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Hawking radiation from a sandwich evaporating Black Hole

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- Hawking radiation (HR) is a result of **QFT in CST**
  $\rightarrow$ Dynamical space-time forming a BH emits radiation that thermalizes at late times.
- *[Hawking'75]* used **Bogoliubov transformations** to obtain it.
- **Back-reaction** $\rightarrow$ **evaporation** (open problem, including possible paradoxes) $\rightarrow$ Quantum Gravity.
- Early stratery (ad-hoc procedure to solve semiclassical eq.):

  guess EBH metric $\rightarrow$ compute HR $\rightarrow$ check if $G_{ab} = 8\pi \langle T_{ab} \rangle$.

  Several **EBH toy models** were proposed, from *[Hiscock'81]* to *regular BH [Hayward'05,Bianchi et al '14, Frolov & Zilnikov '17]*

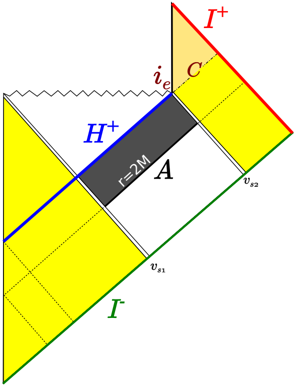
  $\rightarrow$ HR is computed by regularization of $\langle T_{ab} \rangle$.

Hawking result for evaporating BH

Sandwich Vaydia metric

$$ds^2 = -\left(1 - \frac{2M(v)}{r}\right) dv^2 + 2dvdr + r^2 d\Omega^2, \quad M(v) = \begin{cases} 0 & , v < v_{s1}, v > v_{s2} \\ M & , v_{s2} > v > v_{s1} \end{cases}$$

- Two incoming null-thin shells of energy M (at $v = v_{s1}$) and $-M$ (at $v = v_{s2}$).
- H^+: event horizon, it ends at i_e.
- ***A***: apparent horizon (outermost trapped surface)
- C: Cauchy horizon growing from i_e.



The **exterior of the** *BH* is **not globally hyperbolic** due to C.
$\rightarrow$ invalidates Bogoliubov transformations to compute HR ¿or does it?

Hawking result for evaporating BH

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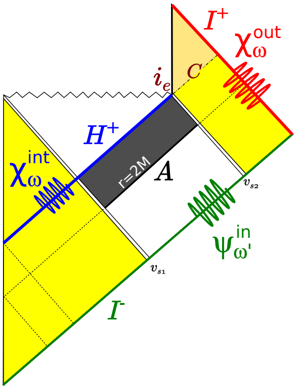
- Consider *s-waves* of K-G equation ($\Box\phi = 0$) ouside H^+:

  $\rightarrow \chi_\omega^{out}$: radiative modes at I^+, vanishing at H^+ and i_e.
  $\rightarrow \chi_\omega^{int}$: complement of *out modes*.
  $\rightarrow \psi_{\omega'}^{in}$: radiative modes at I^-.



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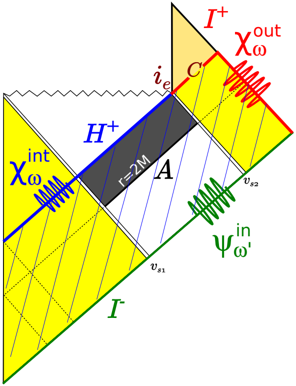
- Positive frequency modes $\rightarrow$ inequivalent cuantizations in the dashed region (correspond to vacua $|0\rangle_{in}$ and $|0\rangle_{out}$)
- In *[RE'22]* s-waves HR is computed in the **near horizon approx.** $\partial_u \partial_v \phi \sim 0$

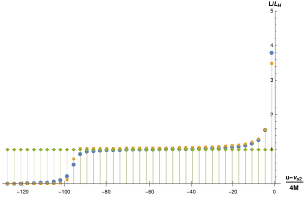


- Assuming the field in the state $|0\rangle_{in}$ (Heisenberg picture)

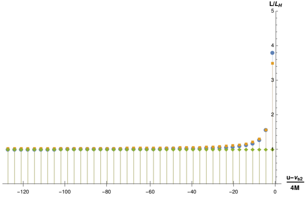
$$\rho_{\omega_j\omega_k} = \langle\omega_j|\, Tr_{int}\left(|0\rangle\,\langle 0|_{in}\right)|\omega_k\rangle = \int_0^{+\infty} d\omega' \beta_{\omega_j,\omega'}\left(\beta_{\omega_k,\omega'}\right)^*$$

with $\beta_{\omega_j,\omega'} = -\left\langle \chi^{out}_{\omega_j}, (\psi^{in}_{\omega'})^* \right\rangle_{K-G}$ (β Bogoliubov coefficient)

- Luminosity:
  $\rightarrow$ divergence at evaporation (*thunderbolt [Hawking, Stewart'92]*)
  $\rightarrow$ numerical agreement with other methods (regularization of T_{ab} using conformal anomaly).
- A *late time* approximation is identified ($v_{s2} - v_{s1} >> 4M$).



Full profile

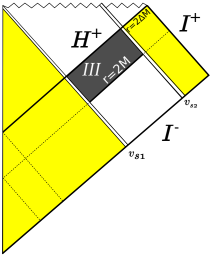


Late-time approximation

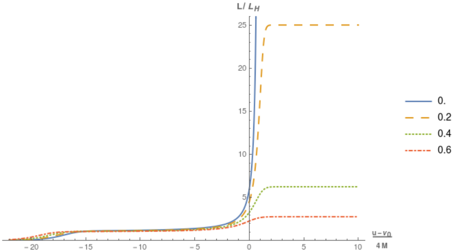
Results

Reason 1 to compute Bogoliubov coefficients

- Bogoliubov coefficients contain **complete information** about the radiation (within restricting hypothesis).
  $\rightarrow$ Thermal and non-thermal contributions are identified.
  $\rightarrow$ an **effective temperature** is computed. It drives the divergence at evaporation interpolating between Hawking temperature of the BH and Hawking temperature of a zero mass BH remnant (comparison with remnant scenario when $\Delta M \rightarrow 0$).



Remnant of mass ΔM



Profile for different ΔM

Results

Reason 2 to compute Bogoliubov coefficients

Because quantum gravity:

- The reduced phase space of spherically symmetric gravity + thin shell (midi-superspace) has been described in ADM variables (*[Louko et al.'92]* and in Ashtekar variables *[Campiglia et al'16]*)
- Mass of the BH (M) and positions of the shell (v_s) are found to be conjugated Dirac observables

$$\{M, v_s\} = 1$$

- In *[RE et al.'17]* M and v_s are promoted to operators with the algebra $\left[\hat{M}, \hat{v}_s\right] = \hbar \hat{I}$. $\rightarrow$ Bogoliubov coeff. are promoted to operators $\hat{\beta}_{\omega_j,\omega_k}(\hat{M}, \hat{v}_s)$.
- Horizon fluctuations introduce important corrections to Hawking result in *scrambling time* scales $[M \ln(M/M_P)]$
- Same procedure applies for two shells. We expect a similar effect could modify the thunderbolt singularity (similar to regular BHs) $\rightarrow$ work in progress.

The End

Thanks!

Thermal spectrum 1

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$$\rho_{\omega_j \omega_k}(n) = \frac{1}{(4\pi)^2} \frac{\epsilon}{\sqrt{\omega_j \omega_k}} \iint_{-\infty}^{\infty} dt d\bar{t} A(u_n, t, \bar{t}) B(\omega_j, \omega_k, t, \bar{t}), \quad (1)$$

$$A(u_n, t, \bar{t}) = \mathrm{sinc}\left(\left\{\frac{u_n - v_{s2}}{4M} + [1 + W(z[t])]\right\} 2M\epsilon\right) \mathrm{sinc}\left(\left\{\frac{u_n - v_{s2}}{4M} + \left[1 + W\left(z[\bar{t}]^*\right)\right]\right\} 2M\epsilon\right) sech\left[\frac{t - \bar{t}}{2}\right]^2 \quad (2)$$

$$B(\omega_j, \omega_k, t, \bar{t}) = \exp\left(i4M\omega_j [1 + W(z[t])] - i4M\omega_k \left[1 + W\left(z[\bar{t}]^*\right)\right]\right), \quad (3)$$

$$\rho^H_{\omega_j \omega_k} = \frac{1}{(4\pi)^2} \frac{\epsilon}{\sqrt{\omega_j \omega_k}} \iint_{-\infty}^{\infty} dt d\bar{t} A^H(u_n, t, \bar{t}) B^H(\omega_j, \omega_k, t, \bar{t}), \quad (4)$$

$$A^H(u_n, t, \bar{t}) = \mathrm{sinc}\left(\left[\frac{u_n - v_0}{4M} + t + \frac{\pi}{2} i\right] 2M\epsilon\right) \mathrm{sinc}\left(\left[\frac{u_n - v_0}{4M} + \bar{t} - \frac{\pi}{2} i\right] 2M\epsilon\right) sech\left[\frac{t - \bar{t}}{2}\right]^2, \quad (5)$$

$$B^H(\omega_j, \omega_k, t, \bar{t}) = \exp\left(-2M[\omega_j + \omega_k]\pi + i4M[\omega_j t - \omega_k \bar{t})\right]. \quad (6)$$

Thermal spectrum
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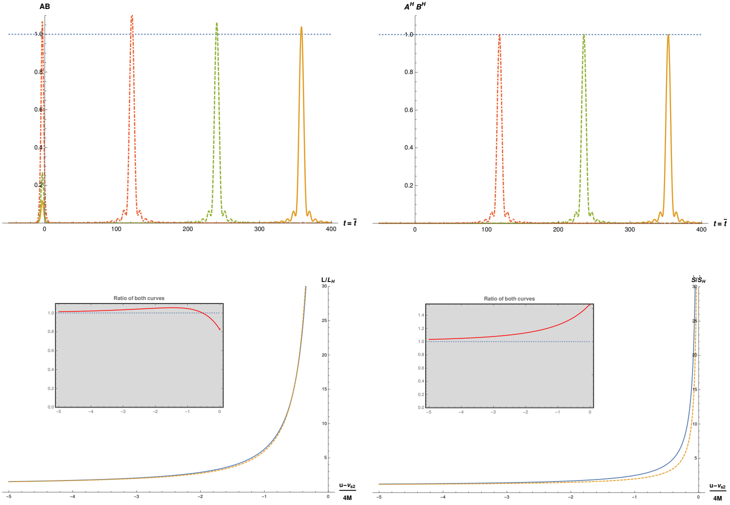


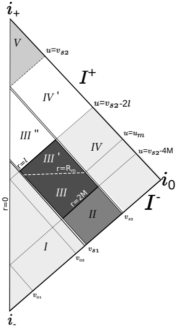
Regular BH models

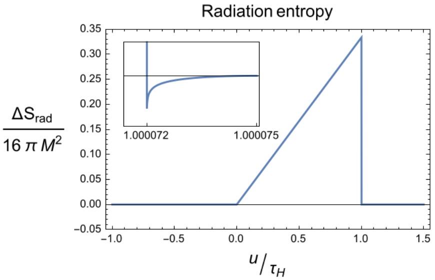
Bianchi et al. 2014

$$ds^2 = -F(r,v)dv^2 + 2dvdr + r^2 d\Omega^2$$

where

$$F(r,v) = \begin{cases} 1, & v < v_{s1} \\ 1 - 2M/r, & \{v_{s2} > v > v_{s1}, r > R_m\} \\ 1 - (r/l)^2, & \{v_{s2} > v > v_{s1}, r \leq R_m\} \\ 1, & v > v_{s2}. \end{cases}$$





Regular BH models

Frolov and Zelnikov 2017

$$ds^2 = -\alpha^2 f(r,p,\alpha)dv^2 + 2\alpha dv dr + r^2 d\Omega^2$$

where

$$f(r,p,\alpha) = \frac{(r-p)(r-1)(r+p/(p+1))}{r^3 + p^2/(p+1)}.$$

$$\alpha = \frac{r^6+1}{r^6+1+p^4}$$