On the continuum limit of spin foams: graviton dynamics and area metric induced corrections.

Athanasios Kogios

Perimeter Institute

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[Dittrich and Kogios, 2022]

Motivation

- No previous systematic study of Area Regge calculus [Asante, Dittrich, and Haggard, 2018; Rovelli, 1993; Barrett, Rocek, and Williams, 1999].

- Formerly, thought that Area Regge imposes flatness $\rightarrow$ incorrect semi-classical limit of spin foams.

- Previous results were based on a lattice with a singular Area Regge action. Treating them involved tilting the lattice and in the limit of vanishing tilt certain dof are suppressed. [Dittrich, 2021]

- These dof are the ones leading to the area metric.
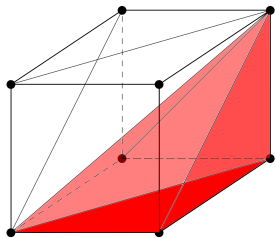
Length vs Area Regge
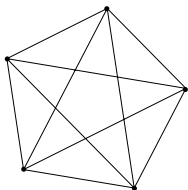
- 10 edges
- 10 areas
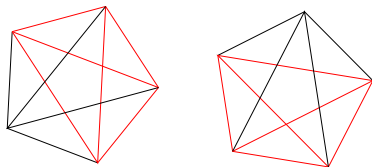
- $10 + 10 - 6 = 14$ edges
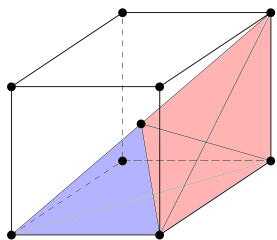- $10 + 10 - 4 = 16$ areas

Many more dof's in Area Regge than Length Regge. What is their geometrical interpretation?

Previous Setup [Dittrich, 2021]

- Every 4-simplex in the hypercube has the same geometry.

- Jacobian from lengths to areas (in given 4-simplex) is degenerate.

- "Hypercubical constraints".

- Suppressed dof (ones leading to area metric).

Our Setup

- Added an inner vertex.

- Jacobian becomes invertible!

Computation

- Computation of Hessian on new inner-vertex lattice.

- Integrating out dof associate to bulk triangles in hypercubes.

- Lattice Fourier Transform: important to determine scaling.

- Identification of (coarse grained) geometrical variables: Area metric variables determine the first two leading orders (next-to-leading order previously suppressed).

Key Mechanism

- Key mechanism: different scaling of various blocks in the Hessian with respect to momenta (equivalently derivatives).

- In particular only length metric degrees of freedom are massless (Hessian block scales with second derivatives).

- Integrate out the χ variables to determine the effective action for the length metric (h).

	h	χ
h	$\mathcal{O}(k^2)$	$\mathcal{O}(k^2)$
χ	$\mathcal{O}(k^2)$	$\mathcal{O}(k^0)$

Table 1: Scaling of the blocks in the Regge action Hessian with the momenta.

Results I

- Leading order: Recover GR!

- Lowest order correction: Weyl squared!

Results II

- Why not Riemann?

- Area metric splits to:

  - trace part (corresponding to massless length dof giving GR)

  - traceless part (giving higher order corrections).

Conclusion

- First systematic results on Area Regge calculus

- As long as Area Regge action appears (possibly after integrating out additional variables), one can hope to obtain GR in continuum limit.

- Continuum limit ensures universal behavior (also in Barrett-Crane model whose semi-classical limit is directly connected to Area Regge action).

- Continuum dynamics can be interpreted in terms of area metrics.

- Enlargement to area metrics leads to the Weyl squared correction for the effective length action.

Bibliography I

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