Black Hole as a Bound state of Semi-classical Degrees of Freedom

RIKEN iTHEMS

Yuki Yokokura

To appear in the next week!

previous results:
[Kawai-Matsuo-Yokokura 2013,
Kawai-Yokokura 2015,16,17,20,22]

2022 July 18 @ Loops22

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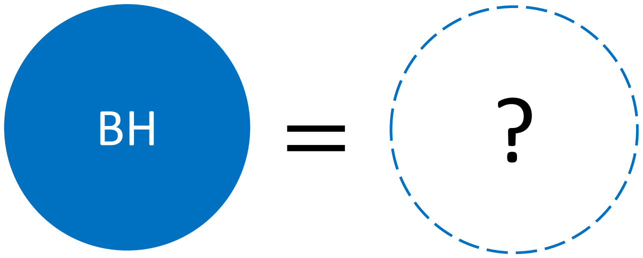
Black hole entropy

- In quantum theory, we don’t know yet what a black hole is.



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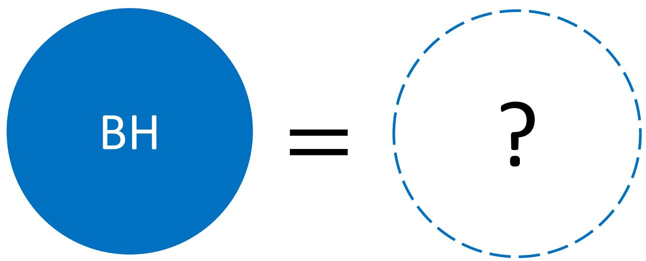

- In quantum theory, the notion of geometry should emerge only under a certain limit.

⇒ Horizon is just an approximated property of black holes.

- The notion of information is covariant and quantum.

⇒A black hole should be characterized by

$$S = \frac{A}{4l_p^2}.$$

BH=bound state of many d.o.f.

- What is the origin of $S = \frac{A}{4l_p^2}$?

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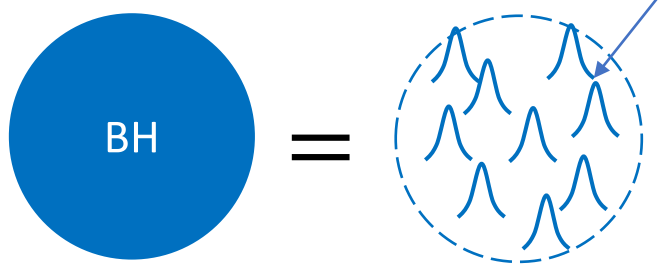
⇒Black hole = gravitational bound state of some d.o.f.

responsible for $S = \frac{A}{4l_p^2}$

Strings and D-branes?
[Strominger-Vafa,...]

discrete spacetime units?
[Ashtekar-Baez-Corichi-Krasnov,...]



semi-classical
dynamical modes?
[Barvinski-Frolov-Zelnikov,...]

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[Dvali-Gomez,...]

+ more approaches...

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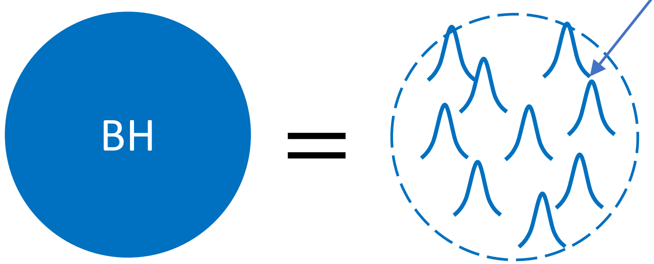

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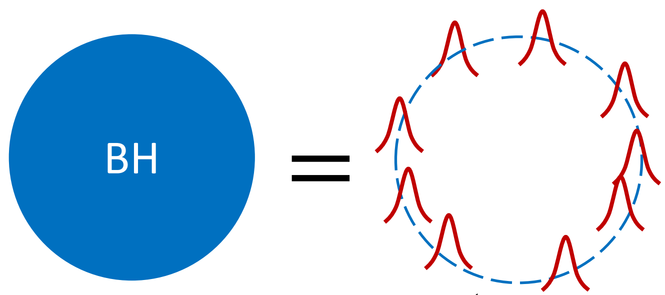

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⇒***Where do the d.o.f. live?***

(i) Around the “horizon”

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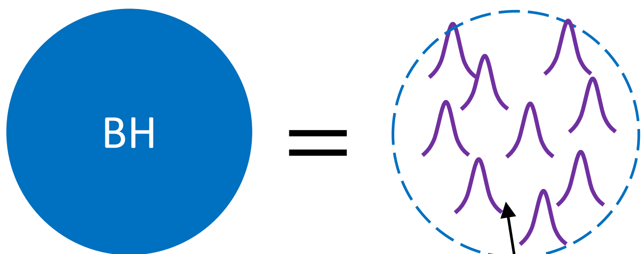

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⇒***Where do the d.o.f. live?***

(ii) Inside somewhere?

⇒We try to consider case (ii) today.

Bound state of semi-classical d.o.f. (1/2)

- Consider a spherical static BH as a bound state of any semi-classical d.o.f. satisfying

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(i) The d.o.f. are distributed inside uniformly in r-direction.

(ii) The acceleration required to stay at r is semi-classically maximum.

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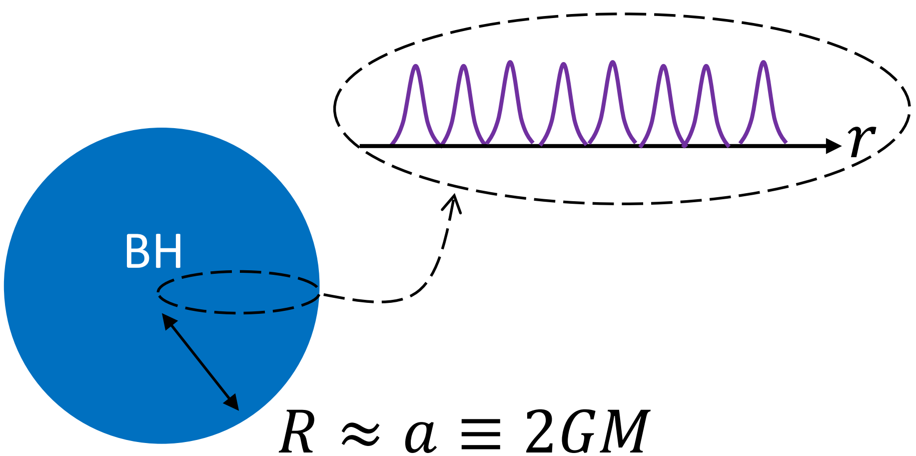
<u>(i) Uniform distribution of information</u>



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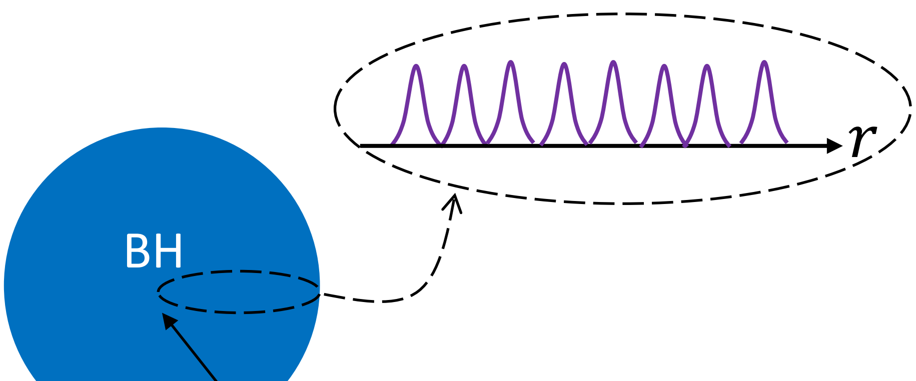

$R \approx a \equiv 2GM$

- Uniform distribution
- $(\Delta M)_{1bit} \sim \frac{\hbar}{r}$ [Bekenstein]
- $S = \int_{\sim l_p}^{\sim a} dr\sqrt{g_{rr}(r)}s(r) \sim \frac{a^2}{l_p^2}$

$$\Rightarrow s(r) \approx const. \sim \frac{\sqrt{N}}{l_p}$$

Bound state of semi-classical d.o.f. (2/2)

(ii) semi-classically maximum acceleration

- BH should have a maximum gravity.
- The minimum resolution of spacetime should be $l_p \equiv \sqrt{\hbar G}$.

(⇒We will explain later how (i) and (ii) hold as a result of dynamics.)

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<u>(ii) semi-classically maximum acceleration</u>

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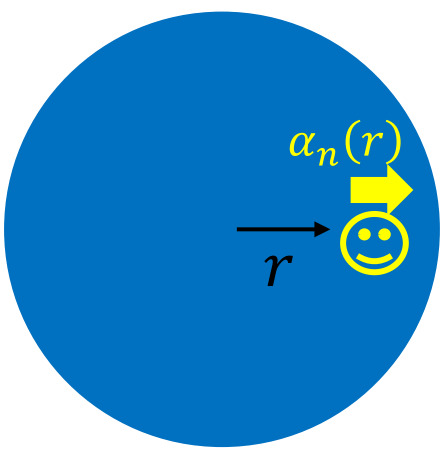
⇒The bound state should have semi-classically maximum acceleration:



(i) uniform cond.

$$\alpha_n(r) \doteq const = O\left(\frac{1}{C\, l_p}\right)$$

$C = O(1) \gg 1$

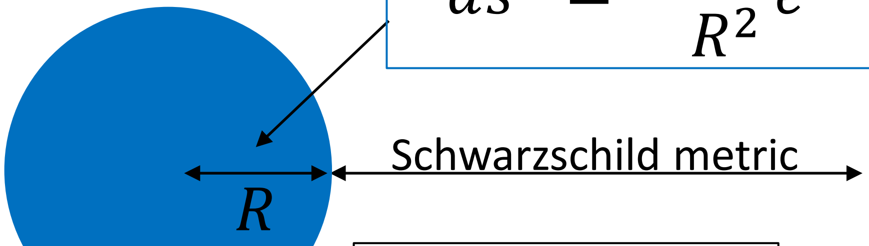
proper acceleration
required to stay at r

(⇒We will explain later how (i) and (ii) hold as a result of dynamics.)

Interior metric

- From (i) (ii), we can derive the interior metric:

$$ds^2 = -\frac{2\sigma}{R^2} e^{-\frac{R^2 - r^2}{2\sigma\eta}} dt^2 + \frac{r^2}{2\sigma} dr^2 + r^2 d\Omega^2$$



$$\sigma \sim N l_p^2 \text{ (with } N \gg 1)$$
$$1 \leq \eta < 2,$$

Size:

$$R = a + \frac{2\sigma}{a} > a$$

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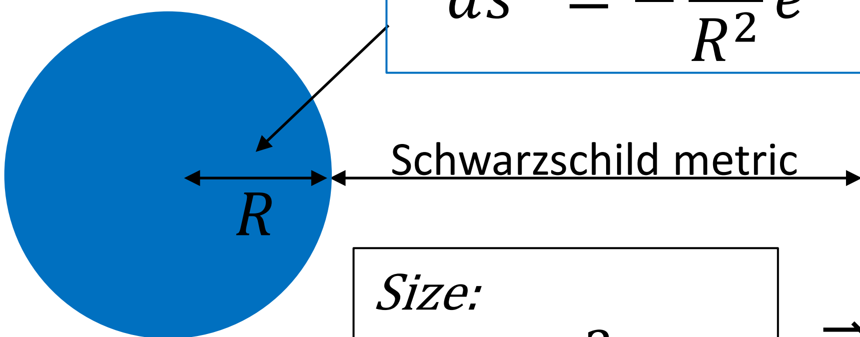

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- No singularity because
  - Leading values of the curvatures for $r \gg l_p$:

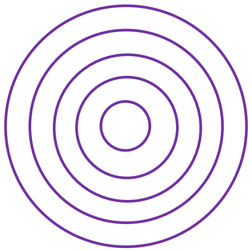
$$R, \sqrt{R_{\mu\nu} R^{\mu\nu}}, \sqrt{R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}} = O\left(\frac{1}{N l_p^2}\right) \ll O\left(\frac{1}{l_p^2}\right) \text{ for } N \gg 1$$

semi-classically maximum

  - $(\Delta M)_{0 \leq r \leq l_p} \sim m_p$ ⇒ the center is represented by a QG state.

Interior structure

- We can show that the interior has a structure like a concentric and continuous stack of AdS_2 (of L) $\times$ S^2(of r).



$$R = -\frac{2}{L^2} + O(r^{-2})$$

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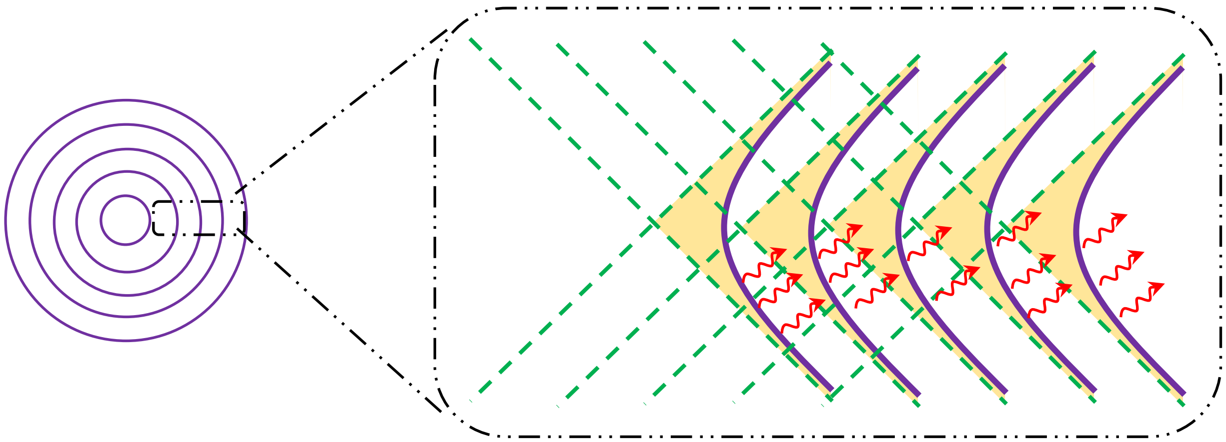

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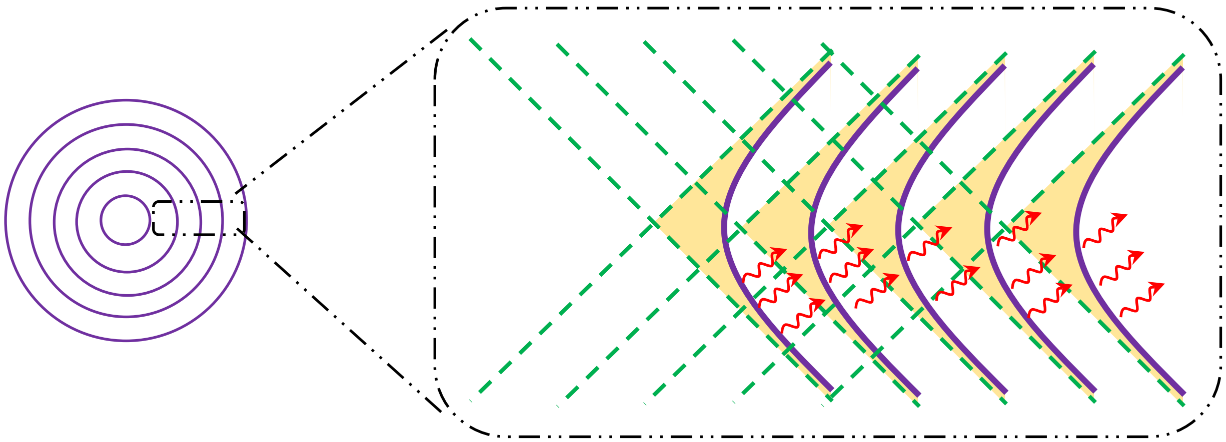

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- Each small region behaves like a subsystem at local temperature

$$T_{loc} = \frac{\hbar}{2\pi L}.$$

Entropy again

- We can use $T_{loc} = \frac{\hbar}{2\pi L}$, thermodynamic relations and $G_{\mu\nu} = 8\pi G \langle \psi | T_{\mu\nu} | \psi \rangle$ to evaluate the entropy density:

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⇒Integrating it over the volume reproduces the area law:

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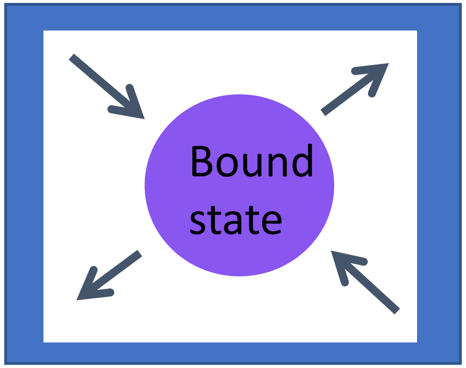
- Also, we can derive Hawking temperature:

$$T_{loc} = \frac{\hbar}{2\pi L} \to T_H = \frac{\hbar}{4\pi a}$$

Consistent with [Gibbons-Hawking]



How to obtain the configuration?

- Q1: Does this metric satisfy $G_{\mu\nu} = 8\pi G\langle\psi|T_{\mu\nu}|\psi\rangle$?

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[Kawai-Yokokura 2020]

⇒Yes. Indeed, (σ, η) exist satisfying it.

  - e.g. For conformal matter, 4D Weyl anomaly fixes

$$\sigma = \frac{8\pi l_p^2 c_W}{3\eta^2} \text{ with } c_W \gg 1.$$

  - Direct evaluation of $\langle\psi|T_{\mu\nu}|\psi\rangle$ can determine η.

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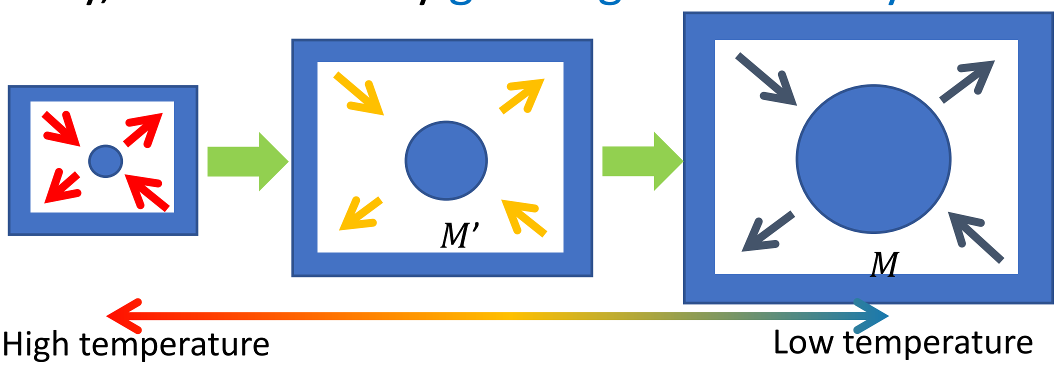
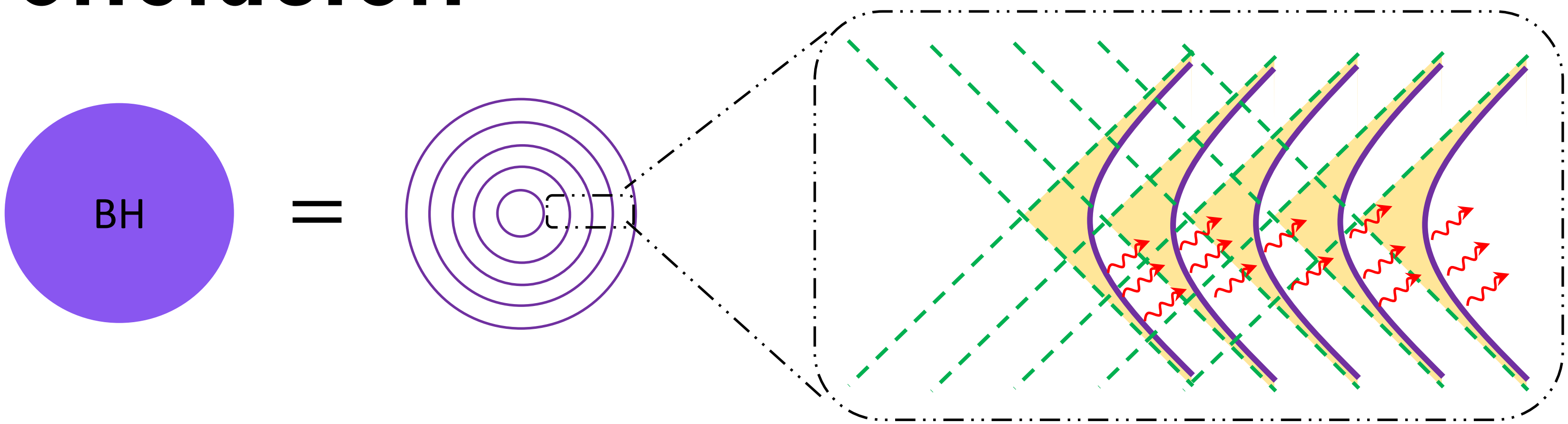
- Q2: How is this uniform configuration formed? [Kawai-Yokokura 2015,22]

⇒As one possibility, it is formed by growing adiabatically in the heat bath.



⇒Most typical BH in various formation processes

Conclusion



- We represent a BH as a bound state of semi-classical d.o.f. with semi-classically maximal gravity.
- The interior has a structure like a continuous stack of $AdS_2 \times S^2$ with $\mathcal{R} \sim \frac{1}{N l_p^2}$.
- The state $|\psi\rangle$ behaves like a thermal state with $T_{loc} = \frac{\hbar}{2\pi L}$.
- The entropy comes from the inside.
- Non-perturbative 4D dynamics of matter fields plays a key role in a highly-curved spacetime.

Thanks!