Radiation in Quantum Gravitational Collapse

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Motivation

- Classical singularities resulting from gravitational collapse are not so *satisfactionary*.
- Quantum fields incorporated into black hole backgrounds induces Hawking radiation. It leads to *the black hole information paradox*, which ~~remains unsolved~~ still is the subject of ongoing debates.
- Non-singular gravitational collapse and related black-to-white hole transition seem to be a more *satisfactionary* description in the above context. How can this be realised?
- *Quantum* treatment (LQC) of the Oppenheimer-Snyder collapse scenario may model the non-singular collapse. What are properties of *radiation* there?

Plan

1. Effective interior geometry from LQC
2. Extracting Exterior Geometry
3. Radiation entropy in the resulting space-time

Effective interior geometry from LQC

Classical Oppenheimer-Snyder Collapse

The collapse of a dust ball. Interior metric: dust FRW solution. Exterior metric: Schwarzschild solution. Two regions are smoothly joined at the surface of the collapsing body.

Quantum treatment for the interior (Parvizi et al., 2021)

- Background geometry coupled to the dust and the massless scalar field $S = \int d^4x\sqrt{-g}\left[\frac{\mathcal{R}}{16\pi G} + \mathcal{L}_T\right] + S_\phi$
- Imposing $ds^2 = -dx_0^2 + a^2(x_0)\, d\mathbf{x}^2$ and fixing $x_0 = T$.
- Quantization: $\mathscr{H}_{\mathrm{kin}} = \mathscr{H}_{\mathrm{grav}} \otimes \mathscr{H}_T \otimes \mathscr{H}_\phi$, gravitational sector: polymer quantization, evolution $i\hbar\partial_T\Psi(v,\phi,T) = \left(\hat{H}_{\mathrm{grav}} + \hat{H}_\phi\right)\Psi(v,\phi,T)$ with
  $\hat{H}_{\mathrm{grav}} = \frac{3\pi G}{8\alpha_o}\sqrt{\hat{v}}\left(\hat{N}^2 - \hat{N}^{-2}\right)^2\sqrt{\hat{v}},\ \hat{N} = \exp\left(i\hat{b}/2\right),\ [\hat{b},\hat{v}] = 2i,$
  $\hat{H}_\phi = \sum_{\mathbf{k}\in\mathcal{L}}\hat{H}_{\mathbf{k}} = \frac{1}{2}\sum_{\mathbf{k}\in\mathcal{L}}\left[\hat{V}^{-1}\otimes\hat{P}_{\mathbf{k}}^2 + k^2\hat{V}^{1/3}\otimes\hat{Q}_{\mathbf{k}}^2\right]$

Quantum treatment for the interior (Parvizi et al., 2021)

Restriction to a single scalar field mode: $\hat{H}_\phi \to \hat{H}_{\mathbf{k}}$.
Solving the evolution equation: Born-Oppenheimer approximation.

Scale factor: $a(T) \approx \langle \hat{V} \rangle^{1/3}$ Finally I write the line element for the interior

$$\mathrm{d}s_-^2 = -\mathrm{d}T^2 + a^2(T)\mathrm{d}r^2 + r^2 a^2(T)\mathrm{d}\Omega^2 \tag{1}$$

Scalar field neglected: $a(T) = A(1 + BT^2)^{1/3}$, where $A > 0$, $B > 0$, the bounce at $T = 0$.

Extracting Exterior Geometry

Ingoing or outgoing Vaidya in the exterior region

Exterior region is described by $x^\alpha = (v, X, \theta, \phi)$

$$\mathrm{d}s_+^2 = -F(X)\mathrm{d}v^2 + 2\mathrm{d}v\mathrm{d}X + X^2\mathrm{d}\Omega^2 \tag{2}$$

or $x^\alpha = (u, X, \theta, \phi)$

$$\mathrm{d}s_+^2 = -F(X)\mathrm{d}u^2 - 2\mathrm{d}u\mathrm{d}X + X^2\mathrm{d}\Omega^2 \tag{3}$$

with yet-to-be-extracted $F(X)$. **Assumptions made:** $F(v, X) = F(X)$, $F(u, X) = F(X)$

Exterior and interior geometries need to be matched. Boundary Σ: the surface of the dust ball parametrized by $(T, r = r_b = \text{const.})$ and *from outside* by (v = V(T), X = R(T)). Let $y^a = (T, \theta, \phi)$ be a coord. system on Σ. Then

$$\mathrm{d}s_-^2\Big|_\Sigma := h^-_{ab}\mathrm{d}y^a\mathrm{d}y^b = -\mathrm{d}T^2 + r^2 a(T)^2\mathrm{d}\Omega^2 \tag{4}$$

and

$$\mathrm{d}s_+^2\Big|_\Sigma := h^+_{ab}\mathrm{d}y^a\mathrm{d}y^b = -(F\dot{V}^2 - 2\dot{V}\dot{R})\mathrm{d}v^2 + R(T)^2\mathrm{d}\Omega^2 \tag{5}$$

Four-velocity of an observer comoving with the surface: $l^\alpha = \partial x^\alpha / \partial T$

Normal n^α to Σ: $n^\alpha n_\alpha = 1$ and $n^\alpha l_\alpha = 0$, chosen so that it points outwards

Conditions for a smooth joining interior and exterior regions at Σ

- continuity of induced metrics: $h^-_{ab} = h^+_{ab}$
- continuity of extrinsic curvatures $K^-_{ab} = K^+_{ab}$, where $K_{ab} := n_{\alpha;\beta} e^\alpha_a e^\beta_b = n_{\alpha;\beta} \frac{\partial x^\alpha}{\partial y^a} \frac{\partial x^\beta}{\partial y^b}$

Then

$$h^-_{TT} = h^+_{TT} \longrightarrow (F\dot{V}^2 - 2\dot{V}\dot{R}) = 1 \tag{6}$$

$$h^-_{\theta\theta} = h^+_{\theta\theta} \longrightarrow X\Big|_\Sigma = R(T) = r_b a(T) \tag{7}$$

$$K^-_{TT} = K^+_{TT} \longrightarrow 0 = n_{\alpha;\beta} l^\alpha l^\beta = -n_\alpha l^\alpha_{;\beta} l^\beta \tag{8}$$

$$K^-_{\theta\theta} = K^+_{\theta\theta} \longrightarrow r_b a(T) = X\Big|_\Sigma F n_X + X\Big|_\Sigma n_u \tag{9}$$

Combining junction conditions

(6), (7), (9) and additionally $F - 2\dot{R}/\dot{V} > 0$, $F < 1$ gives

$$F(T) = 1 - \dot{R}^2 \tag{10}$$

and also

$$\dot{V} = (\dot{R} + \sqrt{F + \dot{R}^2})/F \tag{11}$$

Given the form of F, one obtains $l^{\alpha}_{;\beta} l^{\beta} = 0$. **Junction conditions are satisfied.**

F(X)

The scale factor (neglecting the scalar field, the bounce at $T = 0$):
$a(T) = A(1 + BT^2)^{(1/3)}$, where $A > 0$, $B > 0$.
From the relation $X\Big|_{\Sigma} = r_b a(T)$ it is possible to extract $T(X)$. Hence

$$F(X) = 1 - \dot{R}(T(X))^2 = 1 - \frac{c}{X} + \frac{d}{X^4} \tag{12}$$

where $d > c > 0$.

The same procedure is implemented for the outgoing Vaidya line element.
Solving for (ingoing) V and (outgoing) U determines the exterior geometry.

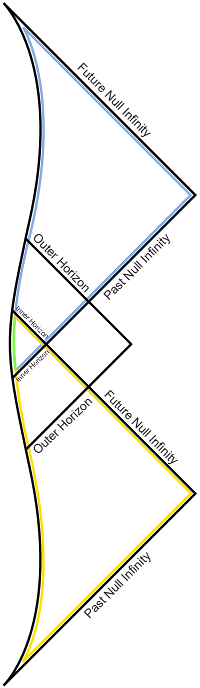
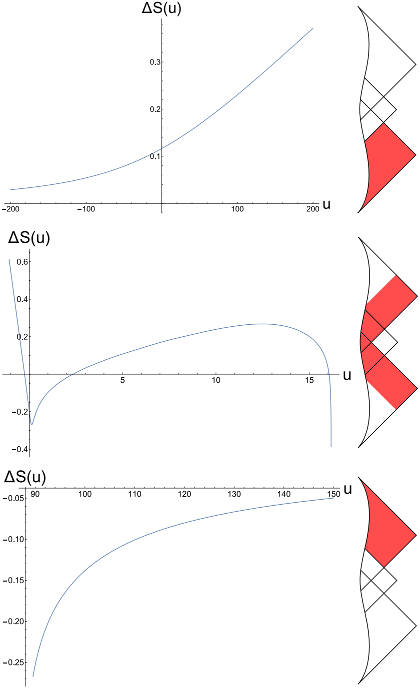


Figure: Space-time Diagram: region covered by ingoing Vaidya line element, region covered by outgoing Vaidya line element, region covered by both and unknown region.



Radiation entropy also called *Page Curve* or just *logarithmic redshift of outgoing rays* (Bianchi et al., 2014)

$$\Delta S_{\rm rad}(u) = -\frac{1}{12} \log w'(u) \qquad (13)$$

Figure: Radiation entropy. Scalar field is neglected. Mass of the dust ball $\approx 13.89 m_{Pl}$. The bounce at $T = 0$.

Conclusions

- Given the assumed propierties of the exterior metric, it was possible to extract the exterior geometry of the collapsing matter which includes quantum geometry modifications.
- *The black hole information paradox* still exists in this model. This is manifestly visible from the time dynamics of the *radiation entropy*.
- This may be due to radical simplifications made within the model. For example imposing at the start for Vaidya line element $F(v, X) = F(X)$.

Thank you for your attention!