Towards ``Matter matters'' in spin foams

in collaboration with Masooma Ali [2206.04076]

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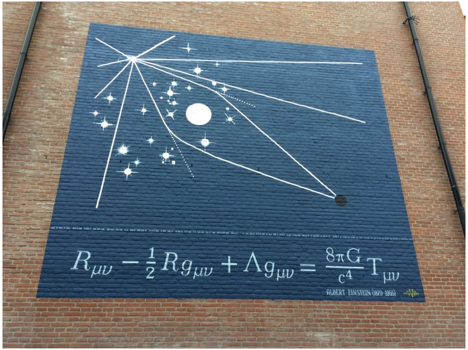
Matter matters in quantum gravity



Lessons from **Asymptotic Safety**: quantum gravity **restricts matter content** [Donà, Eichhorn, Percacci '14]

Consistency check: recover quantum field theory on fixed (background) space-time

Matter in spin foams

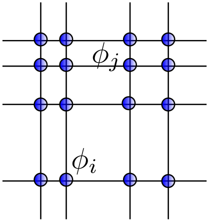
- How to **incorporate matter** in spin foam quantum gravity?
  - Matter on top of spin foam [Oriti, Pfeiffer '03, Mikovic '03, Speziale '07, Bianchi, Han, Rovelli, Wieland, Magliaro, Perini '13]
  - Unification scenarios [Crane '00, Smolin '09]
  - Massless scalar field [Lewandowski, Sahlmann '15, Kisielowski, Lewandowski '18]
  - LQG path integral [Han, Liu '19] + fermions [Zhang, Liu, Han '22]
- **Computational challenges** for pure spin foams:
  - **Numerical algorithm** for EPRL/FK model [Dona, Fanizza, Sarno, Speziale '19, Dona, Gozzini, Sarno '20]
  - **Effective spin foams** [Asanta, Dittrich, Haggard PRL '20, Asante, Dittrich, Padua-Argüelles '21]
  - MCMC on **Lefshetz thimbles** [Han, Huang, Liu, Qu, Wan '20]
  - (Idea of) **Hybrid algorithm** [Asante, Simão, S.St. '22] $\longrightarrow$ **José Simão's talk on Friday**

What is the **dynamics** of the coupled system?

First test: **Massive scalar field** on a **restricted 4D spin foam**

Scalar field coupled to cuboid spin foams

- **"Minimal coupling" ansatz**
  - Define scalar lattice field theory for each (semi-classical) spin foam state
  - Leave spin foam amplitude unchanged
  - Summing over spin foam states $\to$ **superposition of scalar LTFs**
- **Assumption**: Scalar field action diagonalized by spin foam state

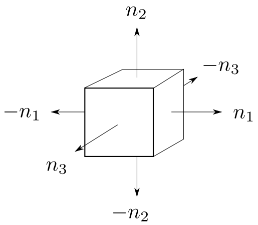


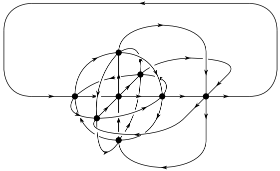
- Restricted **cuboid spin foams** [Bahr, S.St. '15]
  - Superposition of hypercuboidal lattices
- **Discrete exterior calculus** to define scalar field action
- **Markov Chain Monte Carlo** algorithm to compute observables

Massive, free scalar field on a **superposition of hypercubodial lattices** weighted by **spin foam amplitudes**.

Cuboid spin foams [Bahr, S.St. '15]

- **Quantum cuboids**: 4D Riemannian EPRL model [Engle, Pereira, Rovelli, Livine '08] on hypercubic 2-complex [Lewandowski, Kaminski, Kisielowski '09']





- **Restrict to coherent** $\mathrm{SU}(2)$-intertwiner [Livine-Speziale '07]
  - Peaked on cuboid shape
- **Semi-classical formula**:

$$\hat{\mathcal{A}}_v(j_1, j_2, j_3, j_4, j_5, j_6) \sim j_i^{2\alpha} \left(\frac{1}{\sqrt{D}} + \frac{1}{\sqrt{D^*}} \right)^2$$

- **Face amplitude** $\mathcal{A}_f \sim (2j+1)^{2\alpha}$
- Spins $\{j_i\}_{i=1,2,\ldots,6} \;\rightarrow\;$ **edge lengths** $\{l_k\}_{k=1,2,3,4}$

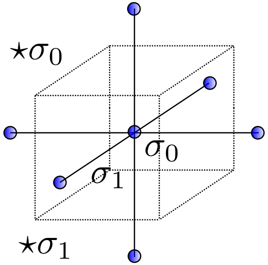
Single parameter α:

large $\alpha \rightarrow$ more weight on large lengths

Scalar field on an irregular lattice

- Scalar field action in terms of **forms**: **Discrete exterior calculus** [Desbrun, Hirani, Leok, Marsden '05, Arnold, Falk, Winther '09, McDonal, Miller '10, Sorkin '75, Calcagni, Oriti, Thürigen '12]

$$S = \frac{1}{2}\int d\phi \wedge *d\phi + \frac{M_0^2}{2}\int \phi \wedge *\phi \;\rightarrow\; S = \frac{1}{2}\sum_{\sigma_1}\langle d\phi | d\phi\rangle + \frac{M_0^2}{2}\sum_{\sigma_0}\langle \phi|\phi\rangle \,.$$



Scalar field action: $S = \frac{1}{2}\phi(\sigma_0^i)\, K_{ij}\, \phi(\sigma_0^j)$ [Hamber, Williams '93]:

$$K_{ij} = \begin{cases} \sum_{\sigma_1 \supset \sigma_0^i} \frac{V_{\star\sigma_1}}{V_{\sigma_1}} + M_0^2\, V_{\star\sigma_0} & i = j \\ -\sum_{\sigma_1 \supset \sigma_0^i} \frac{V_{\star\sigma_1}}{V_{\sigma_1}} & i \neq j \end{cases}$$

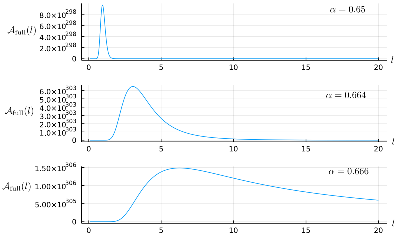
- $V_{\star\sigma_0}$: 4-volume dual to vertex σ_0
- $V_{\star\sigma_1}$: 3-volume dual to edge σ_1.
- V_{σ_1}: length of edge σ_1.

Analytical study: regular lattices

- **Partition function** reads:

$$Z = \int \prod_{\sigma_1} dl_{\sigma_1} \prod_{\sigma_0} d\phi(\sigma_0) \prod_{\sigma_4} \hat{A}_{\sigma_4}(\alpha, \{l_{\sigma_1}\}) \text{ exp} \left(-\frac{1}{2} \phi(\sigma_0^i)\, K_{ij}(\{l_{\sigma_1}\}, M_0)\, \phi(\sigma_0^j) \right)$$

- Restrict edge lengths to **regular lattice**
- Integrate out scalar fields: **probability distribution for lengths**

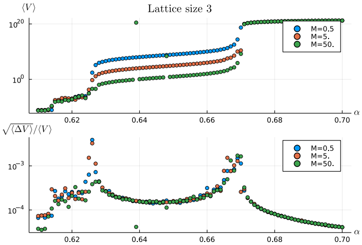


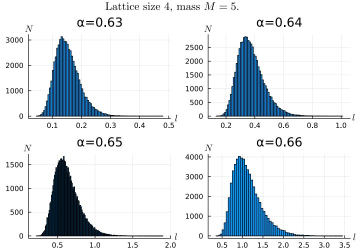
$$\sim \frac{1}{Z} \frac{l^{N(24\alpha - 14)}}{\sqrt{l^{2N} \left(\sum_{i=1}^{N} a_i l^{2i} M^{2i} \right)}}$$

- Peaked on **finite lengths** (for a range of α)

Peak position and width depends on mass M_0 and α.

Geometric observables

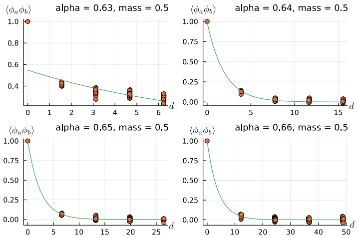


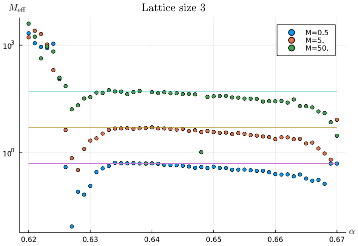


- Total 4-volume $\langle V \rangle$ and (normalized) variance $\frac{\sqrt{\langle \Delta V \rangle}}{\langle V \rangle}$
- Recognize **three distinct regimes**
  - “Plateau”: Finite volume for intermediate α
  - Small / large α: cut-off dependent
- **Normalized variance has two peaks**
  - Localized at end of plateau
- In plateau: lengths **sharply peaked**
- Scalar field **mass** has **strong effect** on volume and average lattice spacing

On average: Regular lattice with **emergent lattice spacing**

Correlations and correlation lengths





- Measure pair **correlations** $\phi_i\phi_j$ and **distance** d_{ij}
  - Plot correlation over distance (forget labels)
  - **Relational observable**
- We can only measure at **probable** distances
  - MCMC algorithm: cannot set distance manually.
- Correlations like **scalar field on regular lattice**
  - Mass changes average spin foam.
- **Correlation length** in plateau regime $\sim M^{-1}$
  - Slight change probably not physical.

Effectively: scalar field on a **fixed, regular lattice**

Scalar field **barely aware of fluctuating spin foam**

Summary & Outlook

- **Massive scalar field coupled to cuboid spin foams**
  - Scalar field on irregular lattice via **discrete exterior calculus**
- **Effectively**: scalar field theory on a **fixed regular lattice**
  - Spin foam peaked on regular lattice
  - **Lattice spacing emergent**, dependent on mass of scalar field
  - Correlation lengths: good agreement with mass
- **Many assumptions / simplifications**
  - **Triangulations**: effective spin foams [Asante, Dittrich, Haggard '20] or hybrid method [Asante, Simão, S.St. '22]
  - **Yang-Mills lattice gauge theory** [Asante, S.St. w.i.p.]
- **Conceptual questions**:
  - Measure correlations at **arbitrary distances**? [Ambjorn, Goerlich, Jurkiewicz, Loll '12]
  - **Renormalization / continuum limit** of matter-spin foam system?

Thank you for your attention!