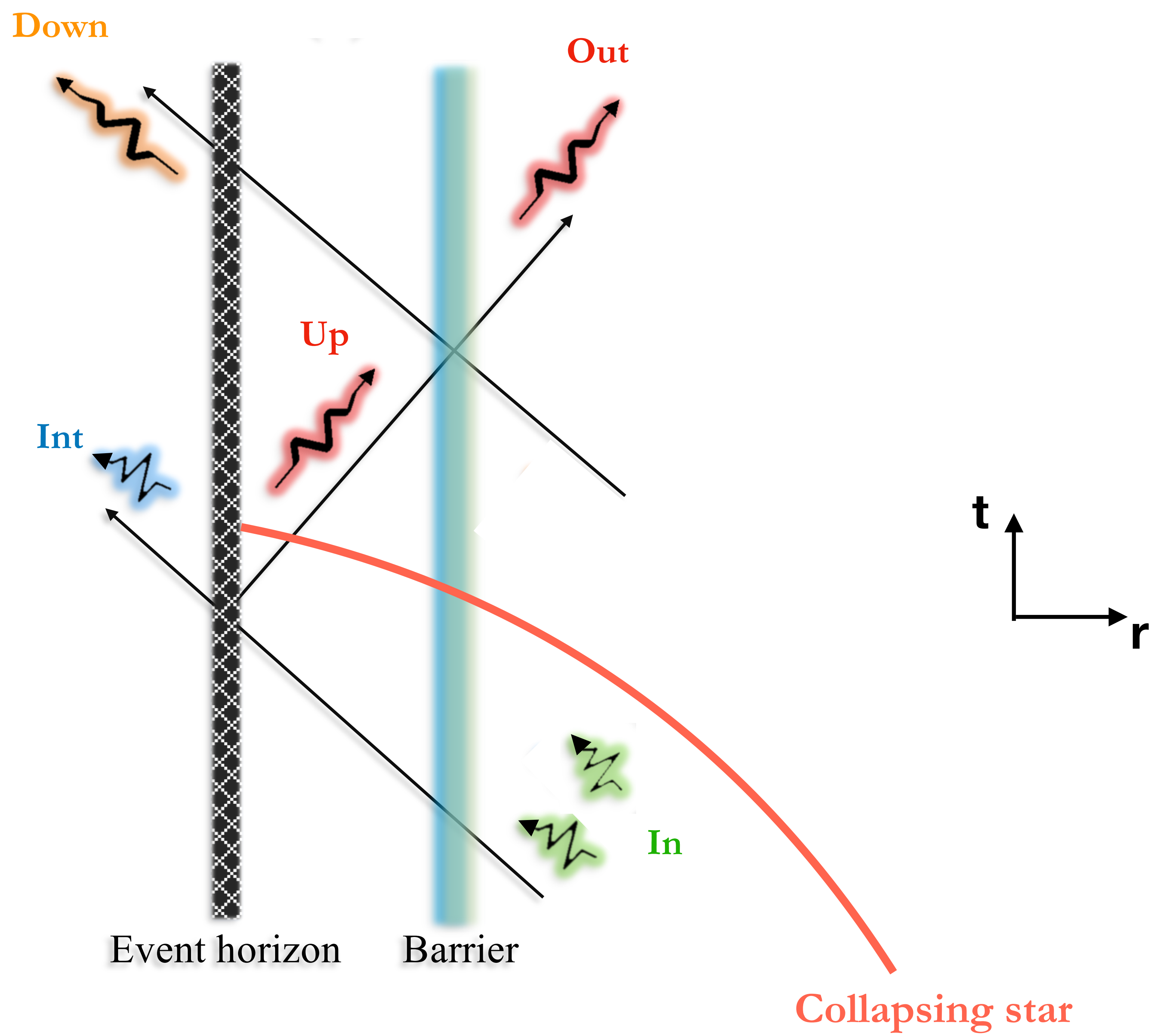
Hawking's squeezing intensity



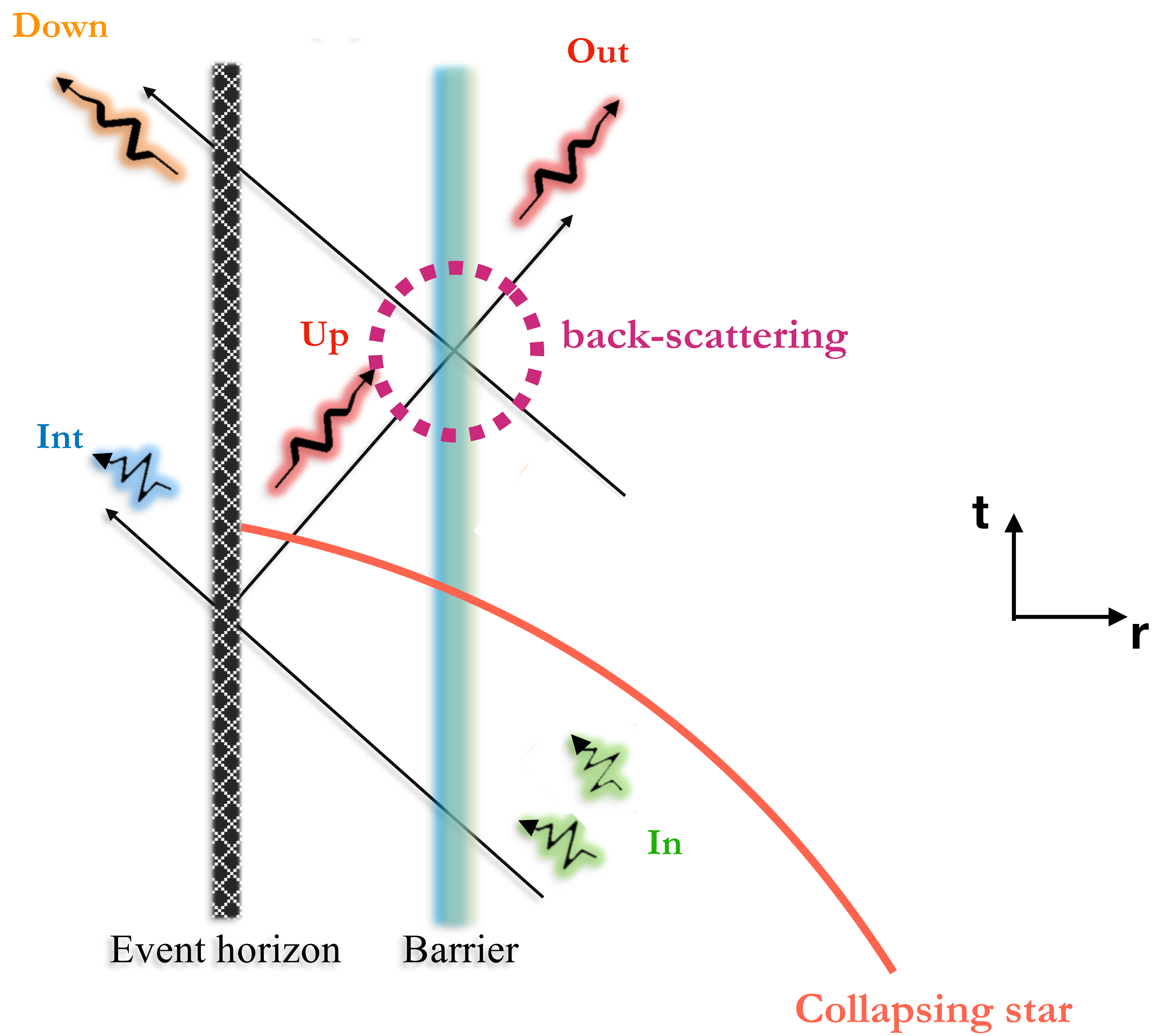
Asymptotic future
Partner
$\hat{a}_I^{\rm int}$
Up mode
$\hat{a}_w^{\rm up}$
$\hat{a}_{II}^{\rm in}$
$\hat{a}_I^{\rm in}$
Asymptotic past
Progenitors
=
$\hat{a}_w^{\rm up}$
$\hat{a}_I^{\rm int}$
Sqz
$\hat{a}_{II}^{\rm in}$
$\hat{a}_I^{\rm in}$
Event horizon
t
r

Including back-scattering

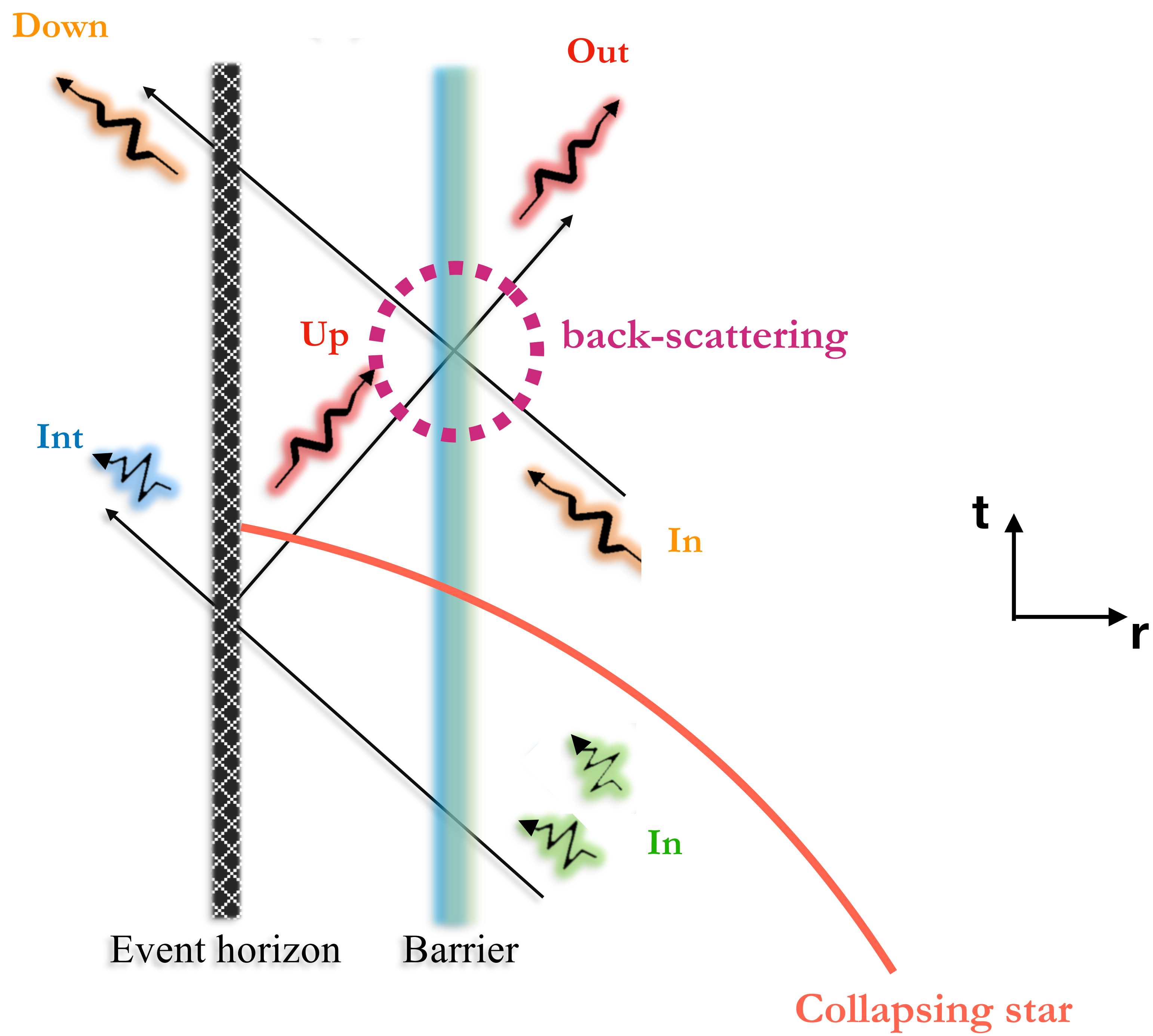
Including back-scattering

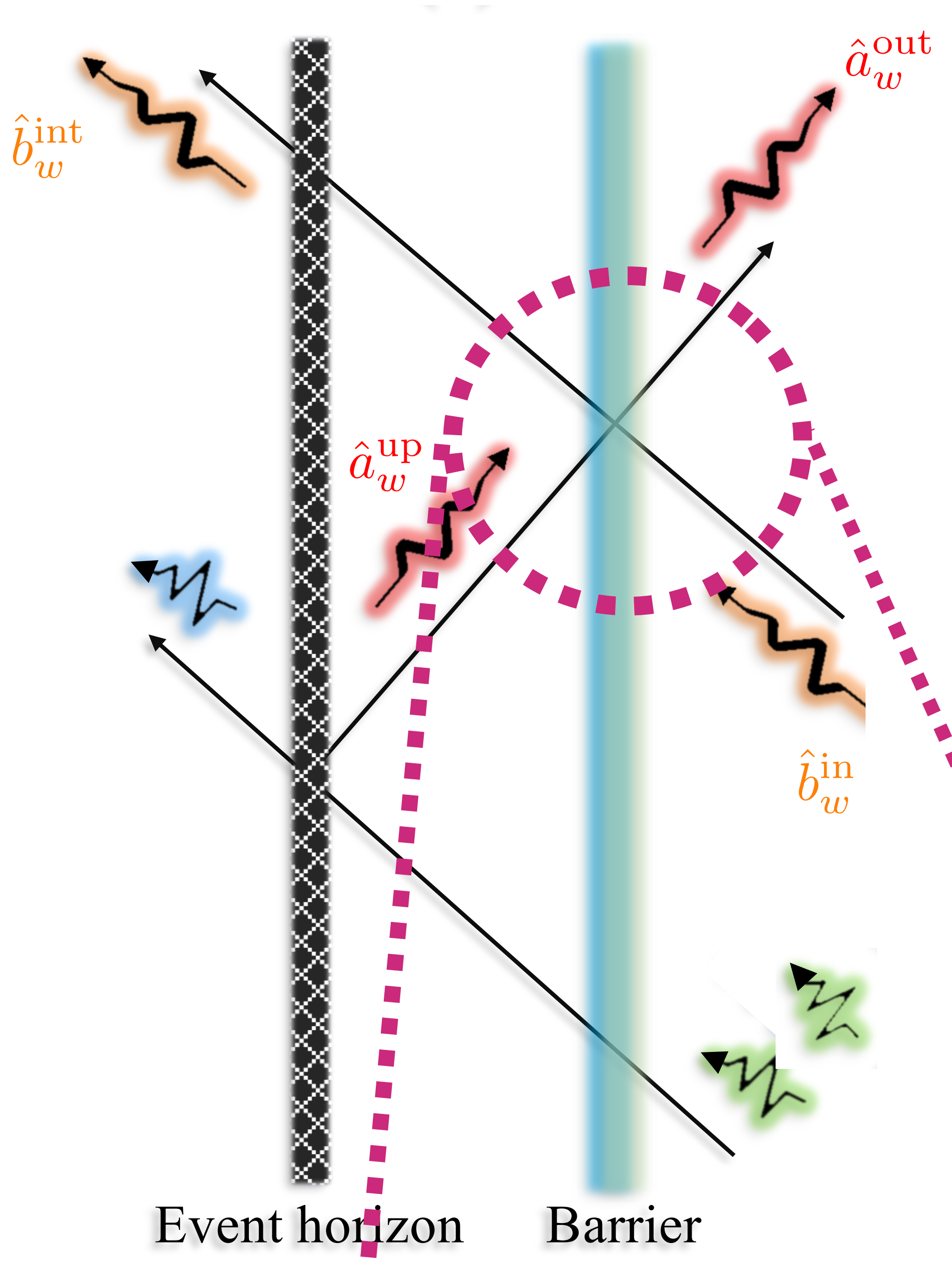


Including back-scattering



Including back-scattering





BEAM SPLITTER

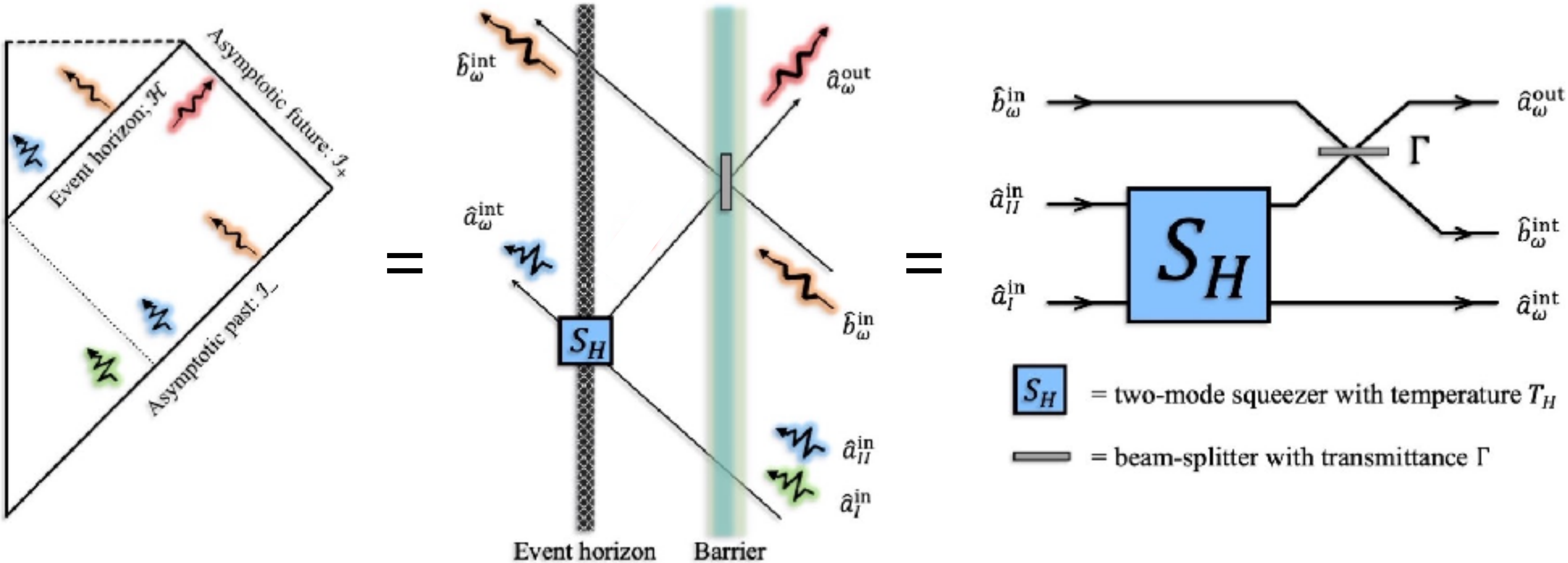
$$\hat{a}_w^{\text{up}} \rightarrow \hat{a}_w^{\text{out}} = \hat{a}_w^{\text{up}} \cos\theta + \hat{b}_w^{\text{in}} \sin\theta$$

$$\hat{b}_w^{\text{in}} \rightarrow \hat{b}_w^{\text{int}} = -\hat{a}_w^{\text{up}} \sin\theta + \hat{b}_w^{\text{in}} \cos\theta$$

Where: $\Gamma_\ell(w) = \cos^2\theta$

Transmission coeffs. **(Page'76)**

To summarize



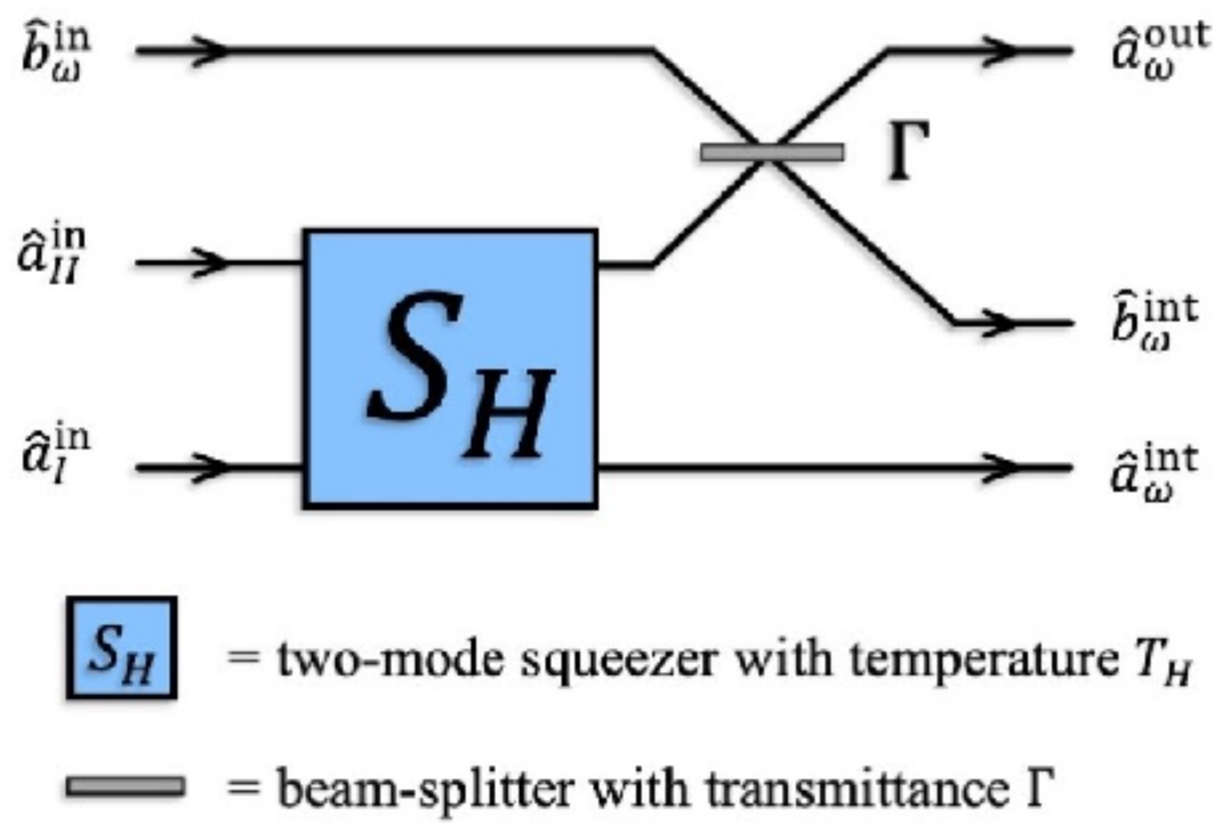
Hawking effect:

For a fixed w = Two-mode Squeezer + Beam splitter

3 modes to 3 modes

Evolution does not mix different w-sectors

This allows direct application of techniques for Gaussian quantum states discussed before



Evolution matrix: $S_{\text{tot}} = S_{\text{BS}} \cdot S_H$

Evolution of "in" state to "out" state: $(\vec{\mu}_{\text{in}},\, \sigma_{\text{in}}) \longrightarrow (\vec{\mu}_{\text{out}} = S_{\text{tot}} \cdot \vec{\mu}_{\text{in}},\, \sigma_{\text{out}} = S_{\text{tot}} \cdot \sigma_{\text{in}} \cdot S_{\text{tot}}^{\top})$

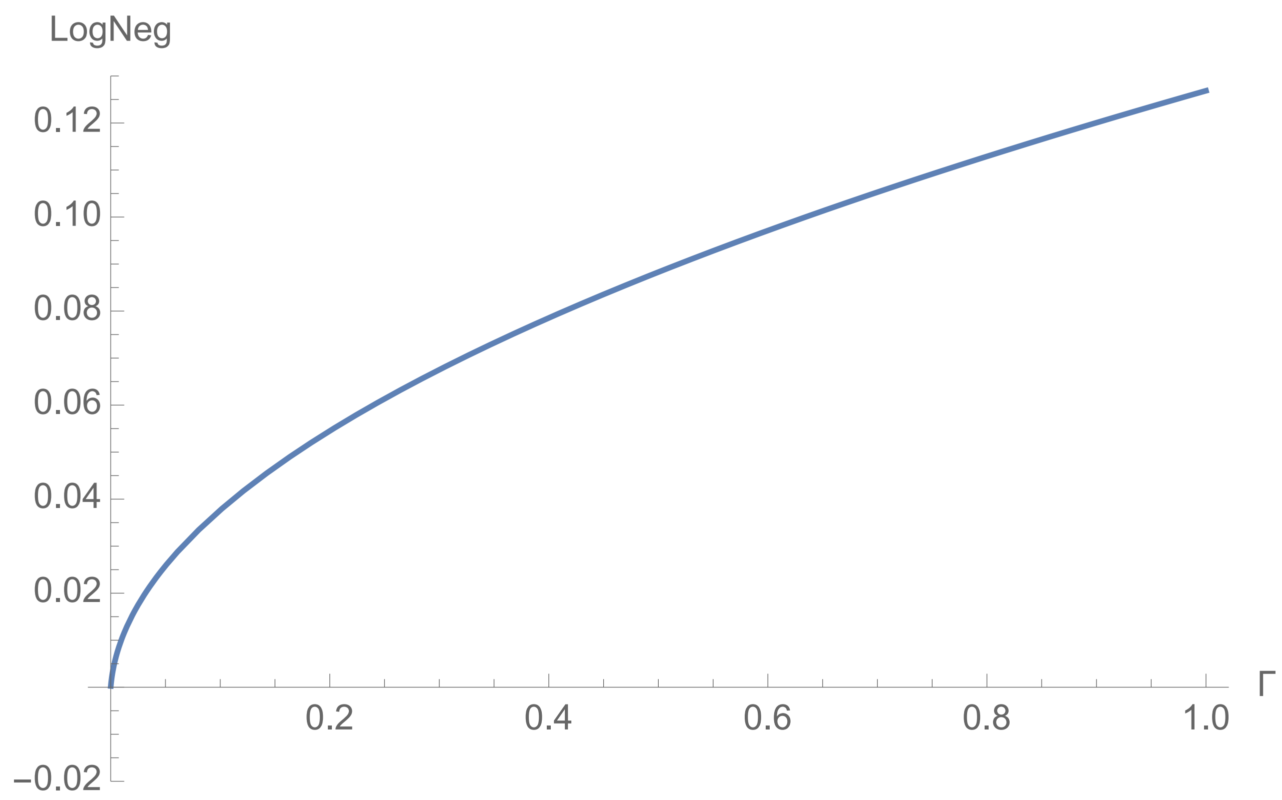
Vacuum Input

$$\vec{\mu}_{\text{in}} = \vec{0} \qquad \sigma_{\text{in}} = \mathbb{I}_6$$

Result of evolution:

$$\langle \hat{n}_{\rm out}(w) \rangle = \Gamma_\ell(w)\, \sinh^2 r_H(w) = \frac{\Gamma_\ell(w)}{e^{w/T_H} - 1}$$

$\text{LogNeg}[\hat{a}_w^{\rm out} | (\hat{a}_I^{\rm int}, \hat{b}_w^{\rm int})]$



Plot corresponding to Schwarzschild BH, $\ell = 1,\ w = 0.25\, M$

The potential barrier degrades the entanglement carried out to infinity

Coherent state input

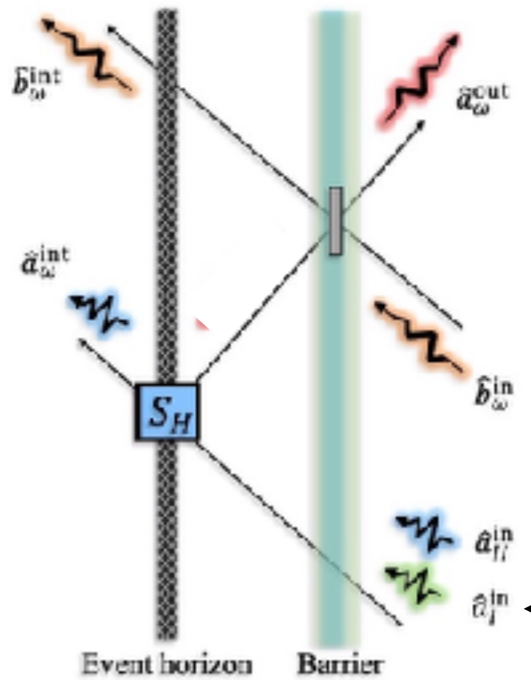
$$\vec{\mu}_{\mathrm{in}} \neq \vec{0} \qquad \sigma_{\mathrm{in}} = \mathbb{I}_6$$

We obtain:

- $$\langle \hat{n}_{\rm out}(w)\rangle = \Gamma(w)\,\sinh^2 r_H(w) + \mathrm{Tr}[\vec{\mu}_{\rm in}^{\top}\, S_{\rm tot}^{\top}\, \vec{\mu}_{\rm in}\, S_{\rm tot}]$$ **(stimulated Hawking radiation)**
- **LogNeg remains exactly the same as for vacuum input**

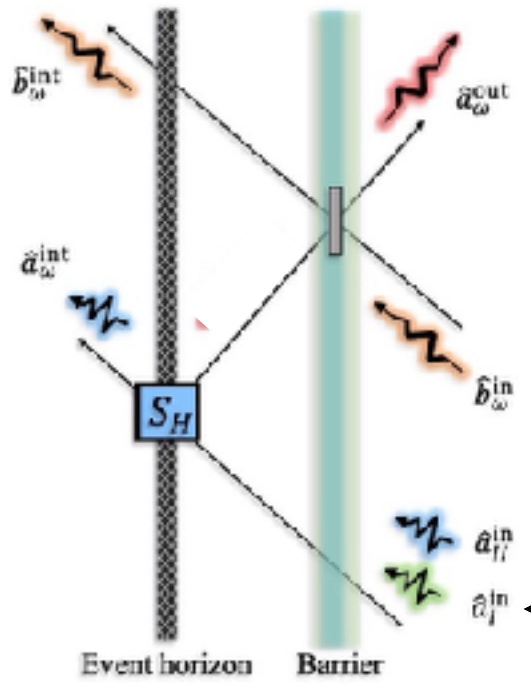
Ok with Standard Lore: Stimulated Hawking radiation is intrinsically classical

Single-mode squeezed input



Illuminate with a single mode squeezed input (academic exercise)

Initial state: $\vec{\mu}_{\rm in} = \vec{0}$ $\qquad \sigma_{\rm in} = \begin{pmatrix} e^{2\,r_I} & 0 & 0 \\ 0 & e^{-2\,r_I} & 0 \\ 0 & 0 & \mathbb{I}_4 \end{pmatrix}$

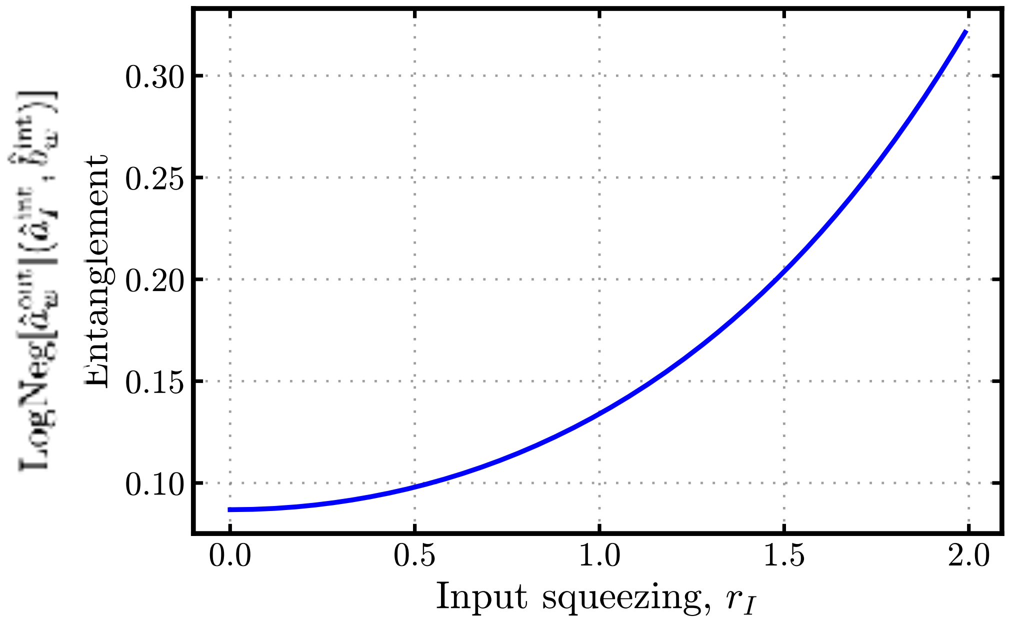


Illuminate with a single mode squeezed input (academic exercise)

Initial state: $\vec{\mu}_{\rm in} = \vec{0}$ $\qquad \sigma_{\rm in} = \begin{pmatrix} e^{2\,r_I} & 0 & 0 \\ 0 & e^{-2\,r_I} & 0 \\ 0 & 0 & \mathbb{I}_4 \end{pmatrix}$

Final state: $\langle \hat{n}_{\rm out}(w) \rangle = \Gamma(w)\, \sinh^2 r_H(w)\, \cosh^2 r_I$

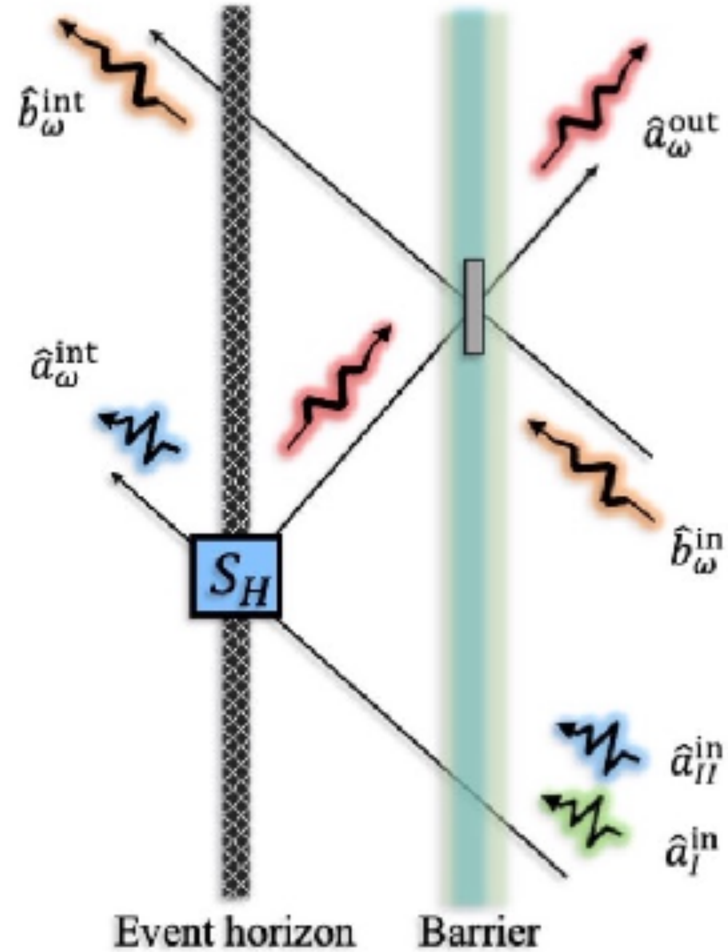
stimulated radiation



Both number of Hawking quanta and LogNeg can be tuned up!!

This is not physically viable for astrophysical BH's

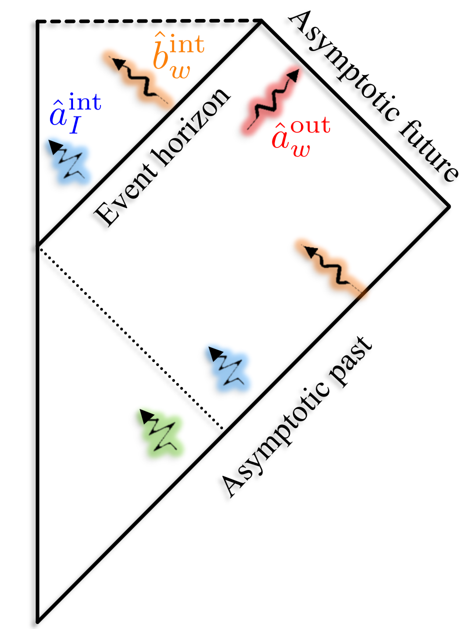
Thermal input

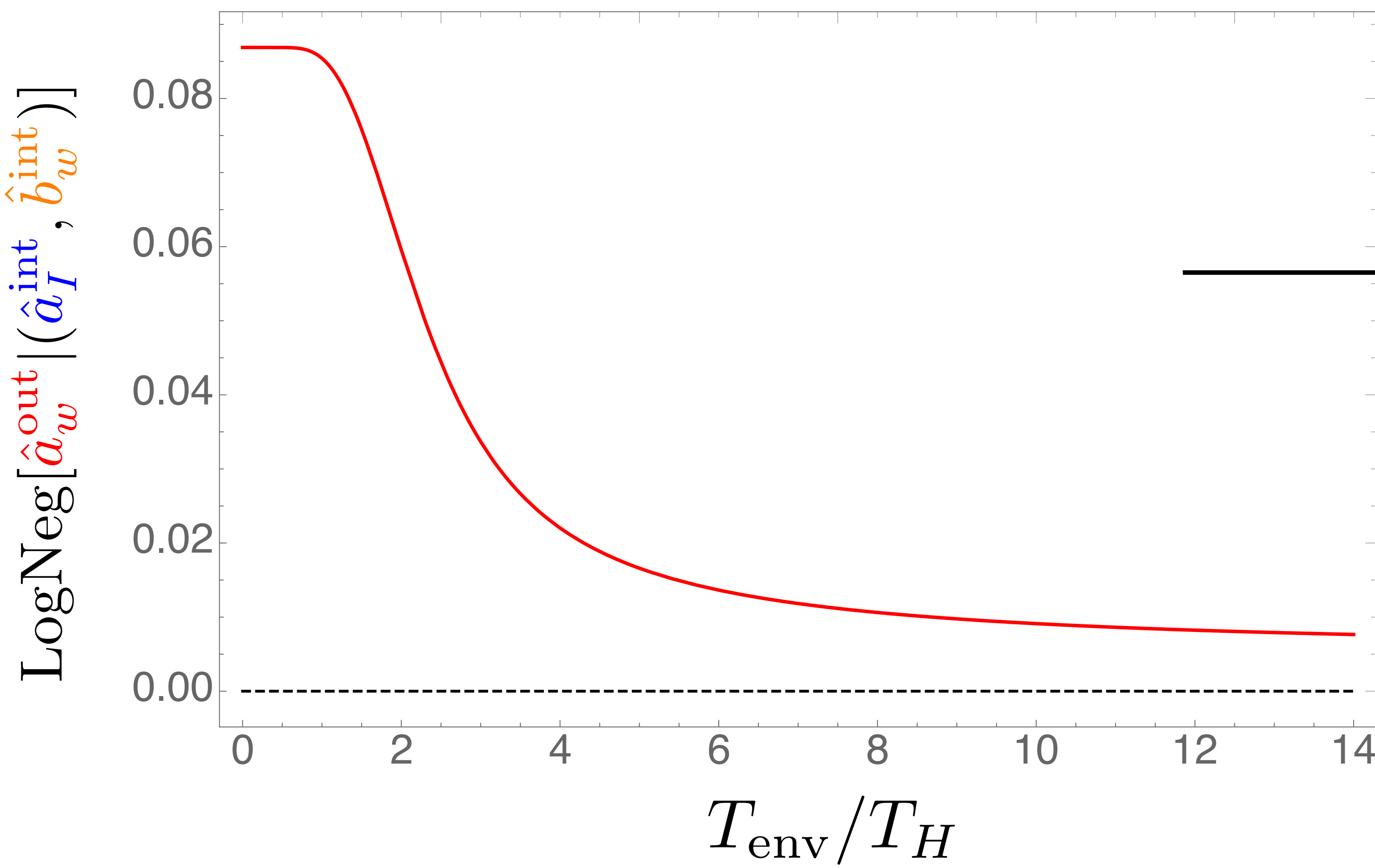


Thermal input. E.g., the CMB.

Initial State: $\vec{\mu}_{\rm in} = \vec{0}$ $\qquad \sigma_{\rm in} = \begin{pmatrix} \mathbb{I}_4 & 0 \\ 0 & \boxed{(1 + 2\, n_{th})\, \mathbb{I}_2} \end{pmatrix}$

Entanglement:

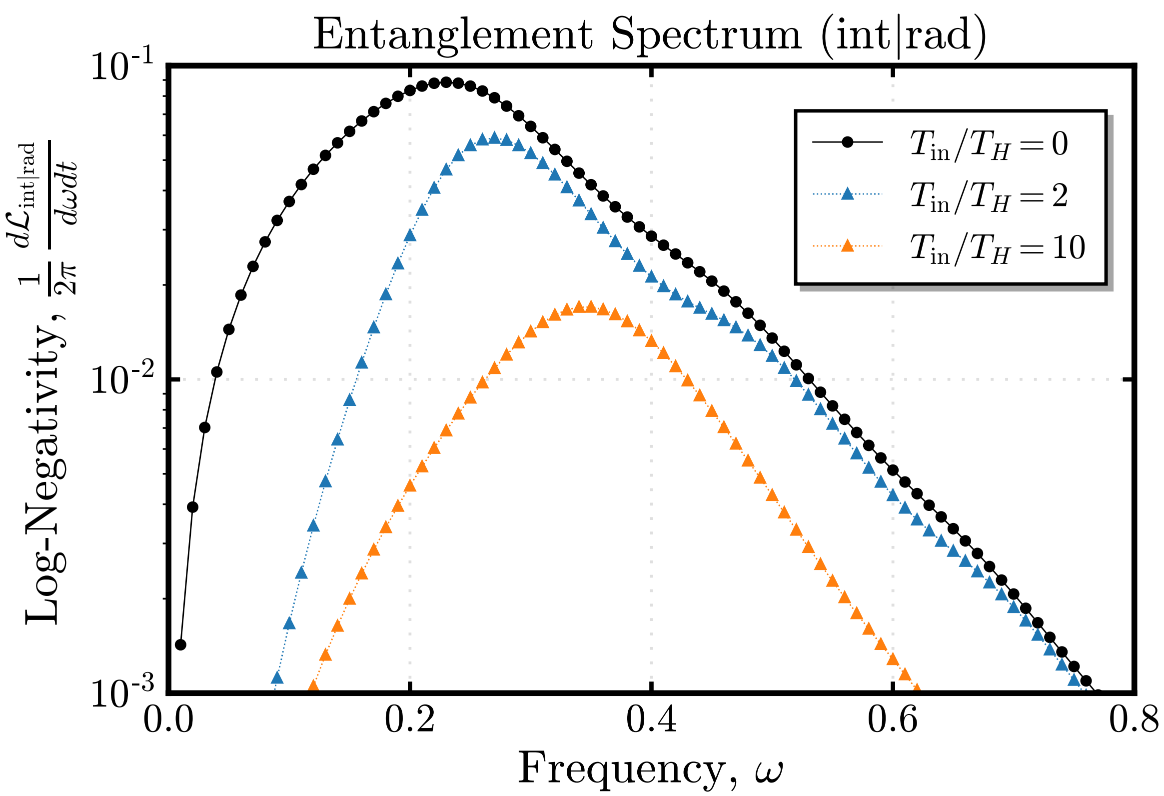




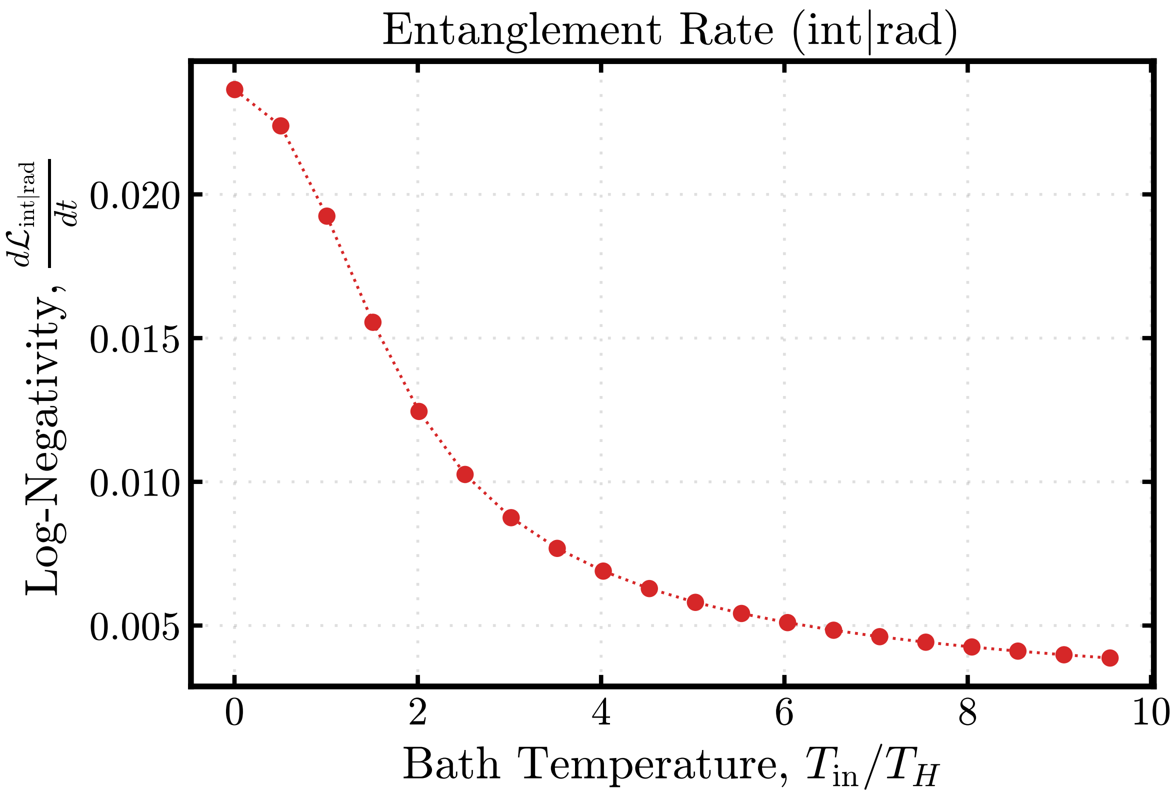
Ambient thermal noise degrades the generation of entanglement

90% reduction

Sum over all modes: total entanglement produce per unit of retarded time

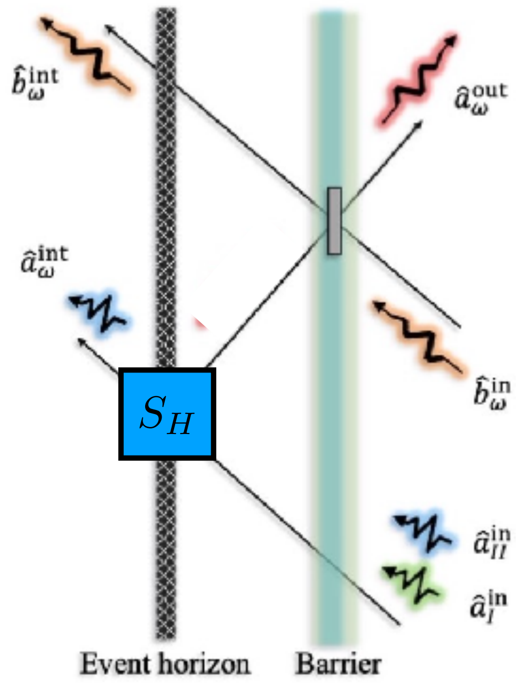


Sum over ℓ, m



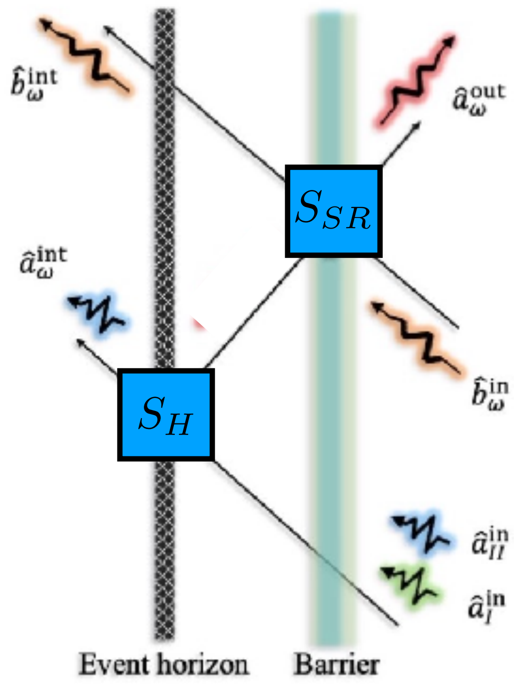
Sum over ℓ, m and w

Rotating BH's



Non-Super-radiant modes

$w > m\,\Omega_H$



Super-radiant modes

$w < m\,\Omega_H$

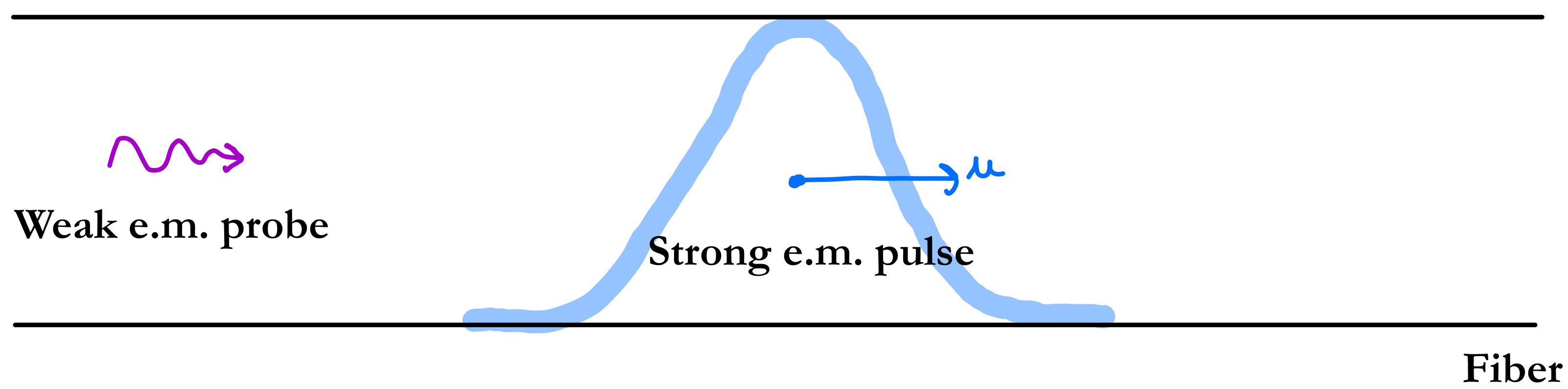
Hawking radiation in analog optical models

Optical analogs

Based on Kerr effect: $n_{eff}(t, \vec{x}) = n + \alpha \, |E_{\mathrm{strong}}(t, \vec{x})|^2$

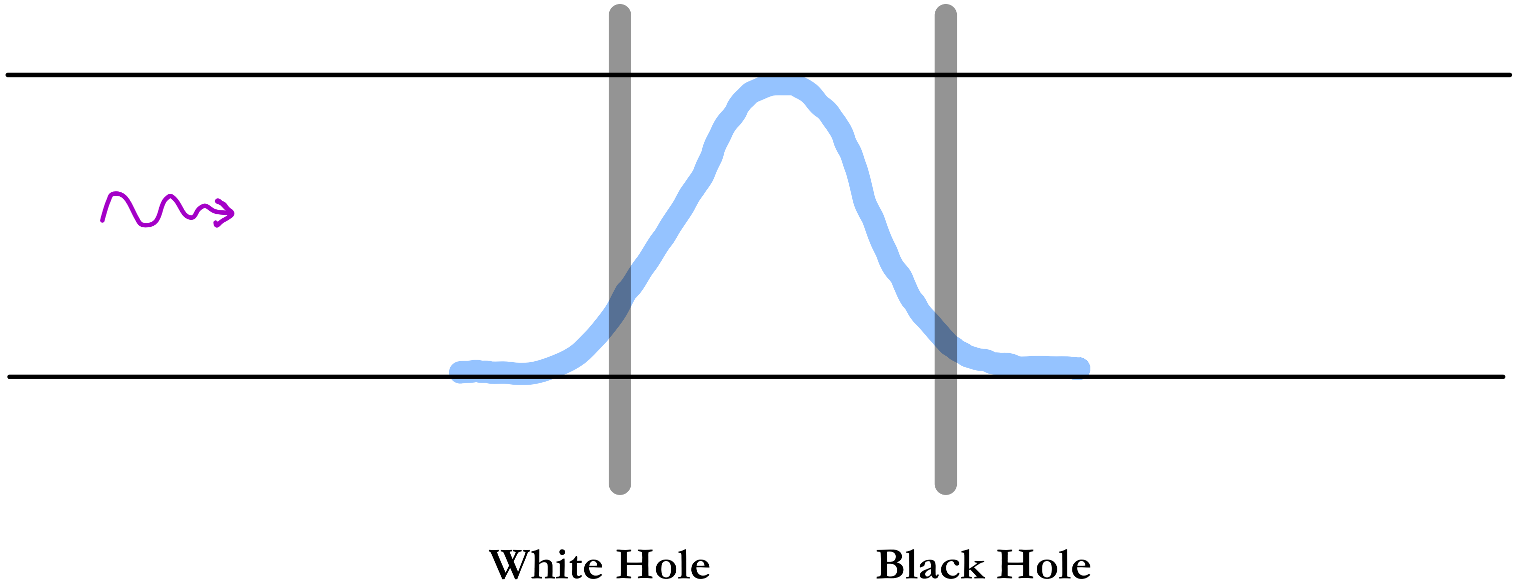
Optical analogs

Based on Kerr effect: $n_{eff}(t,\vec{x}) = n + \alpha\,|E_{\text{strong}}(t,\vec{x})|^2$

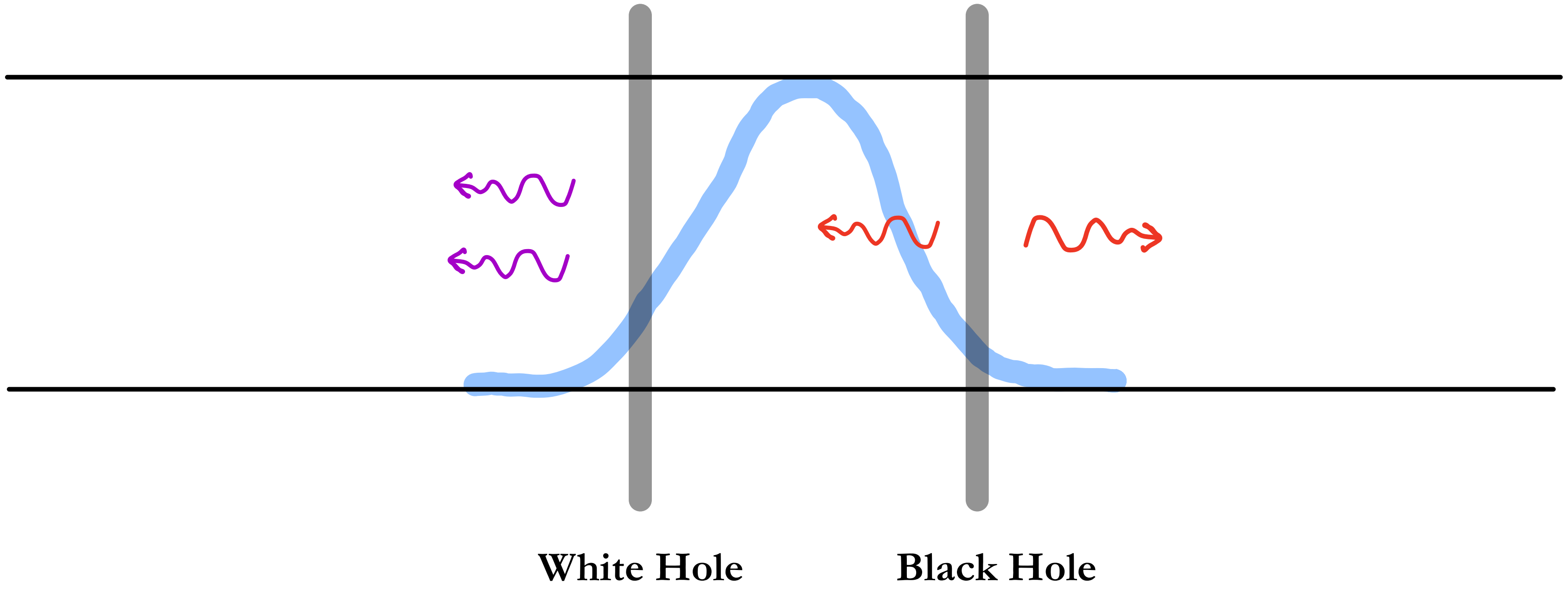


In the frame comoving with the pulse…

Frame comoving with the strong pulse

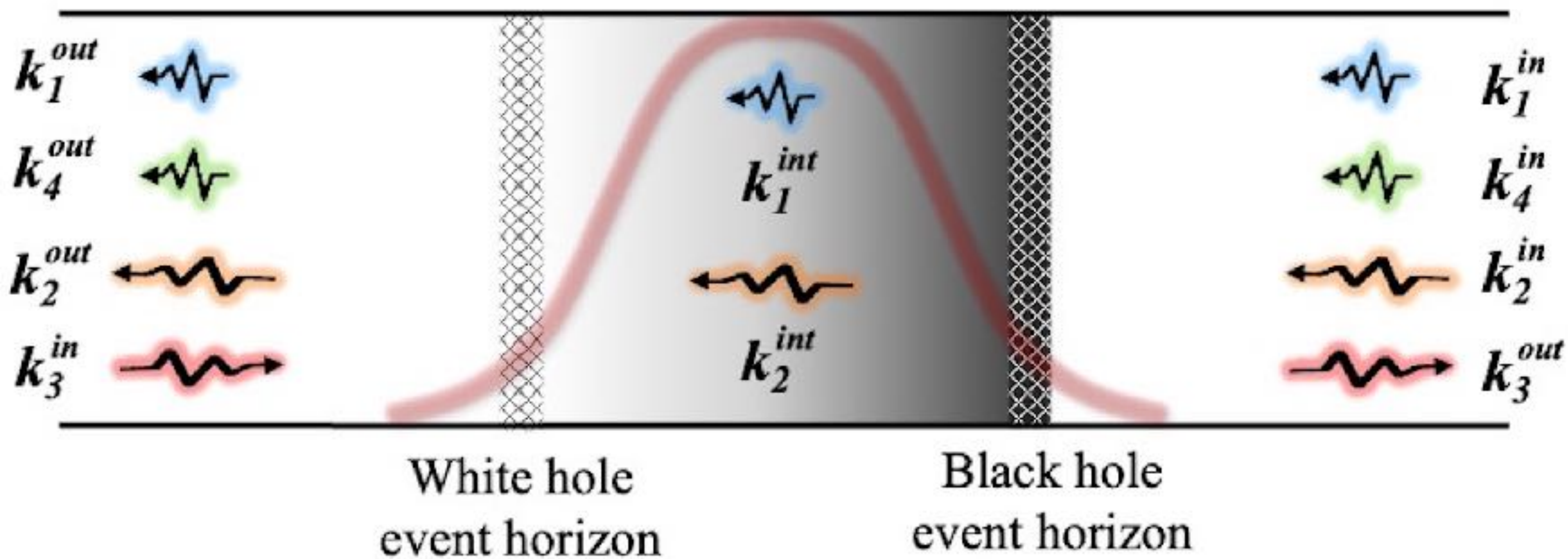


Hawking radiation!



Comoving frame

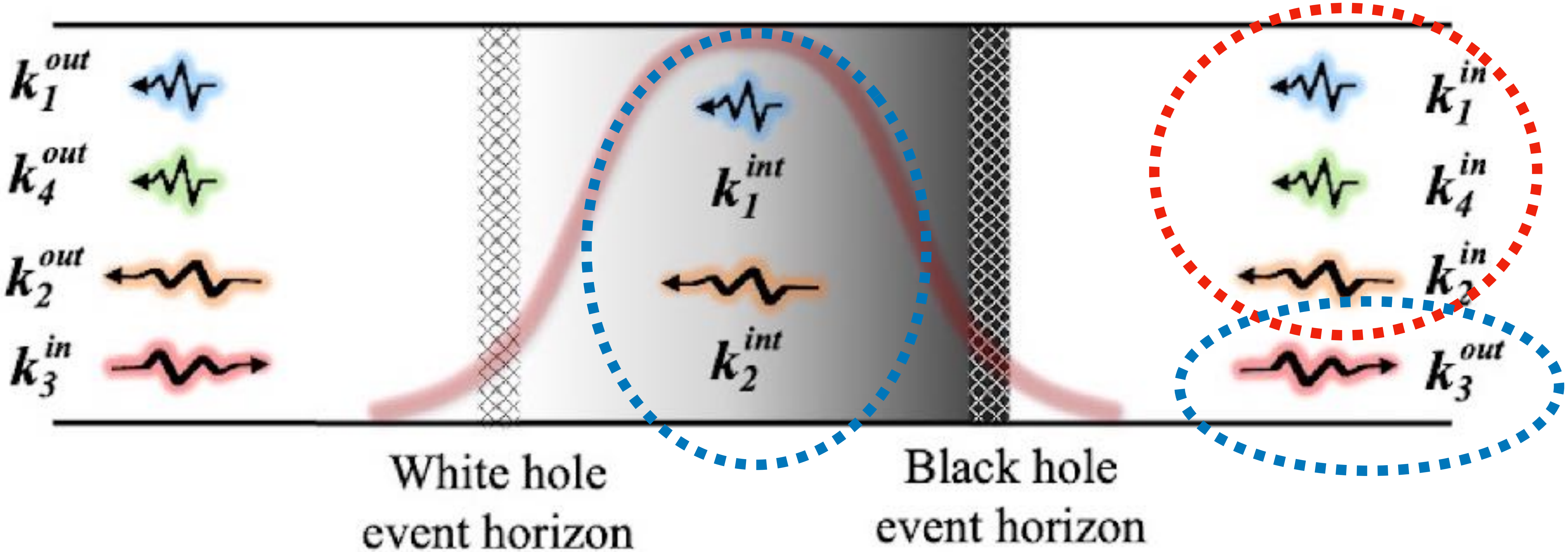
Modes for a single comoving frequency w



(See Linder, Schutzhold, Unruh 2016 for details)

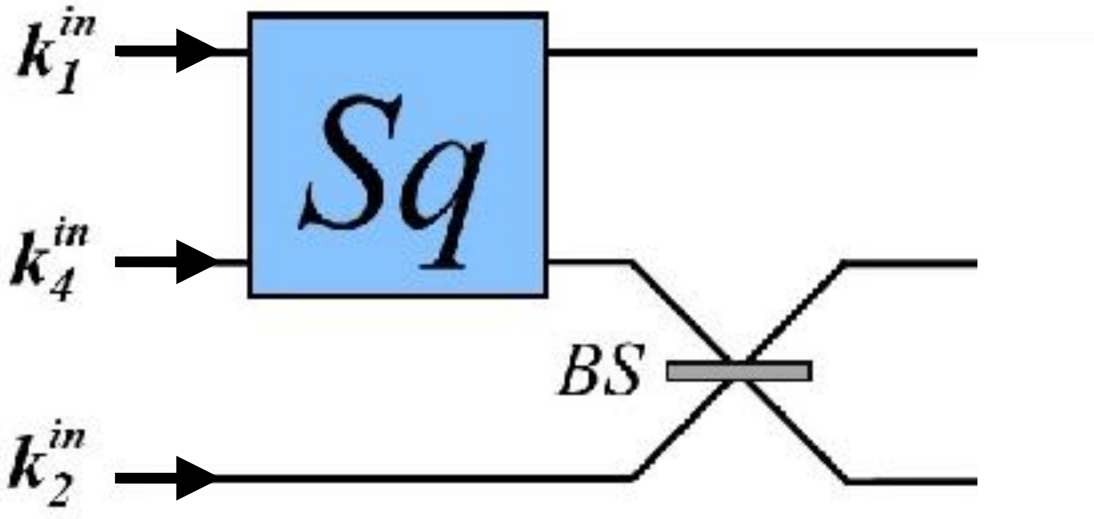
Comoving frame

Modes for a single comoving frequency w



(See Linder, Schutzhold, Unruh 2016 for details)

Circuit for the White-Black hole pair:

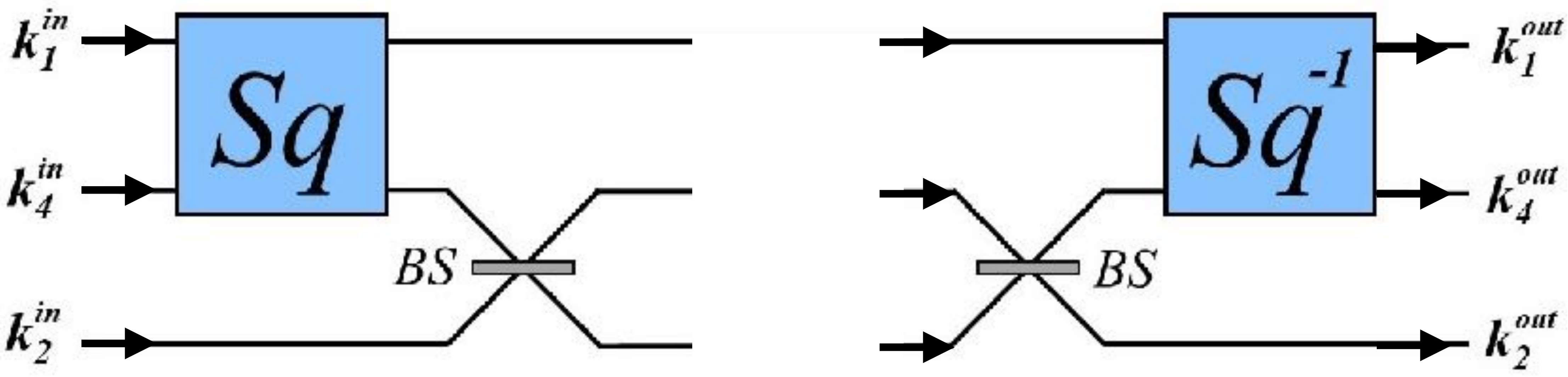


Evolution matrix: $S_{\text{total}} = S_{\text{WH}} \cdot S_{\text{BS}_2} \cdot S_{\text{BS}_1} \cdot S_{\text{BH}}$

From this, we can compute every aspect of the evolution of any Gaussian state

We add the effect of ambient noise, losses and detector inefficiencies

Circuit for the White-Black hole pair:

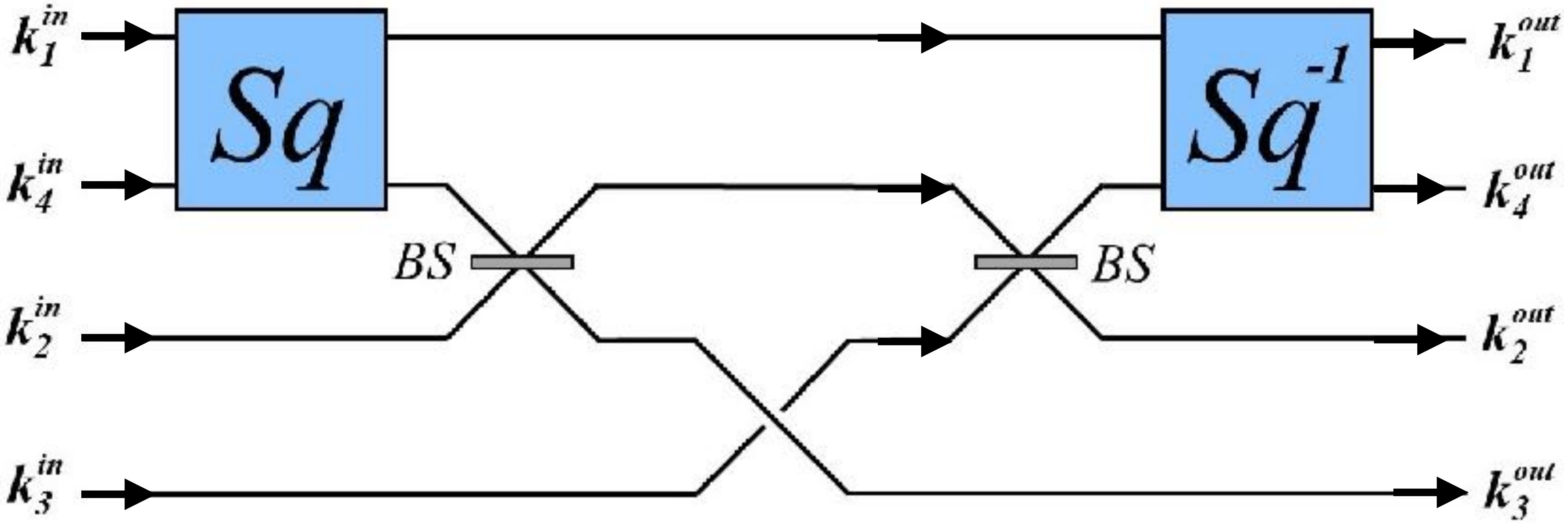


Evolution matrix: $S_{\text{total}} = S_{\text{WH}} \cdot S_{\text{BS}_2} \cdot S_{\text{BS}_1} \cdot S_{\text{BH}}$

From this, we can compute every aspect of the evolution of any Gaussian state

We add the effect of ambient noise, losses and detector inefficiencies

Circuit for the White-Black hole pair:



Evolution matrix: $S_{\text{total}} = S_{\text{WH}} \cdot S_{\text{BS}_2} \cdot S_{\text{BS}_1} \cdot S_{\text{BH}}$

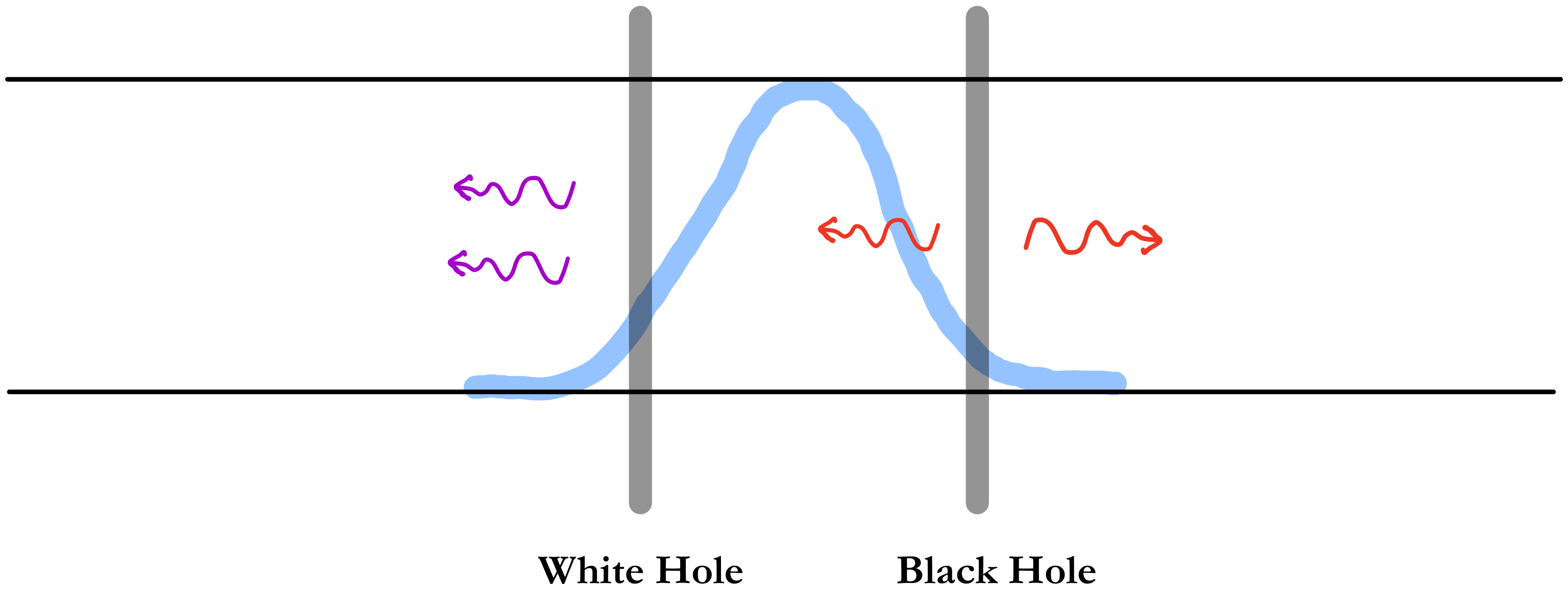
From this, we can compute every aspect of the evolution of any Gaussian state

We add the effect of ambient noise, losses and detector inefficiencies

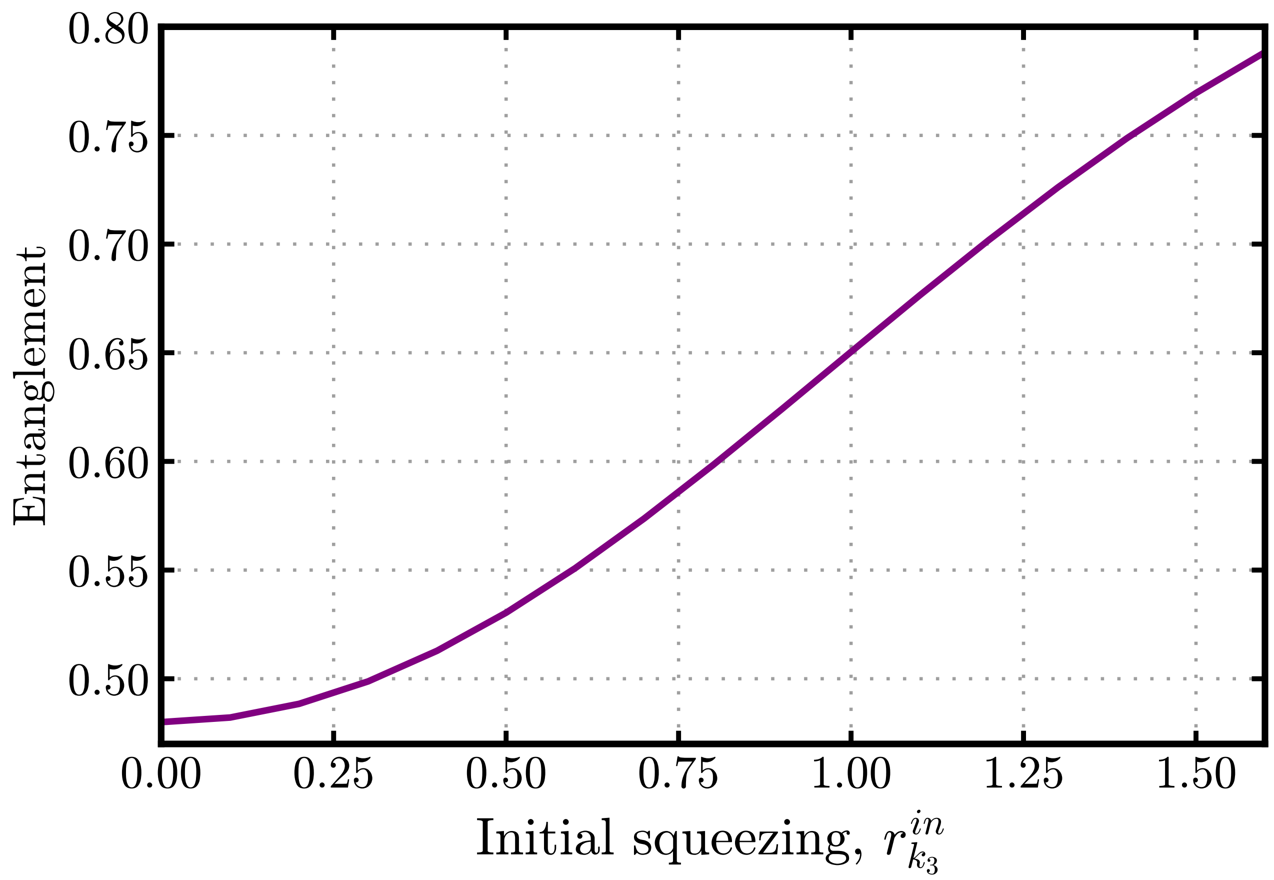
Our contribution:

Stimulated Hawking radiation is classical because the use of coherent states and because entanglement-degrading environmental effects (ambient thermal noise and losses)

Proposal: use instead squeezed states ⟶ Amplification of entanglement



Entanglement between Hawking pairs emitted by the white hole:



We argue that this mechanisms is a sharp tool to:

- **Increase the observability of the Hawking effect**
- **Be able to confirm the radiation observed has quantum origin**
- **We provide a concrete protocol to materialize these ideas in the laboratory (including the effects of thermal noise and detector inefficiencies)**

  See D. Kranas' talk.

Conclusions

(1) The Gaussian formalism for continuous-variable systems and symplectic circuits: power tools to study Hawking-like effects

(1) The Gaussian formalism for continuous-variable systems and symplectic circuits: power tools to study Hawking-like effects

(2) Hawking process = two-mode squeezer

(1) The Gaussian formalism for continuous-variable systems and symplectic circuits: power tools to study Hawking-like effects

(2) Hawking process = two-mode squeezer

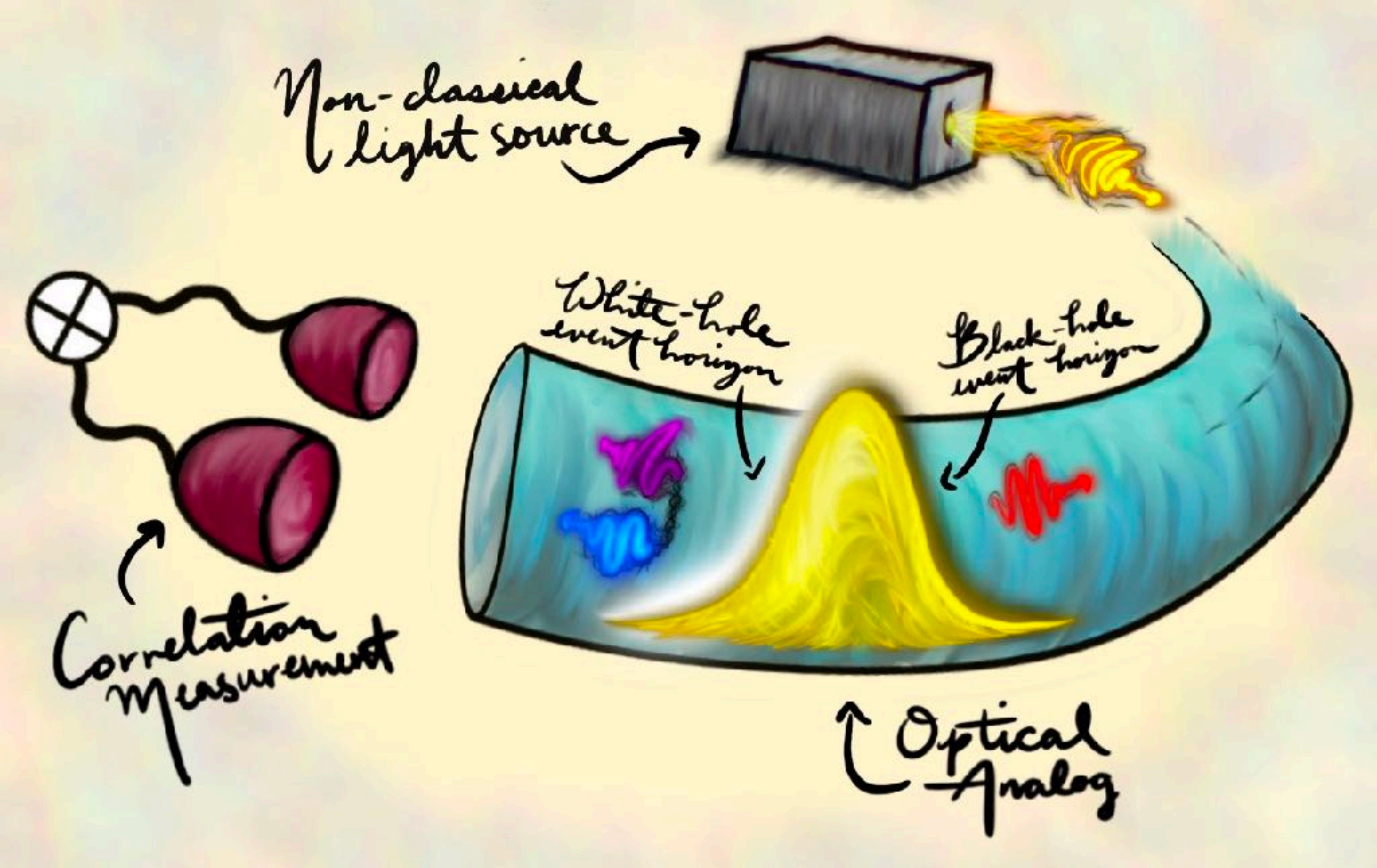
(3) We have applied the tools to astrophysical BH's as well as to optical BH-WH pairs

(1) The Gaussian formalism for continuous-variable systems and symplectic circuits: power tools to study Hawking-like effects

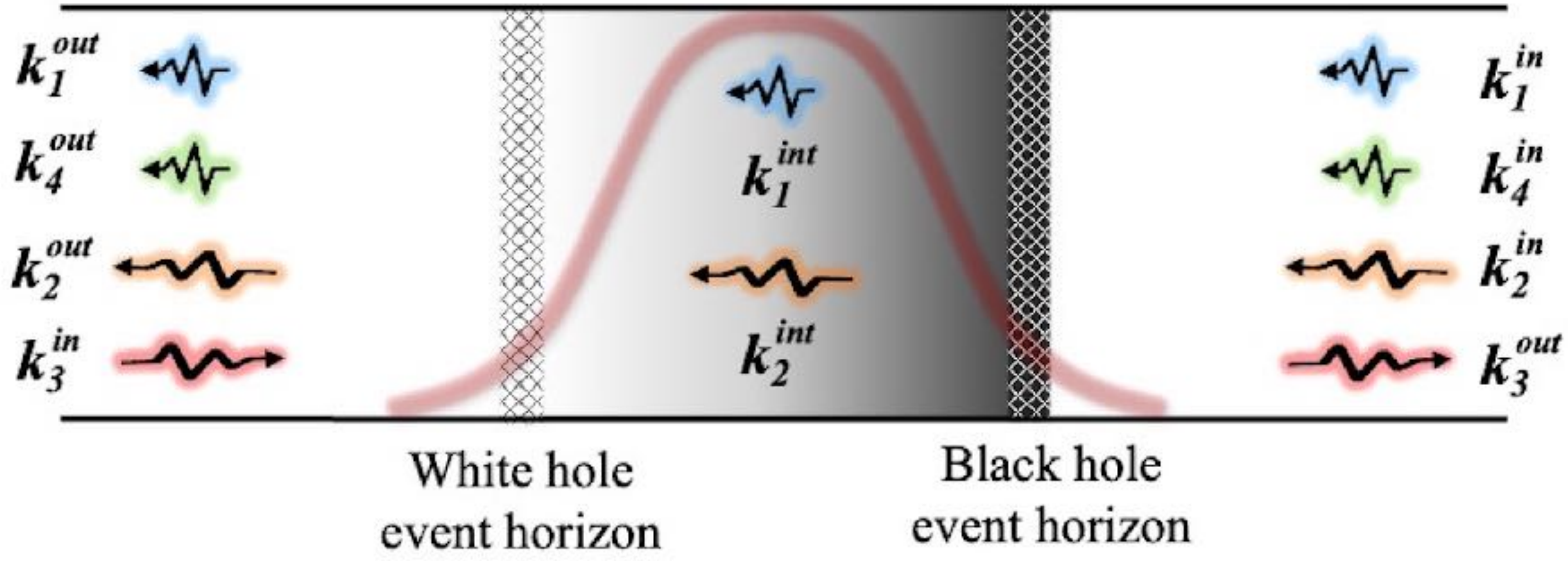
(2) Hawking process = two-mode squeezer

(3) We have applied the tools to astrophysical BH's as well as to optical BH-WH pairs

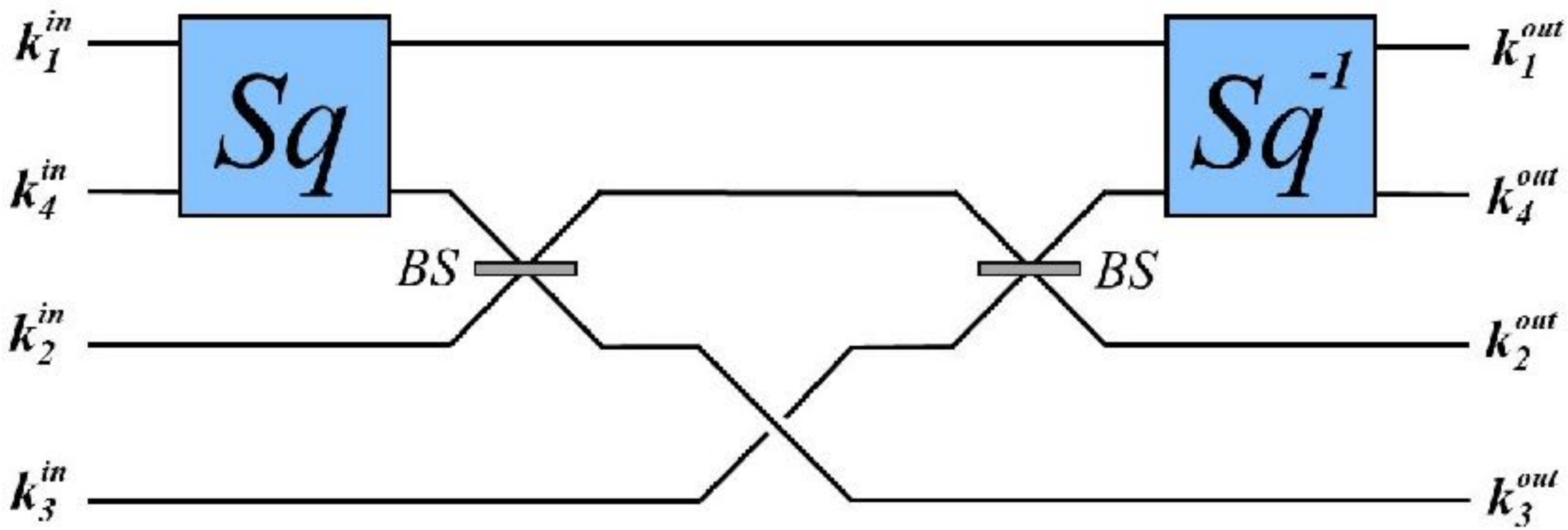
(4) Stimulated or induced process: interesting strategy to increase the observability of quantum aspects of Hawking radiation



Credits: Anthony Brady



k_1^{out}
k_4^{out}
k_2^{out}
k_3^{in}
k_1^{int}
k_2^{int}
k_1^{in}
k_4^{in}
k_2^{in}
k_3^{out}
White hole event horizon
Black hole event horizon



From this circuit $\longrightarrow$ S_{WB} **written in terms of** r_H, ϕ, θ

Solutions of the dispersion relation

Fixed frequency w