# Exploring black holes in modified gravity

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ENS LYON

Partially based on
J. BA, D. Langlois, K. Noui [JHEP '16]
J. BA, H. Liu [PRD '18]
J. BA, S. Mukohyama, H. Liu [JCAP '19]
J. BA, H. Liu, H. Motohashi, S. Mukohyama, K. Noui, [JCAP '20]

# Context and Motivations

**A successful theory of gravity: General Relativity**

- GR has passed all observational tests with flying colors: weak field regime (solar system), strong field regime (GRAVITY, EHT), radiative regime (LIGO, VIRGO, LISA)
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- Need to embed GR in a larger space of theories to confront it to future observations
- Construct a parametrization of the possible deviations for the coming observational tests (Brans Dicke theory still viable theory ...)

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**Motivation from cosmology**

- Explaining the recent acceleration of the cosmic expansion - dark energy
- Can we have a single model which account for both early cosmology data and recent measurement of the Hubble rate ?
- As it stands, the standard model of cosmology ($\Lambda$-CDM) seems less and less accurate to do the job !
  $H_0$ tension getting more and more severe ...
- What would be the effect of additional scalar, vector modes ? What is their impact on the cosmological history and compact objects ?

From now on, focus on scalar-tensor theories of gravity

# Context and Motivations

**Strategy to explore alternatives ST theories**

- Find universal criteria to construct the most general scalar-tensor theory : degeneracy criteria
- Find a way to explore their solution space : disformal transformation

DHOST theories and the degeneracy criterion

## Degeneracy criteria: one ring to find them all

- Early attempt to modify GR based on scalar-tensor theories with first order derivative lagrangian : Brans-Dicke (1961), generalization to K-essence ...

- In general, going to higher derivative introduces an Ostrograsky ghost (OG) which spoil the theory: unbounded hamiltonian

- Horndeski gravity provided a first example of a second order Lagrangian without OG ... a unique healthy theory ? [Horndeski '74]
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- Lead to new landscape of healthy theories
  [**BA**, Crisostomi, Koyama, Langlois, Noui, Tasinato '16]
- Applicable beyond scalar-tensor: construction of higher spin modified theories of gravity [Heisenberg '18]

- Example of quadratic degenerate higher order scalar tensor (DHOST) theories lagrangian:

$$S[g,\phi] = \int \mathrm{d}^4 x \sqrt{|g|} \left[ f(\phi, X)\mathcal{R} + \sum_i A_i(\phi, X)\mathcal{L}_i \right]$$

with

$$L_1 = \phi_{\mu\nu}\phi^{\mu\nu} \ , \ \ L_2 = (\Box\phi)^2 \ , \ \ L_3 = \Box\phi\phi^\rho\phi^\sigma\phi_{\rho\sigma} \ , \ \ L_4 = \phi_{\mu\nu}\phi^{\nu\rho}\phi^\mu\phi_\rho \ , \ \ L_5 = (\phi_{\rho\sigma}\phi^\rho\phi^\sigma)^2$$

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- Allowed classes of degenerate theories:

$$\text{Class I: } A_1 + A_2 = 0\ ,\quad f \neq 0$$

$$\text{Class II: } Xf\,(2A_1 + XA - 4 + 4f_X) - 2f^2 - 8X^2 f_X^2 = 0\ ,\quad f \neq 0$$

$$\text{Class III: } f = 0$$

- How can we explore their solution space ?

Disformal transformation as a swiss-knife

- Consider the following transformation: $(g, \phi, X) \to (\tilde{g}, \tilde{\phi}, \tilde{X})$

$$\tilde{g}_{\mu\nu} = A(\phi, X) g_{\mu\nu} + B(\phi, X)\phi_\mu \phi_\nu \ , \qquad \tilde{X} = \frac{X}{A + BX} \ , \qquad \tilde{\phi} = \phi \tag{1}$$

where $X = \phi_\mu \phi^\mu$ and $\phi_\mu = \partial_\mu \phi$. [Bekenstein '92]

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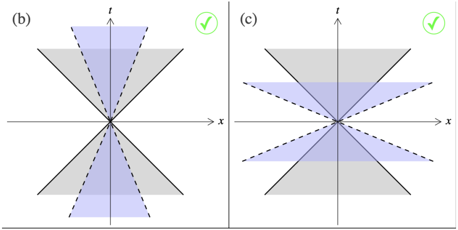
**Main features of disformal transformation**

- It changes the light cone: thigher if $B < 0$, larger if $B > 0$

$$g_{\mu\nu} k^\mu k^\nu = 0 \,, \qquad \to \qquad \tilde{g}_{\mu\nu} k^\mu k^\nu = B \left(\phi_\mu k^\mu\right)^2$$



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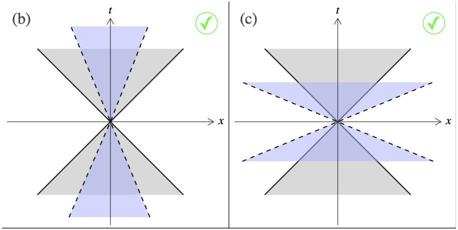


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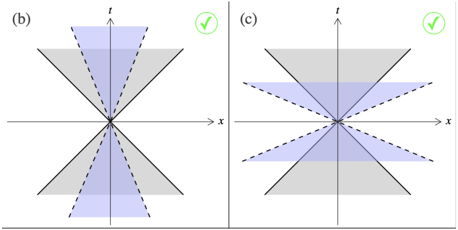


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- In general, change Petrov type of the seed [**BA**, De Felice, Gorji, Mukohyama, Pookilath '21]

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- Each degeneracy class is stable under a general invertible disformal transformation [Zumalacarregui, Garcia-Bellido '14 ][**BA**, Langlois, Noui '16]

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- Provide a simple but powerful way to construct new exact solutions of DHOST theories
- Field redefinition for space-time, but non-trivial if matter couples to $\tilde{g}_{\mu\nu}$
- lead to new observational signatures

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Let see some explicit examples

Disformal spherically symmetric black hole solutions

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- Idead: dress Schwarzschild with a scalar field

$$T_{\mu\nu}^{\text{effective}} = 0 \qquad \Rightarrow \qquad \phi(t, r) := qt + \psi(r) \ , \qquad X := X_0 < 0$$

  with a non-gravitating scalar field : *stealth solution*
  [Ayeon-Beato, Martinez, Zanelli '06] [**BA**, Liu '18, Hayato, Minamitsuji '19 '20]
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- Reduces to Schwarzschild black hole with a rescaled mass : $\tilde{X}_0 = -q^2$
- or give a new black hole with a linear Misner sharp mass: : $\tilde{X}_0 > -q^2$
- Remain Petrov type D [**BA**, Liu, Mukohyama '19]

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where the effective metric $\tilde{G}_{\mu\nu}$ is obtained as:

$$\tilde{G}_{\mu\nu} = F_1(A_i)\tilde{g}_{\mu\nu} + F_2(A_i)\phi_\mu\phi_\nu \tag{4}$$

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- Gravitons and photons do not propagate on the same background in general. Lead to new observational signatures of these theories ... and constraints on the free functions of the lagrangian

Constructing rotating black hole solutions

- Construct a deformed Kerr solution [**BA**, Hayato, Liu, Mukohyama, Noui '20]

$$\mathrm{d}s^2 = \mathrm{d}s^2_{\mathrm{Kerr}} - B_0 m^2 \left( \mathrm{d}t \pm \frac{\sqrt{2Mar(r^2+a^2)}}{\Delta} \mathrm{d}r \right)^2$$

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- The geometry is no longer circular: $\mathrm{d}r\mathrm{d}t$ term cannot be absorbed by a coordinate change
- The geometry is asymptotically flat. At 1.5 PN, rescaled mass and angular momentum

$$\tilde{M} = \frac{M}{1+B_0}\,, \qquad \tilde{a} = a\sqrt{1+B_0}$$

- At 2PN, leading order contributions of the Newtonian quadrupole moment depends on $B_0$:

$$\left(\tilde{M}, \tilde{a}, B_0\right) \qquad \Rightarrow \qquad \text{Violation of the no-hair theorem}$$

- Can we put constraints on $B_0$ using the GRAVITY mission (Orbits of S2 around Sagitarius $A^*$) ? Senstitive only to 1PN order ... so still viable

Conclusion and perspectives

# Conclusion

**DHOST theories**

- Construction relies on the degeneracy criterion to avoid the Ostrogradsky ghost
- Divided into degeneracy classes stable under disformal transformations
- DHOST theories provide the most general scalar tensor theories constructed so far

**Black holes solutions**

- Stealth sector provides a way to dress GR solutions with a non-gravitating scalar mode: provide starting point to explore their solution space !
- Disformal solution-generating method allows to construct non-stealth Schwarzschild and Kerr black hole with many new properties
- Main effects of disformal transformations: modify Petrov type, rescaled mass and spin at 1.5 PN for Kerr, violation of no-hair theorem at 2 PN
- Photons and (axial) gravitons propagate on a different background (with different causal structrure) : new observational signatures

- Radiative solution can be constructed: framework to investigate new memory effects ?
- Regular black holes can be constructed: framework to test their stability ?

DHOST theories provide a framework to construct exotic geometries beyond GR and parametrize possible deviations to be confronted to future observations.

Thank you

# Additional materials

Constructing radiative solutions

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**Motivation**

- Can we construct explicit solutions with non-linear gravitational waves in DHOST ?

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- Can we construct explicit solutions with non-linear gravitational waves in DHOST ?
- Investigate how higher order terms modify the memory effects of GR
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$$\lim_{u,r\to+\infty} \phi_r(u, r_*) \sim \frac{\sqrt{2}C}{Ur^2}\,, \tag{7}$$

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- But subtle sub-leading effects should exist in the memories : modification to the geodesic deviation