Dynamical frame covariance

Philipp Höhn

Okinawa Institute of Science and Technology

OIST

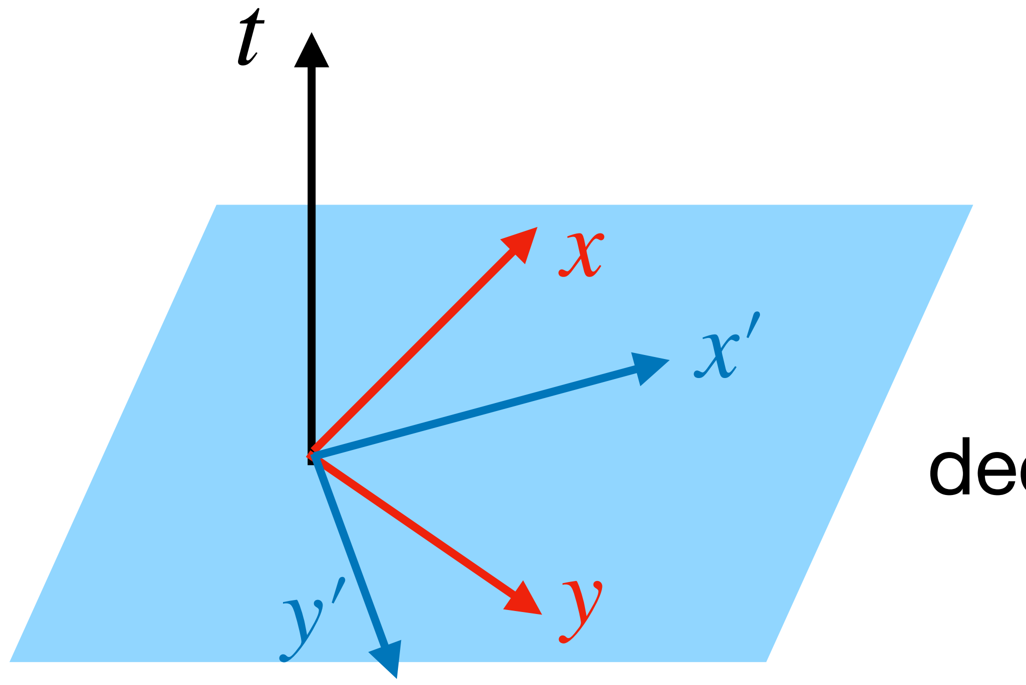
Loops '22 @ Lyon
July 19th, 2022

synthesis of work with: Ahmad, Carrozza, de la Hamette, Eccles, Galley, Goeller, Kirklin, Kotecha, Krumm, Lock, Loveridge, Mele, Müller, Smith, …

Relativities

Galilean

"All the laws of mechanics the same in every inertial frame"

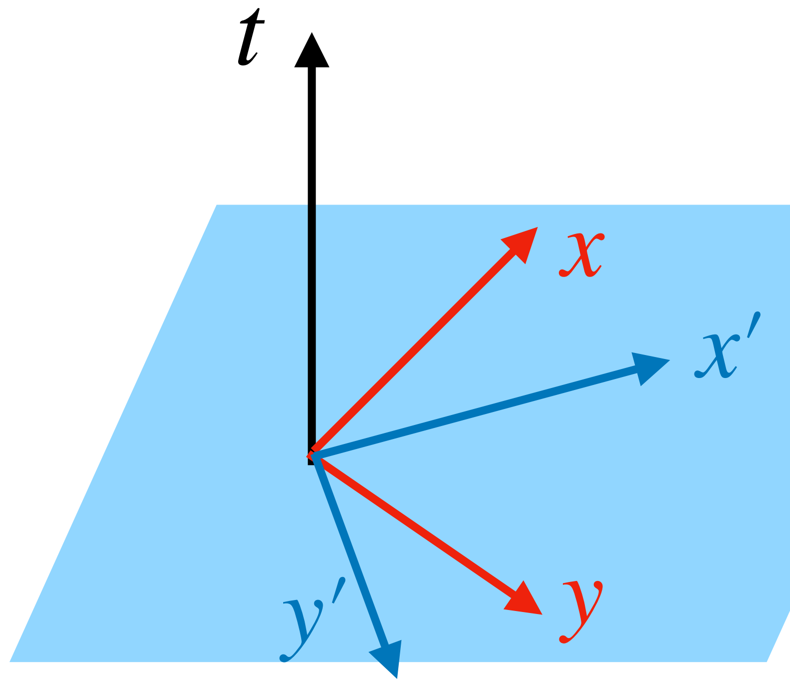


frame-dependent decomposition of space

Relativities

Galilean

"All the laws of mechanics the same in every inertial frame"

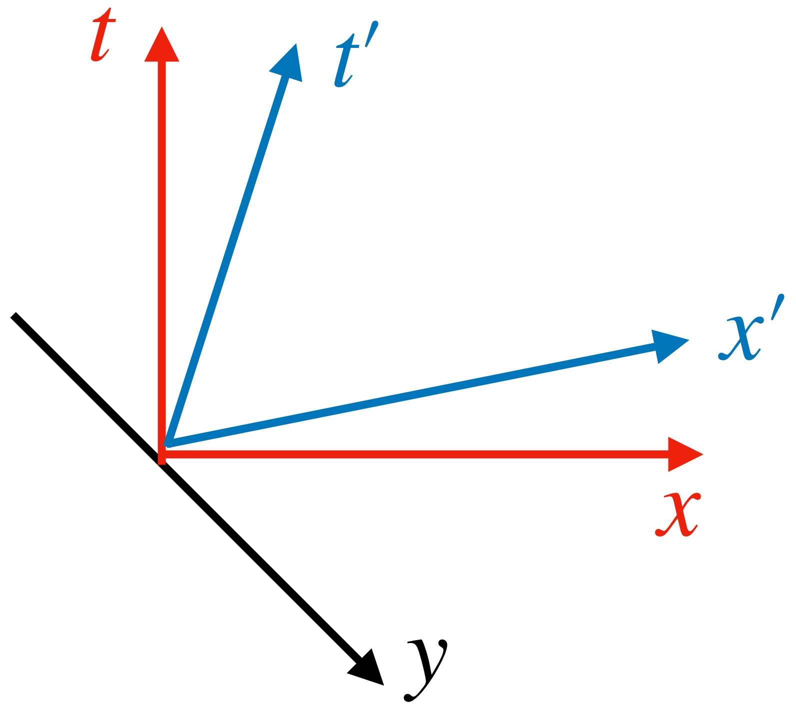


frame-dependent decomposition of space

$$\frac{1}{c} \neq 0$$

Special

"All the laws of physics (exc. Newt. gravity) the same in every inertial frame"

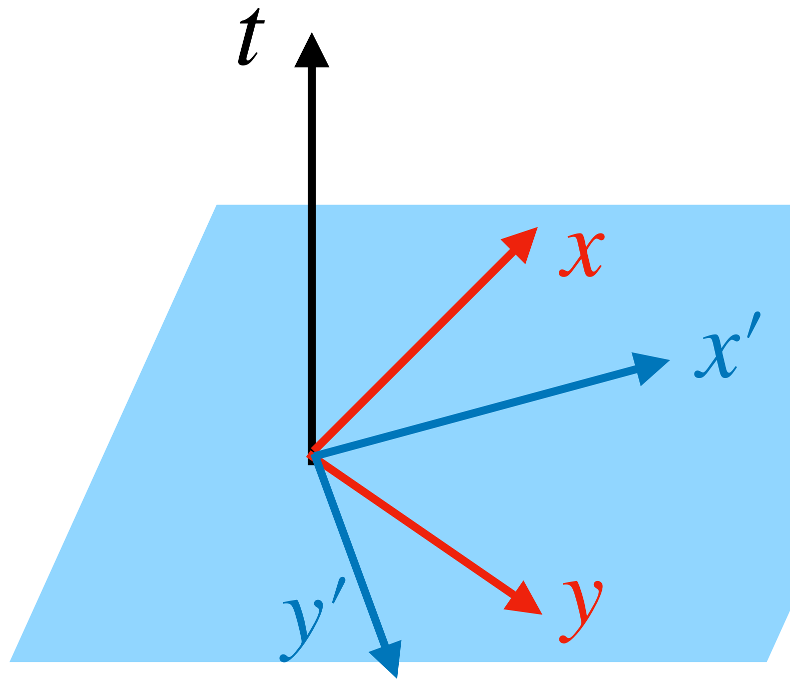


frame-dependent decomposition of spacetime into space and time

Relativities

Galilean

"All the laws of mechanics the same in every inertial frame"

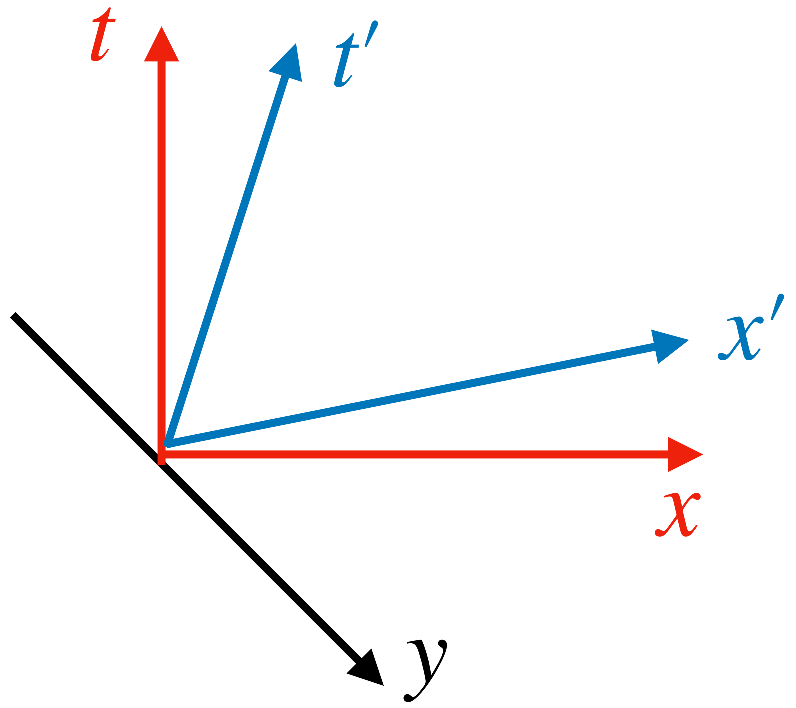


frame-dependent decomposition of space

$$\frac{1}{c} \neq 0$$

Special

"All the laws of physics (exc. Newt. gravity) the same in every inertial frame"

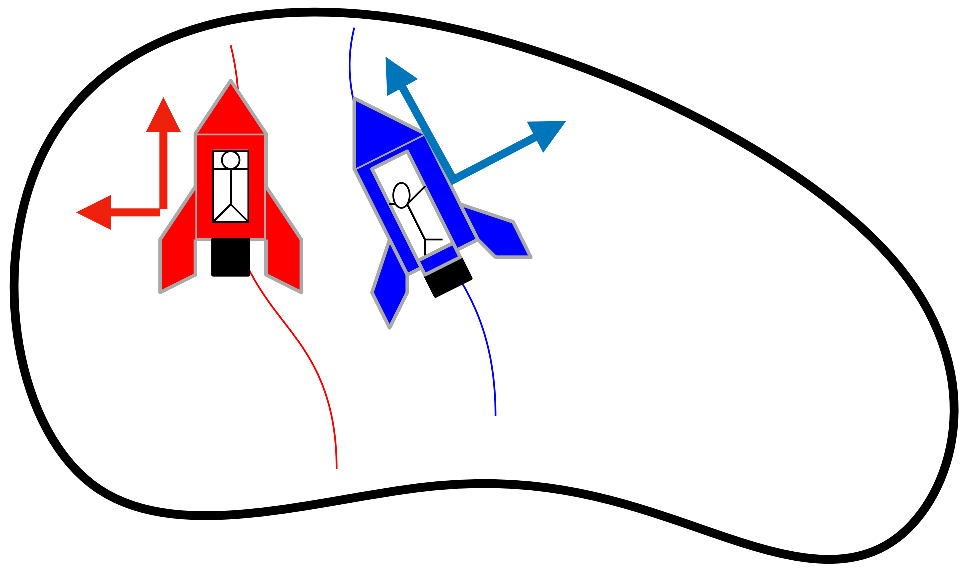


frame-dependent decomposition of spacetime into space and time

$$G \neq 0$$

General

"All the laws of physics the same in every frame"

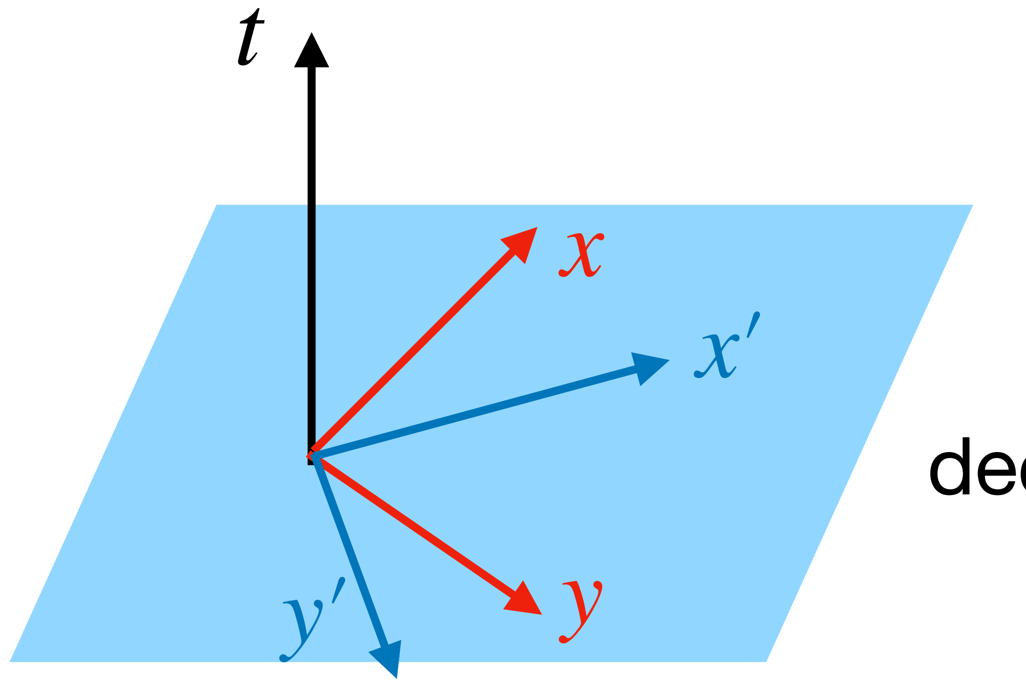


frame-dependent local decomposition of spacetime into space and time

Relativities

Galilean

"All the laws of mechanics the same in every inertial frame"

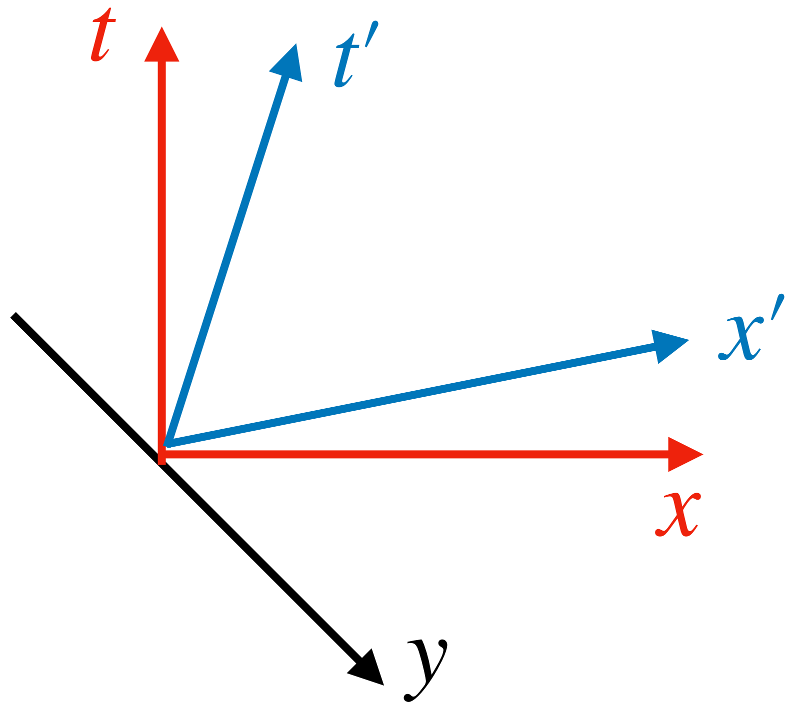


frame-dependent decomposition of space

$$\frac{1}{c} \neq 0$$

Special

"All the laws of physics (exc. Newt. gravity) the same in every inertial frame"



frame-dependent decomposition of spacetime into space and time

$$G \neq 0$$

$$\hbar \neq 0$$

Quantum?

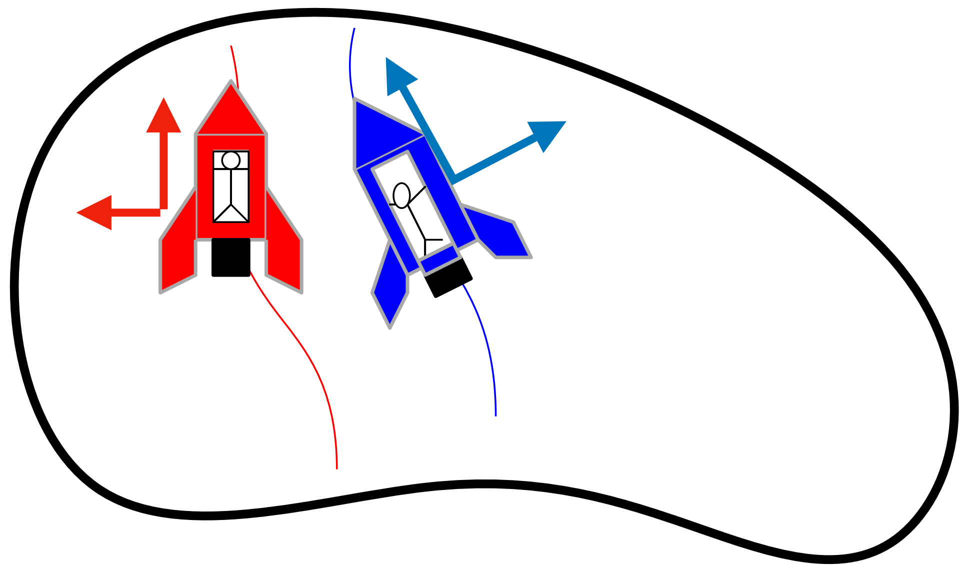
"All the … laws of … the same in every … QRF"

?

QRF-dependent decomposition of ….

General

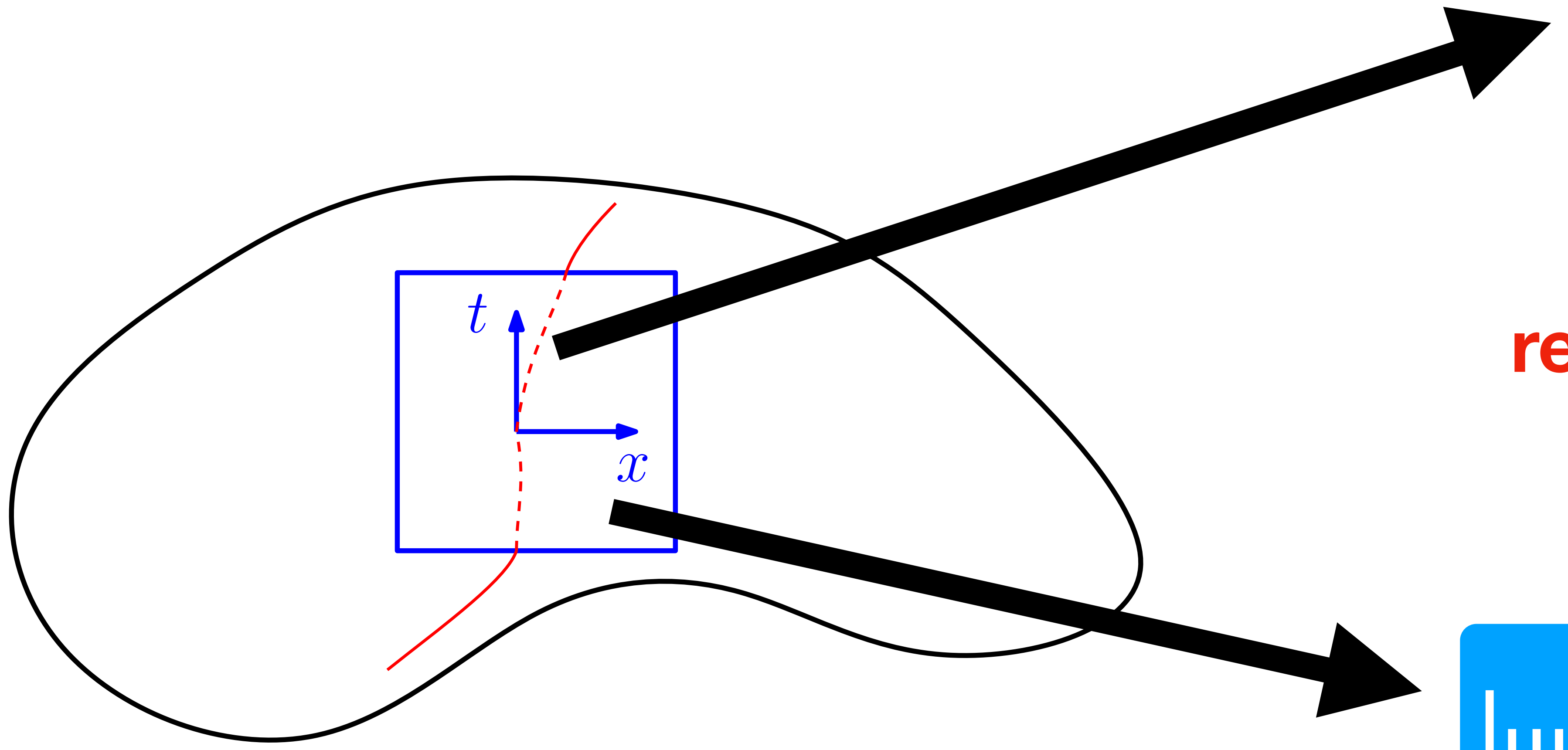
"All the laws of physics the same in every frame"



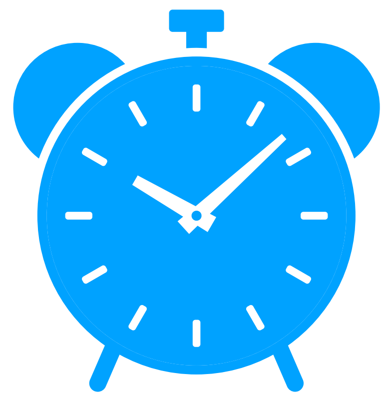
frame-dependent local decomposition of spacetime into space and time

Quantum reference frames

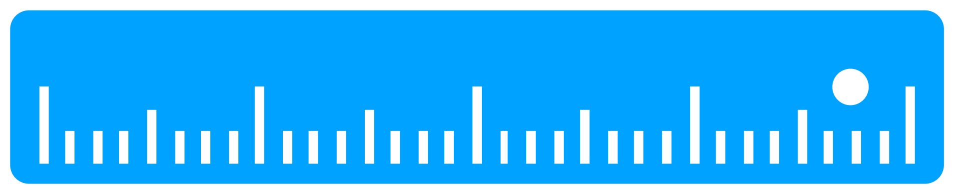
internalize frames



spacetime



reference frames always physical systems



- no background structure
- universality of QT (ext. of Heisenberg cut) $\Rightarrow$ RF subject to QM itself

"RFs in relative superposition"

Why dynamical frames?

E.g., gravity: tension between usual notion of bulk locality (in terms of fixed event labeling) and gauge-invariance

- **scalar field $\varphi(x)$ under diffeos:**

$$f_*\varphi(x) = \varphi(f^{-1}(x)) \qquad \Rightarrow \qquad \varphi(x) \text{ only gauge-inv. if } \begin{cases} x \in \partial\mathscr{M}, \text{ as } f^{-1}(x) = x \\ \text{or } \varphi = const\,. \end{cases}$$

$\Rightarrow$ notion of locality that fails is one based on fixed, non-dynamical — and hence unphysical — reference frames

Why dynamical frames?

E.g., gravity: tension between usual notion of bulk locality (in terms of fixed event labeling) and gauge-invariance

- scalar field $\varphi(x)$ under diffeos:

$$f_*\varphi(x) = \varphi(f^{-1}(x)) \qquad \Rightarrow \qquad \varphi(x) \text{ only gauge-inv. if } \begin{cases} x \in \partial\mathscr{M}, \text{ as } f^{-1}(x) = x \\ \text{or } \varphi = const\,. \end{cases}$$

$\Rightarrow$ notion of locality that fails is one based on fixed, non-dynamical — and hence unphysical — reference frames

$\Rightarrow$ instead, want dynamical (field-dep.) coordinatization $x[\phi]$ of spacetime s.t. for bulk diffeos: $\quad x[f_*\phi] = f(x[\phi])$

$$\Rightarrow \varphi(x[\phi]) = \varphi(f^{-1} \circ f(x[\phi])) = f_*\varphi(x[f_*\phi])$$

is gauge-inv. & local rel. to dynamically defined event

The story in pictures

RFs and symmetries

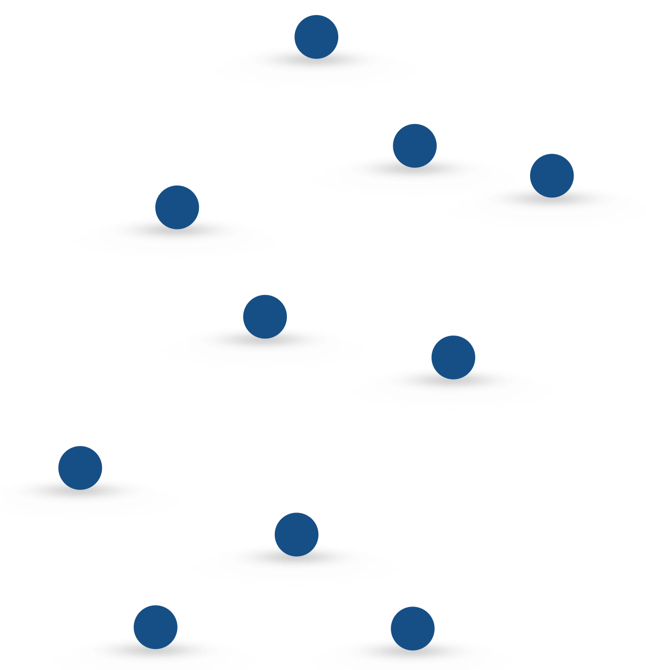
Premise:

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

RFs and symmetries

<u>Premise:</u>

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

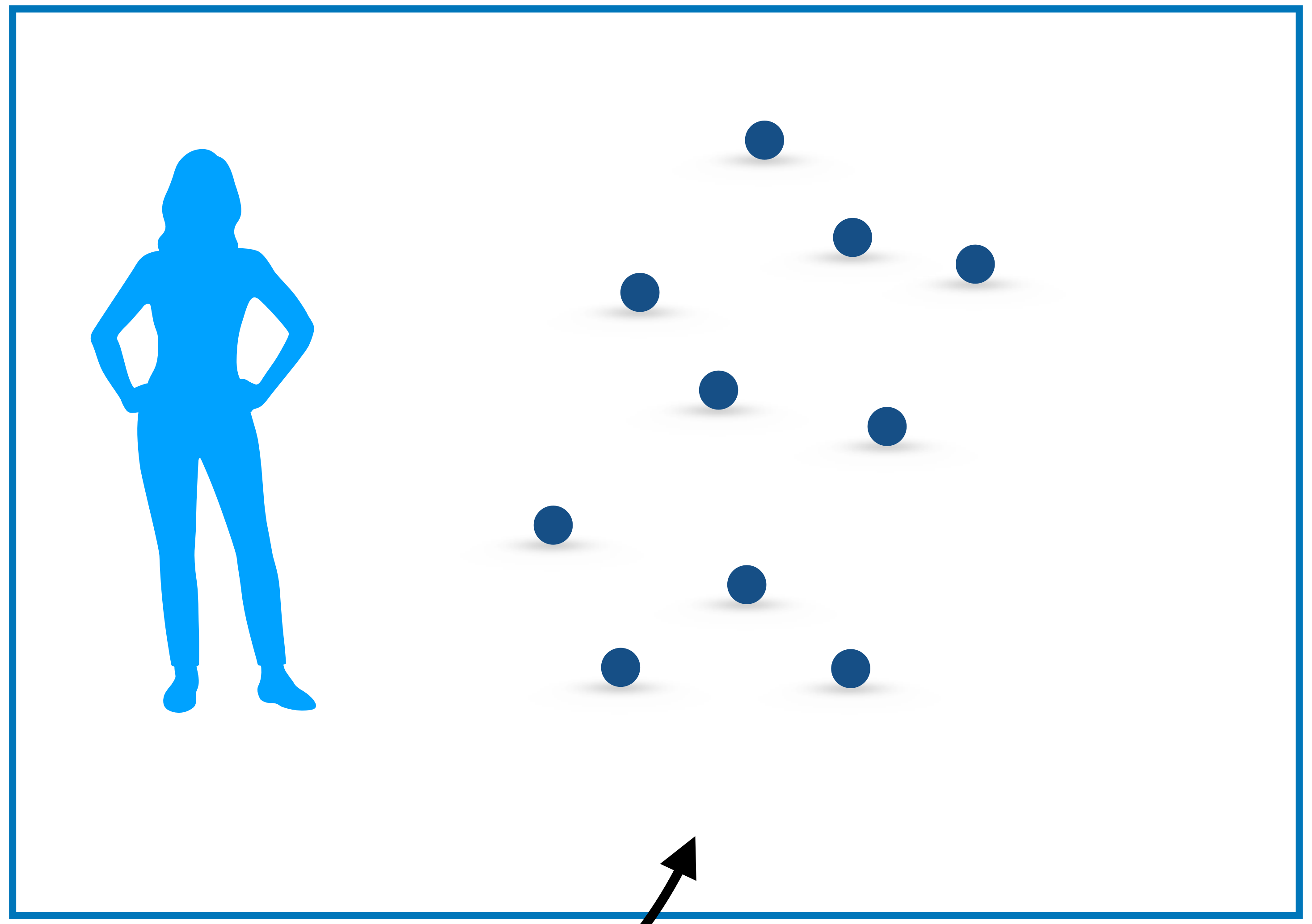


<u>pair (G, S) could be, e.g.:</u>

- spatial symmetry + group of particles
- gauge group + gauge field in some region
- diffeos + all dynamical fields in spacetime

RFs and symmetries

external frame



quantum information/foundations:
lab frame

gauge theories/gravity:
fictitious or edge modes (boundaries)

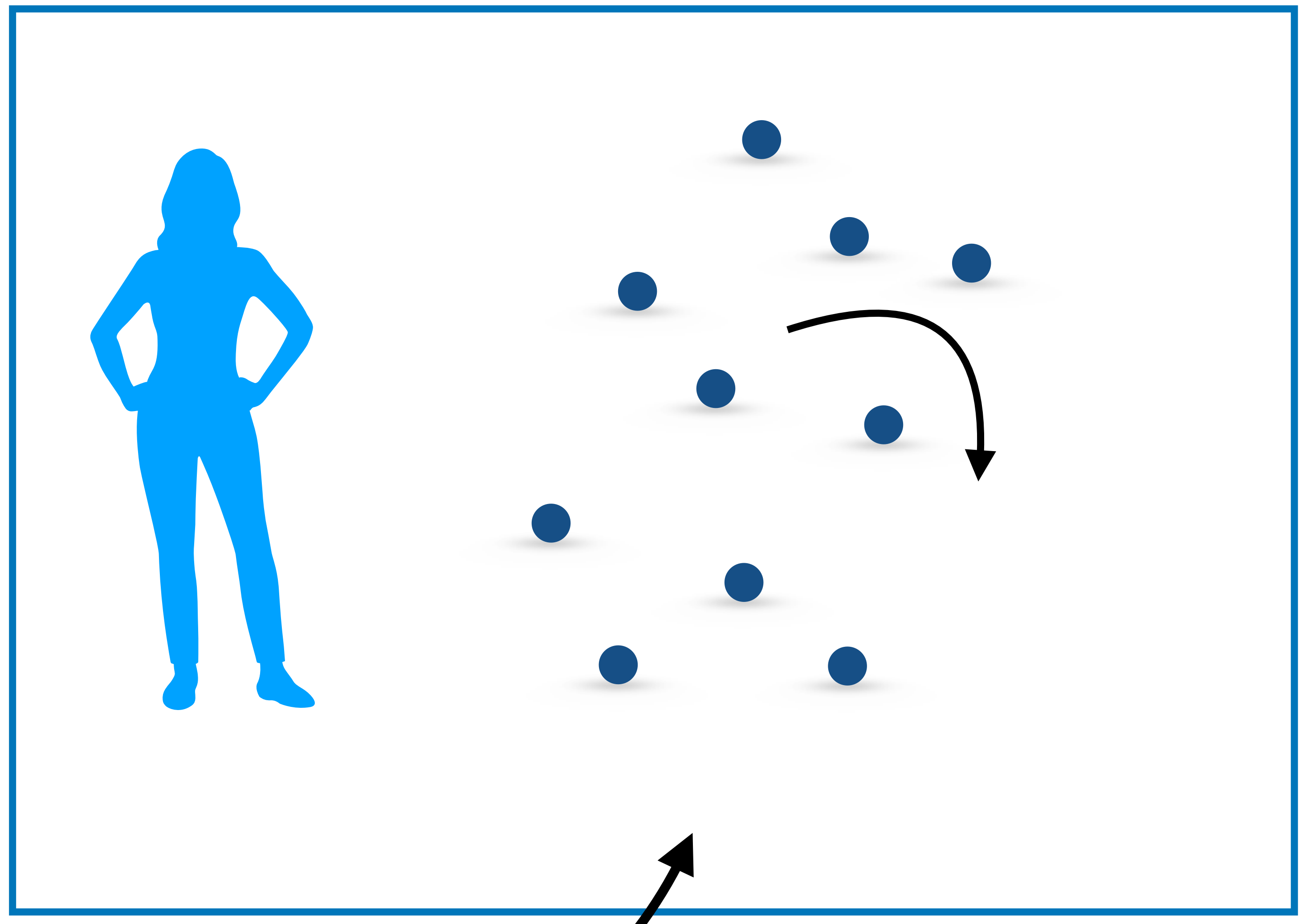
Premise:
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

pair (G, S) could be, e.g.:

- spatial symmetry + group of particles
- gauge group + gauge field in some region
- diffeos + all dynamical fields in spacetime

RFs and symmetries

external frame



quantum information/foundations:
lab frame

gauge theories/gravity:
fictitious or edge modes (boundaries)

<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

<u>pair (G, S) could be, e.g.:</u>

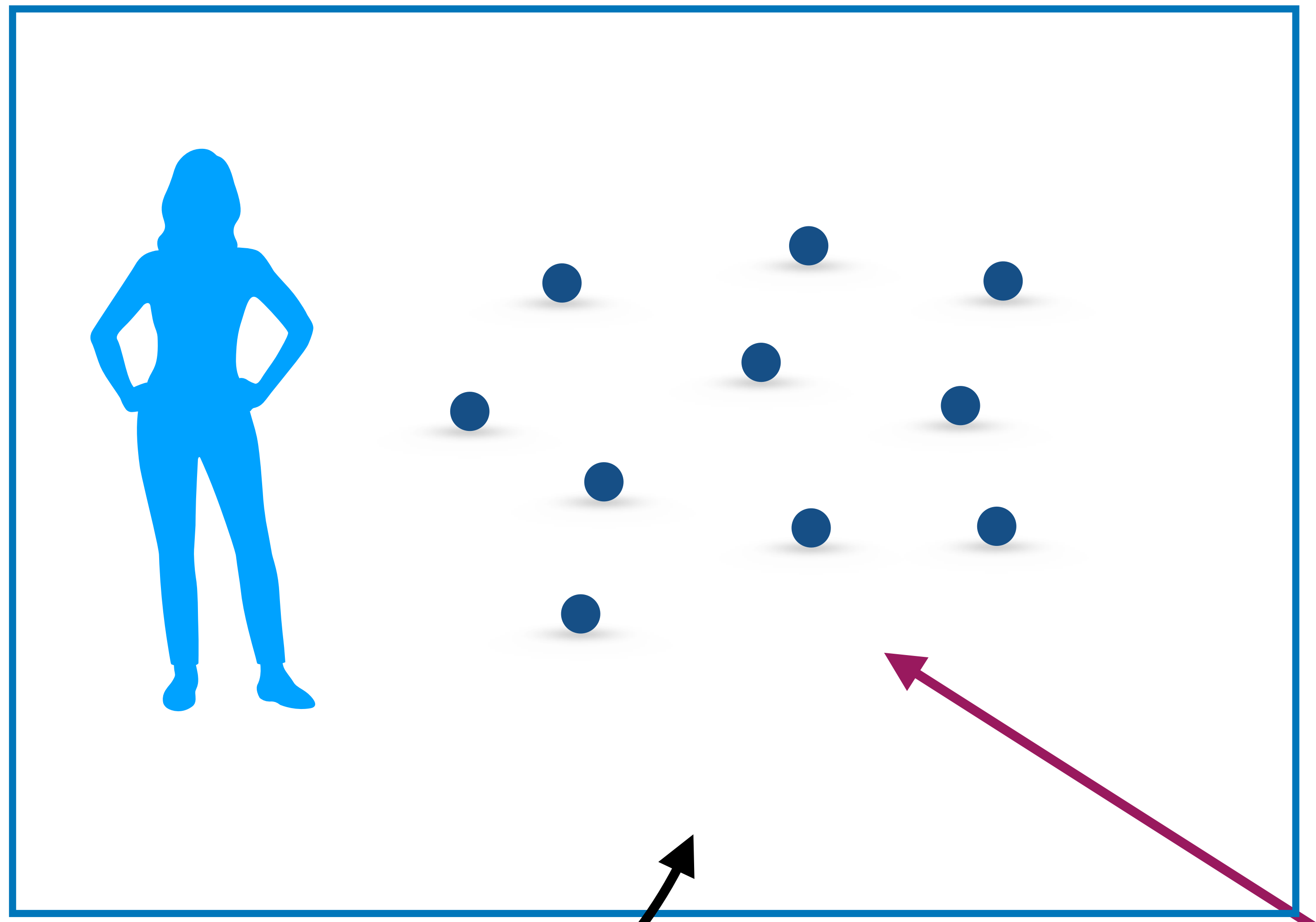
- spatial symmetry + group of particles
- gauge group + gauge field in some region
- diffeos + all dynamical fields in spacetime

RFs and symmetries

external frame

quantum information/foundations:
lab frame

gauge theories/gravity:
fictitious or edge modes (boundaries)



distinguishable rel. to external frame
“space of externally distinguishable states”

<u>Premise:</u>

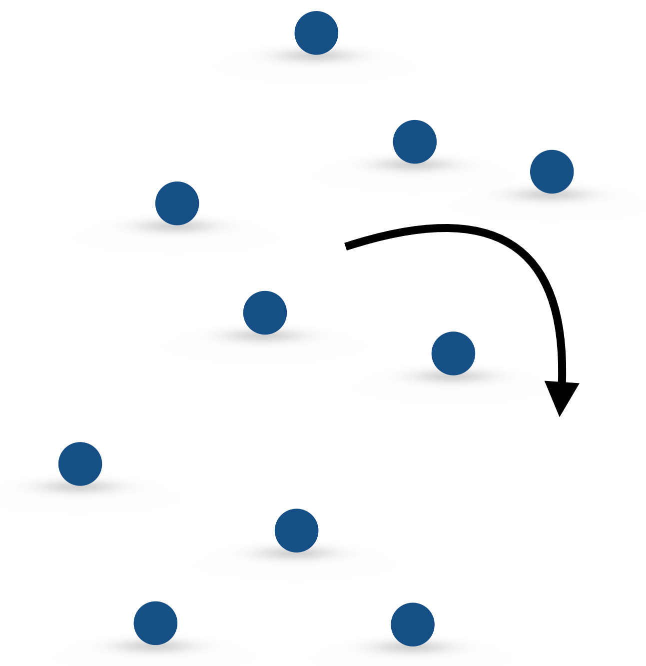
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

<u>pair (G, S) could be, e.g.:</u>

- spatial symmetry + group of particles
- gauge group + gauge field in some region
- diffeos + all dynamical fields in spacetime

RFs and symmetries

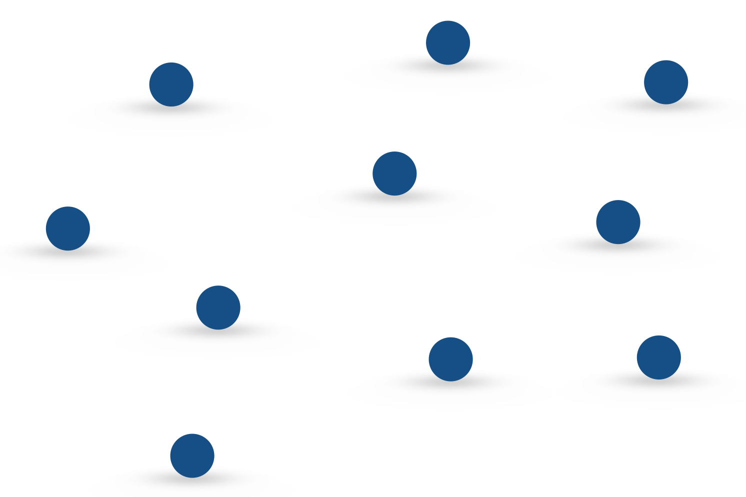
<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



RFs and symmetries

<u>Premise:</u>

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



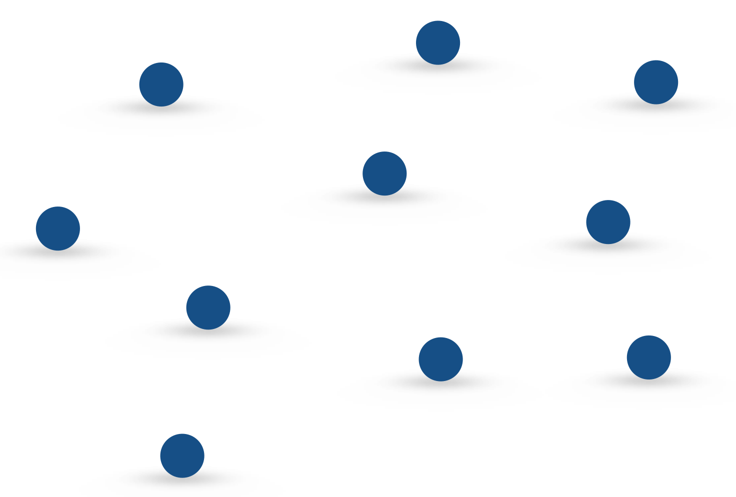
RFs and symmetries

G-transformations = ext. frame transf. = “gauge transformations”

$\Rightarrow$ “gauge inv.” = external frame indep.

Premise:

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



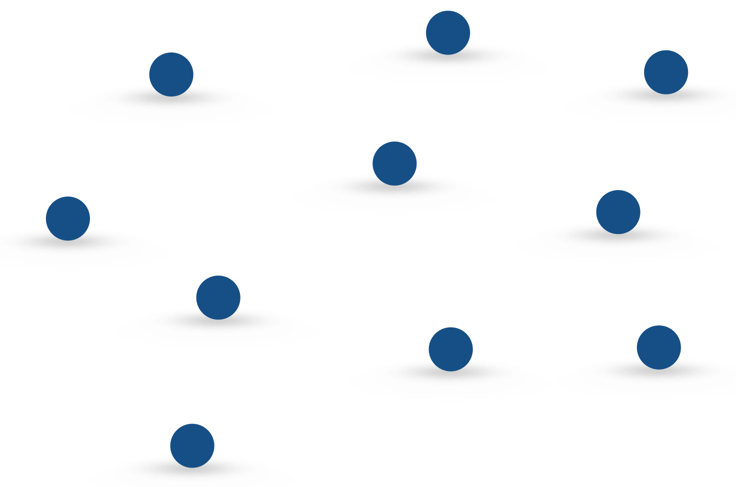
internally indistinguishable

RFs and symmetries

G-transformations = ext. frame transf. = “gauge transformations”

$\Rightarrow$ “gauge inv.” = external frame indep.

<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



internally indistinguishable

quantum information/foundations:
change of ext. frame

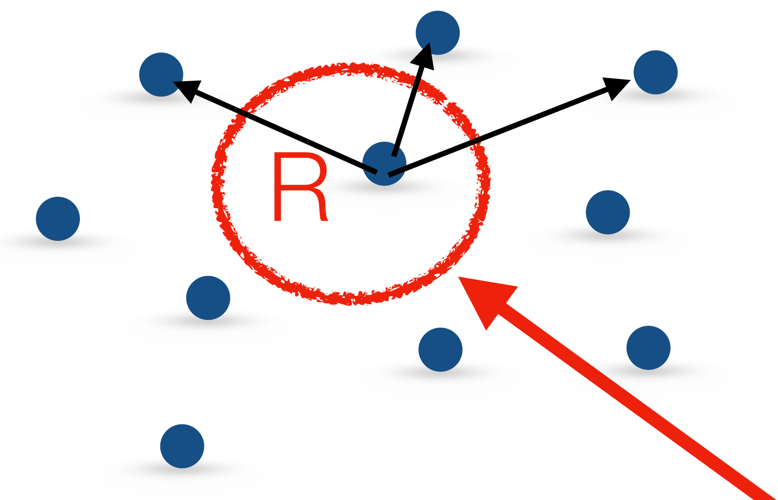
gauge theories/gravity:
gauge transf./diffeos

RFs and symmetries

G-transformations = ext. frame transf. = "gauge transformations"

$\Rightarrow$ "gauge inv." = external frame indep.

<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



internally indistinguishable

Describe S relative to internal reference subsystem R

quantum information/foundations:
change of ext. frame

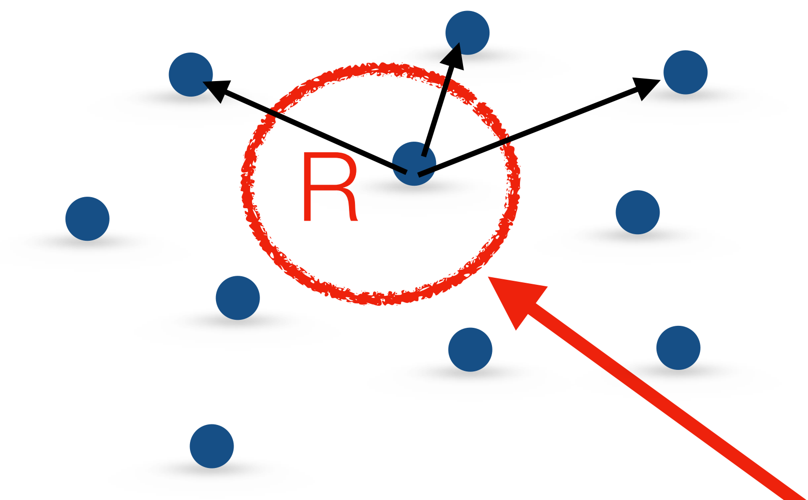
gauge theories/gravity:
gauge transf./diffeos

RFs and symmetries

G-transformations = ext. frame transf. = "gauge transformations"

$\Rightarrow$ "gauge inv." = external frame indep.

<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g\cdot\rho$ are Indistinguishable for all $g\in G$ when S considered in isolation



internally indistinguishable

quantum information/foundations:
change of ext. frame

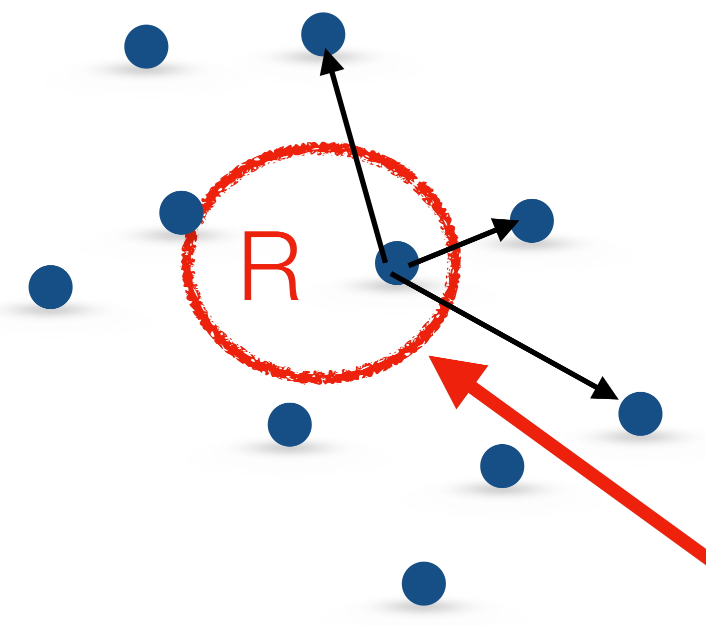
gauge theories/gravity:
gauge transf./diffeos

Describe S relative to internal reference subsystem R

should transform "nicely" (covariantly) under G

RFs and symmetries

<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



internally indistinguishable

Describe S relative to internal reference subsystem R

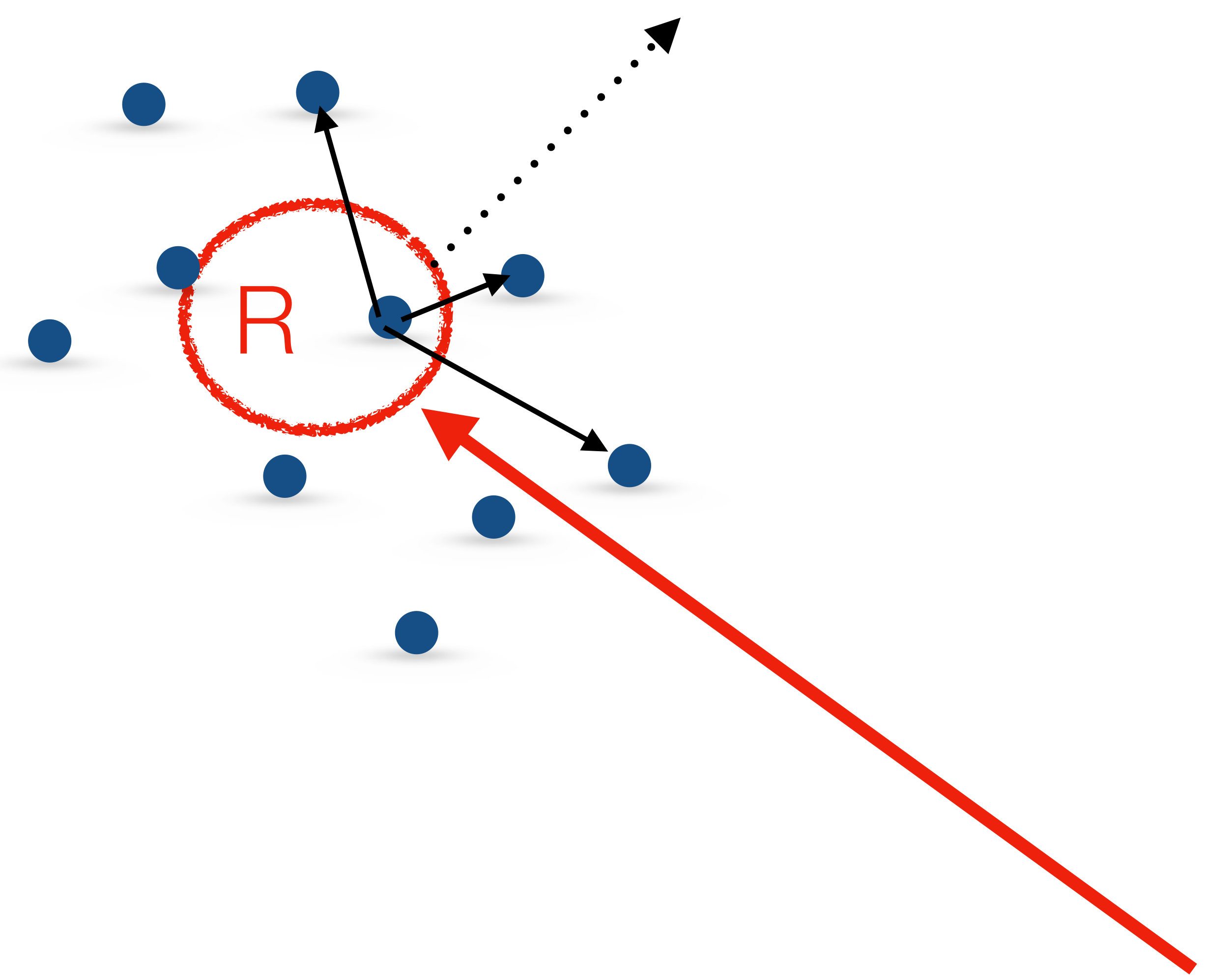
should transform "nicely" (covariantly) under G

ρ and $g \cdot \rho$ members of same relational equivalence class of states,
<u>different descriptions</u> of same relational state

RFs and symmetries

Premise:

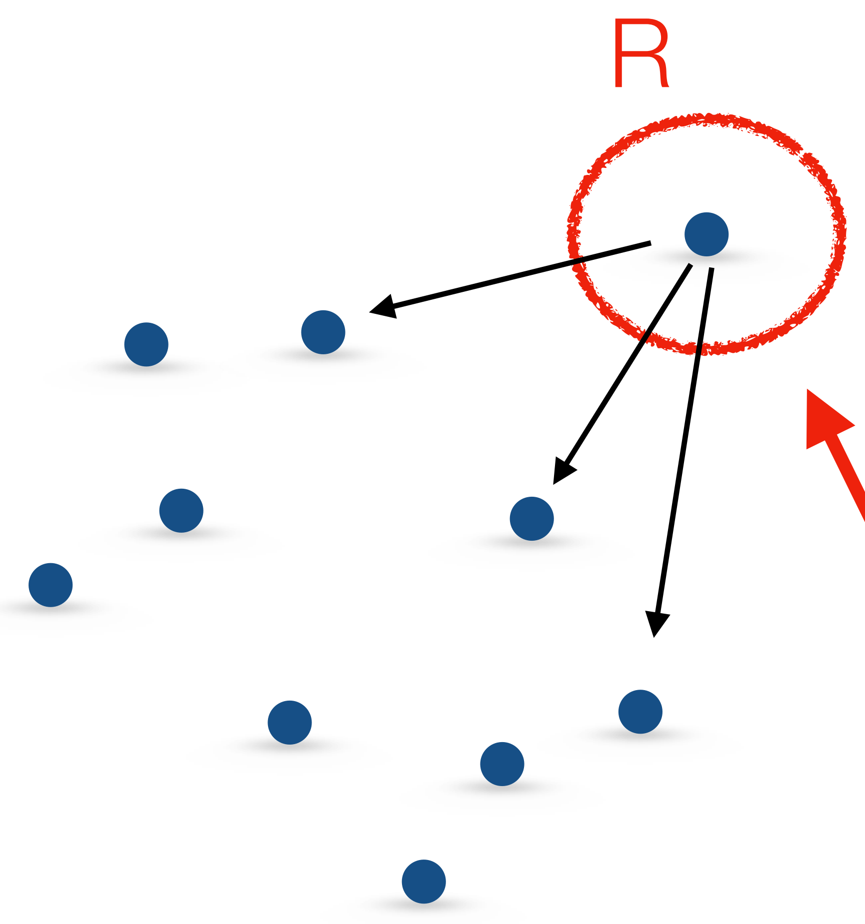
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



Describe S relative to internal reference subsystem R

Interested in internally distinguishable (relational) states/observables

RFs and symmetries



internally distinguishable
(relations changed)

$\Rightarrow$ **internal frame reorientation**
("symmetry")

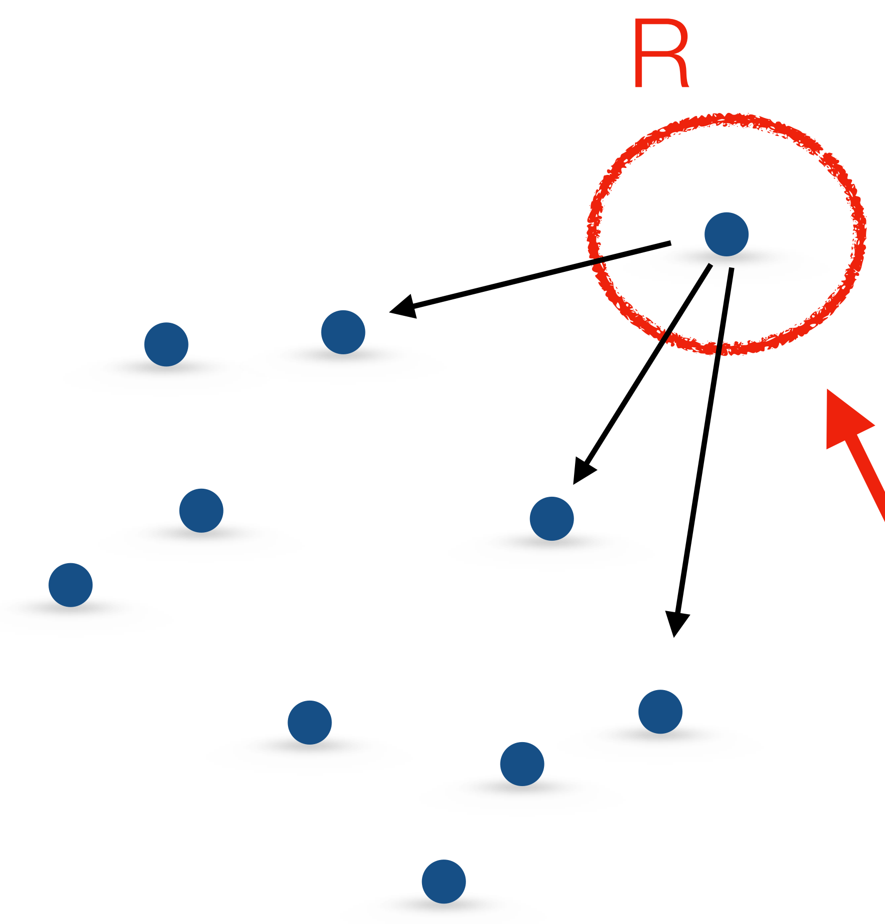
<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

Describe S relative to internal reference subsystem R

Interested in internally distinguishable (relational) states/ observables

RFs and symmetries

<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



internally distinguishable
(relations changed)

$\Rightarrow$ **internal frame reorientation (“symmetry”)**

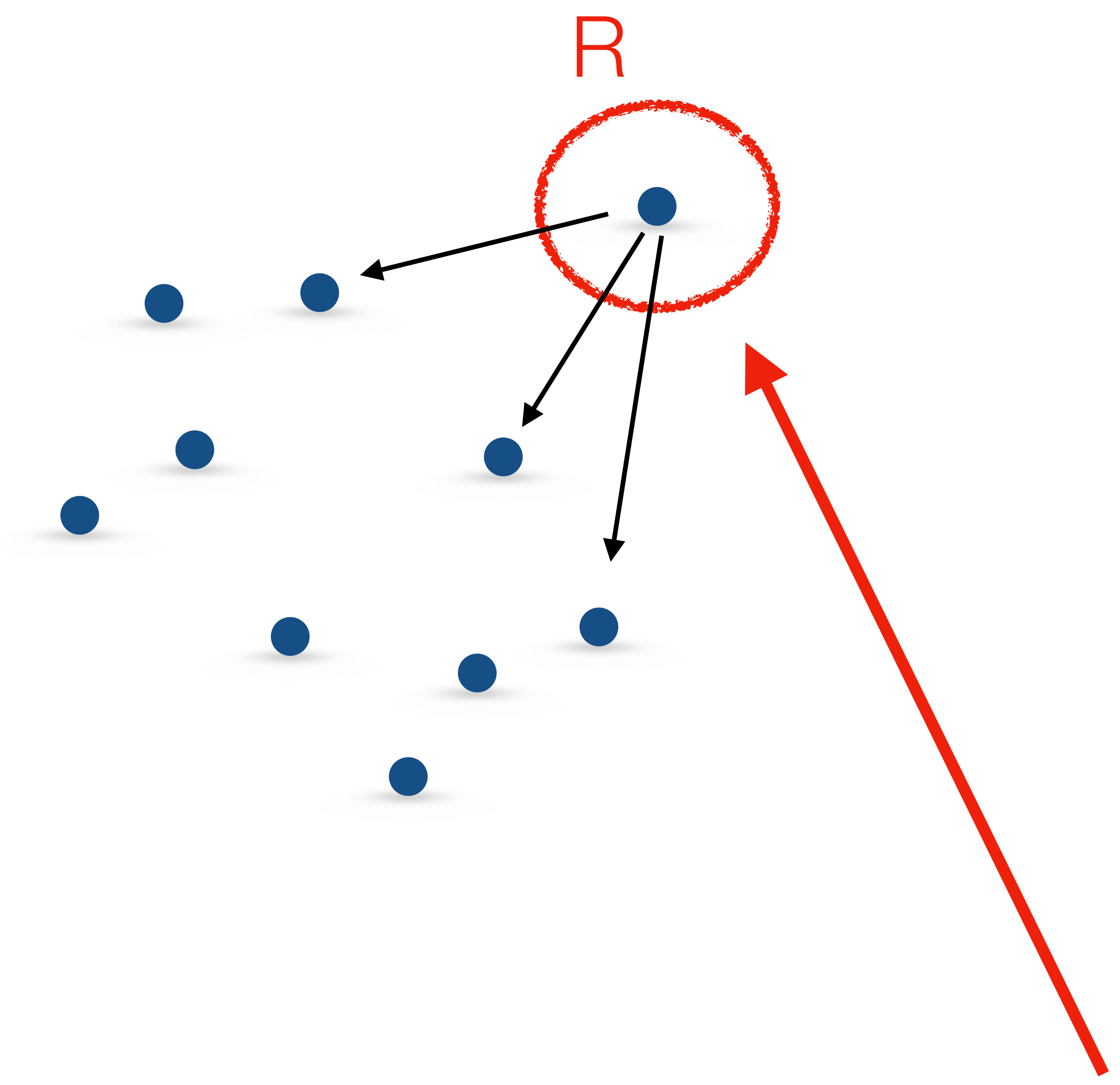
gauge theories/gravity: e.g. corner symmetries

Describe S relative to internal reference subsystem R

Interested in internally distinguishable (relational) states/ observables

2 ways of “jumping into a RF perspective”

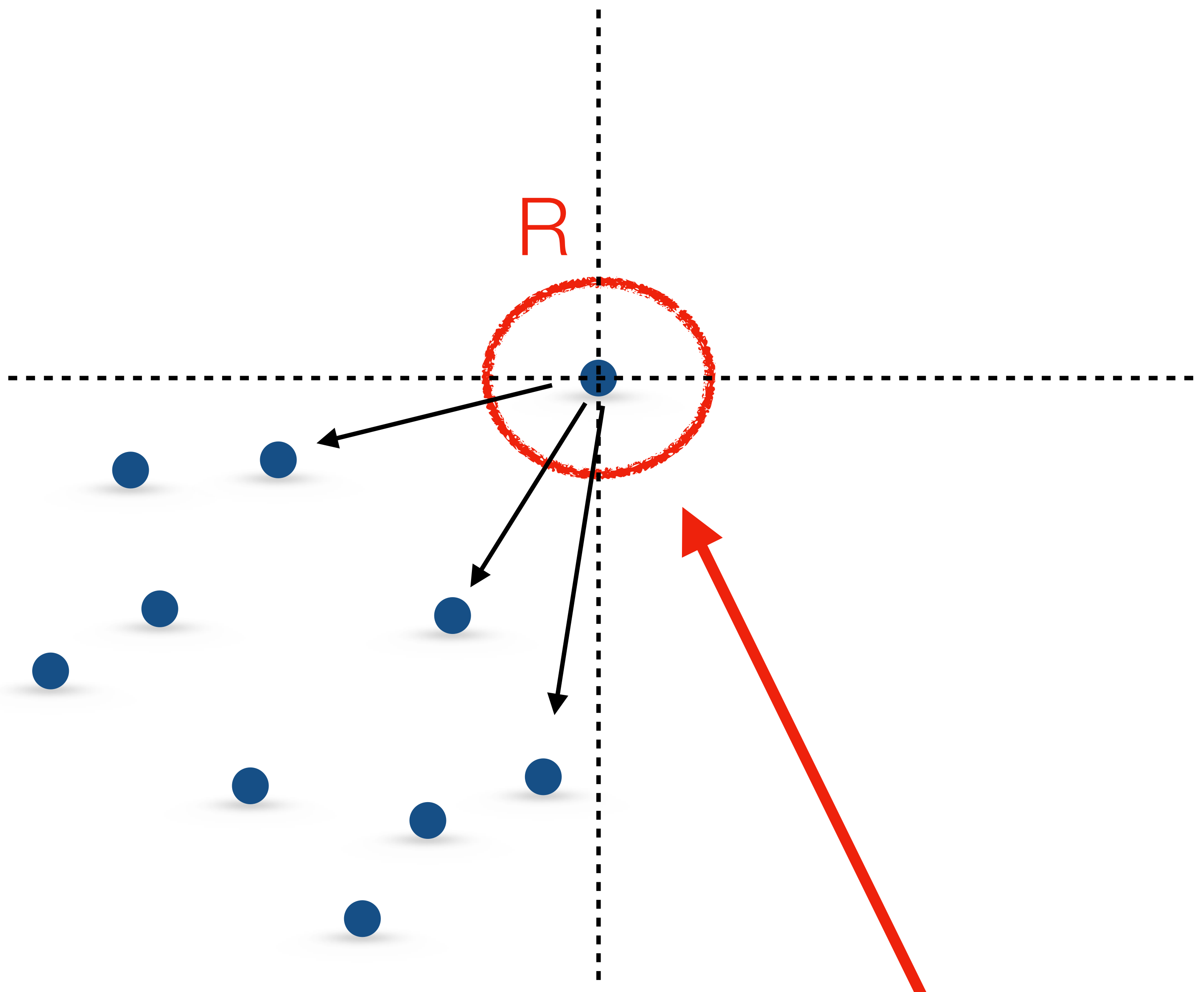
<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



Describe S relative to internal reference subsystem R

1. relational observables relative to R (gauge inv.)

2 ways of “jumping into a RF perspective”



Premise:

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

Describe S relative to internal reference subsystem R

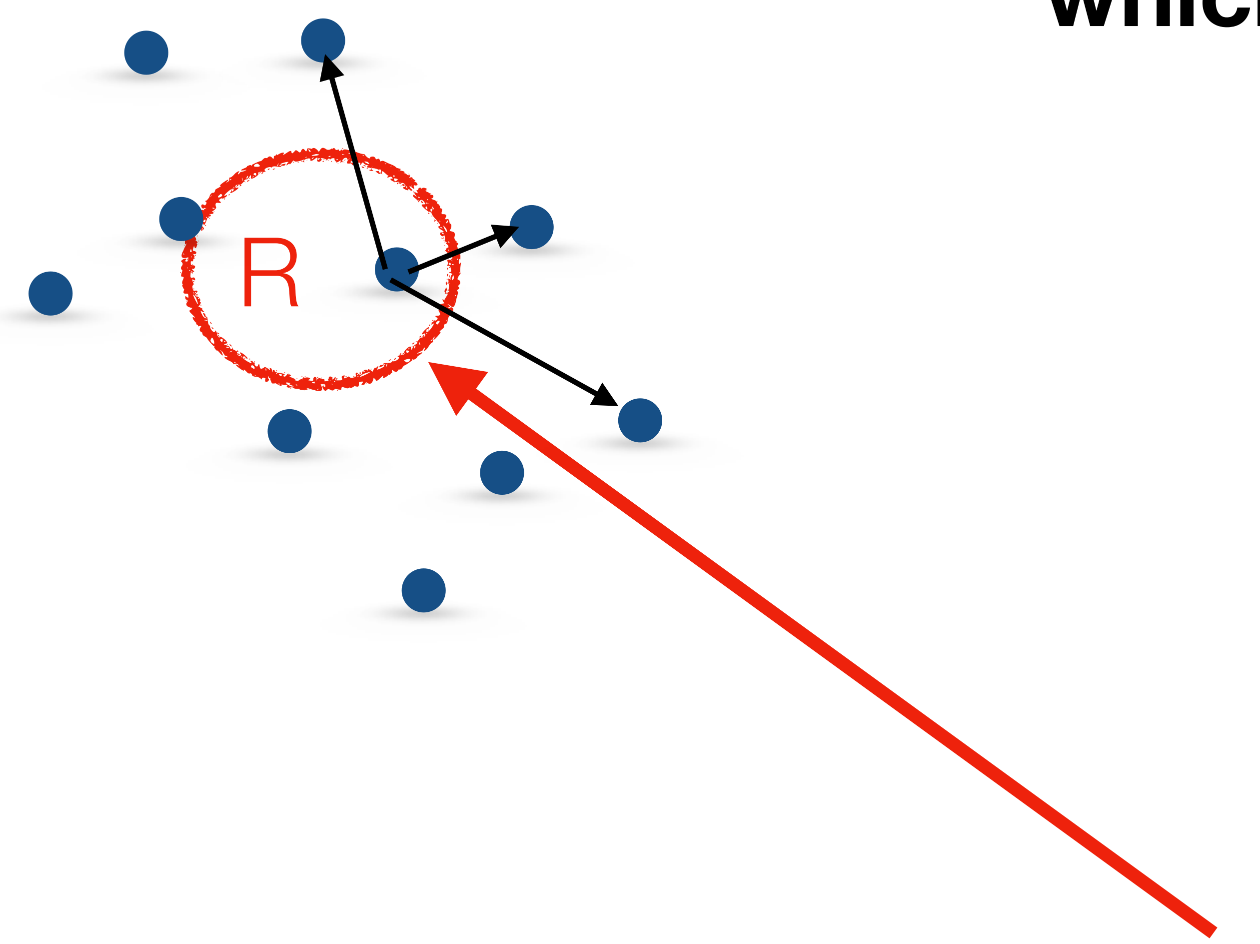
1. relational observables relative to R (gauge inv.)

2. put R into “origin” (gauge fix)

The multiple choice problem

Premise:

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



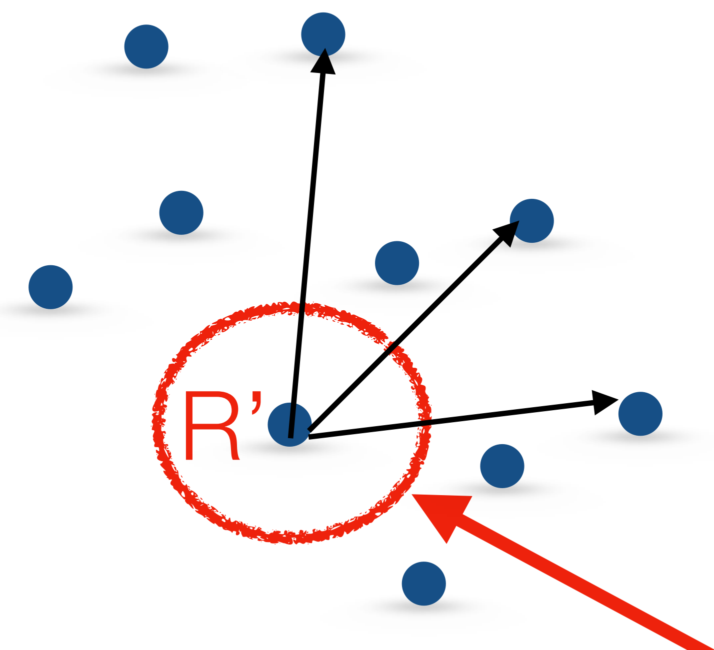
which frame to choose?

Describe S relative to internal reference subsystem R

The multiple choice problem

Premise:

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

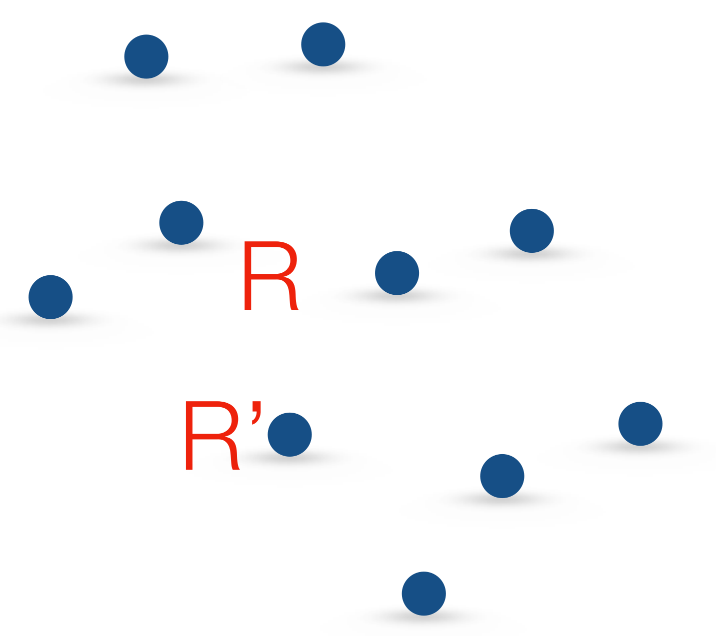


which frame to choose?

Describe S relative to internal reference subsystem R'

2 ways of changing RF

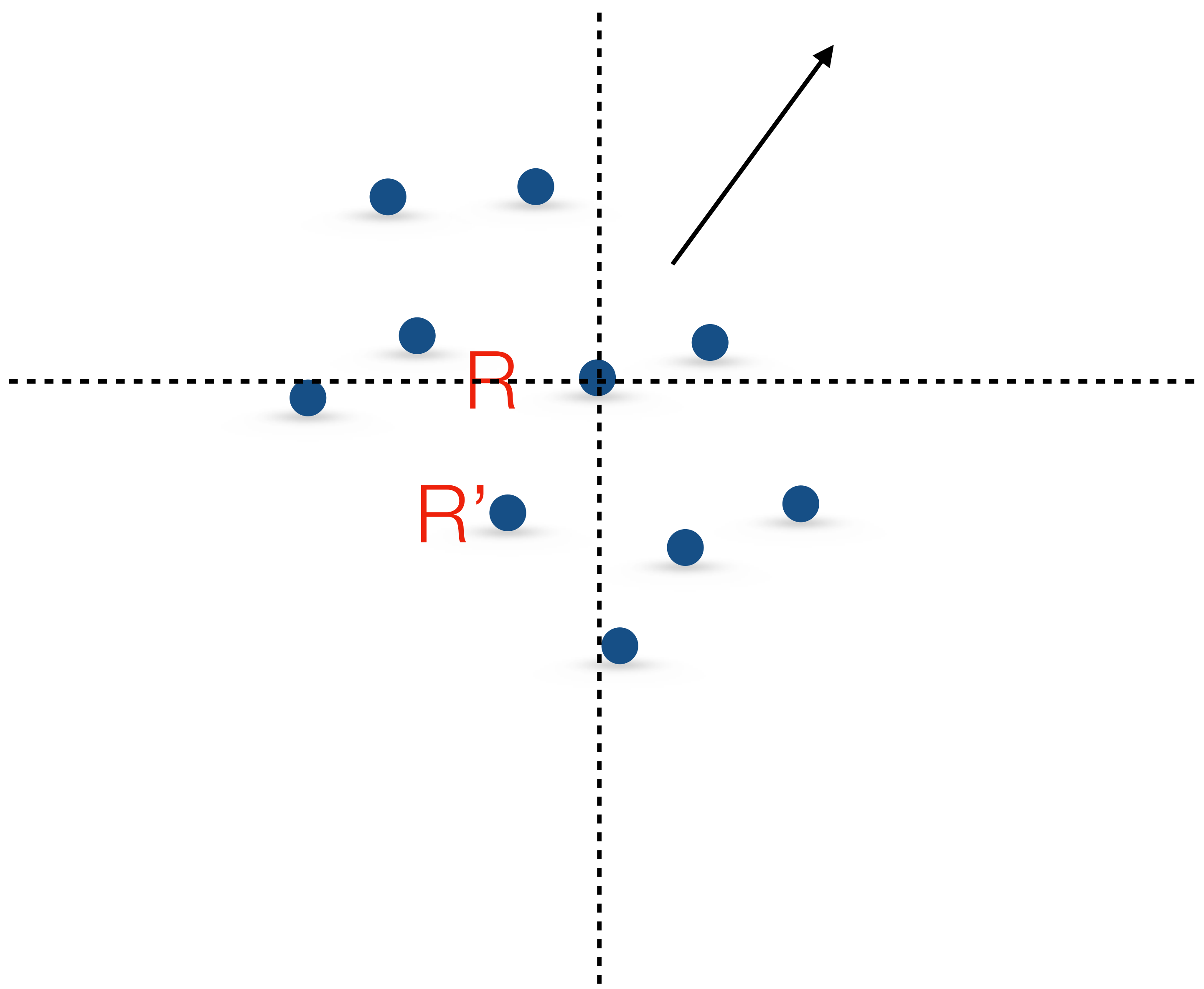
Premise:
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



2 ways of changing RF

Premise:

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

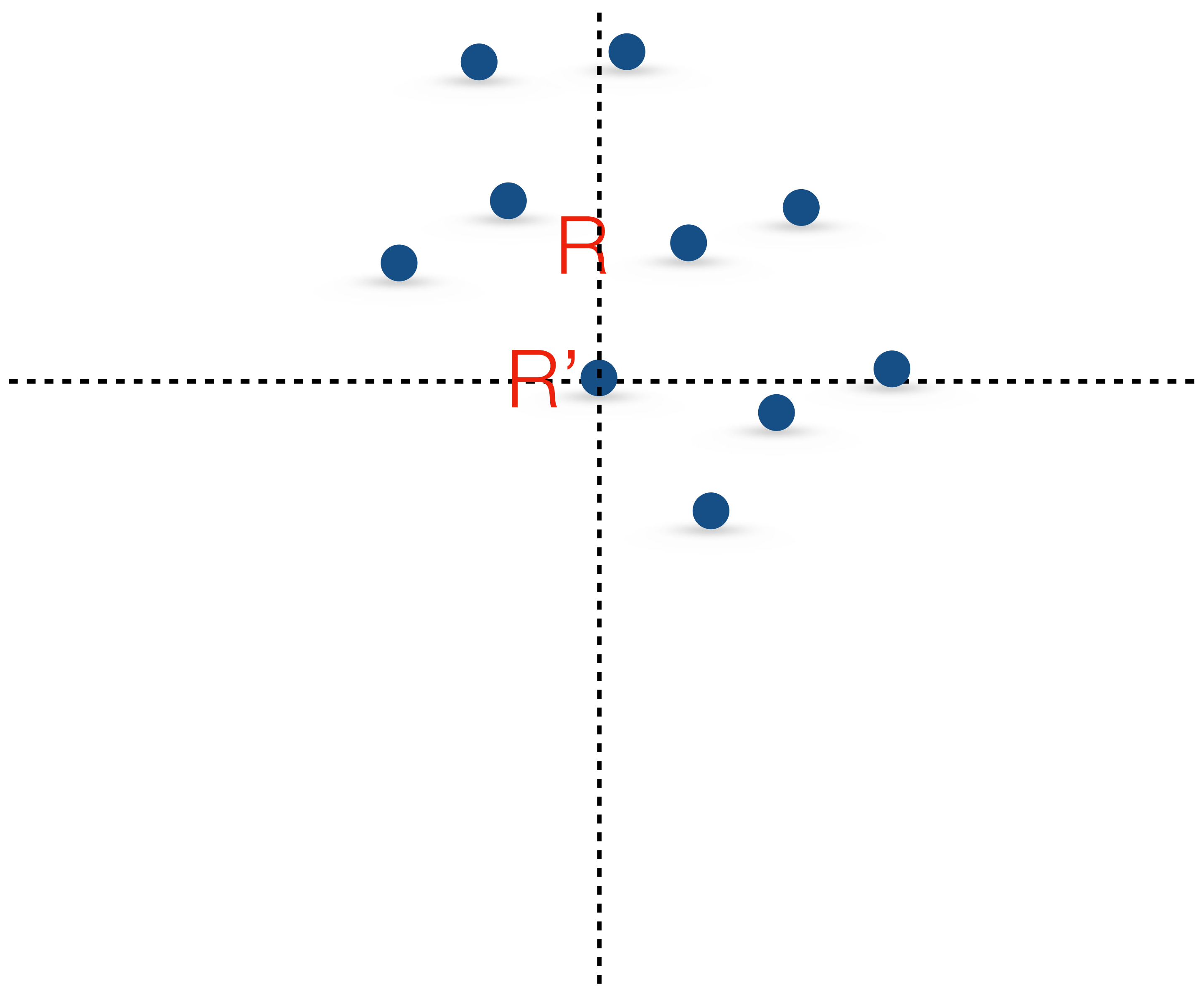


1. relation-conditional gauge transf.

2 ways of changing RF

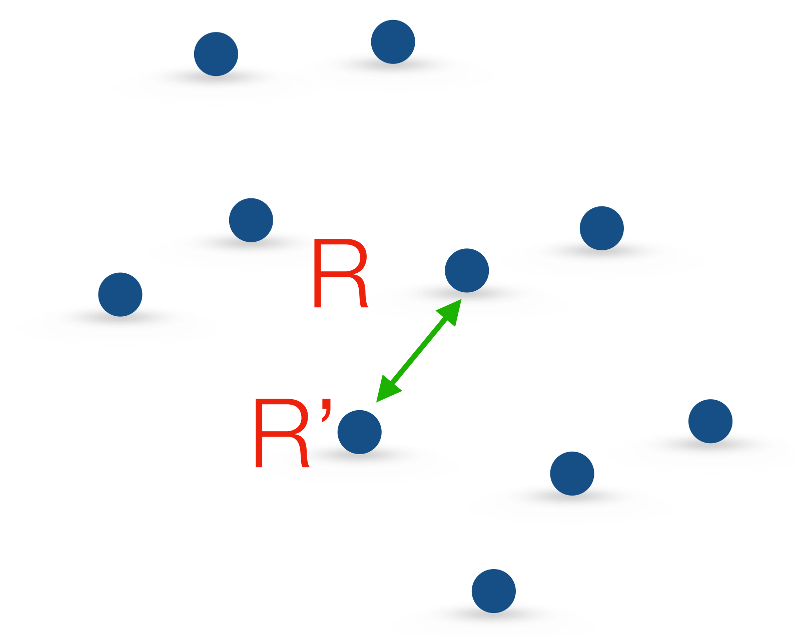
Premise:

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



1. relation-conditional gauge transf.

2 ways of changing RF



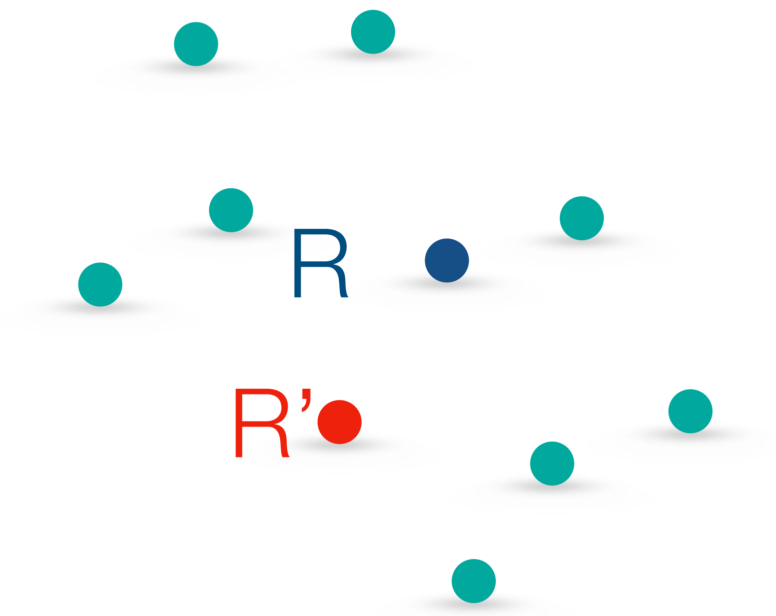
Premise:

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

1. **relation-conditional gauge transf.**
2. **relation-conditional reorientation**

kinematical vs. relational subsystems

green balls: subsystem S'



<u>Premise:</u>

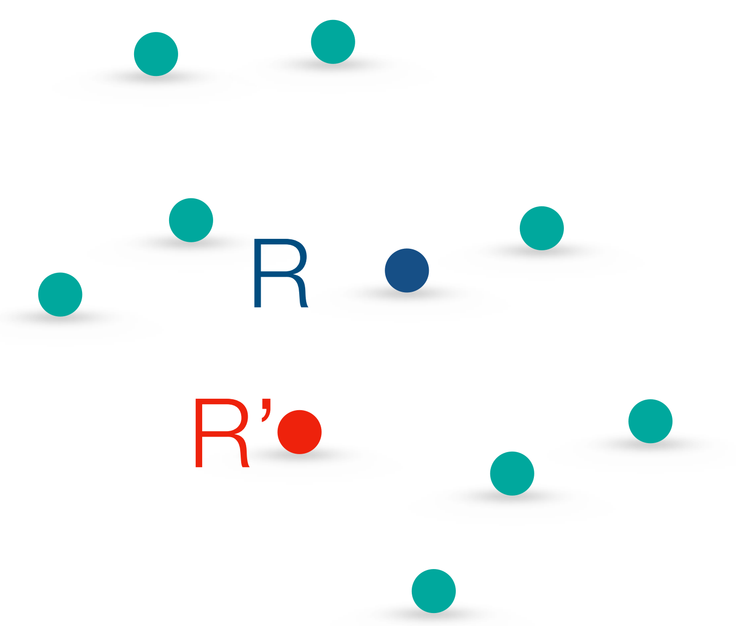
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

kinematical vs. relational subsystems

green balls: subsystem S'

<u>Premise:</u>

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation



1. **kinematical and relational (gauge inv.) notion of subsystem distinct**

kinematical vs. relational subsystems

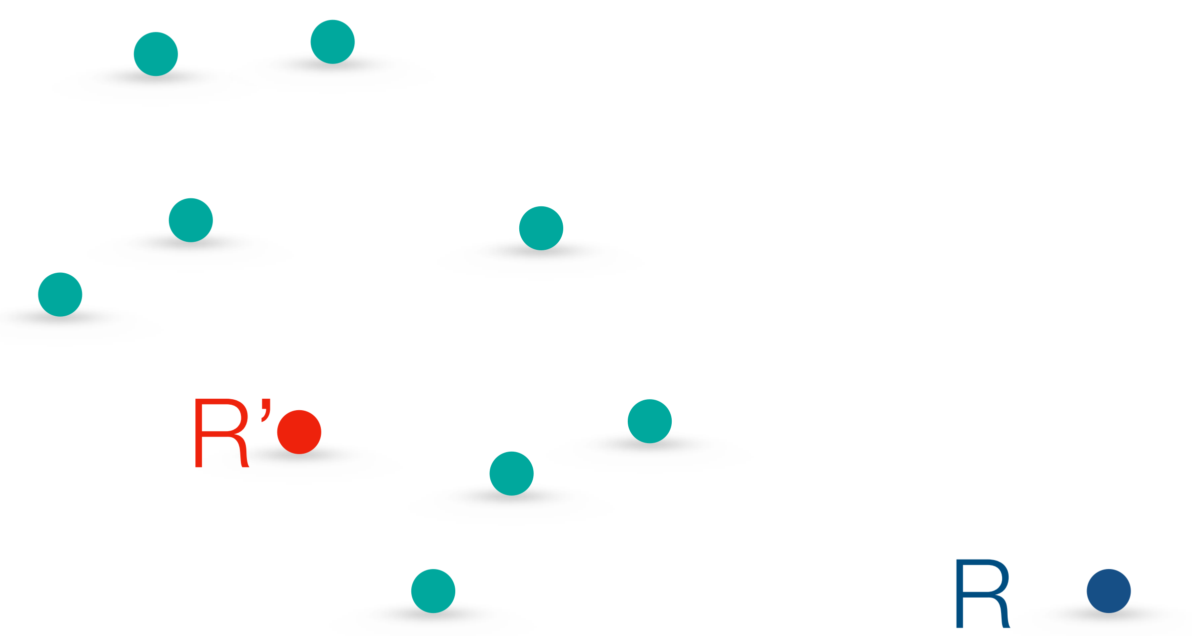
green balls: subsystem S'

Premise:
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

1. **kinematical and relational (gauge inv.) notion of subsystem distinct**

kinematical vs. relational subsystems

green balls: subsystem S'



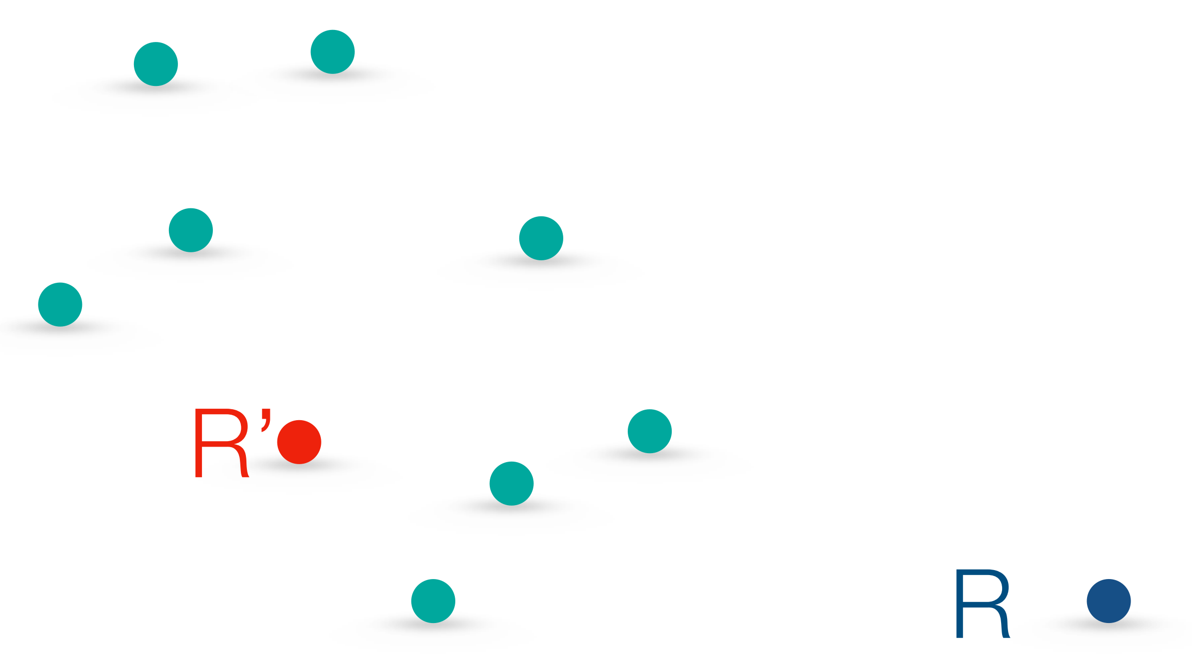
<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

leaves description of S' rel. to external frame invariant, but changes description relative to frame R

1. **kinematical and relational (gauge inv.) notion of subsystem distinct**

kinematical vs. relational subsystems

green balls: subsystem S'



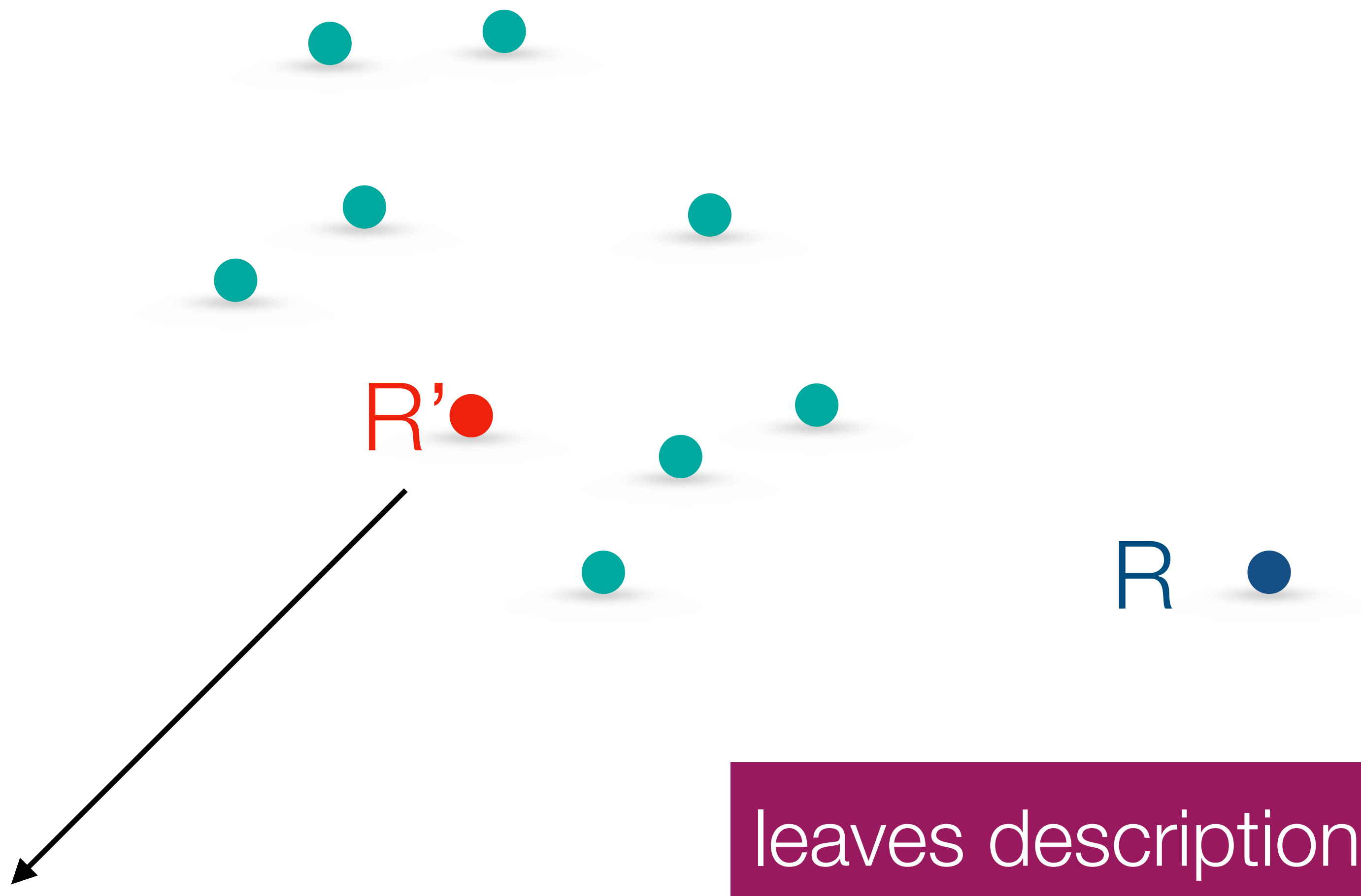
Premise:
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

leaves description of S' rel. to external frame invariant, but changes description relative to frame R

1. **kinematical and relational (gauge inv.) notion of subsystem distinct**
2. **gauge inv. notion of subsystem depends on choice of RF**

kinematical vs. relational subsystems

green balls: subsystem S'



<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

leaves description of S' rel. to external frame invariant, but changes description relative to frame R

1. **kinematical and relational (gauge inv.) notion of subsystem distinct**
2. **gauge inv. notion of subsystem depends on choice of RF**

kinematical vs. relational subsystems

leaves description of S' relative to R invariant, but changes it relative to R'

green balls: subsystem S'

<u>Premise:</u>

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

leaves description of S' rel. to external frame invariant, but changes description relative to frame R

1. **kinematical and relational (gauge inv.) notion of subsystem distinct**
2. **gauge inv. notion of subsystem depends on choice of RF**

kinematical vs. relational subsystems

leaves description of S' relative to R invariant,
but changes it relative to R'

green balls: subsystem S'

<u>Premise:</u>

System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

leaves description of S' rel. to external frame invariant,
but changes description relative to frame R

1. **kinematical and relational (gauge inv.) notion of subsystem distinct**
2. **gauge inv. notion of subsystem depends on choice of RF**

$\Rightarrow$ **gauge inv. correlations, thermal properties, … are RF dependent**

kinematical vs. relational subsystems

leaves description of S' relative to R invariant, but changes it relative to R'

green balls: subsystem S'

<u>Premise:</u>
System S subject to symmetry group G, s.t. states ρ and $g \cdot \rho$ are Indistinguishable for all $g \in G$ when S considered in isolation

leaves description of S' rel. to external frame invariant, but changes description relative to frame R

1. **kinematical and relational (gauge inv.) notion of subsystem distinct**
2. **gauge inv. notion of subsystem depends on choice of RF**

   ⇒ **gauge inv. correlations, thermal properties, … are RF dependent**

gauge theories/gravity:
e.g. S' subregion, R, R' edge mode frames

The story in equations

What's a dynamical reference frame?

Dynamical frame R always associated with (gauge) symmetry group G

R configurations: orientations o of the frame

[e.g. orientation of a tetrad, reading of a clock, …]

<u>restrict here (for now) to:</u>

- complete (group valued) frames: space of frame orientations $\simeq G$
- gauge covariant frames: $o(g'g) = g' \cdot o(g)$

[can also treat more general situations: Carrozza, PH '21; de la Hamette, Galley, PH, Loveridge, Müller '21; Goeller, PH, Kirklin '22]

use orientations to parametrize/gauge-fix G-orbits

Unifying picture of dynamical frame covariance

	Special relativity	Quantum relativity	Gauge relativity	General relativity
frame orientations (group valued frame)				
gauge transf./ gauge covariance				
frame reorientations/ "symmetries" (act only on the frame)				
relational observables $O_{F,R}(g)$ "what's value of F when RF is in orientation g?"				
"jumping into RF perspective" (gauge fixing)				
RF change 1 (rel. cond. gauge transf.)				
RF change 2 (rel. cond. symmetry) (1st frame relative to 2nd)				

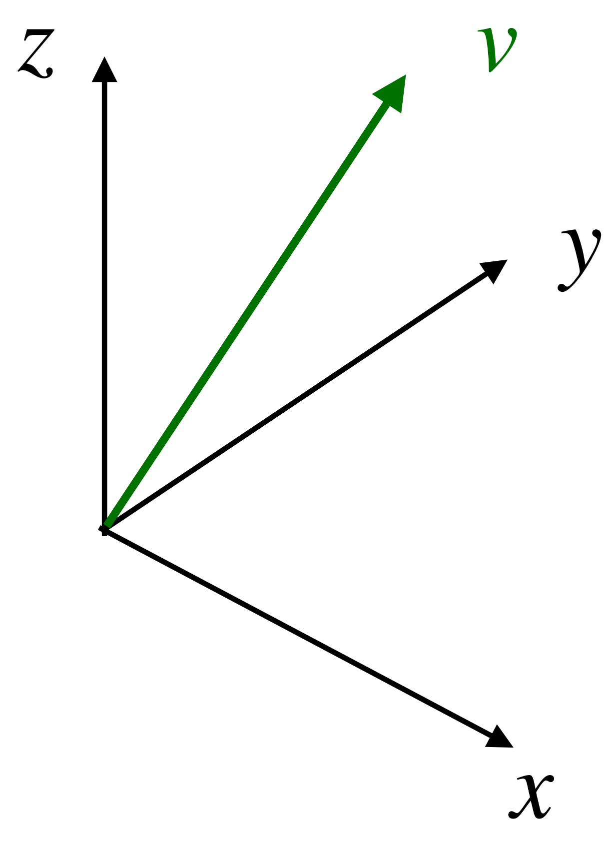
2 commuting group actions

Unifying picture of dynamical frame covariance

	Special relativity	Quantum relativity	Gauge relativity	General relativity
frame orientations (group valued frame)				
gauge transf./ gauge covariance				
frame reorientations/ "symmetries" (act only on the frame)				
relational observables $O_{F,R}(g)$ "what's value of F when RF is in orientation g?"				
"jumping into RF perspective" (gauge fixing)				
RF change 1 (rel. cond. gauge transf.)				
RF change 2 (rel. cond. symmetry) (1st frame relative to 2nd)				

2 commuting group actions

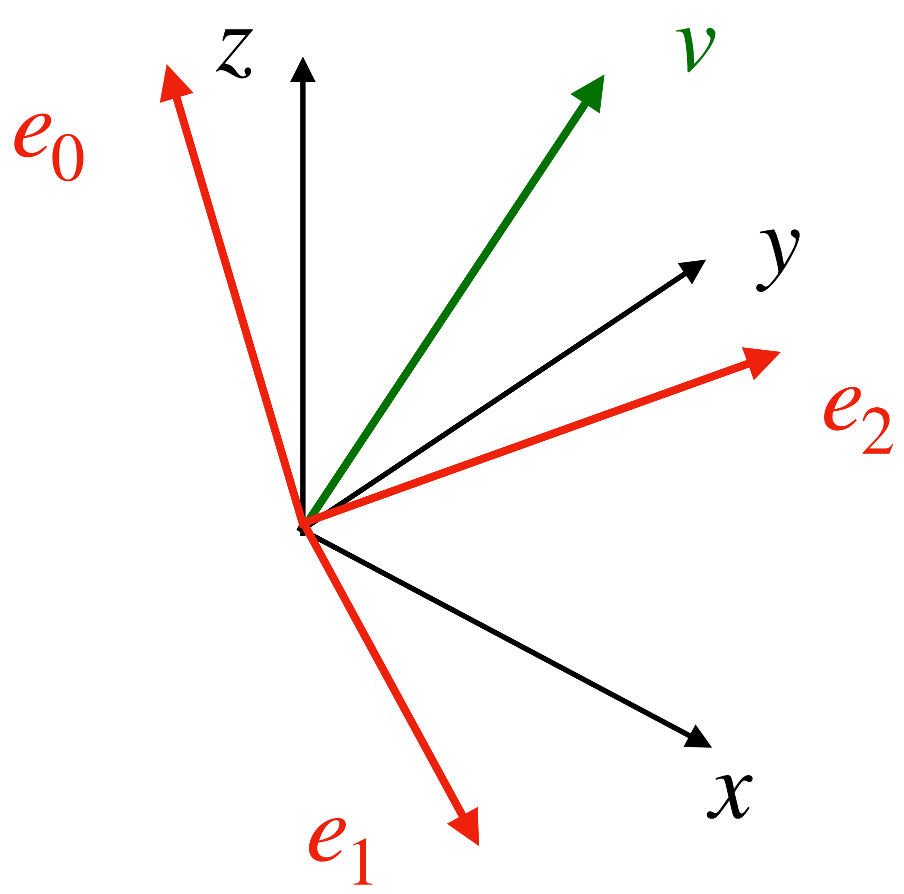
Warmup: Special relativity with internal frames



fictitious/external coord. frame

$v^\mu \mapsto \Lambda^\mu{}_\nu v^\nu \qquad \Lambda \in \mathrm{SO}_+(3,1),$ **internally indistinguishable**

Warmup: Special relativity with internal frames



fictitious/external coord. frame

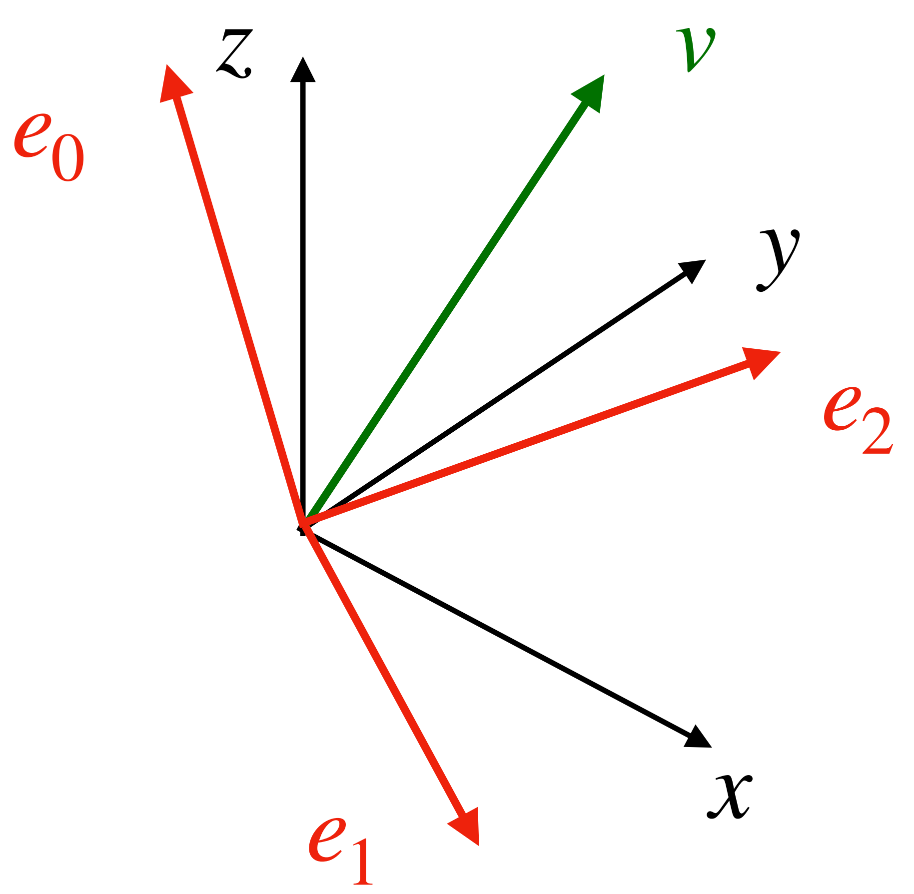
$v^\mu \mapsto \Lambda^\mu{}_\nu v^\nu \qquad \Lambda \in \mathrm{SO}_+(3,1)$, **internally indistinguishable**

introduce internal frame (tetrad)

$e_a^\mu \qquad \mu = t, x, y, z$ spacetime index, $\quad a = 0,1,2,3$ frame index

frame orientations

Warmup: Special relativity with internal frames



fictitious/external coord. frame

$v^\mu \mapsto \Lambda^\mu{}_\nu v^\nu \qquad \Lambda \in \mathrm{SO}_+(3,1),$ **internally indistinguishable**

introduce internal frame (tetrad)

$e_a^\mu \qquad \mu = t, x, y, z$ spacetime index, $\qquad a = 0,1,2,3$ frame index

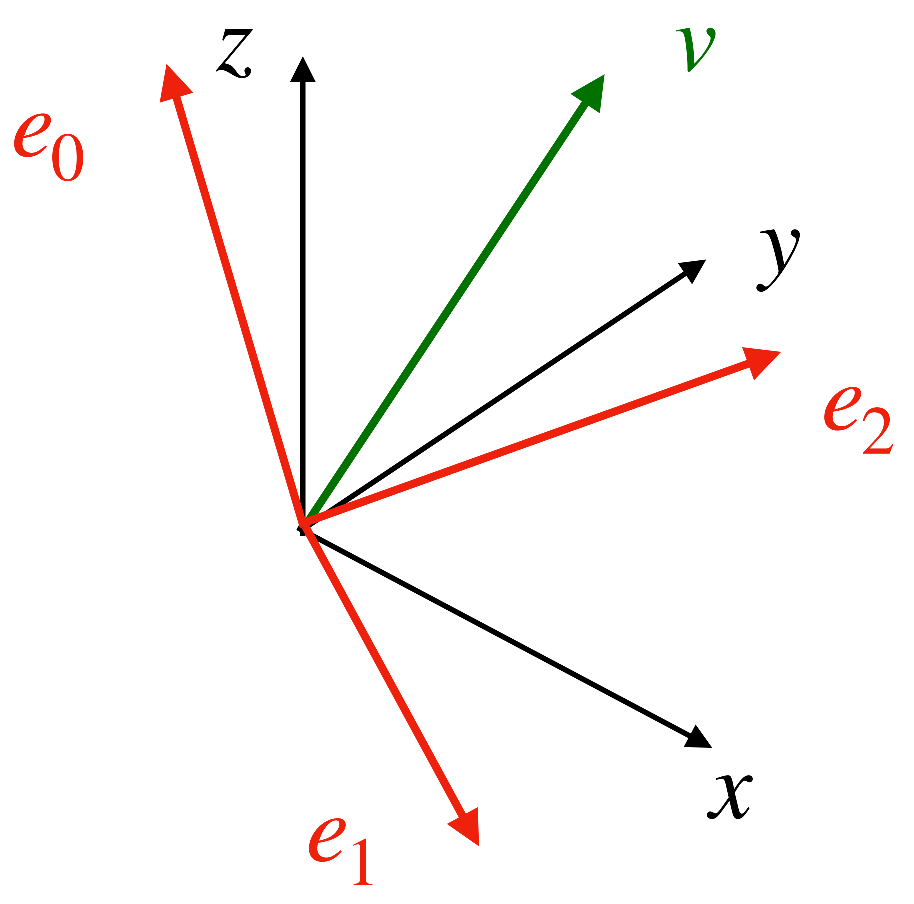
frame orientations

2 indices, 2 commuting group actions:

- **"gauge transformations":** $\Lambda^\mu{}_\nu \, e_a^\nu \qquad \Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$
- **"symmetries" (frame reorientations):** $\Lambda_a{}^b \, e_b^\mu \qquad \Lambda_a{}^b \in \mathrm{SO}_+(3,1)$

only acts on frame

Warmup: Special relativity with internal frames



fictitious/external coord. frame

$v^\mu \mapsto \Lambda^\mu{}_\nu v^\nu \qquad \Lambda \in SO_+(3,1)$, **internally indistinguishable**

introduce internal frame (tetrad)

$e^\mu_a \qquad \mu = t, x, y, z$ spacetime index, $\quad a = 0,1,2,3$ frame index

frame orientations

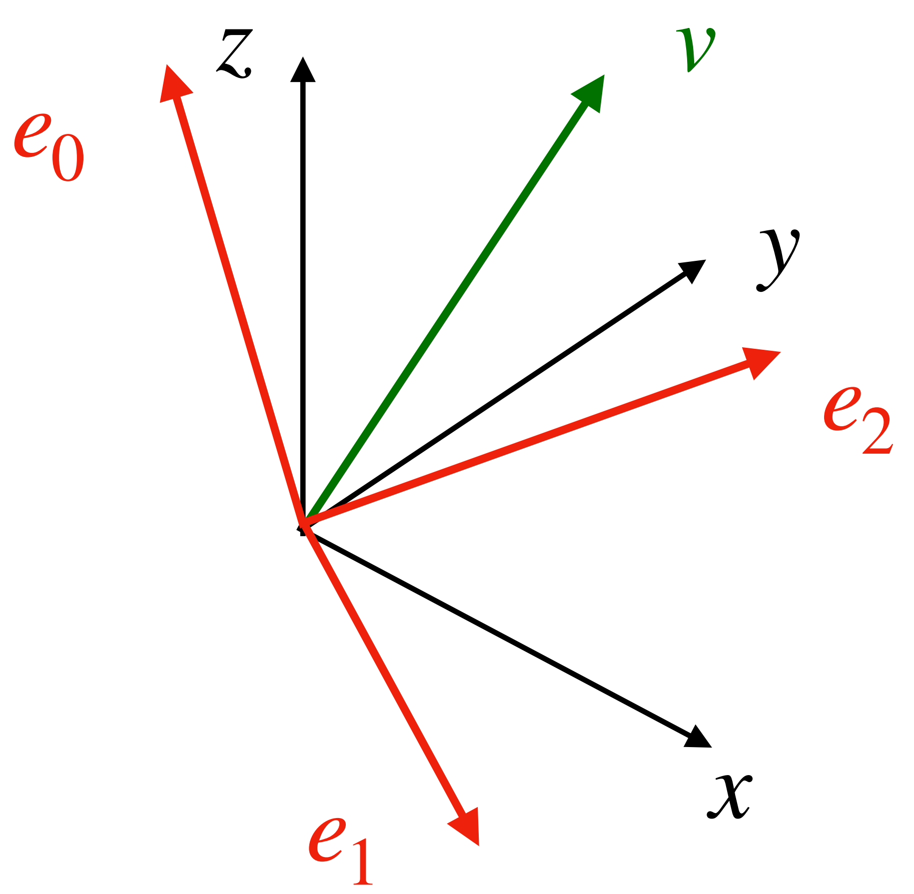
2 indices, 2 commuting group actions:

- **"gauge transformations":** $\quad \Lambda^\mu{}_\nu\, e^\nu_a \qquad \Lambda^\mu{}_\nu \in SO_+(3,1)$
- **"symmetries" (frame reorientations):** $\quad \Lambda_a{}^b\, e^\mu_b \qquad \Lambda_a{}^b \in SO_+(3,1)$

only acts on frame

$\Rightarrow$ **group acts on itself since** $\quad \eta_{ab} = e^\mu_a\, e^\nu_b\, \eta_{\mu\nu} \quad \Rightarrow \quad e^\mu_a \in SO_+(3,1)$ **group valued frame orientations**

Warmup: Special relativity with internal frames



fictitious/external coord. frame

$v^\mu \mapsto \Lambda^\mu{}_\nu v^\nu \qquad \Lambda \in \mathrm{SO}_+(3,1),$ **internally indistinguishable**

introduce internal frame (tetrad)

$e^\mu_a \qquad \mu = t, x, y, z$ spacetime index, $\qquad a = 0,1,2,3$ frame index

frame orientations

2 indices, 2 commuting group actions:

- **"gauge transformations":** $\qquad \Lambda^\mu{}_\nu \, e^\nu_a \qquad \Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$
- **"symmetries" (frame reorientations):** $\qquad \Lambda_a{}^b \, e^\mu_b \qquad \Lambda_a{}^b \in \mathrm{SO}_+(3,1)$

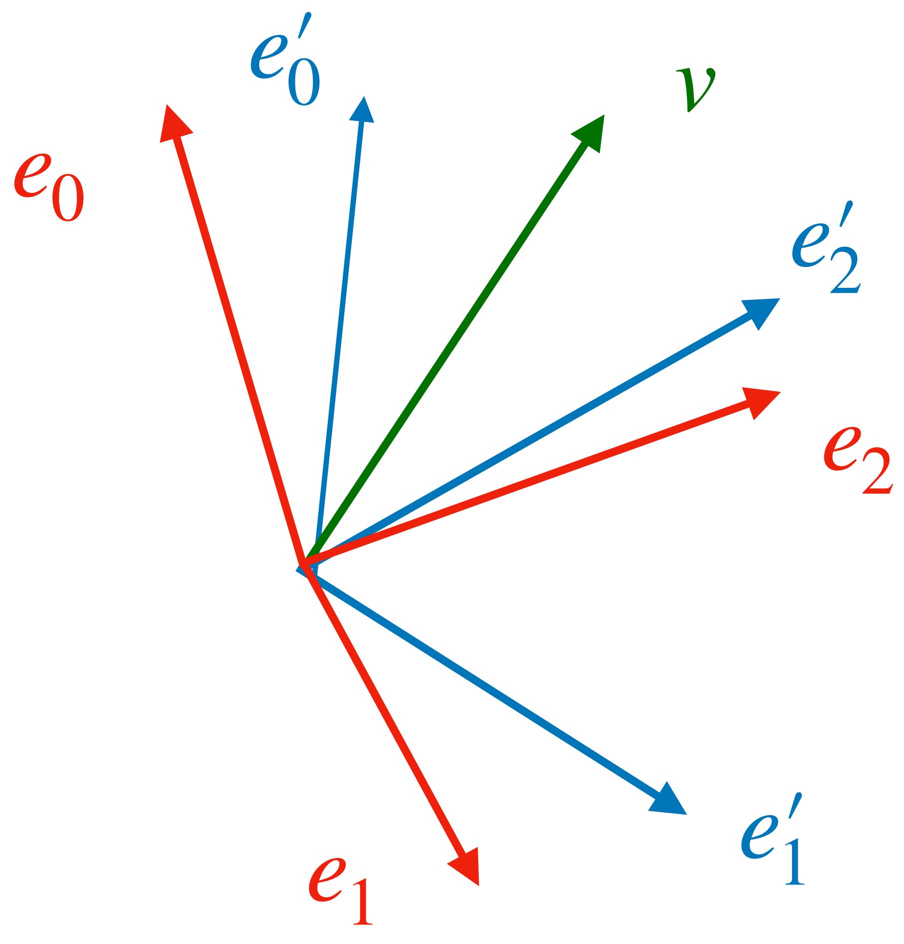
only acts on frame

$\Rightarrow$ **group acts on itself since** $\qquad \eta_{ab} = e^\mu_a \, e^\nu_b \, \eta_{\mu\nu} \qquad \Rightarrow \qquad e^\mu_a \in \mathrm{SO}_+(3,1)$ **group valued frame orientations**

$\Rightarrow$ **"gauge-invariant" description of v:** $\qquad v_a = (v, e_a) = \eta_{\mu\nu} v^\mu e^\nu_a \qquad$ **"relational/frame dressed observables"**

(describes v relative to frame)

Warmup: Special relativity with internal frames



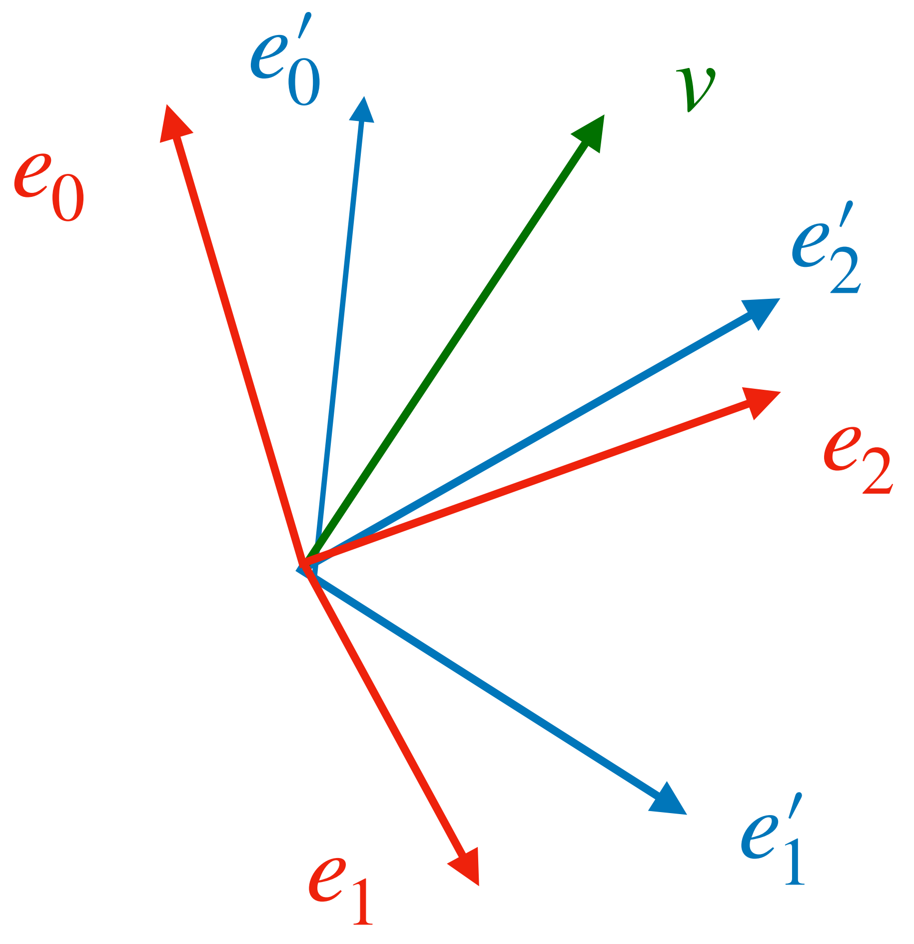
introduce second internal frame $e'_{a'}$

$$v_a = v^\mu \eta_{\mu\nu} e^\nu_a = v^\mu e'_{\mu a'} e'^{a'}_\nu e^\nu_a = v_{a'} \Lambda^{a'}{}_a$$

relational observable rel. to e

relational observable rel. to e'

Warmup: Special relativity with internal frames



introduce second internal frame $e'_{a'}$

$$v_a = v^\mu \eta_{\mu\nu} e^\nu_a = v^\mu e'_{\mu a'} e'^{a'}_\nu e^\nu_a = v_{a'} \Lambda^{a'}{}_a$$

relational observable rel. to e

relational observable rel. to e'

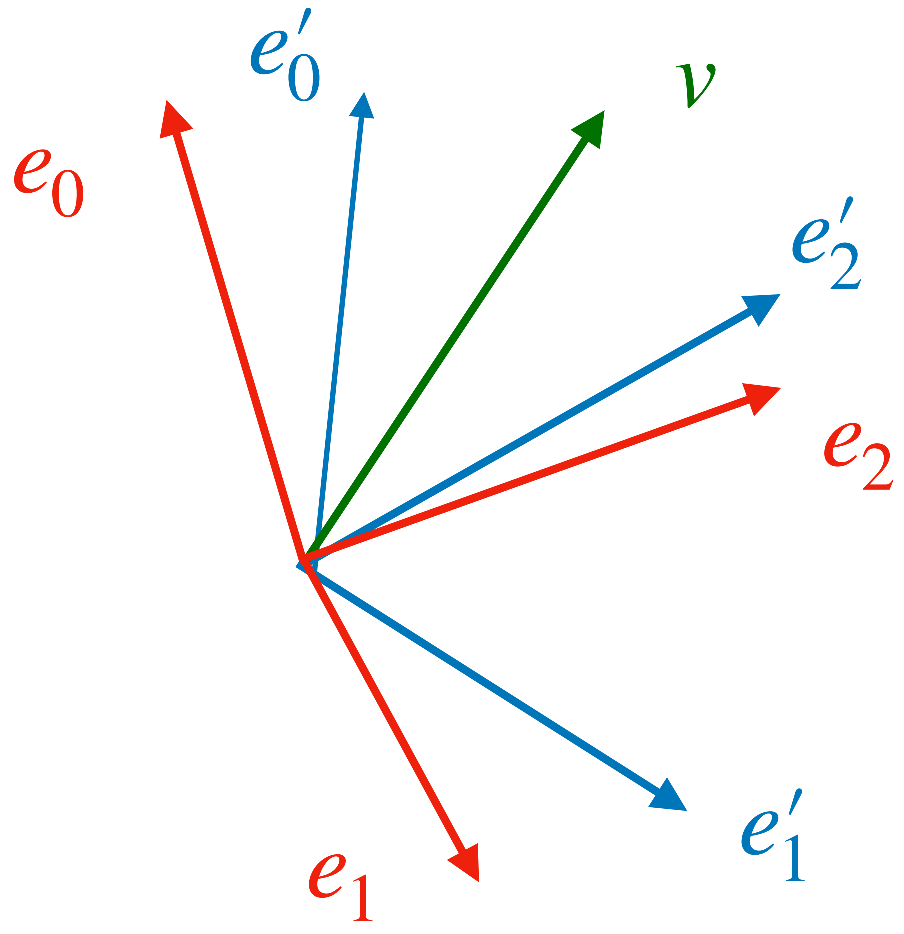
symmetry induced RF transformation:

$$\Lambda^{a'}{}_a = e'^{a'}_\mu e^\mu_a \in \mathrm{SO}_+(3,1)$$

is relational observable describing 1st rel. to 2nd frame

$$\begin{tikzcd} & v^\mu \arrow[dl, "e^\mu_a"'] \arrow[dr, "e'^\mu_{a'}"] & \\ v_a = \eta_{\mu\nu} e^\mu_a v^\nu & & v_{a'} = \eta_{\mu\nu} e'^\mu_{a'} v^\nu \arrow[ll, "\Lambda^{a'}{}_a \in \mathrm{SO}_+(3,1)"'] \end{tikzcd}$$

Warmup: Special relativity with internal frames



introduce second internal frame $\quad e'_{a'}$

$$v_a = v^\mu \eta_{\mu\nu} e^\nu_a = v^\mu e'_{\mu a'} e'^{a'}_\nu e^\nu_a = v_{a'} \Lambda^{a'}{}_a$$

relational observable rel. to e $\qquad$ relational observable rel. to e'

symmetry induced RF transformation:

$\Lambda^{a'}{}_a = e'^{a'}_\mu e^\mu_a \in \mathrm{SO}_+(3,1)$

is relational observable describing 1st rel. to 2nd frame

$$\begin{tikzcd} & v^\mu \arrow[dl, "e^\mu_a"'] \arrow[dr, "e'^\mu_{a'}"] & \\ v_a = \eta_{\mu\nu} e^\mu_a v^\nu & & v_{a'} = \eta_{\mu\nu} e'^\mu_{a'} v^\nu \arrow[ll, "\Lambda^{a'}{}_a \in \mathrm{SO}_+(3,1)"'] \end{tikzcd}$$

gauge induced RF transformation:

$\Lambda^{\nu'}{}_\mu \in \mathrm{SO}_+(3,1)$ **looks the same as** $\Lambda^{a'}{}_a$

coordinate change via gauge fixings

$$\begin{tikzcd} & v_a = \eta_{\mu\nu} e^\nu_a v^\mu \arrow[dl, "e^\nu_a = \delta^\nu_a"'] \arrow[dr, "e'^{\nu'}_{a'} = \delta^{\nu'}_{a'}"] & \\ v_a \hat{=} v_\mu & & v_a \hat{=} \Lambda^{\nu'}{}_\mu v_{\nu'} \arrow[ll, "\Lambda^{\nu'}{}_\mu \in \mathrm{SO}_+(3,1)"'] \end{tikzcd}$$

Unifying picture of dynamical frame covariance

	Special relativity	Quantum relativity	Gauge relativity	General relativity
frame orientations (group valued frame)	tetrad $e^\mu_a \in \mathrm{SO}_+(3,1)$			
gauge transf./ gauge covariance	spacetime Lorentz tr. $\Lambda^\mu{}_\nu \, e^\nu_a \quad \Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$			
frame reorientations/ "symmetries" (act only on the frame)	frame Lorentz tr. $\Lambda_a{}^b \, e^\mu_b \quad \Lambda_a{}^b \in \mathrm{SO}_+(3,1)$			
relational observables $O_{F,R}(g)$ "what's value of F when RF is in orientation g?"	$F_{\mu\ldots} e^\mu_a \Lambda_b{}^a(g^{-1})\cdots$			
"jumping into RF perspective" (gauge fixing)	$e^\mu_a = \Lambda^\mu{}_a(g)$			
RF change 1 (rel. cond. gauge transf.)	$\Lambda^\mu{}_\nu(e,e') = \Lambda^\mu{}_{a'}(g') e'^{a'}_\kappa \, e^\kappa_b \Lambda_\nu{}^b(g)$			
RF change 2 (rel. cond. symmetry) (1st frame relative to 2nd)	$\Lambda^{a'}{}_b(e,e') = e'^{a'}_\mu \, e^\mu_b$			

2 commuting group actions

Unifying picture of dynamical frame covariance

2 commuting group actions

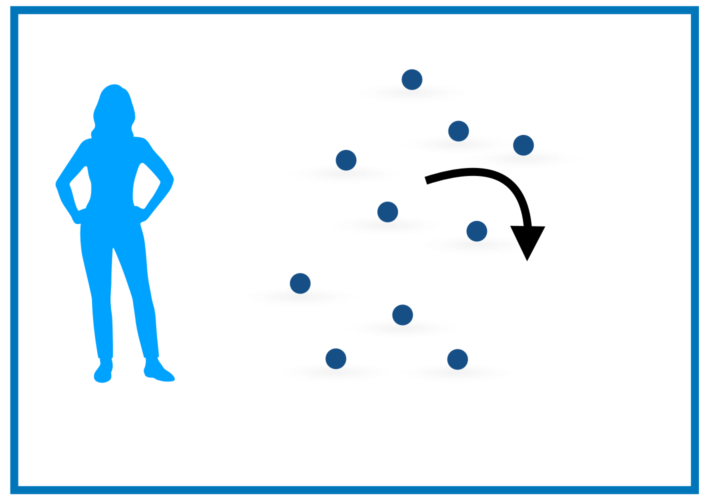
	Special relativity	Quantum relativity	Gauge relativity	General relativity
frame orientations (group valued frame)	tetrad $e^\mu_a \in \mathrm{SO}_+(3,1)$			
gauge transf./ gauge covariance	spacetime Lorentz tr. $\Lambda^\mu{}_\nu\, e^\nu_a \quad \Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$			
frame reorientations/ “symmetries” (act only on the frame)	frame Lorentz tr. $\Lambda_a{}^b\, e^\mu_b \quad \Lambda_a{}^b \in \mathrm{SO}_+(3,1)$			
relational observables $O_{F,R}(g)$ “what’s value of F when RF is in orientation g?”	$F_{\mu\ldots} e^\mu_a \Lambda_b{}^a(g^{-1})\cdots$			
“jumping into RF perspective” (gauge fixing)	$e^\mu_a = \Lambda^\mu{}_a(g)$			
RF change 1 (rel. cond. gauge transf.)	$\Lambda^\mu{}_\nu(e,e') = \Lambda^\mu{}_{a'}(g')e'^{a'}_\kappa\, e^\kappa_b \Lambda_\nu{}^b(g)$			
RF change 2 (rel. cond. symmetry) (1st frame relative to 2nd)	$\Lambda^{a'}{}_b(e,e') = e'^{a'}_\mu\, e^\mu_b$			

Quantum reference frames

… or frames in relative superposition

Quantum reference frames

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*



setup relative to external (possibly fictitious) frame:

$$\mathscr{H}_{\text{kin}} = \mathscr{H}_R \otimes \mathscr{H}_S$$

space of externally distinguishable states

frame

system

unimodular Lie group as gauge transformations (e.g. Galilei, Poincaré, SU(2), reparametrizations…):

$$U_{RS}(g) = U_R(g) \otimes U_S(g) \qquad g \in G$$

analog of $\Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$ in SR

frame orientations: coherent state system (gauge transf. from left/gauge cov.)

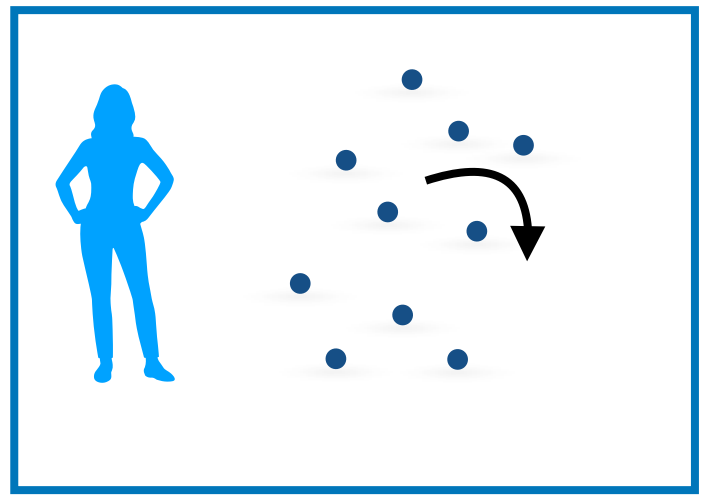
$$|\phi(g)\rangle_R \mapsto U_R(g')\,|\phi(g)\rangle_R = |\phi(g'g)\rangle_R$$

analog of $\Lambda^\mu{}_\nu e^\mu_a$ with $e^\mu{}_a \in \mathrm{SO}_+(3,1)$ in SR

e.g. clock reading, position of reference particle, …

Quantum reference frames

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*



setup relative to external (possibly fictitious) frame:

$$\mathscr{H}_{\mathrm{kin}} = \underbrace{\mathscr{H}_R}_{\text{frame}} \otimes \underbrace{\mathscr{H}_S}_{\text{system}}$$

space of externally distinguishable states

unimodular Lie group as gauge transformations (e.g. Galilei, Poincaré, SU(2), reparametrizations…):

$$U_{RS}(g) = U_R(g) \otimes U_S(g) \qquad g \in G$$

analog of $\Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$ in SR

frame orientations: coherent state system (gauge transf. from left/gauge cov.)

$$|\phi(g)\rangle_R \mapsto U_R(g')\,|\phi(g)\rangle_R = |\phi(g'g)\rangle_R$$

analog of $\Lambda^\mu{}_\nu e^\mu_a$ with $e^\mu{}_a \in \mathrm{SO}_+(3,1)$ in SR

furnish covariant POVM

$$\int_G dg\,|\phi(g)\rangle\langle\phi(g)|_R = \mathbf{1}_R$$

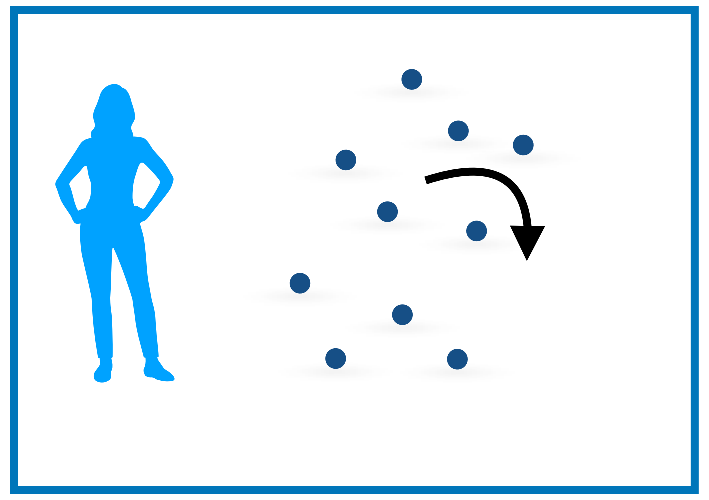
$\Rightarrow$ probabilistic interpretation

orientation states typically not orthogonal $\quad \langle\phi(g)\,|\,\phi(g')\rangle \nsim \delta(g,g')$

$\Rightarrow$ encompasses imperfect QRFs

Quantum reference frames

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*



setup relative to external (possibly fictitious) frame:

$$\mathscr{H}_{\mathrm{kin}} = \mathscr{H}_R \otimes \mathscr{H}_S$$

frame

system

space of externally distinguishable states

unimodular Lie group as gauge transformations (e.g. Galilei, Poincaré, SU(2), reparametrizations…):

$$U_{RS}(g) = U_R(g) \otimes U_S(g) \qquad g \in G$$

analog of $\Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$ in SR

frame orientations: coherent state system (gauge transf. from left/gauge cov.)

$$|\phi(g)\rangle_R \mapsto U_R(g')|\phi(g)\rangle_R = |\phi(g'g)\rangle_R$$

analog of $\Lambda^\mu{}_\nu e^\mu_a$ with $e^\mu{}_a \in \mathrm{SO}_+(3,1)$ in SR

“symmetries/frame reorientations” (act from right):

$$V_R(g')|\phi(g)\rangle_R = |\phi(gg'^{-1})\rangle_R$$

analog of $\Lambda^a{}_b e^\mu_a$ in SR

$\Rightarrow$ 2 commuting group actions, but V_R doesn't exist for all reps

Relational observables

recall relational observables from SR $\quad v_a = v^\mu \, \eta_{\mu\nu} \, e^\nu_a$

Relational observables

recall relational observables from SR $\quad v_a = v^\mu \, \eta_{\mu\nu} \boxed{e^\nu_a}$

frame-orientation conditional gauge transformation

Relational observables

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
PH, Smith, Lock 1912.00033

recall relational observables from SR $v_a = v^\mu \, \eta_{\mu\nu} \boxed{e^\nu_a}$

frame-orientation conditional gauge transformation

<u>Relational observables for general groups through G-twirl:</u>

$$F_{f_S,R}(g) = \int_G d\tilde{g} \, U_{RS}(\tilde{g}) \big(\, |\phi(g)\rangle\langle\phi(g)|_R \otimes f_S \big) \, U^\dagger_{RS}(\tilde{g})$$

"what's the value of f_S when R is in orientation g?"

Relational observables

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
PH, Smith, Lock 1912.00033

recall relational observables from SR $v_a = v^\mu \, \eta_{\mu\nu} \boxed{e_a^\nu}$

frame-orientation conditional gauge transformation

Relational observables for general groups through G-twirl:

$$F_{f_S,R}(g) = \int_G d\tilde{g}\, U_{RS}(\tilde{g}) \big(|\phi(g)\rangle\langle\phi(g)|_R \otimes f_S \big)\, U_{RS}^\dagger(\tilde{g})$$

"what's the value of f_S when R is in orientation g?"

frame-orientation conditional gauge transformation (controlled unitary)

Relational observables

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
PH, Smith, Lock 1912.00033

recall relational observables from SR $v_a = v^\mu \eta_{\mu\nu} \boxed{e^\nu_a}$

frame-orientation conditional gauge transformation

Relational observables for general groups through G-twirl:

$$F_{f_S,R}(g) = \int_G d\tilde{g}\, U_{RS}(\tilde{g}) \Big(|\phi(g)\rangle\langle\phi(g)|_R \otimes f_S \Big)\, U^\dagger_{RS}(\tilde{g})$$

"what's the value of f_S when R is in orientation g?"

frame-orientation conditional gauge transformation (controlled unitary)

$\Rightarrow$ **gauge-inv.** $[F_{f_S,R}, U_{RS}(g')] = 0$

relational observables in the sense of Rovelli, Dittrich, Thiemann, ….

Relational observables

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
PH, Smith, Lock 1912.00033

recall relational observables from SR $\quad v_a = v^\mu \eta_{\mu\nu} \boxed{e^\nu_a}$

frame-orientation conditional gauge transformation

<u>Relational observables for general groups through G-twirl:</u>

$$F_{f_S,R}(g) = \int_G d\tilde{g}\, U_{RS}(\tilde{g}) \big(|\phi(g)\rangle\langle\phi(g)|_R \otimes f_S \big)\, U^\dagger_{RS}(\tilde{g})$$

"what's the value of f_S when R is in orientation g?"

frame-orientation conditional gauge transformation (controlled unitary)

$\Rightarrow$ **gauge-inv.** $\quad [F_{f_S,R}, U_{RS}(g')] = 0$

relational observables in the sense of Rovelli, Dittrich, Thiemann, ….

$\Rightarrow$ **preserves algebraic properties of f_S (*-homomorphism) only on gauge-inv. Hilbert space $\mathcal{H}_{\text{phys}}$:** $\qquad |\psi\rangle_{\text{phys}} = U_{RS}(g)\, |\psi\rangle_{\text{phys}}$

Perspective-neutral formulation of QRF covariance

introduce 2nd frame $$\mathscr{H}_{\text{kin}} = \mathscr{H}_{R_1} \otimes \mathscr{H}_{R_2} \otimes \mathscr{H}_S$$ (orientations again via coherent states)

frame 1 frame 2 system

gauge transformations $$U_{R_1R_2S}(g) = U_{R_1}(g) \otimes U_{R_2}(g) \otimes U_S(g)$$ (factors could carry different reps.)

Perspective-neutral formulation of QRF covariance

introduce 2nd frame $\mathcal{H}_{\text{kin}} = \mathcal{H}_{R_1} \otimes \mathcal{H}_{R_2} \otimes \mathcal{H}_S$ (orientations again via coherent states)

$\mathcal{H}_{R_1}$: frame 1, $\mathcal{H}_{R_2}$: frame 2, $\mathcal{H}_S$: system

gauge transformations $U_{R_1R_2S}(g) = U_{R_1}(g) \otimes U_{R_2}(g) \otimes U_S(g)$ (factors could carry different reps.)

gauge-inv. (external frame-indep.) physical Hilbert space $\mathcal{H}_{\text{phys}}$ with $|\psi\rangle_{\text{phys}} = U_{R_1R_2S}(g)\,|\psi\rangle_{\text{phys}}$

is space of relational equivalence classes of states

$\Rightarrow$ many different ways in describing same invariant $|\psi_{\text{phys}}\rangle$

$\Rightarrow$ associate with different internal QRF choices: redundant = reference DoFs

$\mathcal{H}_{\text{phys}}$ is a perspective-neutral space: description of physics prior to having chosen internal reference system relative to which remaining DoFs are described

gauge-induced QRF changes: quantum coordinate changes

$$
\begin{tikzcd}
 & v_a = \eta_{\mu\nu} e^{\nu}_{a} v^{\mu} \arrow[dl, "e^{\nu}_{a} = \delta^{\nu}_{a}"'] \arrow[dr, "e'^{\nu'}_{a'} = \delta^{\nu'}_{a'}"] & \\
v_a \mathrel{\hat{=}} v_{\mu} & & v_a \mathrel{\hat{=}} \Lambda^{\nu'}{}_{\mu} v_{\nu'} \arrow[ll, "\Lambda^{\nu'}{}_{\mu} \in \mathrm{SO}_{+}(3,1)"']
\end{tikzcd}
$$

recall: “jumping into frame perspective” through gauge-fixing

gauge-induced QRF changes: quantum coordinate changes

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
PH, Smith, Lock 1912.00033

$$\begin{tikzcd}
 & v_a = \eta_{\mu\nu} e^{\nu}_a v^{\mu} \arrow[dl, "e^{\nu}_a = \delta^{\nu}_a"'] \arrow[dr, "e'^{\nu'}_{a'} = \delta^{\nu'}_{a'}"] & \\
v_a \hat{=} v_\mu & & v_a \hat{=} \Lambda^{\nu'}{}_{\mu} v_{\nu'} \arrow[ll, "\Lambda^{\nu'}{}_{\mu} \in \mathrm{SO}_+(3,1)"']
\end{tikzcd}$$

recall: "jumping into frame perspective" through gauge-fixing

Perspective-neutral physical Hilbert space

$\mathscr{H}_{\text{phys}}$

$$\varphi_{R_1}(g_1) = \langle \phi(g_1) |_{R_1} \otimes \mathbf{1}_{R_2 S}$$

States relative to internal perspective of R_1

gauge-induced QRF changes: quantum coordinate changes

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
PH, Smith, Lock 1912.00033

$$\begin{tikzcd}[column sep=small] & v_a = \eta_{\mu\nu} e^{\nu}_{a} v^{\mu} \arrow[dl, "e^{\nu}_{a} = \delta^{\nu}_{a}"'] \arrow[dr, "e'^{\nu'}_{a'} = \delta^{\nu'}_{a'}"] & \\ v_a \hat{=} v_\mu & & v_a \hat{=} \Lambda^{\nu'}{}_{\mu} v_{\nu'} \arrow[ll, "\Lambda^{\nu'}{}_{\mu} \in \mathrm{SO}_+(3,1)"'] \end{tikzcd}$$

recall: “jumping into frame perspective” through gauge-fixing

Perspective-neutral physical Hilbert space

$\mathscr{H}_{\mathrm{phys}}$

$$\varphi_{R_1}(g_1) = \langle \phi(g_1) |_{R_1} \otimes \mathbf{1}_{R_2 S}$$

States relative to internal perspective of R_1

generalized Page-Wootters construction:

$$|\psi_{R_2 S}(g_1)\rangle = \varphi_{R_1}(g_1) |\psi\rangle_{\mathrm{phys}}$$

$$U_{R_2 S}(g) |\psi_{R_2 S}(g_1)\rangle = |\psi_{R_2 S}(g g_1)\rangle$$

gauge-induced QRF changes: quantum coordinate changes

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
PH, Smith, Lock 1912.00033

$$v_a = \eta_{\mu\nu} e^\nu_a v^\mu$$

$e^\nu_a = \delta^\nu_a$

$e'^{\nu'}_{a'} = \delta^{\nu'}_{a'}$

$\Lambda^{\nu'}{}_\mu \in \mathrm{SO}_+(3,1)$

$v_a \hat{=} v_\mu$

$v_a \hat{=} \Lambda^{\nu'}{}_\mu v_{\nu'}$

recall: "jumping into frame perspective" through gauge-fixing

Perspective-neutral physical Hilbert space

$\mathscr{H}_{\text{phys}}$

redundancy

fix/condition on orientation of R_1

$$\varphi_{R_1}(g_1) = \langle \phi(g_1) |_{R_1} \otimes \mathbf{1}_{R_2 S}$$

invertible for perspective-neutral (not kinematical) states!

generalized Page-Wootters construction:

$$|\psi_{R_2S}(g_1)\rangle = \varphi_{R_1}(g_1)|\psi\rangle_{\text{phys}}$$

$$U_{R_2S}(g)|\psi_{R_2S}(g_1)\rangle = |\psi_{R_2S}(gg_1)\rangle$$

States relative to internal perspective of R_1

no redundancy

gauge-induced QRF changes: quantum coordinate changes

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
PH, Smith, Lock 1912.00033

$$\begin{tikzcd}
 & v_a = \eta_{\mu\nu} e^\nu_a v^\mu \arrow[dl, "e^\nu_a = \delta^\nu_a"'] \arrow[dr, "e'^{\nu'}_{a'} = \delta^{\nu'}_{a'}"] & \\
v_a \mathrel{\hat{=}} v_\mu & & v_a \mathrel{\hat{=}} \Lambda^{\nu'}{}_\mu v_{\nu'} \arrow[ll, "\Lambda^{\nu'}{}_\mu \in \mathrm{SO}_+(3,1)"']
\end{tikzcd}$$

recall: "jumping into frame perspective" through gauge-fixing

Perspective-neutral physical Hilbert space

$\mathscr{H}_{\text{phys}}$

$$\varphi_{R_1}(g_1) = \langle \phi(g_1) |_{R_1} \otimes \mathbf{1}_{R_2 S}$$

$$\varphi_{R_2}(g_2) = \langle \phi(g_2) |_{R_2} \otimes \mathbf{1}_{R_1 S}$$

States relative to internal perspective of R_1

$$\varphi_{R_2} \circ \varphi_{R_1}^{-1}$$

states relative to internal perspective of R_2

get unitary QRF changes!

gauge-induced QRF changes: quantum coordinate changes

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
PH, Smith, Lock 1912.00033

$$
\begin{tikzcd}
 & v_a = \eta_{\mu\nu} e^{\nu}_{a} v^{\mu} \arrow[dl, "e^{\nu}_{a} = \delta^{\nu}_{a}"'] \arrow[dr, "e'^{\nu'}_{a'} = \delta^{\nu'}_{a'}"] & \\
v_a \hat{=} v_\mu & & v_a \hat{=} \Lambda^{\nu'}{}_{\mu} v_{\nu'} \arrow[ll, "\Lambda^{\nu'}{}_{\mu} \in \mathrm{SO}_+(3,1)"']
\end{tikzcd}
$$

recall: “jumping into frame perspective” through gauge-fixing

Perspective-neutral algebra (rel. observables)

$\mathscr{A}_{\text{phys}}$

$$\varphi_{R_1}(g_1) = \langle \phi(g_1) |_{R_1} \otimes \mathbf{1}_{R_2 S}$$

$$\varphi_{R_2}(g_2) = \langle \phi(g_2) |_{R_2} \otimes \mathbf{1}_{R_1 S}$$

$$\varphi_{R_1}(g_1)\, F_{f_{R_2 S}, R_1}(g_1)\, \varphi_{R_1}^{-1}(g_1) = f_{R_2 S}$$

Observables relative to internal perspective of R_1

$$\varphi_{R_2} \circ \varphi_{R_1}^{-1}$$

observables relative to internal perspective of R_2

Symmetry-induced QRF changes

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*

changes of relational observables, recall:

$$v_a = \boxed{v_{a'} \Lambda^{a'}{}_{a}}$$

relational observable rel. to e

relational observable rel. to e'

- **RF transformation between two frames is** $\Lambda^{a'}{}_{a} = e'^{a'}_{\mu} e^{\mu}_{a} \in \mathrm{SO}_{+}(3,1)$
  relational observable describing 1st rel. to 2nd frame

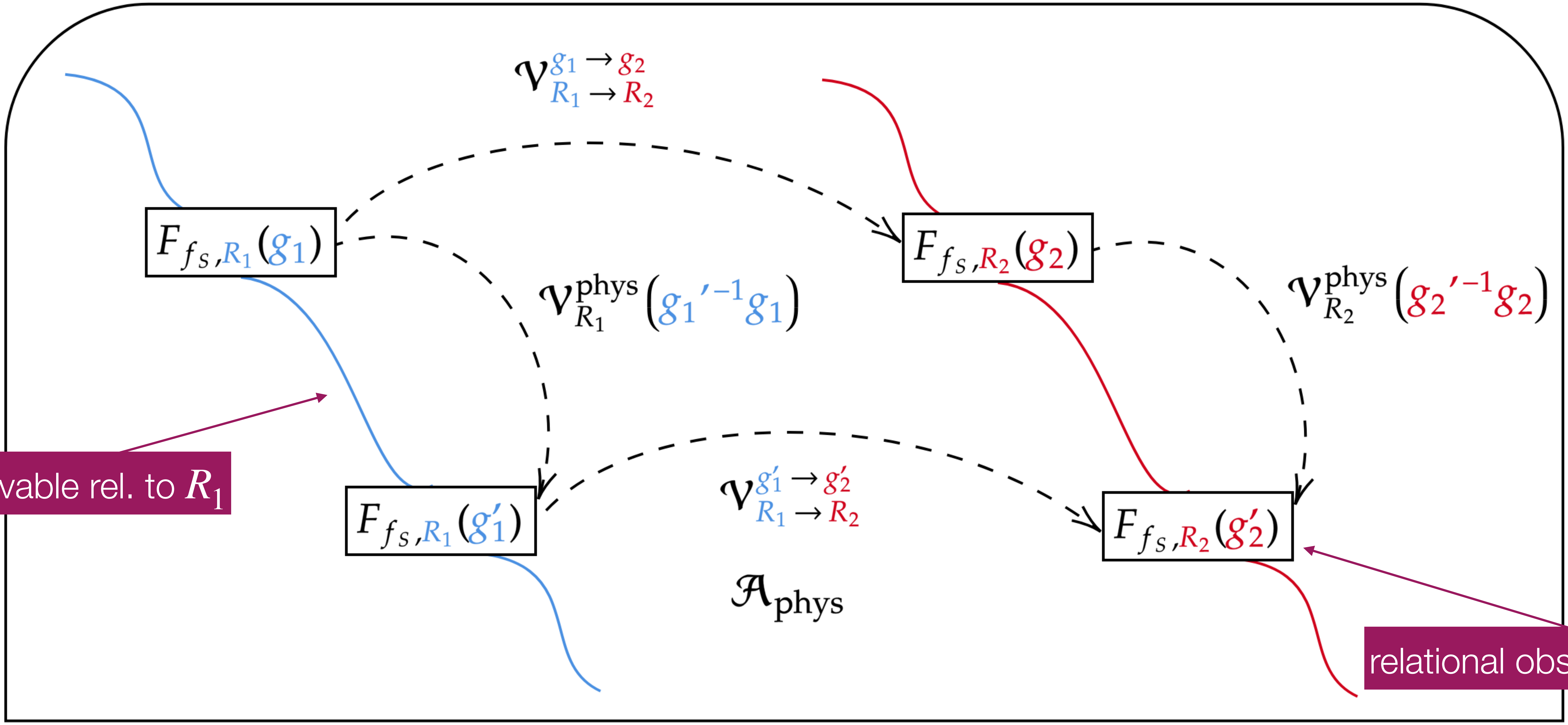
relation-conditional frame reorientation

Symmetry-induced QRF changes

de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*

can do analog in QT: G-twirl for symmetries $V_{R_1}(g) \otimes \mathbf{1}_{R_2 S}$

relation-conditional frame reorientation

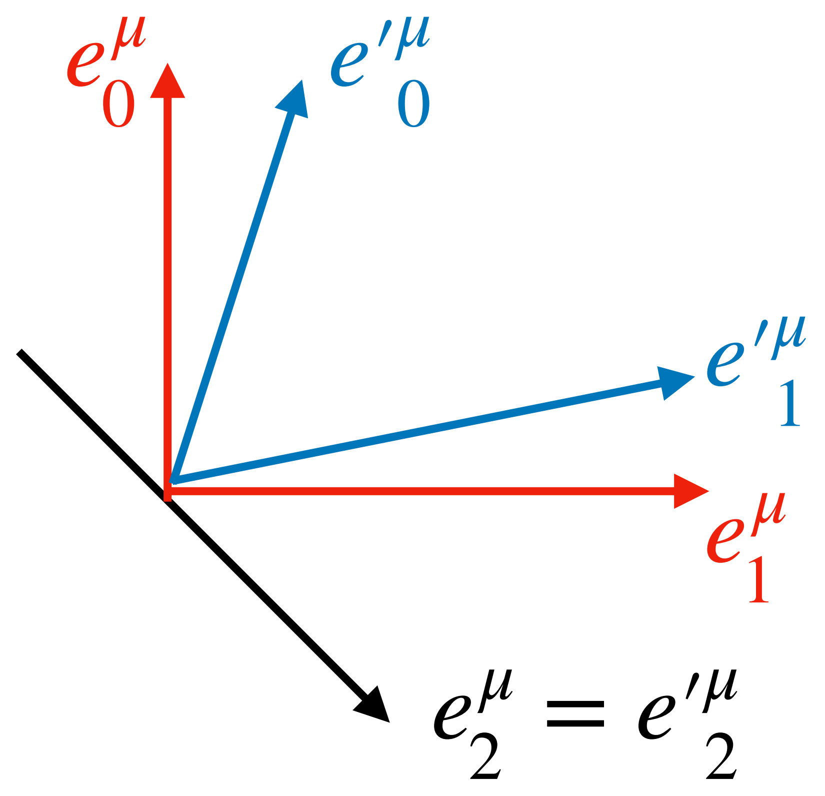


relational observable rel. to R_1

relational observable rel. to R_2

Quantum relativity of subsystems

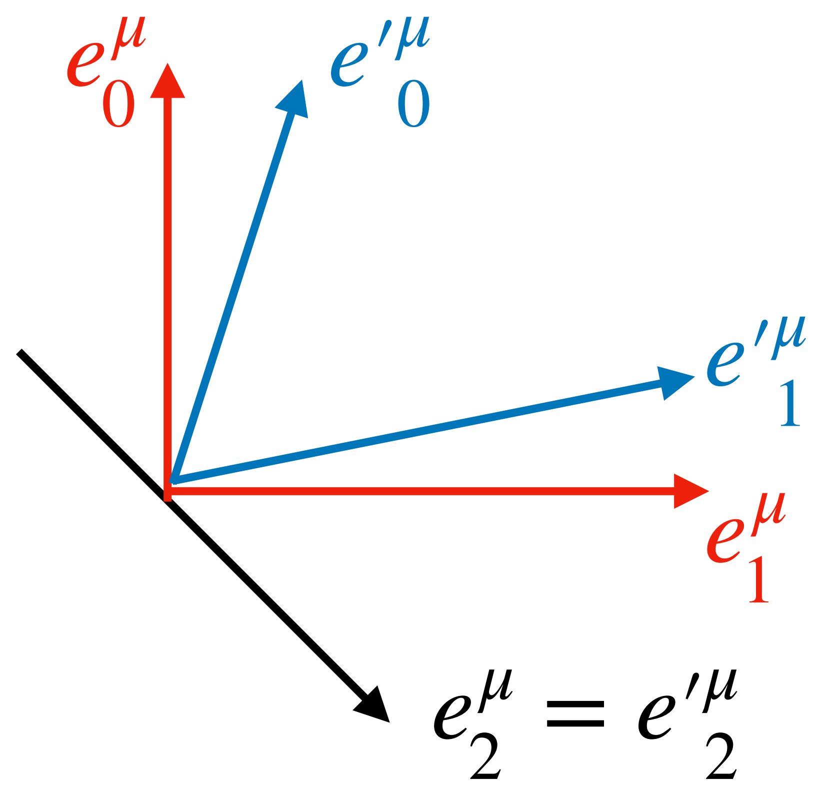
Ahmad, Galley, PH, Lock, Smith PRL ’22;
de la Hamette, Galley, PH, Loveridge, Müller *2110.13824;*
Kotecha, Mele, PH *to appear*



relativity of simultaneity:
different observers decompose space of (relational) length observables in different ways into space and time

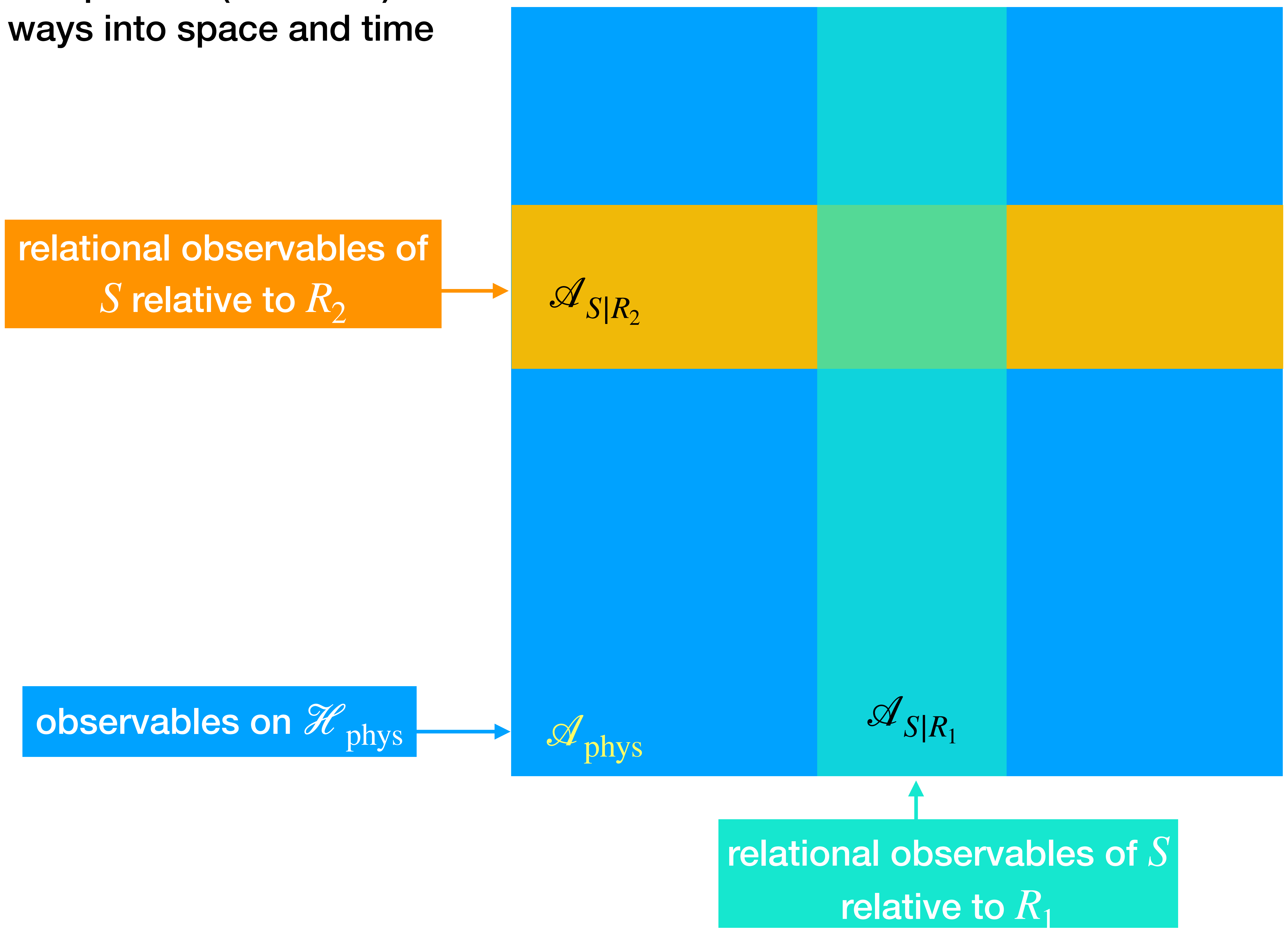
Quantum relativity of subsystems

Ahmad, Galley, PH, Lock, Smith PRL '22;
de la Hamette, Galley, PH, Loveridge, Müller *2110.13824;*
Kotecha, Mele, PH *to appear*



relativity of simultaneity:
different observers decompose space of (relational) length observables in different ways into space and time

3 kinematical subsystems $\mathscr{H}_{\text{kin}} = \mathscr{H}_{R_1} \otimes \mathscr{H}_{R_2} \otimes \mathscr{H}_S$



Recall: kinematical vs. relational subsystems

leaves description of S' relative to R invariant, but changes it relative to R'

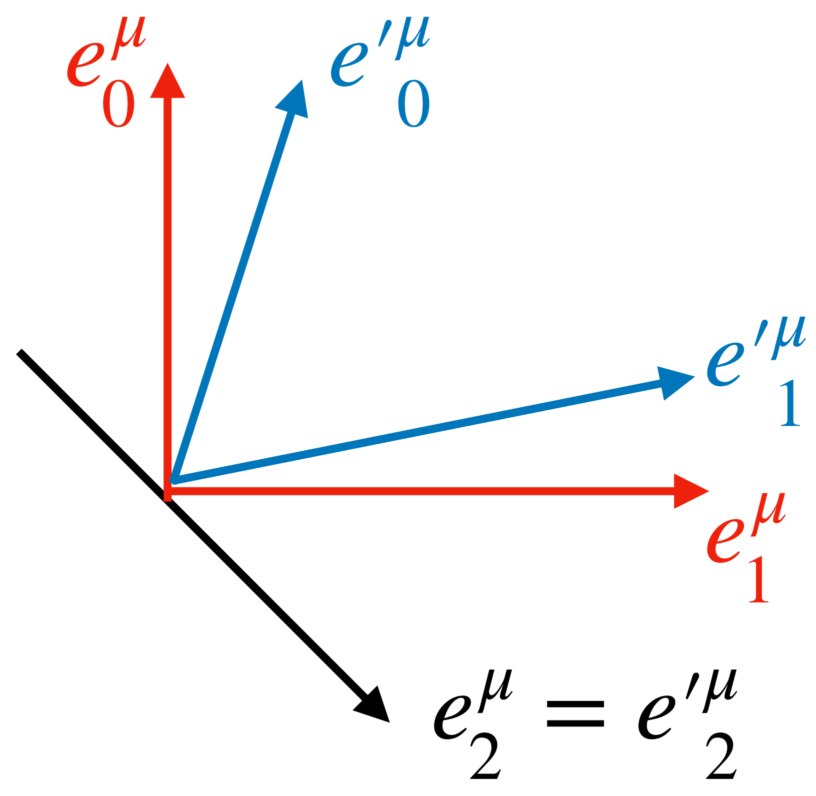
green balls: subsystem S'

leaves description of S' rel. to external frame invariant, but changes description relative to frame R

1. **kinematical and relational (gauge inv.) notion of subsystem distinct**
2. **gauge inv. notion of subsystem depends on choice of RF**

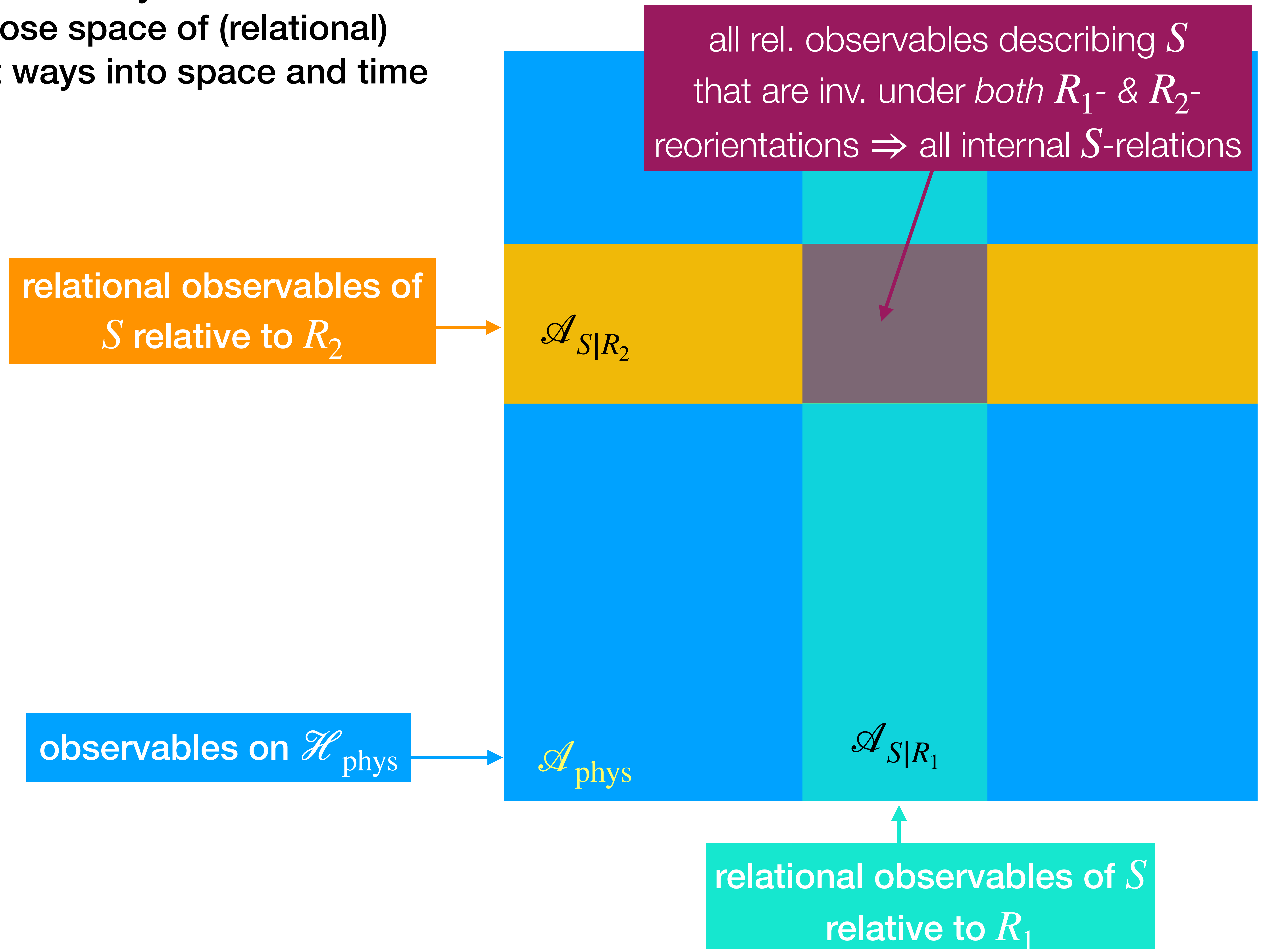
Quantum relativity of subsystems

Ahmad, Galley, PH, Lock, Smith PRL '22;
de la Hamette, Galley, PH, Loveridge, Müller *2110.13824;*
Kotecha, Mele, PH *to appear*



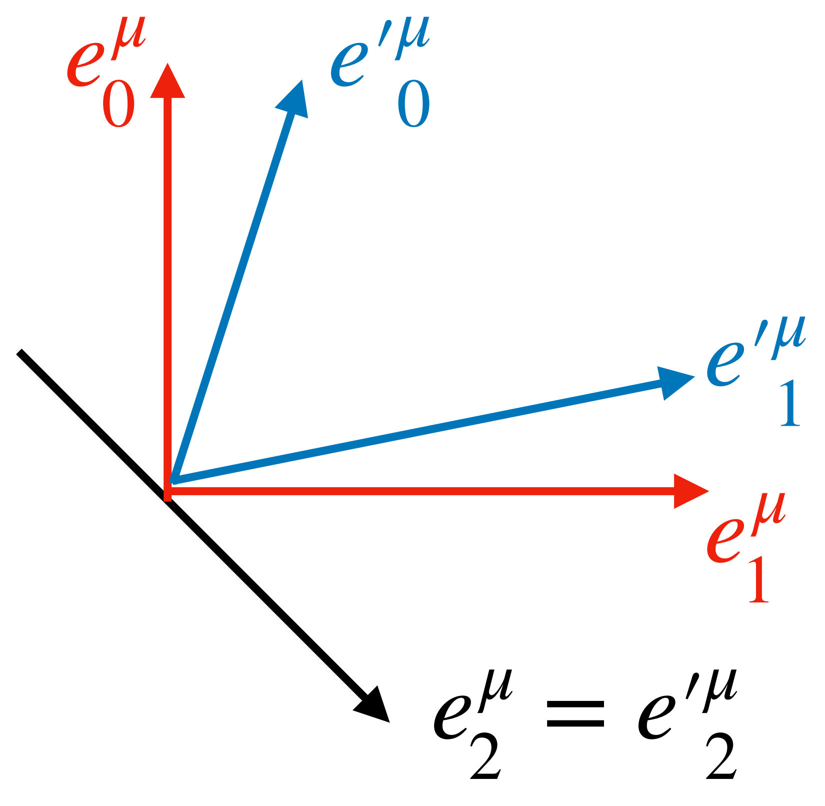
relativity of simultaneity:
different observers decompose space of (relational) length observables in different ways into space and time

3 kinematical subsystems $\mathscr{H}_{\text{kin}} = \mathscr{H}_{R_1} \otimes \mathscr{H}_{R_2} \otimes \mathscr{H}_S$

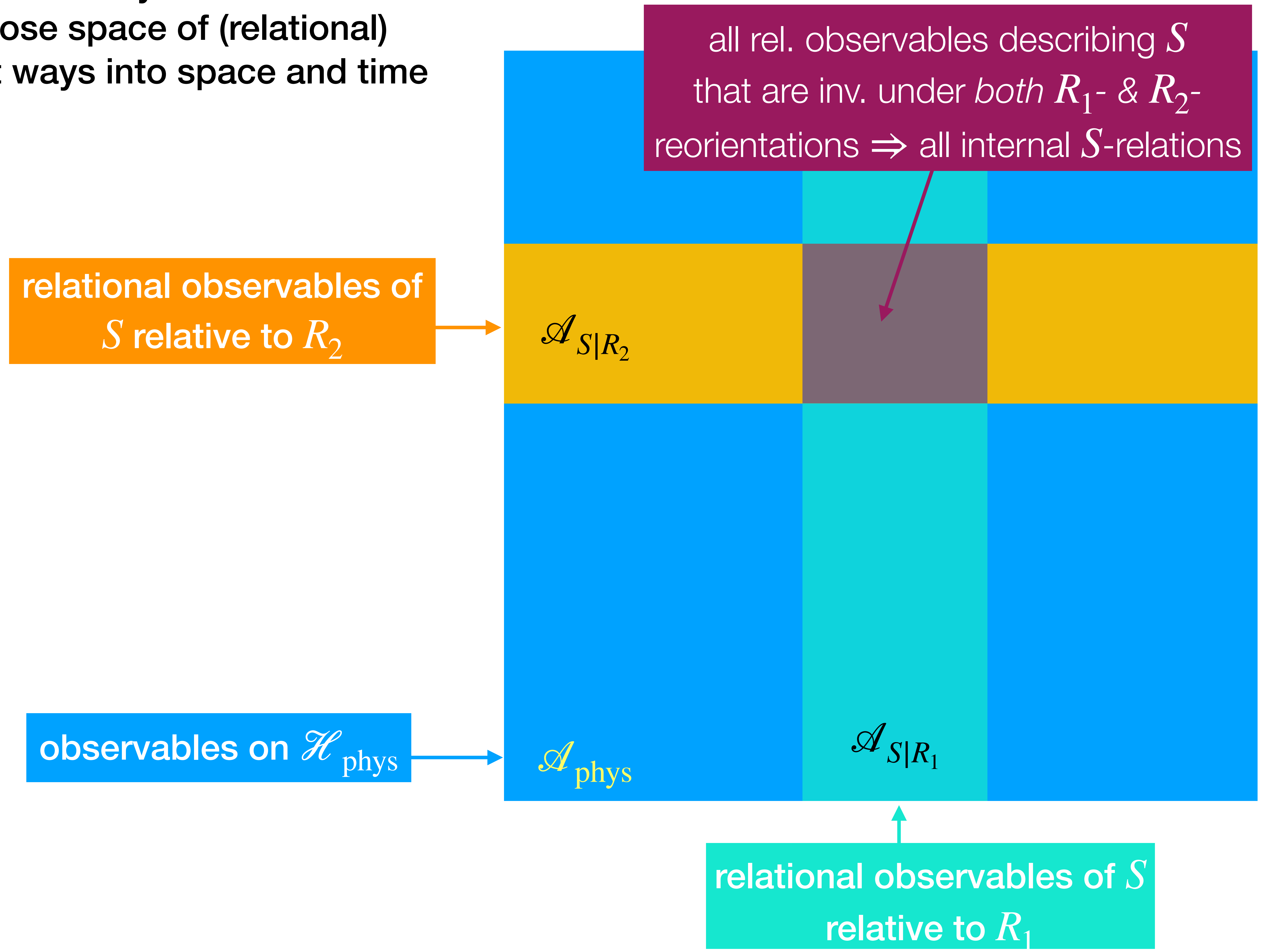


Quantum relativity of subsystems

Ahmad, Galley, PH, Lock, Smith PRL '22;
de la Hamette, Galley, PH, Loveridge, Müller *2110.13824;*
Kotecha, Mele, PH *to appear*



relativity of simultaneity:
different observers decompose space of (relational) length observables in different ways into space and time



3 kinematical subsystems $\mathscr{H}_{\rm kin} = \mathscr{H}_{R_1} \otimes \mathscr{H}_{R_2} \otimes \mathscr{H}_S$

$\Rightarrow$ different relational (inv.) ways to refer to a kinematical subsystem

$\Rightarrow$ different relational observable subalgebras inside total invariant algebra

$\Rightarrow$ induce different gauge-inv. tensor factorizations
(not in general factorization across $R_j | R_i$ and $S | R_i$)

$\Rightarrow$ different appearance of same physics

Upshot: frame-dependent gauge-inv. properties

Ahmad, Galley, PH, Lock, Smith PRL '22;
de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
Kotecha, Mele, PH *to appear*

"frames R_1 and R_2 mean different inv. DoFs when they refer to subsystem S"

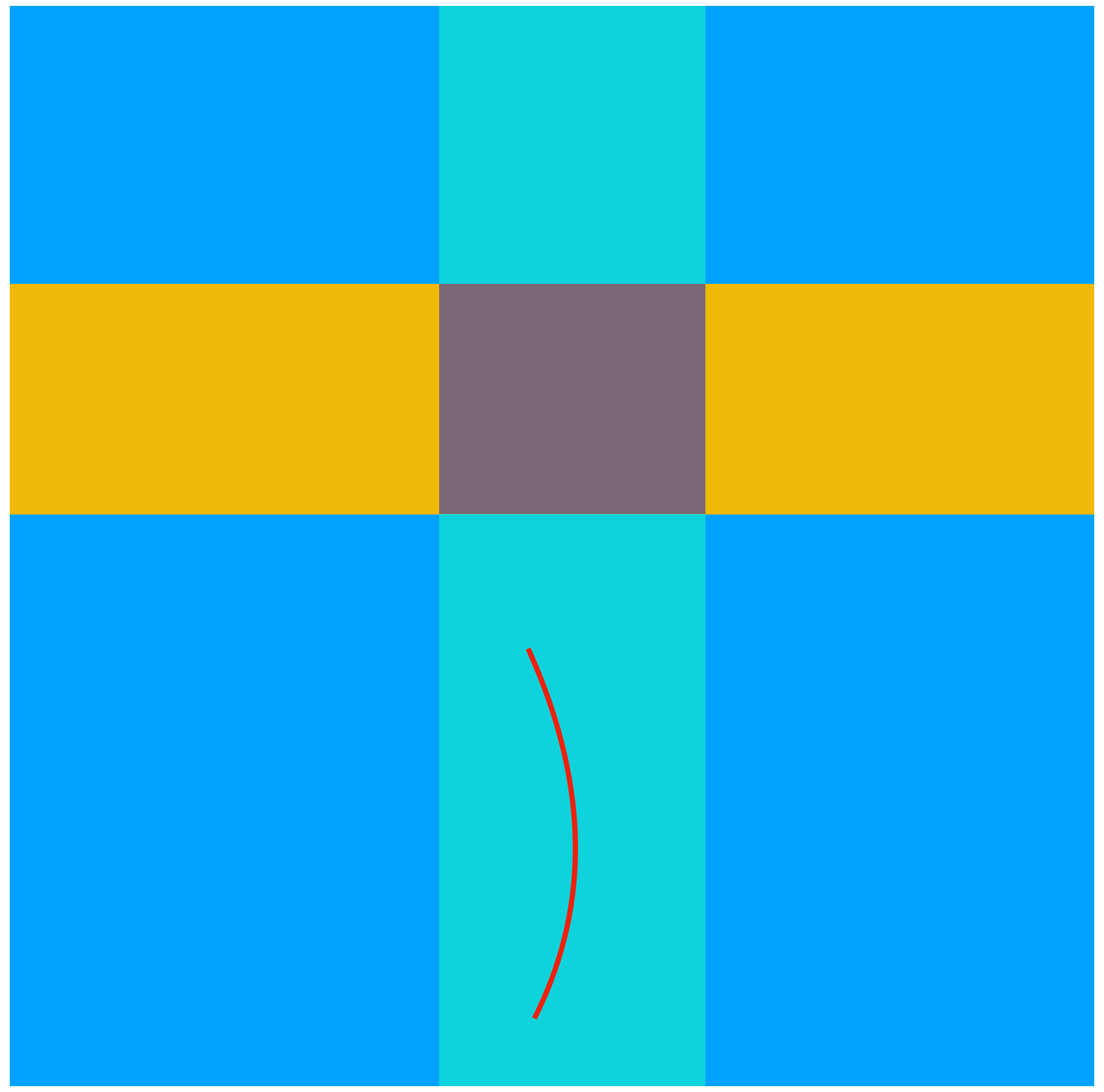
if factorizability in two frame perspectives, i.e.

$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{S|R_1} \otimes \mathscr{A}_{R_2|R_1} \simeq \mathscr{A}_{S|R_2} \otimes \mathscr{A}_{R_1|R_2}$ but $\mathscr{A}_{S|R_2} \neq \mathscr{A}_{S|R_1}$

then correlations/entanglement of S with its complement will in general differ in two perspectives

(see also Giacomini, Castro-Ruiz, Brukner '17; Castro-Ruiz, Oreshkov '21)

$\Rightarrow$ gauge-inv. entanglement entropy in general $S(\rho_{S|R_2}) \neq S(\rho_{S|R_1})$ for same global physical state



Upshot: frame-dependent gauge-inv. properties

Ahmad, Galley, PH, Lock, Smith PRL '22;
de la Hamette, Galley, PH, Loveridge, Müller *2110.13824*
Kotecha, Mele, PH *to appear*

"frames R_1 and R_2 mean different inv. DoFs when they refer to subsystem S"

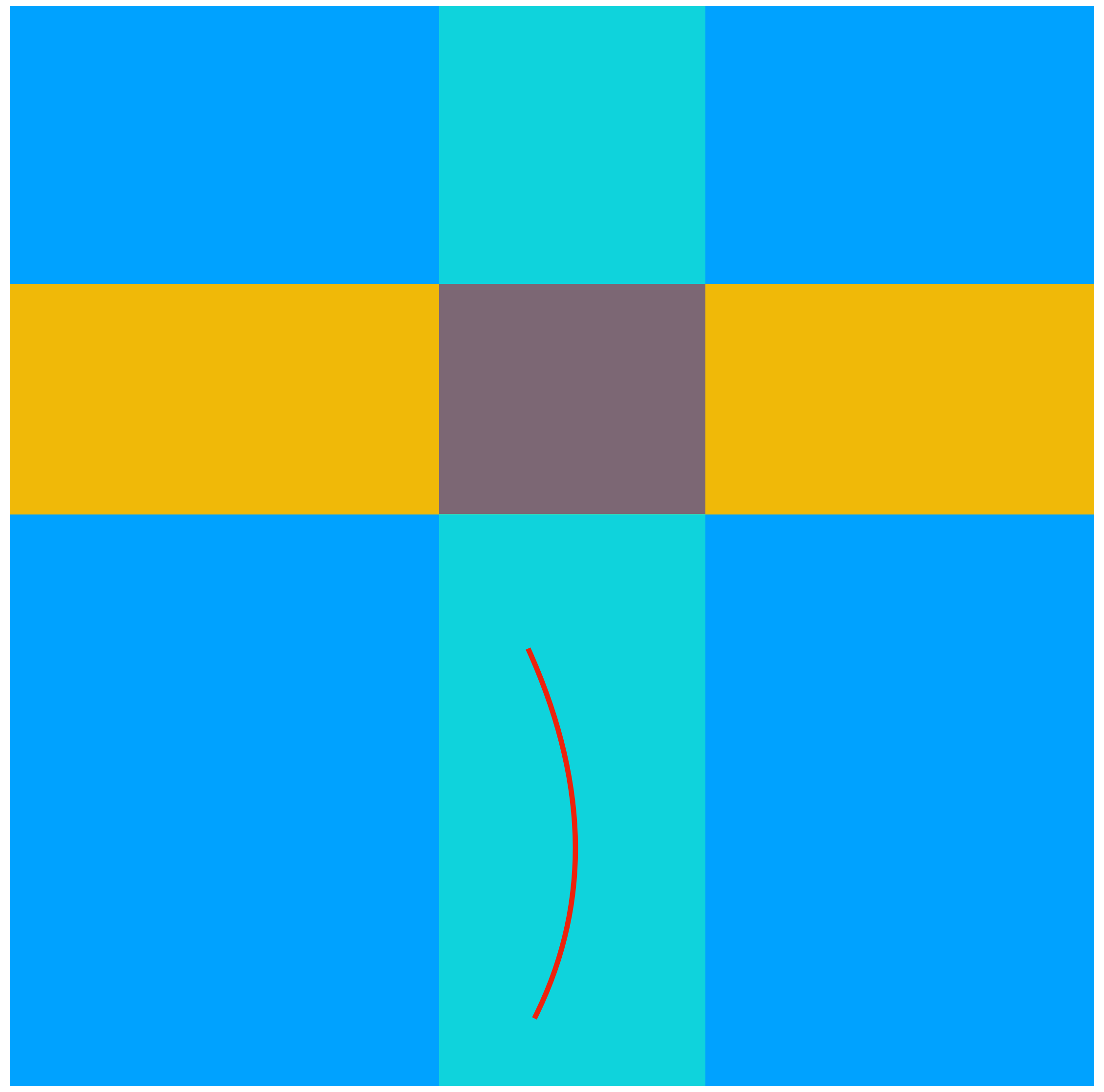
if factorizability in two frame perspectives, i.e.

$\mathscr{A}_{\text{phys}} \simeq \mathscr{A}_{S|R_1} \otimes \mathscr{A}_{R_2|R_1} \simeq \mathscr{A}_{S|R_2} \otimes \mathscr{A}_{R_1|R_2}$ but $\mathscr{A}_{S|R_2} \neq \mathscr{A}_{S|R_1}$

then correlations/entanglement of S with its complement will in general differ in two perspectives

(see also Giacomini, Castro-Ruiz, Brukner '17; Castro-Ruiz, Oreshkov '21)

$\Rightarrow$ gauge-inv. entanglement entropy in general $S(\rho_{S|R_2}) \neq S(\rho_{S|R_1})$ for same global physical state



$\Rightarrow$ dynamics of S can be closed/isolated relative to R_1 and open relative to R_2

(can map unitary dynamics/zero heat & work exchange into open dynamics/ non-zero heat & work exchange in other perspective)

$\Rightarrow$ **QRF relativity of superpositions, correlations, thermodynamics, …**

see F. Mele's talk today!

Unifying picture of dynamical frame covariance

2 commuting group actions

	Special relativity	**Quantum relativity**	**Gauge relativity**	**General relativity**
frame orientations (group valued frame)	tetrad $e^\mu_a \in SO_+(3,1)$	coherent state $\vert\phi(g)\rangle \in \mathscr{H}_R \qquad g \in G$		
gauge transf./ gauge covariance	spacetime Lorentz tr. $\Lambda^\mu{}_\nu\, e^\nu_a \quad \Lambda^\mu{}_\nu \in SO_+(3,1)$	left action: $U(g')\vert\phi(g)\rangle = \vert\phi(g'g)\rangle$		
frame reorientations/ "symmetries" (act only on the frame)	frame Lorentz tr. $\Lambda_a{}^b\, e^\mu_b \quad \Lambda_a{}^b \in SO_+(3,1)$	action from the right $V_R(g')\vert\phi(g)\rangle = \vert\phi(gg'^{-1})\rangle$		
relational observables $O_{F,R}(g)$ "what's value of F when RF is in orientation g?"	$F_{\mu\dots} e^\mu_a \Lambda_b{}^a(g^{-1})\cdots$	$\int_G dg' \hat{U}_{RS}(g')(\vert\phi(g)\rangle\langle\phi(g)\vert \otimes F_S)$		
"jumping into RF perspective" (gauge fixing)	$e^\mu_a = \Lambda^\mu{}_a(g)$	conditioning on frame or. $\varphi(g) = \langle\phi(g)\vert \otimes \mathbf{1}_S$		
RF change 1 (rel. cond. gauge transf.)	$\Lambda^\mu{}_\nu(e,e') = \Lambda^\mu{}_{a'}(g')e'^{a'}_\kappa\, e^\kappa_b \Lambda_\nu{}^b(g)$	$\varphi_2(g') \circ \varphi_1^{-1}(g)$		
RF change 2 (rel. cond. symmetry) (1st frame relative to 2nd)	$\Lambda^{a'}{}_b(e,e') = e'^{a'}_\mu\, e^\mu_b$	long expression (see 2110.13824)		

Unifying picture of dynamical frame covariance

	Special relativity	Quantum relativity	Gauge relativity	General relativity
frame orientations (group valued frame)	tetrad $e^\mu_a \in SO_+(3,1)$	coherent state $\vert\phi(g)\rangle \in \mathscr{H}_R \quad g \in G$		
gauge transf./ gauge covariance	spacetime Lorentz tr. $\Lambda^\mu{}_\nu\, e^\nu_a \quad \Lambda^\mu{}_\nu \in SO_+(3,1)$	left action: $U(g')\vert\phi(g)\rangle = \vert\phi(g'g)\rangle$		
frame reorientations/ "symmetries" (act only on the frame)	frame Lorentz tr. $\Lambda_a{}^b\, e^\mu_b \quad \Lambda_a{}^b \in SO_+(3,1)$	action from the right $V_R(g')\vert\phi(g)\rangle = \vert\phi(gg'^{-1})\rangle$		
relational observables $O_{F,R}(g)$ "what's value of F when RF is in orientation g?"	$F_{\mu\ldots}e^\mu_a\Lambda_b{}^a(g^{-1})\cdots$	$\int_G dg'\hat{U}_{RS}(g')(\vert\phi(g)\rangle\langle\phi(g)\vert \otimes F_S)$		
"jumping into RF perspective" (gauge fixing)	$e^\mu_a = \Lambda^\mu{}_a(g)$	conditioning on frame or. $\varphi(g) = \langle\phi(g)\vert \otimes \mathbf{1}_S$		
RF change 1 (rel. cond. gauge transf.)	$\Lambda^\mu{}_\nu(e,e') = \Lambda^\mu{}_{a'}(g')e'^{a'}_\kappa\, e^\kappa_b \Lambda_\nu{}^b(g)$	$\varphi_2(g')\circ\varphi_1^{-1}(g)$		
RF change 2 (rel. cond. symmetry) (1st frame relative to 2nd)	$\Lambda^{a'}{}_b(e,e') = e'^{a'}_\mu\, e^\mu_b$	long expression (see 2110.13824)		

2 commuting group actions

Edge modes as dynamical frames in gauge theory

edge modes appear at finite boundaries in gauge theories

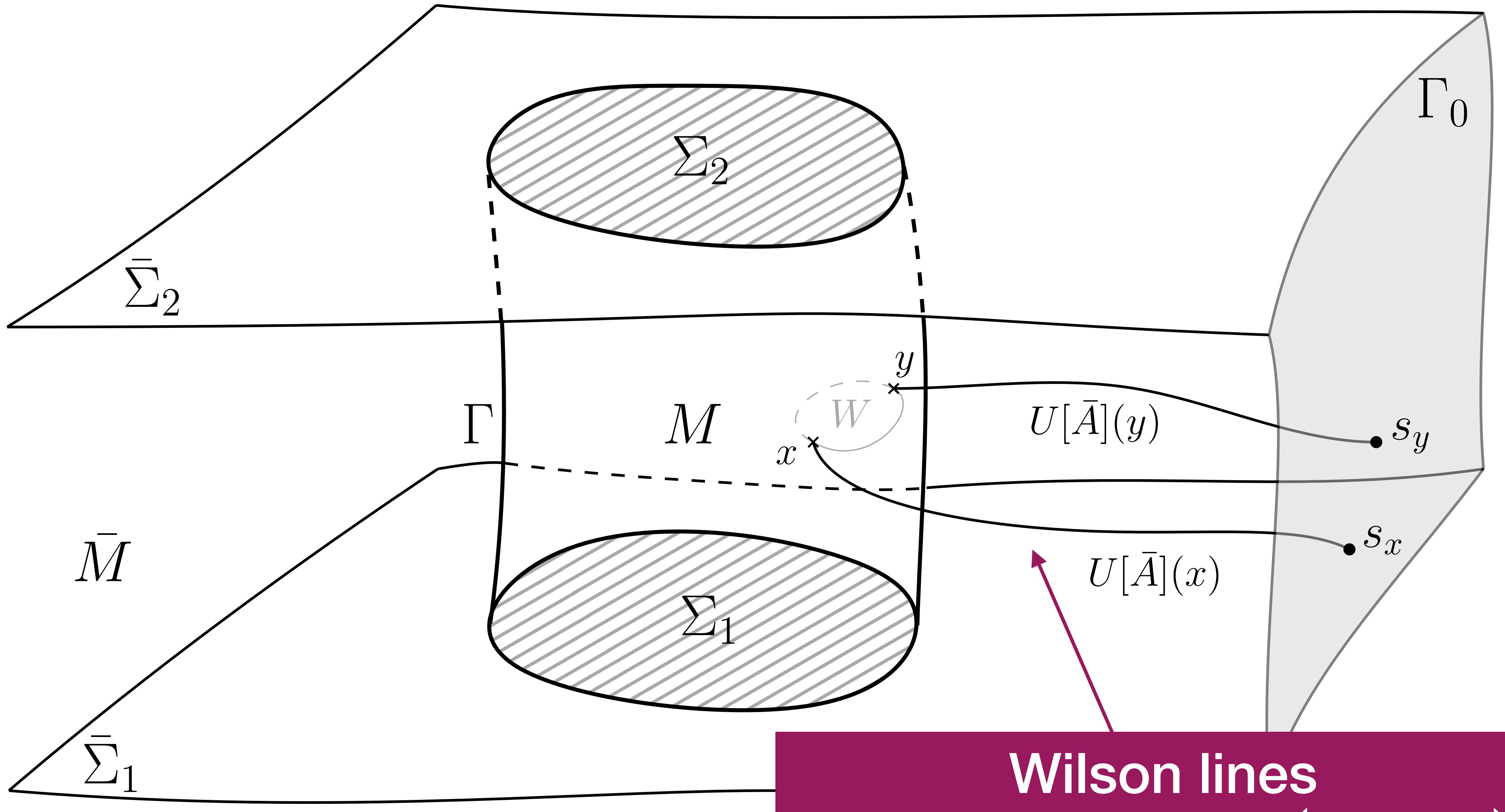
Carrozza, PH 2109.06184

Donnelly, Freidel, Francois, Geiller, Gomes, Pranzetti, Riello, Speranza, Wieland….

can understand them from perspective of global theory:

- not new DoFs that need to be postulated
- as group valued “internalized” external frames via Wilson lines originating in complement
- describe how subregion relates to its complement

[see also Riello ’21]



Wilson lines

$$U[\bar{A}](x) = \mathscr{P}\exp\left(-\int_{\gamma_x}\bar{A}\right)$$

Edge modes as dynamical frames in gauge theory

Carrozza, PH 2109.06184

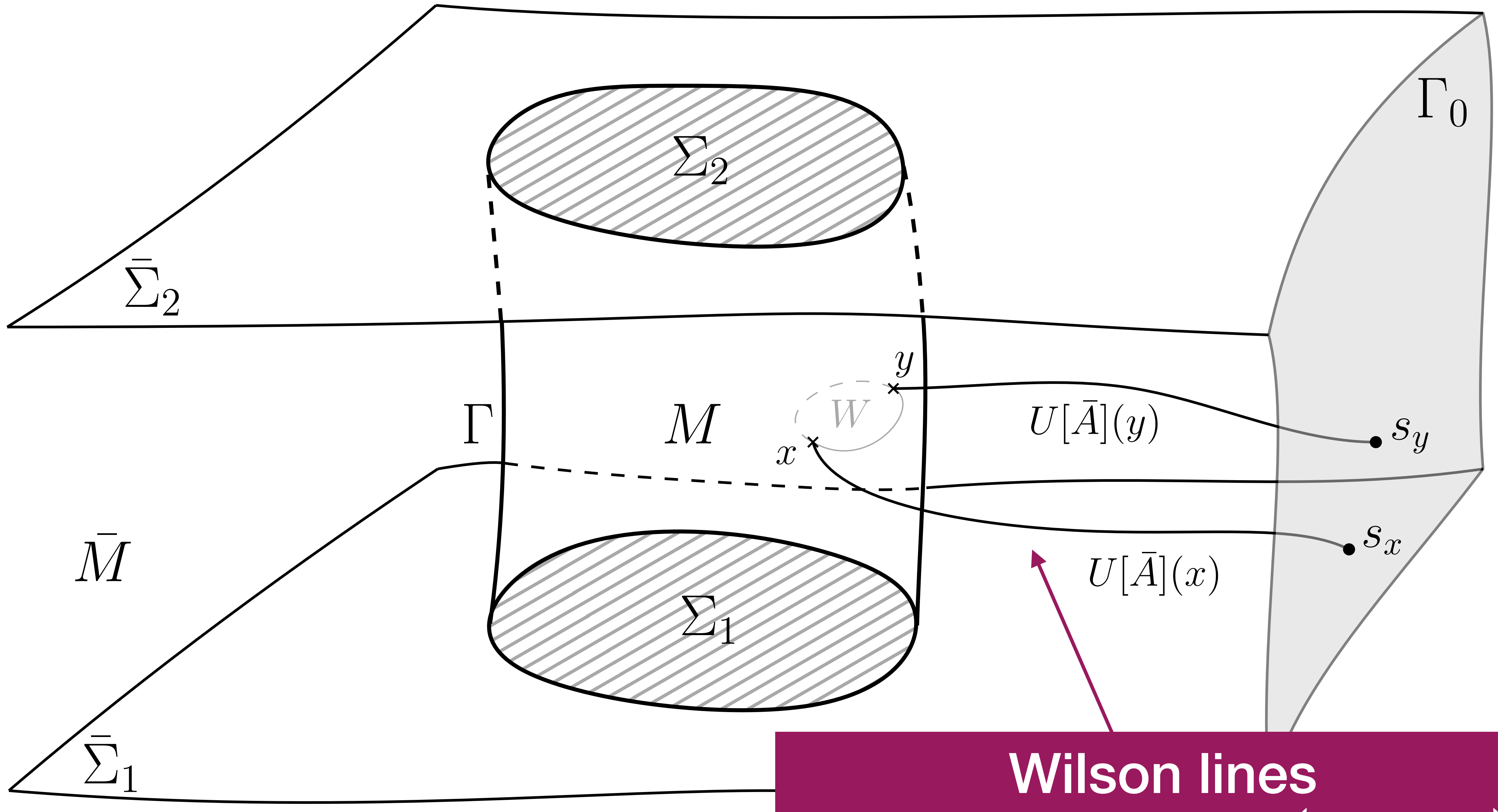
edge modes appear at finite boundaries in gauge theories

Donnelly, Freidel, Francois, Geiller, Gomes, Pranzetti, Riello, Speranza, Wieland….

can understand them from perspective of global theory:

- not new DoFs that need to be postulated
- as group valued “internalized” external frames via Wilson lines originating in complement
- describe how subregion relates to its complement

[see also Riello ’21]



bulk gauge transf. (from left): $g \triangleright U = gU$ gauge cov. frame

asymptotic frame reorientations (from right): $g \odot U = Ug^{-1}$ acts only on frame

Wilson lines

$$U[\bar{A}](x) = \mathscr{P}\exp\left(-\int_{\gamma_x}\bar{A}\right)$$

Edge modes as dynamical frames in gauge theory

Carrozza, PH 2109.06184

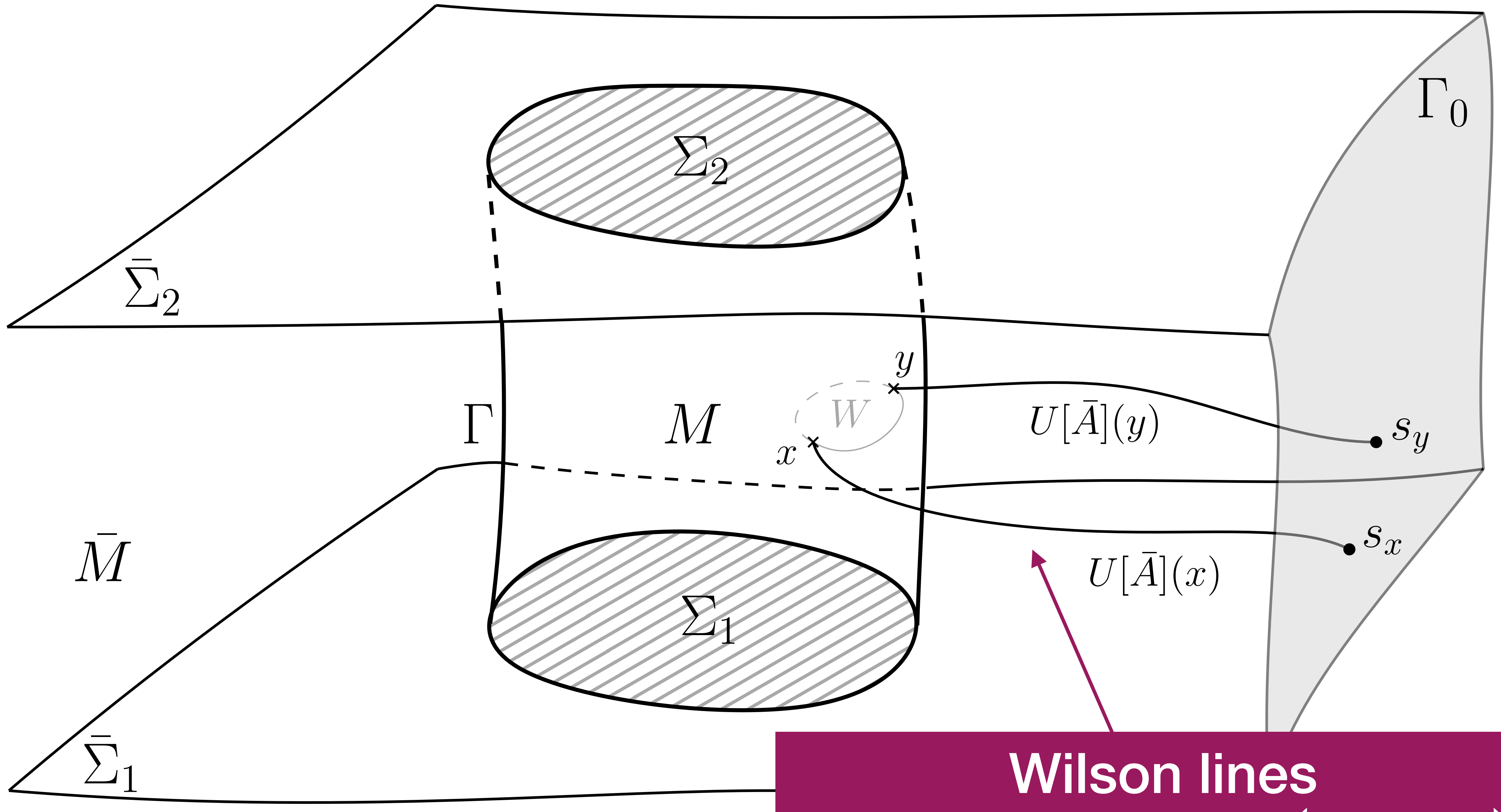
edge modes appear at finite boundaries in gauge theories

Donnelly, Freidel, Francois, Geiller, Gomes, Pranzetti, Riello, Speranza, Wieland….

can understand them from perspective of global theory:

- not new DoFs that need to be postulated
- as group valued “internalized” external frames via Wilson lines originating in complement
- describe how subregion relates to its complement

[see also Riello ’21]



bulk gauge transf. (from left): $g \triangleright U = gU$ gauge cov. frame

asymptotic frame reorientations (from right): $g \odot U = Ug^{-1}$ acts only on frame

Wilson lines

$$U[\bar{A}](x) = \mathscr{P}\exp\left(-\int_{\gamma_x}\bar{A}\right)$$

global → subregion procedure (post-selection) yields unambiguous presymplectic structure:

frame reorientation (symmetry) algebra $\{Q[\rho], Q[\sigma]\} = Q\left[[\rho,\sigma]\right]$ [recovers result from Donnelly, Freidel ’16, but status depends on BCs]

Recall: kinematical vs. relational subsystems

leaves description of S' relative to R invariant, but changes it relative to R'

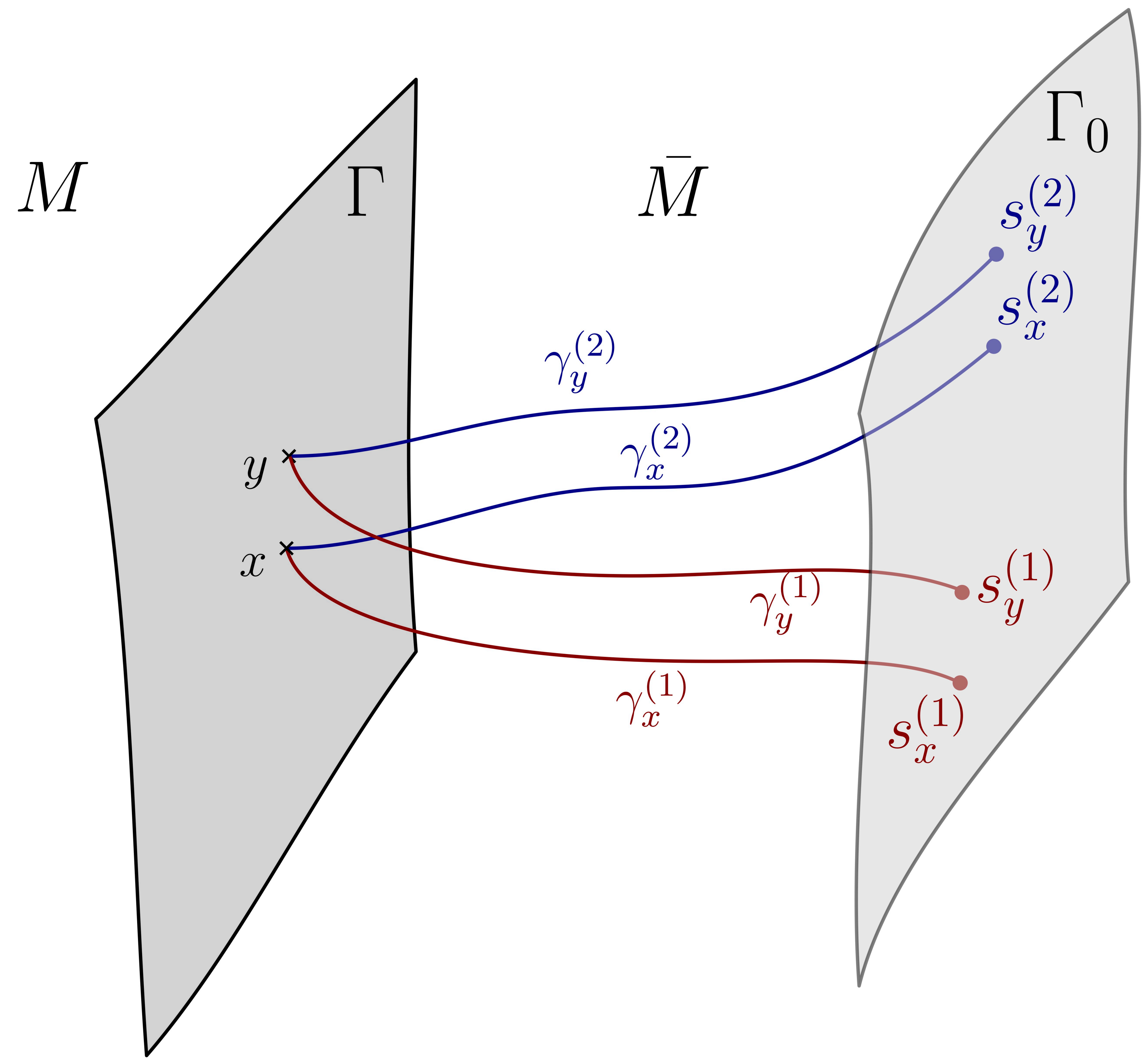
green balls: subsystem S'

leaves description of S' rel. to external frame invariant, but changes description relative to frame R

1. **kinematical and relational (gauge inv.) notion of subsystem distinct**
2. **gauge inv. notion of subsystem depends on choice of RF**

gauge theories/gravity:
e.g. S' subregion, R, R' edge mode frames

No unique edge mode frame field



$\Rightarrow$ **different systems of Wilson lines $\Rightarrow$ different edge mode frame fields**

$\Rightarrow$ **distinct gauge inv. description of subregion (e.g. different thermal properties & corerelations)**

Unifying picture of dynamical frame covariance

2 commuting group actions

	Special relativity	Quantum relativity	Gauge relativity	General relativity
frame orientations (group valued frame)	tetrad $e^\mu_a \in \mathrm{SO}_+(3,1)$	coherent state $\|\phi(g)\rangle \in \mathscr{H}_R \quad g \in G$	field-dep. gauge transf. $U[\phi] : \mathscr{M} \to G$	
gauge transf./ gauge covariance	spacetime Lorentz tr. $\Lambda^\mu{}_\nu\, e^\nu_a \quad \Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$	left action: $U(g')\|\phi(g)\rangle = \|\phi(g'g)\rangle$	left action $g \triangleright U = gU$	
frame reorientations/ "symmetries" (act only on the frame)	frame Lorentz tr. $\Lambda_a{}^b\, e^\mu_b \quad \Lambda_a{}^b \in \mathrm{SO}_+(3,1)$	action from the right $V_R(g')\|\phi(g)\rangle = \|\phi(gg'^{-1})\rangle$	action from the right $g \odot U = Ug^{-1}$	
relational observables $O_{F,R}(g)$ "what's value of F when RF is in orientation g?"	$F_{\mu\ldots}e^\mu_a\Lambda_b{}^a(g^{-1})\cdots$	$\int_G dg'\hat{U}_{RS}(g')(\|\phi(g)\rangle\langle\phi(g)\| \otimes F_S)$	$(U(x)g(x)^{-1})^{-1} \triangleright F(x)$	
"jumping into RF perspective" (gauge fixing)	$e^\mu_a = \Lambda^\mu{}_a(g)$	conditioning on frame or. $\varphi(g) = \langle\phi(g)\| \otimes \mathbf{1}_S$	$U[\phi_0](x) = g(x)$	
RF change 1 (rel. cond. gauge transf.)	$\Lambda^\mu{}_\nu(e,e') = \Lambda^\mu{}_{a'}(g')e'^{a'}_\kappa\, e^\kappa_b \Lambda_\nu{}^b(g)$	$\varphi_2(g') \circ \varphi_1^{-1}(g)$	$g'\, U_2^{-1}[\phi]\, U_1[\phi]\, g^{-1}$	
RF change 2 (rel. cond. symmetry) (1st frame relative to 2nd)	$\Lambda^{a'}{}_b(e,e') = e'^{a'}_\mu\, e^\mu_b$	long expression (see 2110.13824)	$U_2^{-1}[\phi]U_1[\phi]$	

Unifying picture of dynamical frame covariance

2 commuting group actions

	Special relativity	**Quantum relativity**	**Gauge relativity**	**General relativity**
frame orientations (group valued frame)	tetrad $e^\mu_a \in \mathrm{SO}_+(3,1)$	coherent state $\lvert\phi(g)\rangle \in \mathscr{H}_R \quad g \in G$	field-dep. gauge transf. $U[\phi] : \mathscr{M} \to G$	
gauge transf./ gauge covariance	spacetime Lorentz tr. $\Lambda^\mu{}_\nu\, e^\nu_a \quad \Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$	left action: $U(g')\lvert\phi(g)\rangle = \lvert\phi(g'g)\rangle$	left action $g \triangleright U = gU$	
frame reorientations/ "symmetries" (act only on the frame)	frame Lorentz tr. $\Lambda_a{}^b\, e^\mu_b \quad \Lambda_a{}^b \in \mathrm{SO}_+(3,1)$	action from the right $V_R(g')\lvert\phi(g)\rangle = \lvert\phi(gg'^{-1})\rangle$	action from the right $g \odot U = Ug^{-1}$	
relational observables $O_{F,R}(g)$ "what's value of F when RF is in orientation g?"	$F_{\mu\ldots}e^\mu_a\Lambda_b{}^a(g^{-1})\cdots$	$\int_G dg'\hat{U}_{RS}(g')(\lvert\phi(g)\rangle\langle\phi(g)\rvert \otimes F_S)$	$(U(x)g(x)^{-1})^{-1} \triangleright F(x)$	
"jumping into RF perspective" (gauge fixing)	$e^\mu_a = \Lambda^\mu{}_a(g)$	conditioning on frame or. $\varphi(g) = \langle\phi(g)\rvert \otimes \mathbf{1}_S$	$U[\phi_0](x) = g(x)$	
RF change 1 (rel. cond. gauge transf.)	$\Lambda^\mu{}_\nu(e,e') = \Lambda^\mu{}_{a'}(g')e'^{a'}_\kappa\, e^\kappa_b\Lambda_\nu{}^b(g)$	$\varphi_2(g') \circ \varphi_1^{-1}(g)$	$g'\,U_2^{-1}[\phi]\,U_1[\phi]\,g^{-1}$	
RF change 2 (rel. cond. symmetry) (1st frame relative to 2nd)	$\Lambda^{a'}{}_b(e,e') = e'^{a'}_\mu\, e^\mu_b$	long expression (see 2110.13824)	$U_2^{-1}[\phi]U_1[\phi]$	

Generally covariant theories

… dynamical frames for the diffeo group

General dynamical frames

1. want general frame $\Rightarrow$ idea: use gauge-covariant dynamical dressings of local events $\qquad x[f_*\phi] = f \circ x[\phi]$

General dynamical frames

1. **want general frame ⇒ idea: use gauge-covariant dynamical dressings of local events** $x[f_*\phi] = f \circ x[\phi]$
2. **want "space of all frames" ⇒ define *universal dressing space*:**

$$\mathscr{D} := \{x[\phi] : \mathscr{S} \to \mathscr{M} \mid x[f_*\phi] = f \circ x[\phi], \textbf{for } f \textbf{ gauge diffeo}\}$$

space of solutions, so frame subject to EoMs

General dynamical frames

1. want general frame ⇒ idea: use gauge-covariant dynamical dressings of local events $x[f_*\phi] = f \circ x[\phi]$
2. want "space of all frames" ⇒ define ***universal dressing space***:

$$\mathscr{D} := \{x[\phi] : \mathscr{S} \to \mathscr{M} \mid x[f_*\phi] = f \circ x[\phi], \text{for } f \text{ gauge diffeo}\}$$

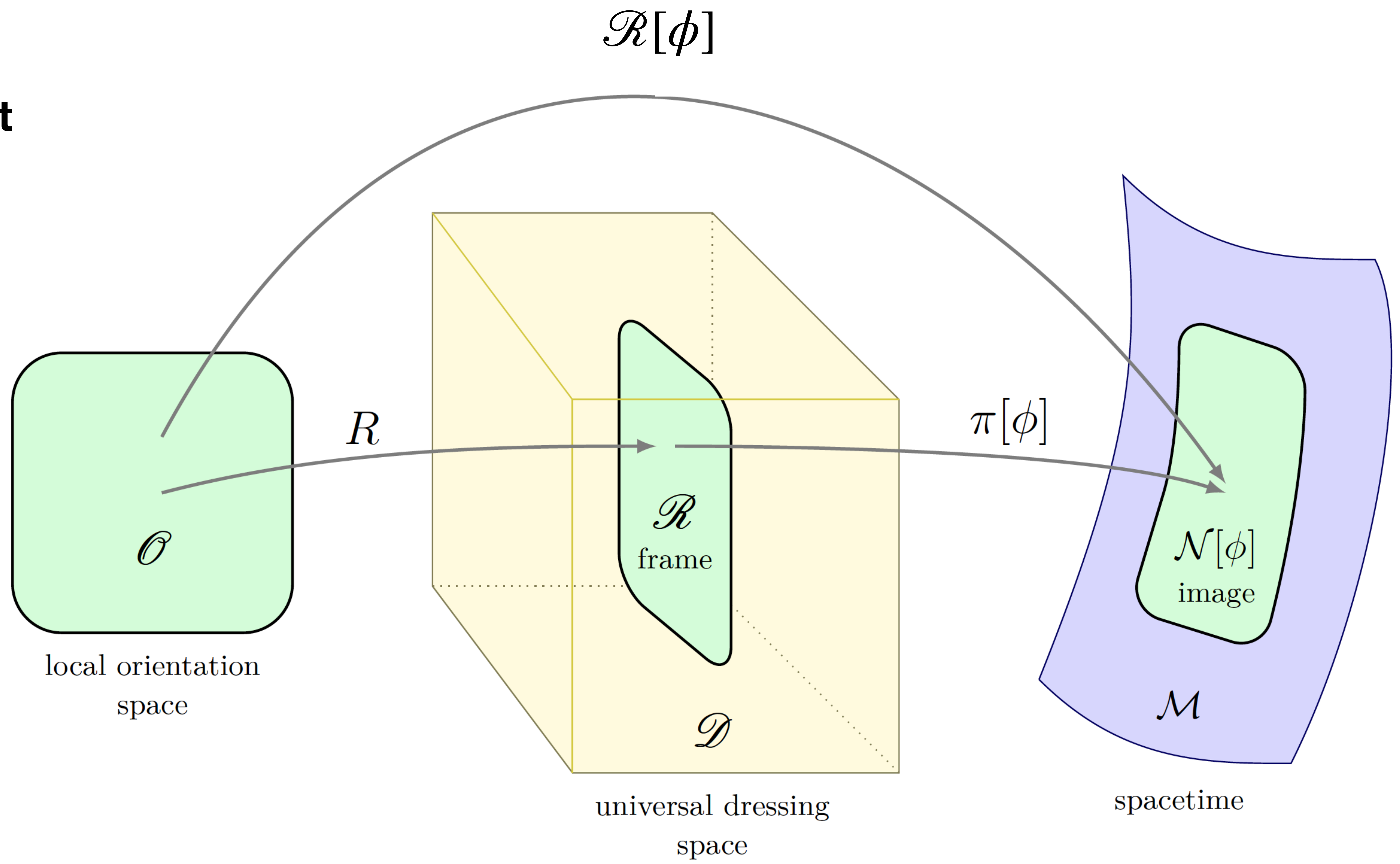
space of solutions, so frame subject to EoMs

⇒ abstractly, can define frame as subset of $\mathscr{D}$, but want parametrisation of it (to coordinatise $\mathscr{D}$ and spacetime)

parametrised frame:

local orientation (parameter) space $\mathscr{O}$ + inj. map $R : \mathscr{O} \to \mathscr{D}$

$$\mathscr{R}[\phi] : \mathscr{O} \to \mathscr{M}$$



General dynamical frames

1. want general frame ⇒ idea: use gauge-covariant dynamical dressings of local events $\quad x[f_*\phi] = f \circ x[\phi]$
2. want "space of all frames" ⇒ define ***universal dressing space***:

$$\mathscr{D} := \{x[\phi] : \mathscr{S} \to \mathscr{M} \mid x[f_*\phi] = f \circ x[\phi], \text{for } f \text{ gauge diffeo}\}$$

space of solutions, so frame subject to EoMs

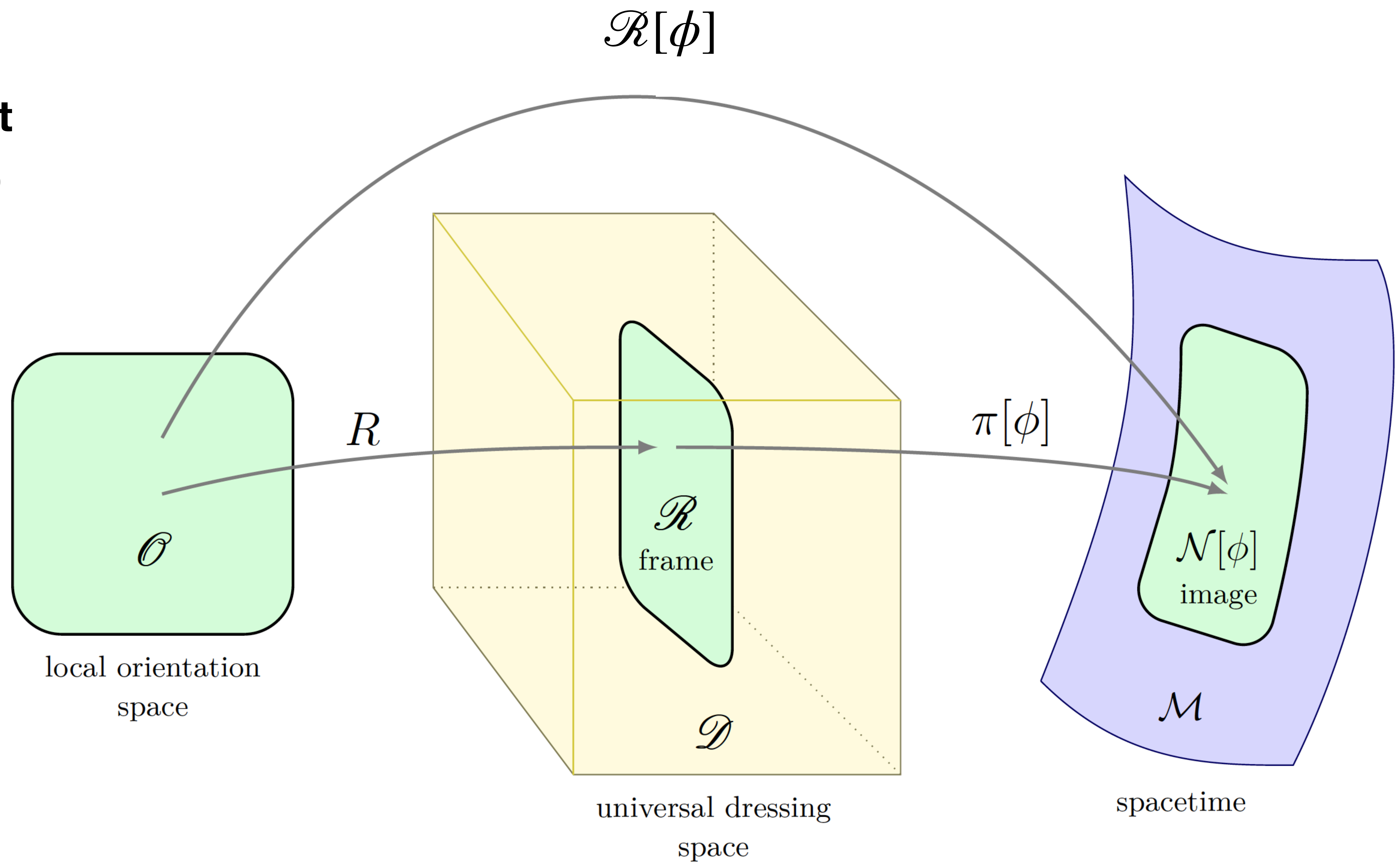
⇒ abstractly, can define frame as subset of $\mathscr{D}$, but want parametrisation of it (to coordinatise $\mathscr{D}$ and spacetime)

<u>parametrised frame:</u>

local orientation (parameter) space $\mathscr{O}$ + inj. map $R : \mathscr{O} \to \mathscr{D}$

$$\mathscr{R}[\phi] : \mathscr{O} \to \mathscr{M}$$

gauge-cov. $\mathscr{R}[f_*\phi] = f \circ \mathscr{R}[\phi]$



General dynamical frames

1. **want general frame ⇒ idea: use gauge-covariant dynamical dressings of local events** $x[f_*\phi] = f \circ x[\phi]$
2. **want "space of all frames" ⇒ define *universal dressing space*:**

$$\mathscr{D} := \{x[\phi] : \mathscr{S} \to \mathscr{M} \mid x[f_*\phi] = f \circ x[\phi], \text{for } f \text{ gauge diffeo}\}$$

space of solutions, so frame subject to EoMs

⇒ abstractly, can define frame as subset of $\mathscr{D}$, but want parametrisation of it (to coordinatise $\mathscr{D}$ and spacetime)

parametrised frame:

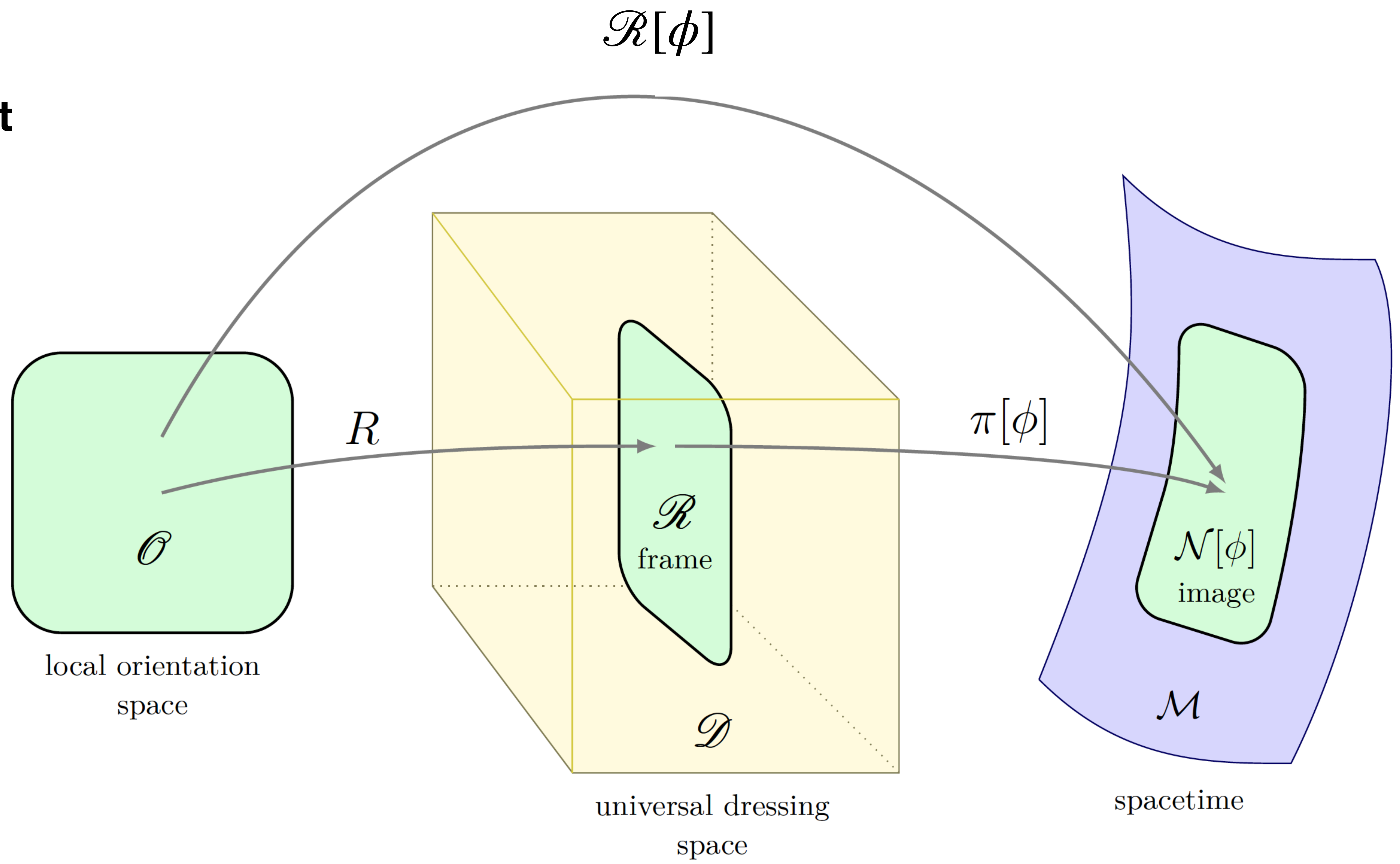
local orientation (parameter) space $\mathscr{O}$ + inj. map $R : \mathscr{O} \to \mathscr{D}$

$$\mathscr{R}[\phi] : \mathscr{O} \to \mathscr{M}$$

gauge-cov. $\mathscr{R}[f_*\phi] = f \circ \mathscr{R}[\phi]$

⇒ if $\mathscr{R}$ injective, can invert on its image

dynamical frame field $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \subset \mathscr{M} \to \mathscr{O}$



General dynamical frames

1. want general frame $\Rightarrow$ idea: use gauge-covariant dynamical dressings of local events $\quad x[f_*\phi] = f \circ x[\phi]$

2. want "space of all frames" $\Rightarrow$ define *universal dressing space*:

$$\mathscr{D} := \{x[\phi] : \mathscr{S} \to \mathscr{M} \mid x[f_*\phi] = f \circ x[\phi], \text{for } f \text{ gauge diffeo}\}$$

space of solutions, so frame subject to EoMs

$\Rightarrow$ abstractly, can define frame as subset of $\mathscr{D}$, but want parametrisation of it (to coordinatise $\mathscr{D}$ and spacetime)

parametrised frame:

local orientation (parameter) space $\mathscr{O}$ + inj. map $R : \mathscr{O} \to \mathscr{D}$

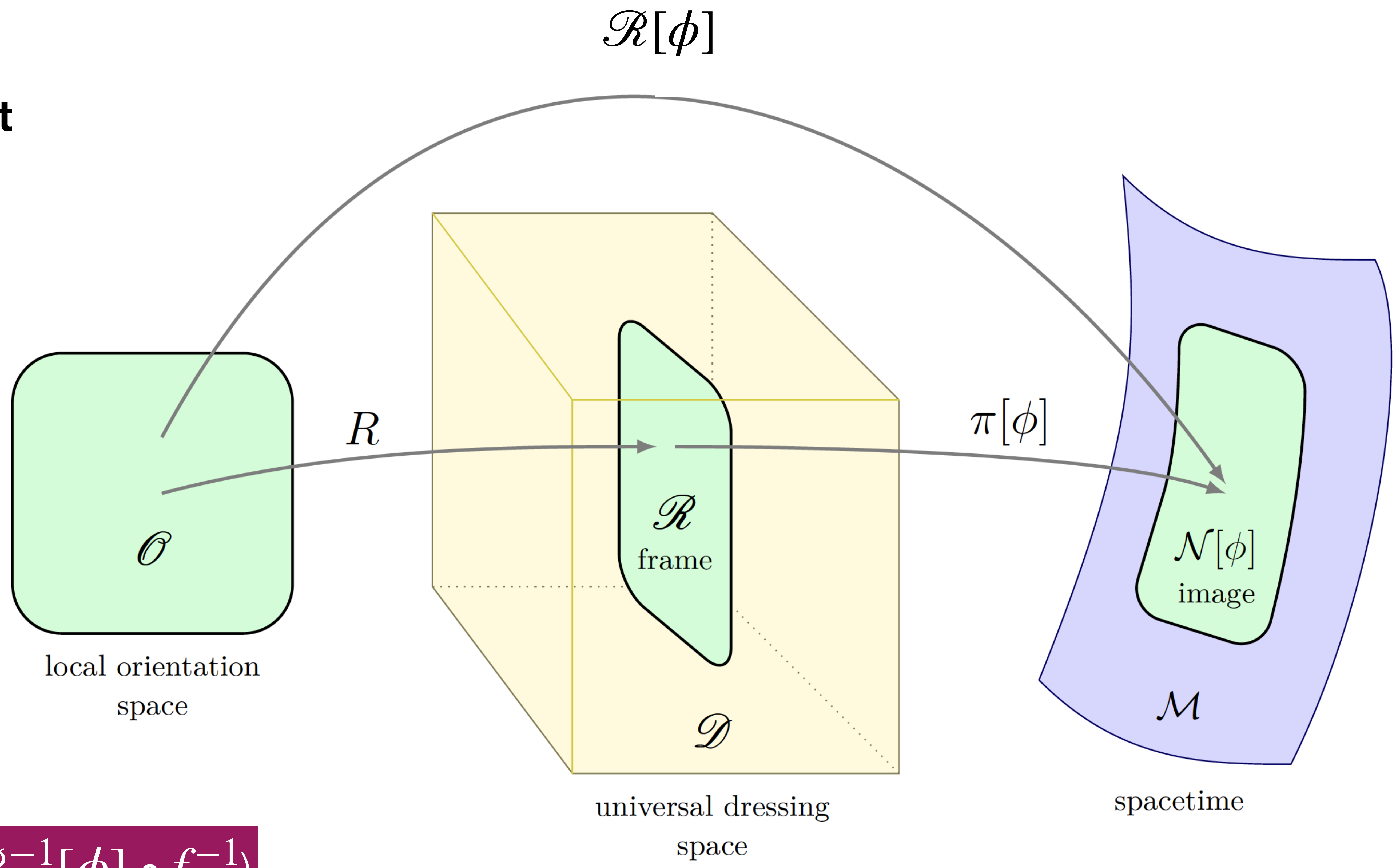
$$\mathscr{R}[\phi] : \mathscr{O} \to \mathscr{M}$$

gauge-cov. $\mathscr{R}[f_*\phi] = f \circ \mathscr{R}[\phi]$

$\Rightarrow$ if $\mathscr{R}$ injective, can invert on its image

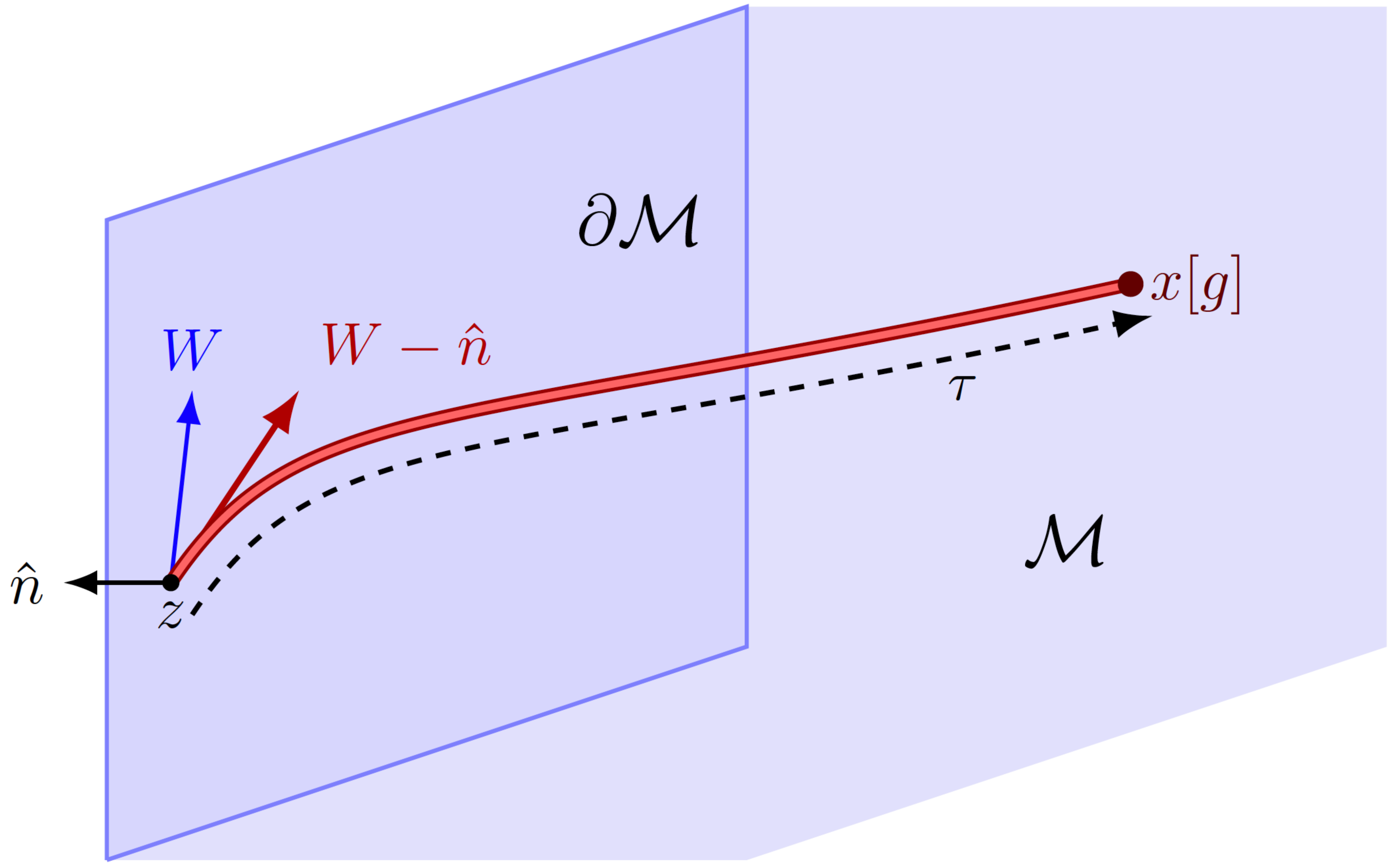
dynamical frame field $\quad \mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \subset \mathscr{M} \to \mathscr{O}$

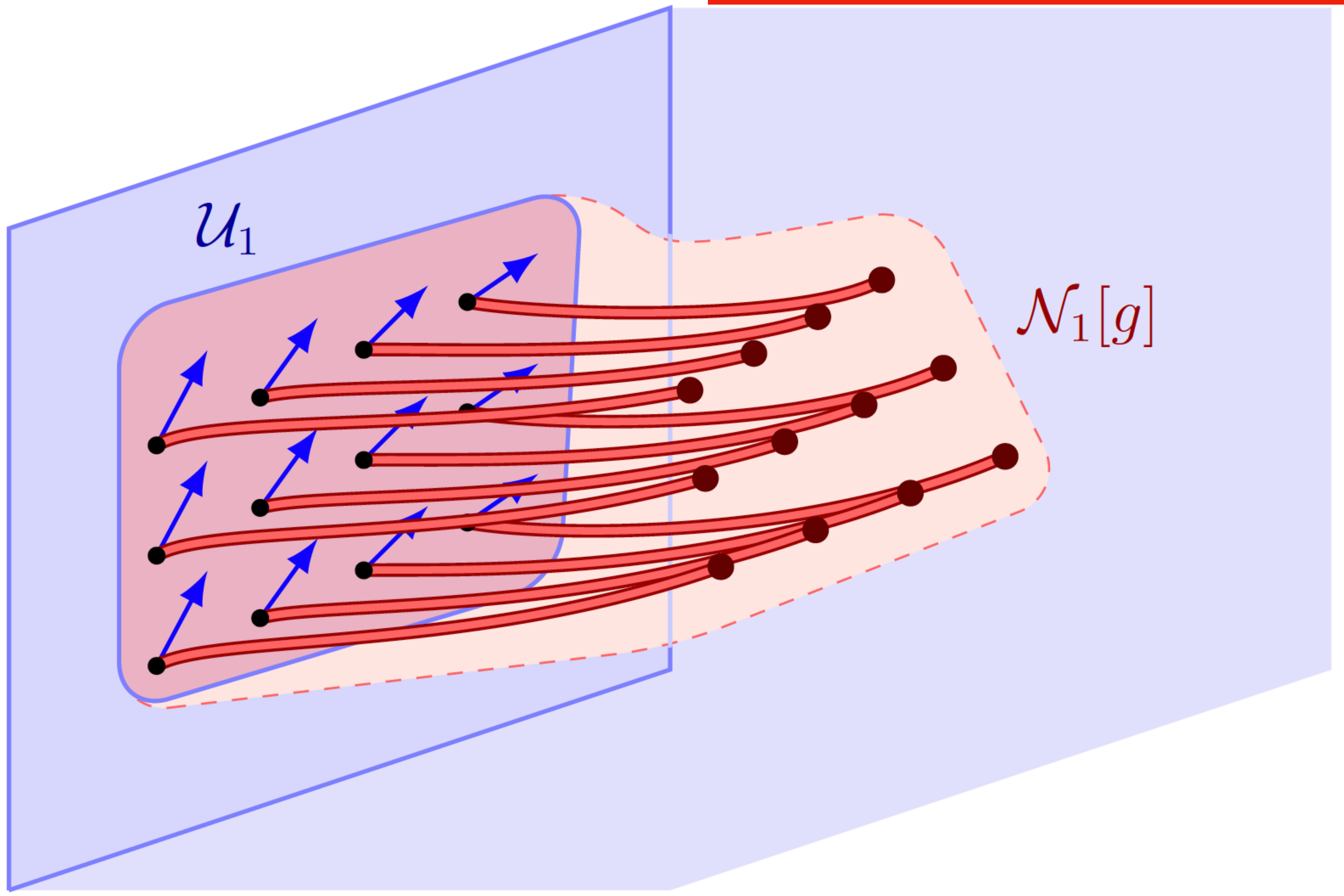
dynamical coord. system (transforms as scalar $\mathscr{R}^{-1}[f_*\phi] = \mathscr{R}^{-1}[\phi] \circ f^{-1}$)



Example: boundary-anchored geodesic frames

see C. Goeller's talk today!





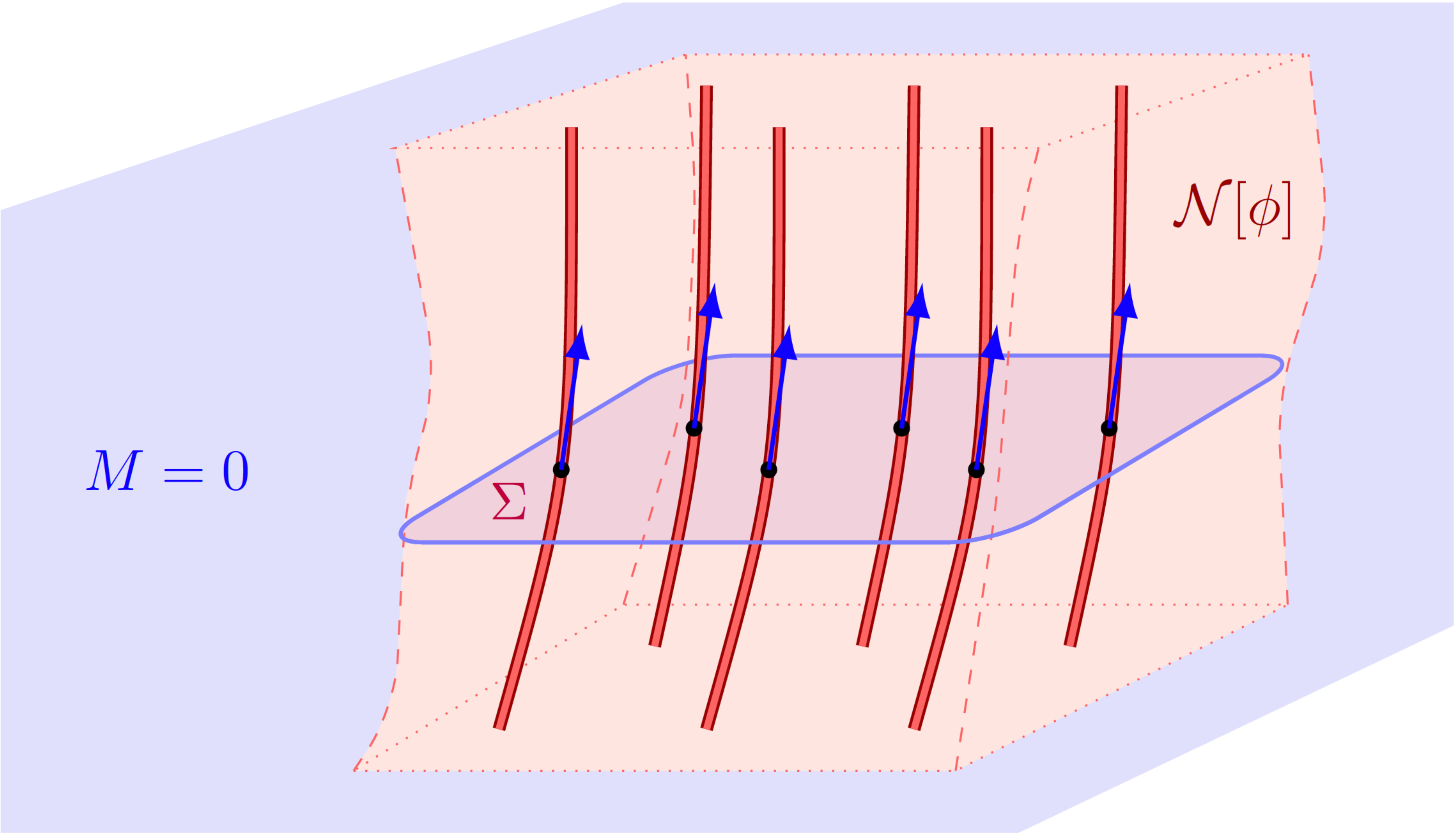
(τ, z, W) **parametrise local orientation space $\mathcal{O}$ of gauge cov. frame**

restrict to $\mathcal{O}_1 \subset \mathcal{O}$ s.t. injective (e.g. W)

$\Rightarrow$ get scalar frame field in some neighbourhood $\quad \mathcal{R}_1^{-1}[g] : \mathcal{N}_1[g] \subset \mathcal{M} \to \mathcal{O}_1$

Example: Brown-Kuchar dust frame

[Brown, Kuchar '95; Goeller, PH, Kirklin '22]



dynamical dust matter frame, works also without bdry

dust flow lines are "Cauchy-surface-anchored" geodesics

$\Rightarrow$ gives rise to dynamical comoving coordinates, given by 4 scalars (T, Z^k) parametrise local orientation space $\mathcal{O}$ (here dust spacetime)

$\Rightarrow$ gauge-covariant frame, construction works similarly to boundary-anchored case

Dressed observables = relational observables

[Goeller, PH, Kirklin ’22]

If $A[f_*\phi] = f_*A[\phi]$ a covariant local field (e.g. tensor field) on spacetime, get frame-dressed observable:

$$O_{A,\mathscr{R}}[\phi] = (\mathscr{R}[\phi])^* A[\phi]$$

pullback has to exist (depends on choice of frame and A)

$\Rightarrow$ **gauge diffeo-inv.** $O_{A,\mathscr{R}}[f_*\phi] = O_{A,\mathscr{R}}[\phi]$

observable on the local frame orientation space $\mathscr{O}$

Dressed observables = relational observables

[Goeller, PH, Kirklin '22]

If $A[f_*\phi] = f_*A[\phi]$ a covariant local field (e.g. tensor field) on spacetime, get frame-dressed observable:

$$O_{A,\mathscr{R}}[\phi] = (\mathscr{R}[\phi])^* A[\phi]$$

pullback has to exist (depends on choice of frame and A)

$\Rightarrow$ **gauge diffeo-inv.** $O_{A,\mathscr{R}}[f_*\phi] = O_{A,\mathscr{R}}[\phi]$

observable on the local frame orientation space $\mathscr{O}$

relational observable

answers "what is the value of (certain component of) A at the event in spacetime, where the frame field $\mathscr{R}^{-1}$ is in local orientation $o \in \mathscr{O}$?"

[in same sense as Rovelli, Dittrich, Thiemann, …, just covariant]

$O_{A,\mathscr{R}}[\phi]$ **is relationally local, local to orientation** $o \in \mathscr{O} \quad \longleftrightarrow \quad x[\phi] \in \mathscr{M}$

field-indep.

field-dep.

Dressed observables = relational observables

[Goeller, PH, Kirklin '22]

If $A[f_*\phi] = f_*A[\phi]$ a covariant local field (e.g. tensor field) on spacetime, get frame-dressed observable:

$$O_{A,\mathscr{R}}[\phi] = (\mathscr{R}[\phi])^*A[\phi]$$

pullback has to exist (depends on choice of frame and A)

$\Rightarrow$ **gauge diffeo-inv.** $O_{A,\mathscr{R}}[f_*\phi] = O_{A,\mathscr{R}}[\phi]$

observable on the local frame orientation space $\mathscr{O}$

relational observable

answers "what is the value of (certain component of) A at the event in spacetime, where the frame field $\mathscr{R}^{-1}$ is in local orientation $o \in \mathscr{O}$?"

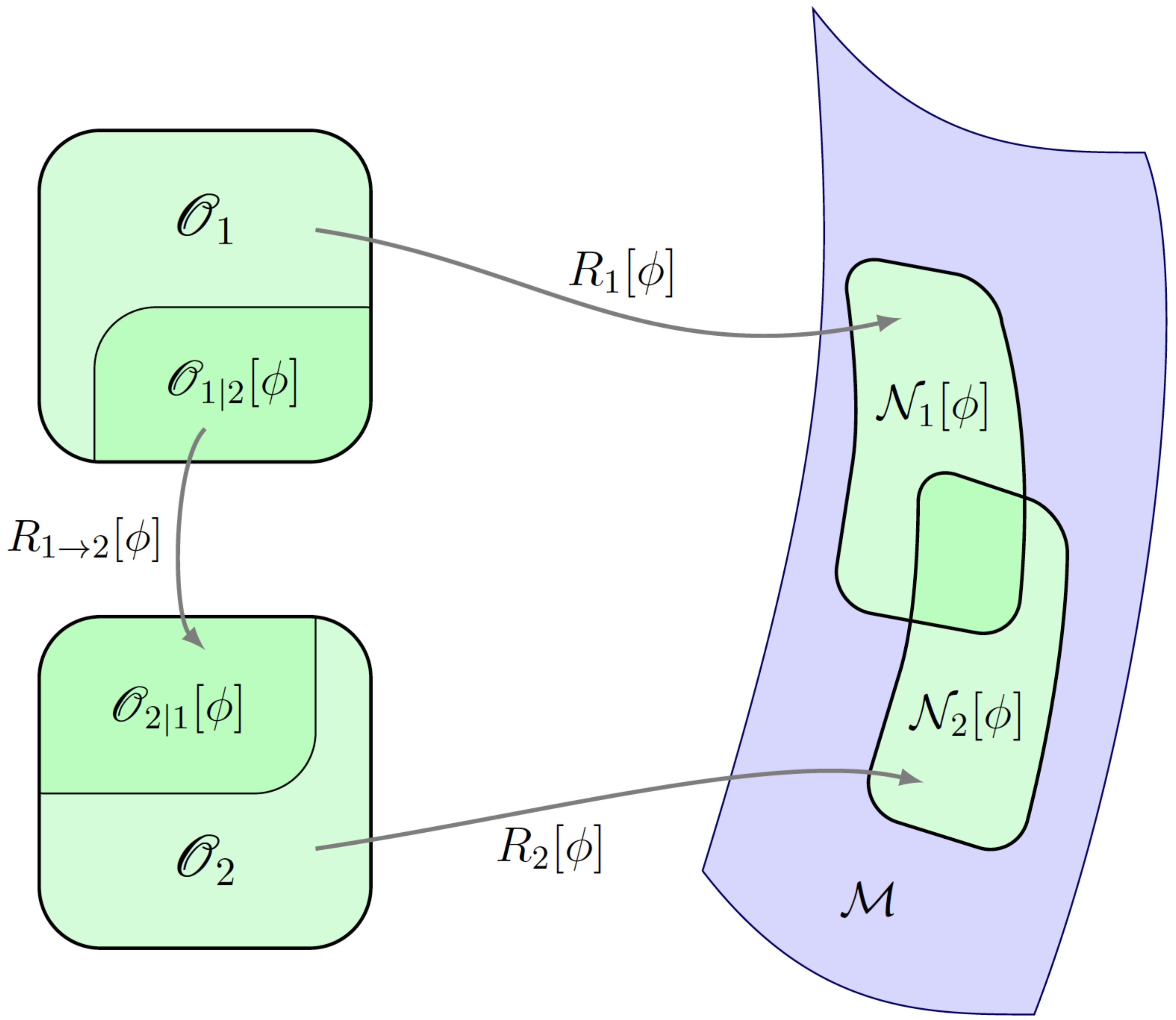
[in same sense as Rovelli, Dittrich, Thiemann, …, just covariant]

$O_{A,\mathscr{R}}[\phi]$ **is relationally local, local to orientation** $o \in \mathscr{O} \longleftrightarrow x[\phi] \in \mathscr{M}$

field-indep. field-dep.

$\Rightarrow$ **unifies and generalises (1) dressed observables** [hep-th community]**,**
(2) power series [Dittrich, …] **& (3) single integral reps** [Marolf, Giddings,…] **of relational observables**

Frame changes and relational atlases

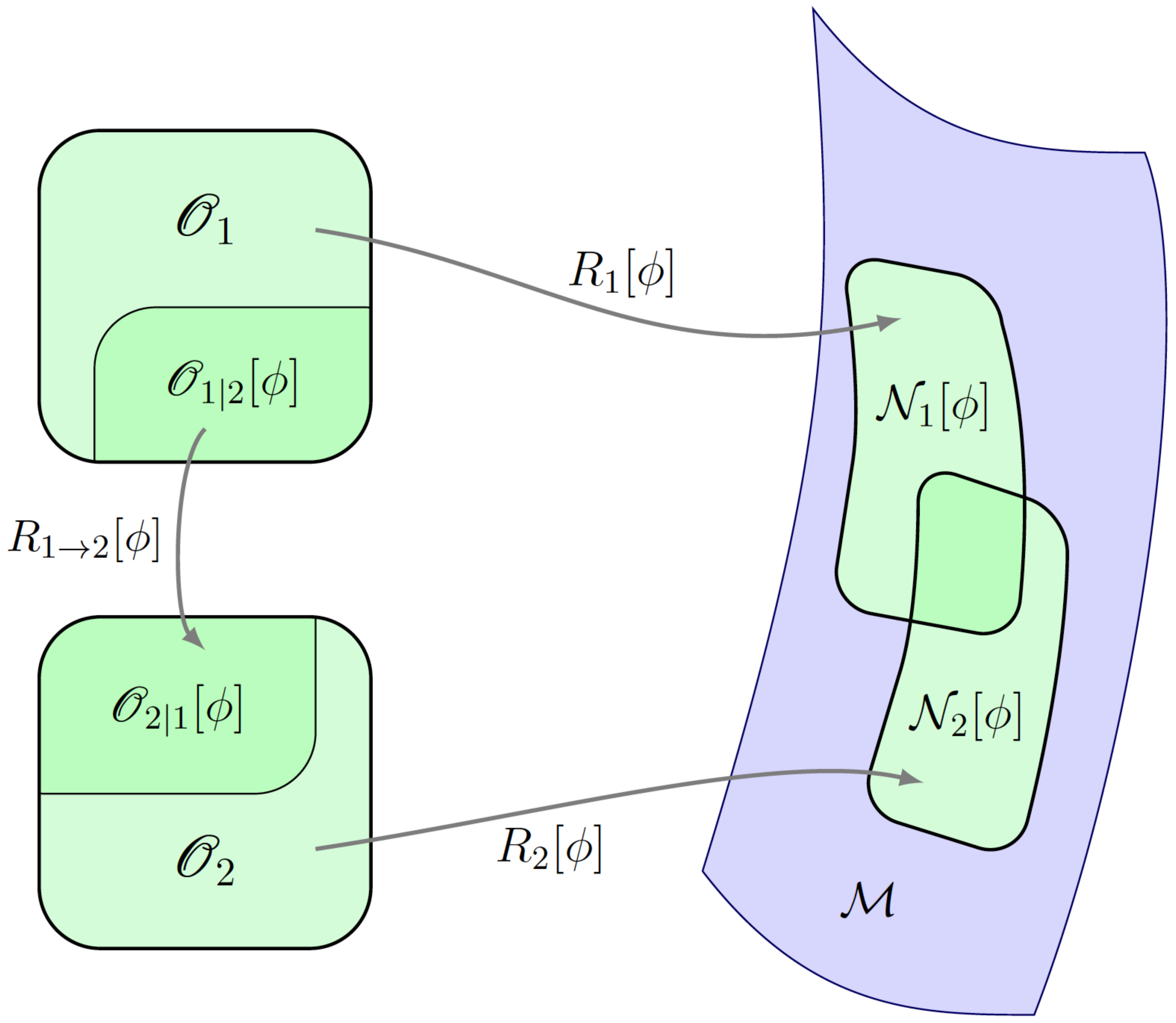


restrict to injective frames with overlapping images $\mathscr{N}_1[\phi] \cap \mathscr{N}_2[\phi] \neq \emptyset$

change of frame map: $\mathscr{R}_{1\to2}[\phi] = \mathscr{R}_2^{-1}[\phi] \circ \mathscr{R}_1[\phi] : \mathscr{O}_1 \to \mathscr{O}_2$

dynamical coord. change

Frame changes and relational atlases



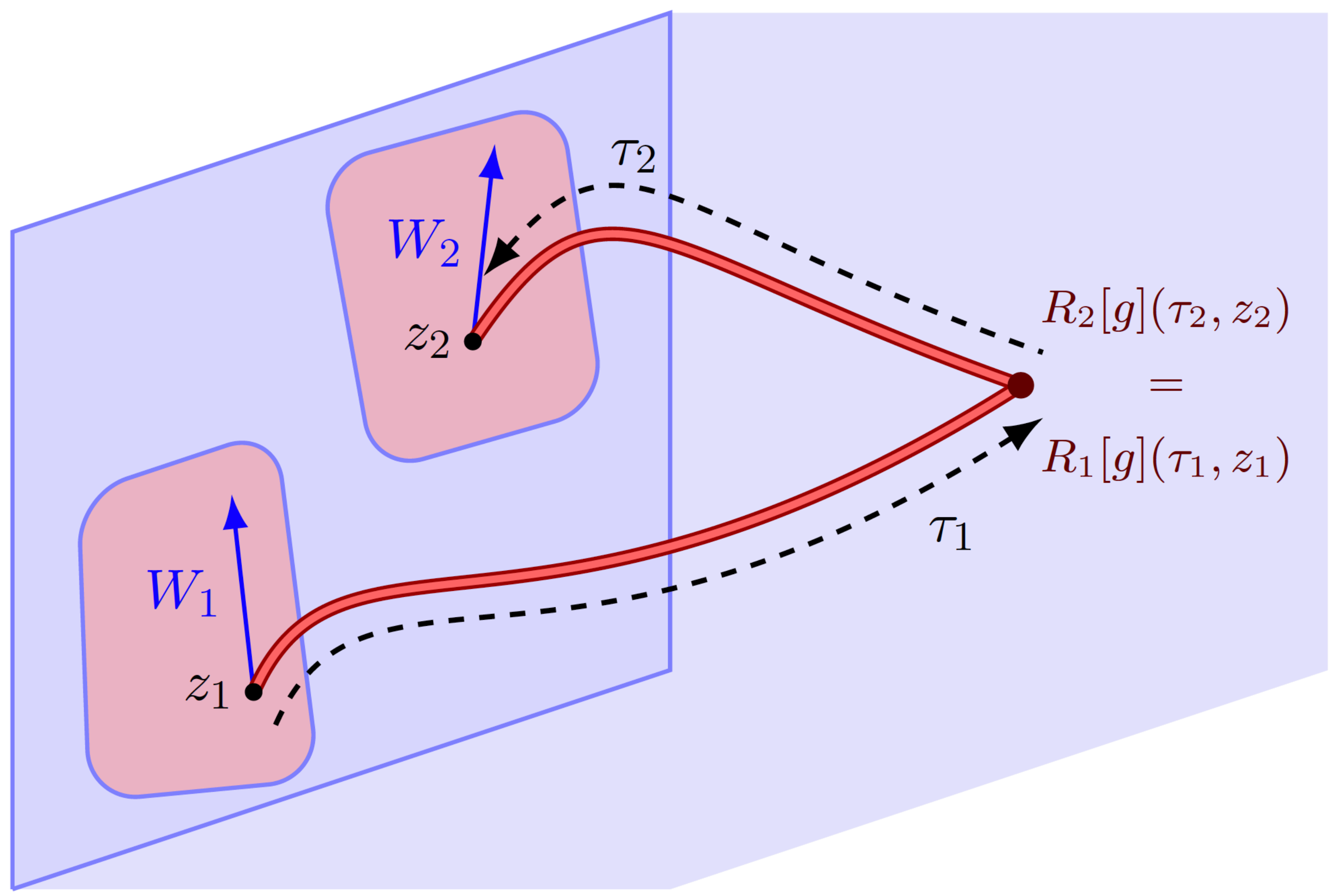
restrict to injective frames with overlapping images $\mathscr{N}_1[\phi] \cap \mathscr{N}_2[\phi] \neq \emptyset$

change of frame map: $\mathscr{R}_{1\to 2}[\phi] = \mathscr{R}_2^{-1}[\phi] \circ \mathscr{R}_1[\phi] : \mathscr{O}_1 \to \mathscr{O}_2$

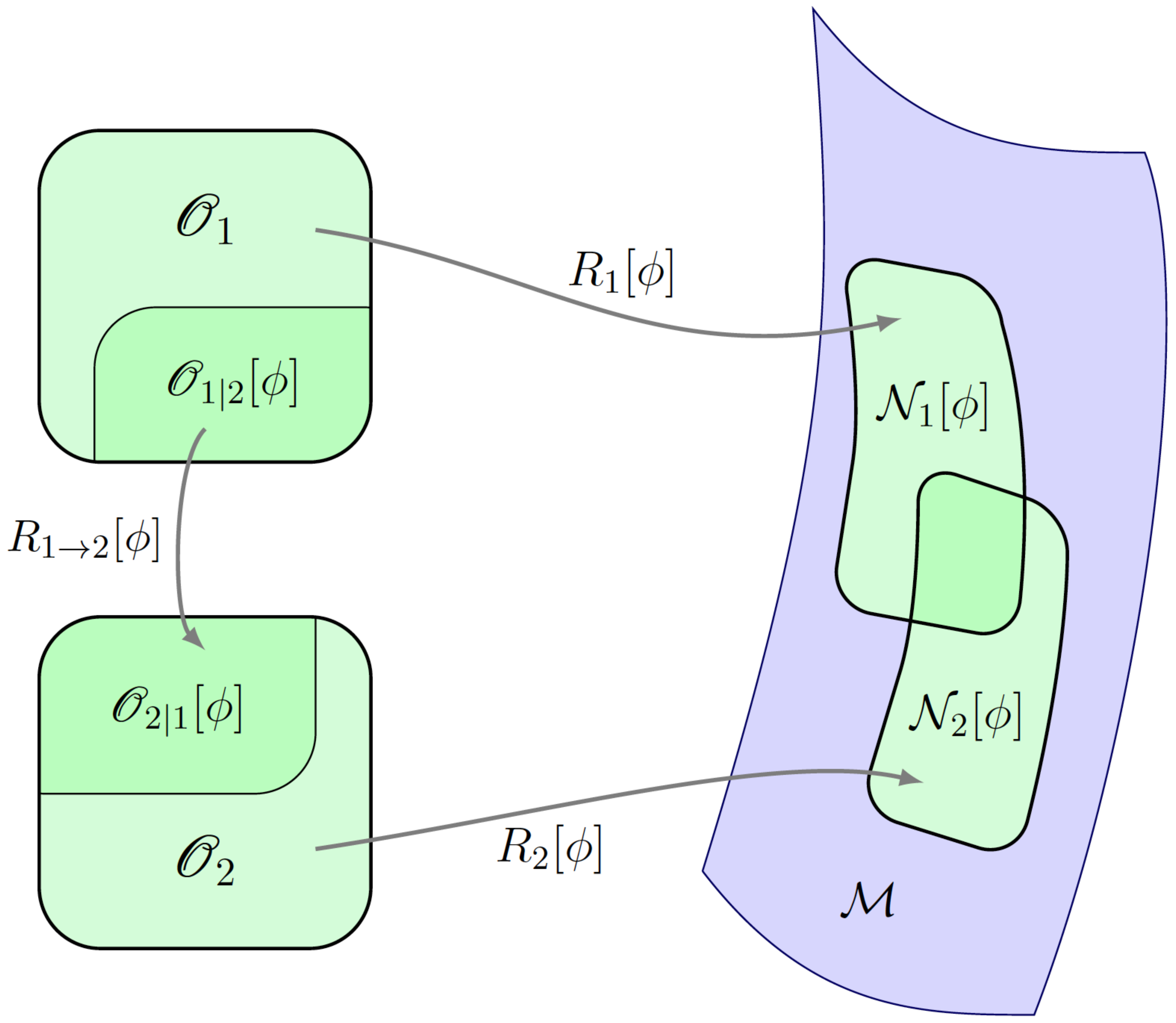
dynamical coord. change

Note: $\mathscr{R}_{1\to 2}[\phi] = (\mathscr{R}_1[\phi])^* \mathscr{R}_2^{-1}[\phi] = O_{\mathscr{R}_2^{-1}, \mathscr{R}_1}[\phi]$

is rel. observable describing 2nd frame rel. to 1st ⇒ gauge-inv.



Frame changes and relational atlases



restrict to injective frames with overlapping images $\mathcal{N}_1[\phi] \cap \mathcal{N}_2[\phi] \neq \emptyset$

change of frame map: $\mathscr{R}_{1\to 2}[\phi] = \mathscr{R}_2^{-1}[\phi] \circ \mathscr{R}_1[\phi] : \mathcal{O}_1 \to \mathcal{O}_2$

dynamical coord. change

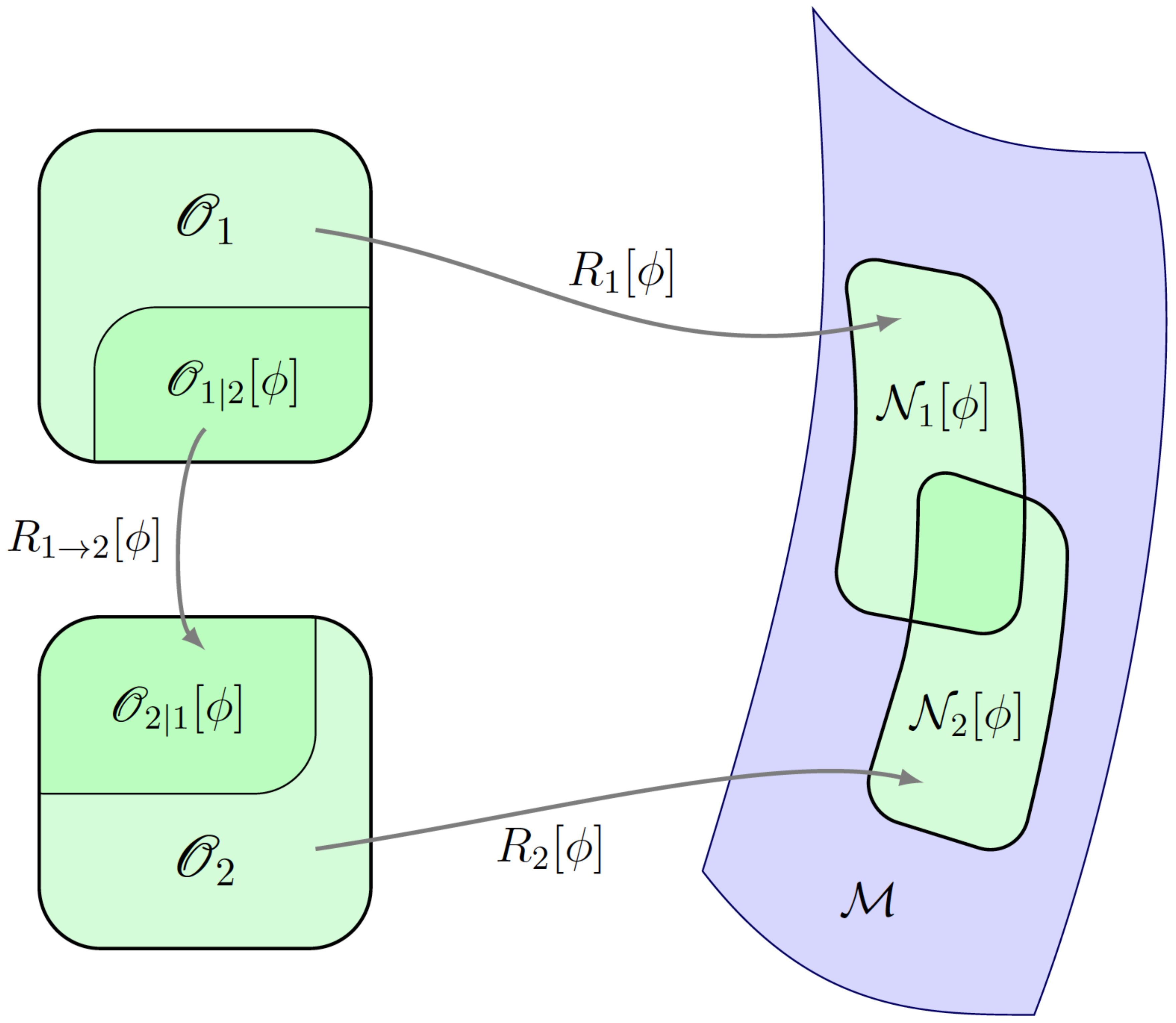
Note: $\mathscr{R}_{1\to 2}[\phi] = (\mathscr{R}_1[\phi])^* \mathscr{R}_2^{-1}[\phi] = O_{\mathscr{R}_2^{-1},\mathscr{R}_1}[\phi]$

is rel. observable describing 2nd frame rel. to 1st $\Rightarrow$ gauge-inv.

$\Rightarrow$ **relational observables transform as** $O_{T,\mathscr{R}_2}[\phi] = (\mathscr{R}_{1\to 2}[\phi])_* O_{T,\mathscr{R}_1}[\phi]$

change of gauge-inv. description of T from internal perspective of frame 1 into internal perspective of frame 2

Frame changes and relational atlases



restrict to injective frames with overlapping images $\mathcal{N}_1[\phi] \cap \mathcal{N}_2[\phi] \neq \emptyset$

change of frame map: $\mathcal{R}_{1\to 2}[\phi] = \mathcal{R}_2^{-1}[\phi] \circ \mathcal{R}_1[\phi] : \mathcal{O}_1 \to \mathcal{O}_2$

dynamical coord. change

Note: $\mathcal{R}_{1\to 2}[\phi] = (\mathcal{R}_1[\phi])^* \mathcal{R}_2^{-1}[\phi] = O_{\mathcal{R}_2^{-1},\mathcal{R}_1}[\phi]$

is rel. observable describing 2nd frame rel. to 1st ⇒ gauge-inv.

⇒ **relational observables transform as** $O_{T,\mathcal{R}_2}[\phi] = (\mathcal{R}_{1\to 2}[\phi])_* O_{T,\mathcal{R}_1}[\phi]$

change of gauge-inv. description of T from internal perspective of frame 1 into internal perspective of frame 2

To cover all of spacetime, need relational atlas $\mathcal{A}$ of (inj.) dyn. frames s.t. $\bigcup_{\mathcal{R}\in\mathcal{A}} \mathcal{R}[\phi](\mathcal{O}) = \mathcal{M}$

⇒ transition fcts. above ⇒ obtain consistent gauge-inv. global description via many local frames

Dynamical frame covariance: a relational update of general covariance

variation of gen. cov. Lagrangian: $\delta L[\phi] = E[\phi] + d\theta[\phi]$

EoM term: $E \approx 0$

bdry term

components of fields in i-coords.

EoMs in local (fixed) coord. system σ_i: $E_i[\phi_i] = (\sigma_i)_* E[(\sigma_i)^* \phi_i]$, **where** $\phi_i = (\sigma_i)_* \phi$

Dynamical frame covariance: a relational update of general covariance

variation of gen. cov. Lagrangian: $\delta L[\phi] = E[\phi] + d\theta[\phi]$

E.g. GR: $E_i[g_i] = G_{\mu\nu}[g_i]\delta g_i^{\mu\nu}\epsilon$

EoM term: $E \approx 0$

bdry term

components of fields in i-coords.

EoMs in local (fixed) coord. system σ_i: $E_i[\phi_i] = (\sigma_i)_* E[(\sigma_i)^*\phi_i]$, **where** $\phi_i = (\sigma_i)_*\phi$

Dynamical frame covariance: a relational update of general covariance

variation of gen. cov. Lagrangian: $\delta L[\phi] = E[\phi] + d\theta[\phi]$

E.g. GR: $E_i[g_i] = G_{\mu\nu}[g_i]\delta g_i^{\mu\nu}\epsilon$

EoM term: $E \approx 0$

bdry term

components of fields in i-coords.

EoMs in local (fixed) coord. system σ_i: $E_i[\phi_i] = (\sigma_i)_* E[(\sigma_i)^* \phi_i]$, **where** $\phi_i = (\sigma_i)_* \phi$

$\Rightarrow$ **general covariance:** $E_1[\bar{\phi}] = 0 \quad \Leftrightarrow \quad E_2[\bar{\phi}] = 0$

“All the laws of physics are the same in every fixed reference frame”

$E_i[\phi_i]$ **is description of laws rel. to fixed frame** i $\quad \Rightarrow \quad$ **gauge-noninv. and (fixed) frame-dependent description of physics**

Dynamical frame covariance: a relational update of general covariance

variation of gen. cov. Lagrangian: $\delta L[\phi] = E[\phi] + d\theta[\phi]$

EoM term: $E \approx 0$

bdry term

E.g. GR: $\tilde{E}_i[\tilde{g}_i] = G_{\mu\nu}[\tilde{g}_i]\delta\tilde{g}_i^{\mu\nu}\tilde{\epsilon}$

components of fields in dyn. i-coords. (rel. observables)

EoMs in local dynamical coord. system $\mathscr{R}_i^{-1}[\phi]$: $\tilde{E}_i[\tilde{\phi}_i] = (\mathscr{R}_i^{-1}[\phi])_* E[(\mathscr{R}_i[\phi])_*\tilde{\phi}_i]$, **where** $\tilde{\phi}_i = (\mathscr{R}_i[\phi])^*\phi$

$\Rightarrow$ **general covariance:** $\tilde{E}_1[\bar{\phi}] = 0 \quad \Leftrightarrow \quad \tilde{E}_2[\bar{\phi}] = 0$

"All the laws of physics are the same in every dynamical reference frame"

$\tilde{E}_i[\tilde{\phi}_i]$ **is description of laws rel. to dynamical frame** i $\Rightarrow$ **gauge-inv. and (dyn.) frame-dependent description of physics**

Dynamical frame covariance: a relational update of general covariance

variation of gen. cov. Lagrangian: $\delta L[\phi] = E[\phi] + d\theta[\phi]$

EoM term: $E \approx 0$

bdry term

E.g. GR: $\tilde{E}_i[\tilde{g}_i] = G_{\mu\nu}[\tilde{g}_i]\delta\tilde{g}_i^{\mu\nu}\tilde{\epsilon}$

components of fields in dyn. i-coords. (rel. observables)

EoMs in local dynamical coord. system $\mathscr{R}_i^{-1}[\phi]$: $\tilde{E}_i[\tilde{\phi}_i] = (\mathscr{R}_i^{-1}[\phi])_* E[(\mathscr{R}_i[\phi])_* \tilde{\phi}_i]$, **where** $\tilde{\phi}_i = (\mathscr{R}_i[\phi])^* \phi$

$\Rightarrow$ **general covariance:** $\tilde{E}_1[\bar{\phi}] = 0 \quad \Leftrightarrow \quad \tilde{E}_2[\bar{\phi}] = 0$

"All the laws of physics are the same in every dynamical reference frame"

$\tilde{E}_i[\tilde{\phi}_i]$ **is description of laws rel. to dynamical frame i** $\Rightarrow$ **gauge-inv. and (dyn.) frame-dependent description of physics that is indep. of any fixed frame (rel. observables indep. of fixed coords.)**

Dynamical frame covariance: a relational update of general covariance

variation of gen. cov. Lagrangian: $\delta L[\phi] = E[\phi] + d\theta[\phi]$

EoM term: $E \approx 0$

bdry term

E.g. GR: $\tilde{E}_i[\tilde{g}_i] = G_{\mu\nu}[\tilde{g}_i]\delta\tilde{g}_i^{\mu\nu}\tilde{\epsilon}$

components of fields in dyn. i-coords. (rel. observables)

EoMs in local dynamical coord. system $\mathscr{R}_i^{-1}[\phi]$: $\tilde{E}_i[\tilde{\phi}_i] = (\mathscr{R}_i^{-1}[\phi])_* E[(\mathscr{R}_i[\phi])_*\tilde{\phi}_i]$, **where** $\tilde{\phi}_i = (\mathscr{R}_i[\phi])^*\phi$

$\Rightarrow$ **general covariance:** $\tilde{E}_1[\bar{\phi}] = 0 \quad \Leftrightarrow \quad \tilde{E}_2[\bar{\phi}] = 0$

"All the laws of physics are the same in every dynamical reference frame"

$\tilde{E}_i[\tilde{\phi}_i]$ **is description of laws rel. to dynamical frame** i $\Rightarrow$ **gauge-inv. and (dyn.) frame-dependent description of physics that is indep. of any fixed frame (rel. observables indep. of fixed coords.)**

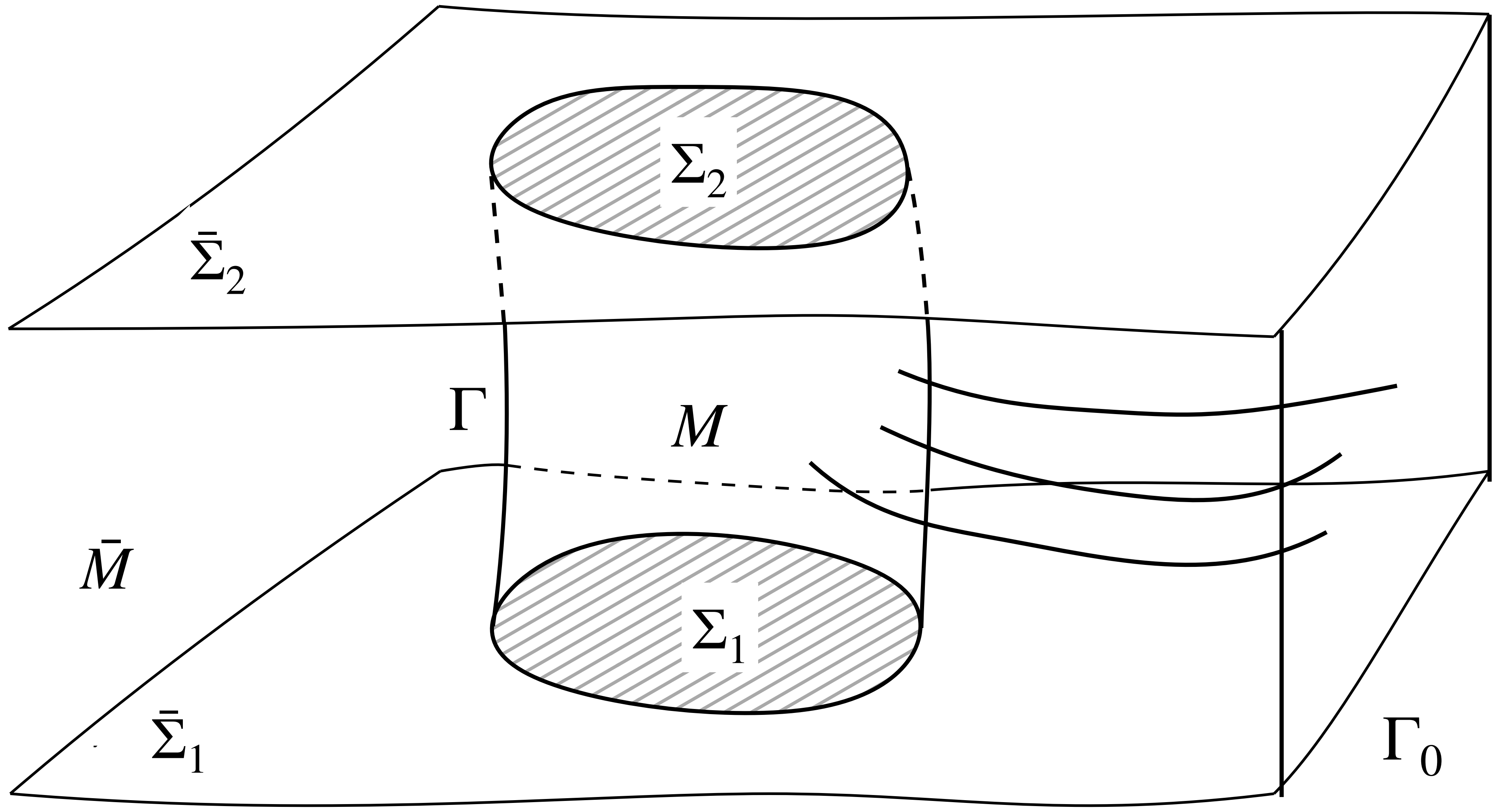
$\Rightarrow$ **dynamical frame covariance provides a dynamical and gauge-inv. (and thus more physical) update of general covariance**

Local subsystems relative to a frame

[Carrozza, Eccles, PH 2205.00913]

gauge-cov. definition of subregion rel. to frame

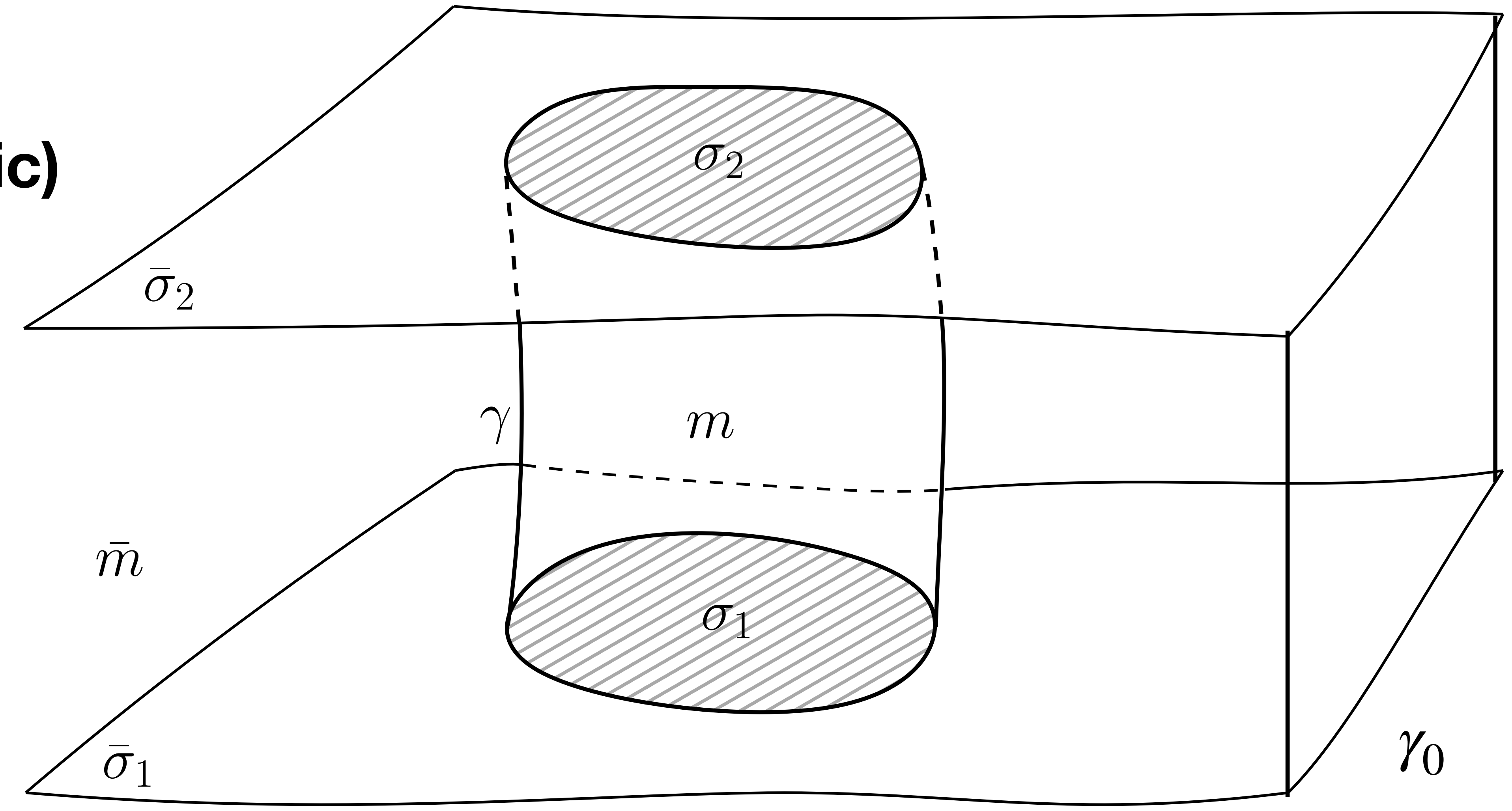
gauge-inv. definition of subregion rel. to frame



spacetime $\mathscr{M}$

gauge-cov. (e.g. geodesic) frame

$\mathscr{R}^{-1}[\phi]$ $\longrightarrow$



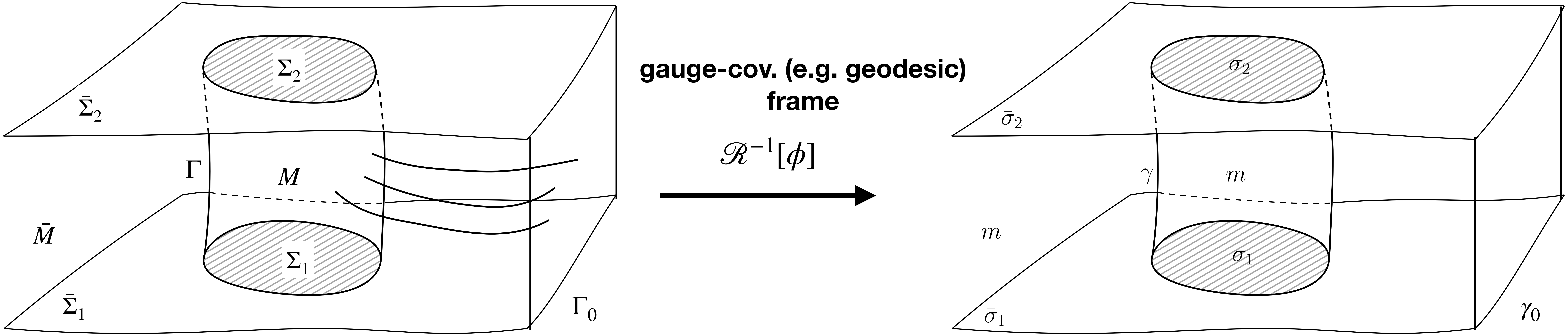
orientation space $\mathscr{O}$

Local subsystems relative to a frame

[Carrozza, Eccles, PH 2205.00913]

gauge-cov. definition of subregion rel. to frame

gauge-inv. definition of subregion rel. to frame



spacetime $\mathscr{M}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

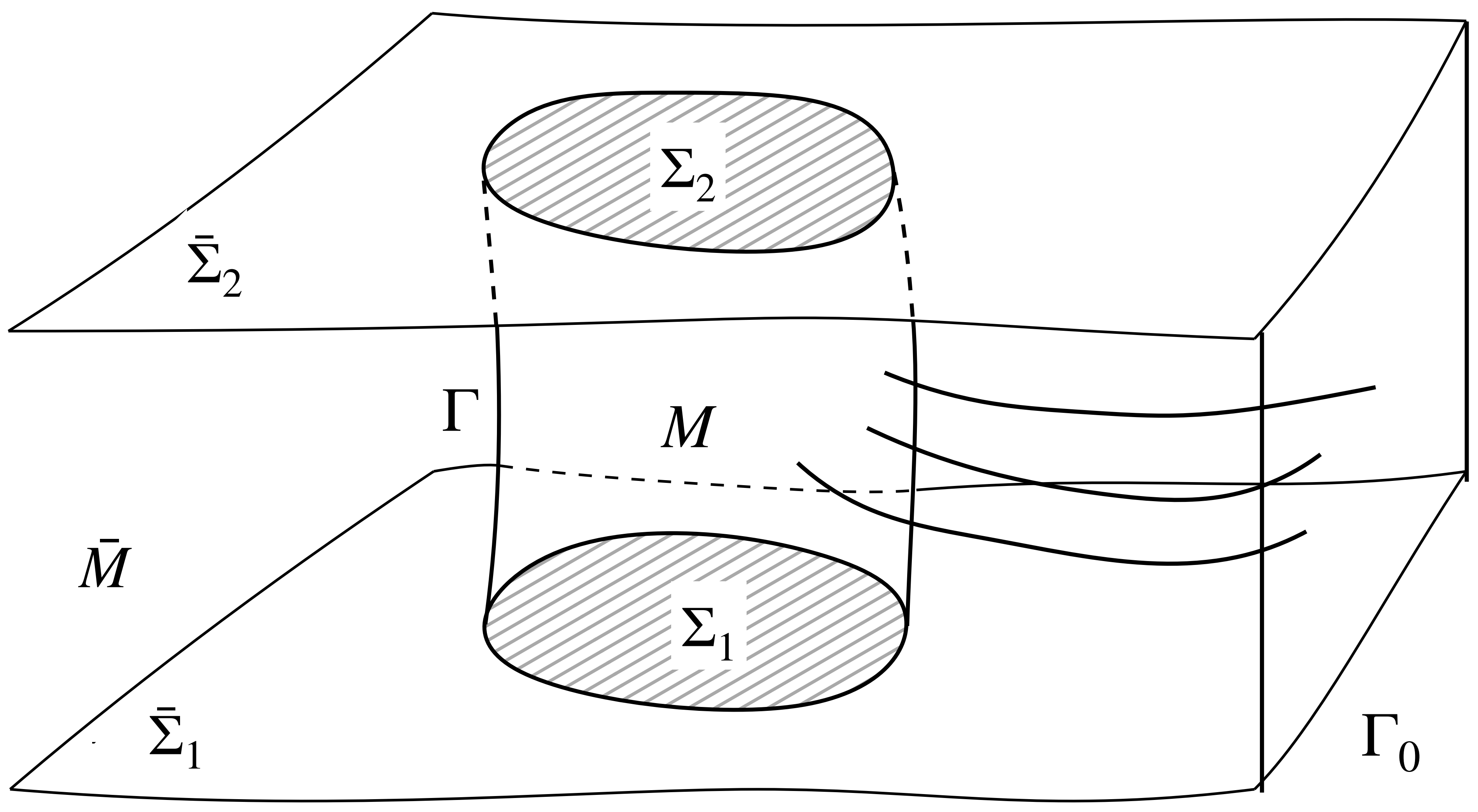
orientation space $\mathscr{O}$

Local subsystems relative to a frame

[Carrozza, Eccles, PH 2205.00913]

gauge-cov. definition of subregion rel. to frame

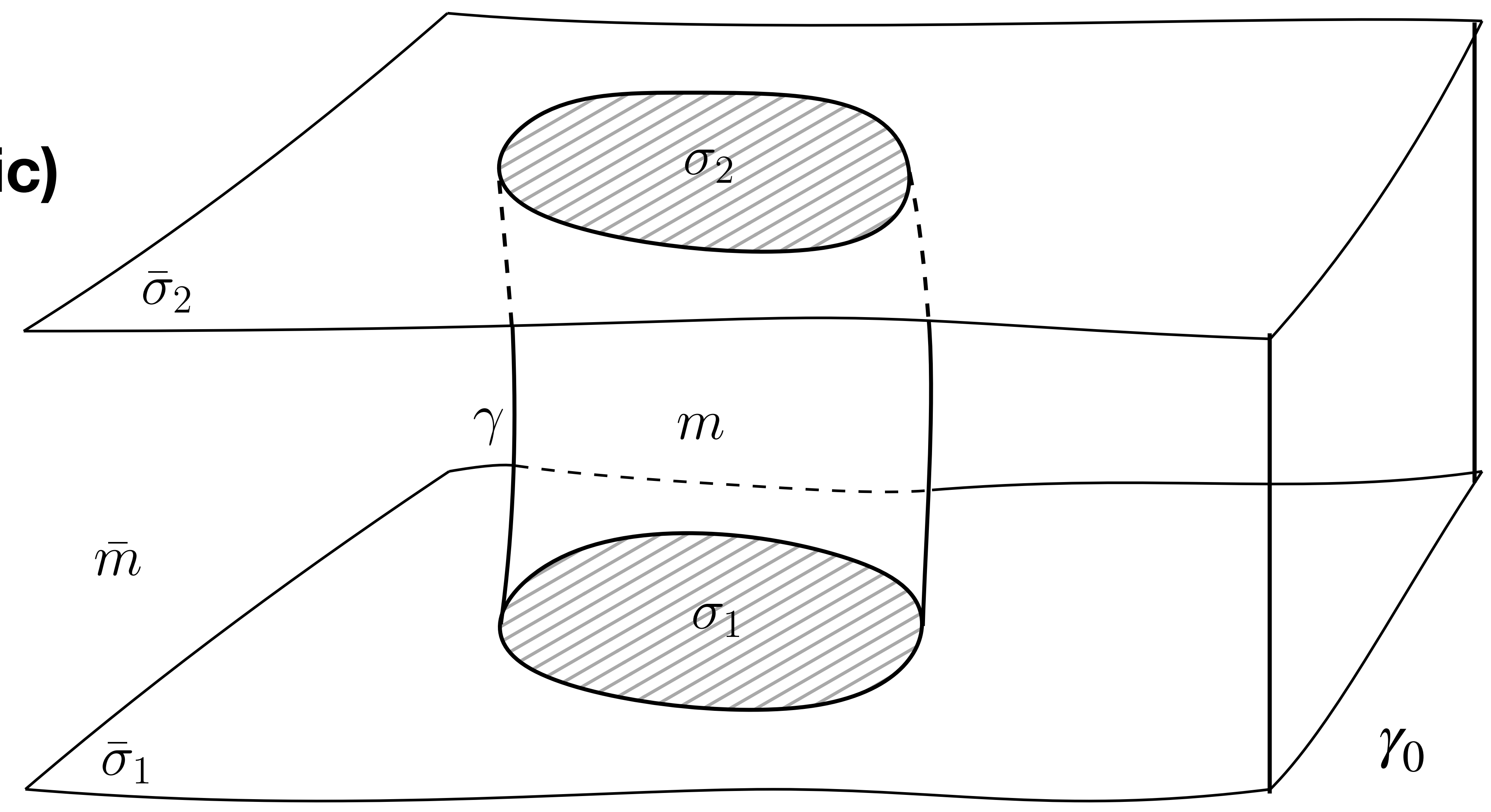
gauge-inv. definition of subregion rel. to frame



spacetime $\mathscr{M}$

gauge-cov. (e.g. geodesic) frame

$\mathscr{R}^{-1}[\phi]$



orientation space $\mathscr{O}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

diffeos act from the right $\mathscr{R}^{-1} \mapsto \mathscr{R}^{-1} \circ f^{-1}$

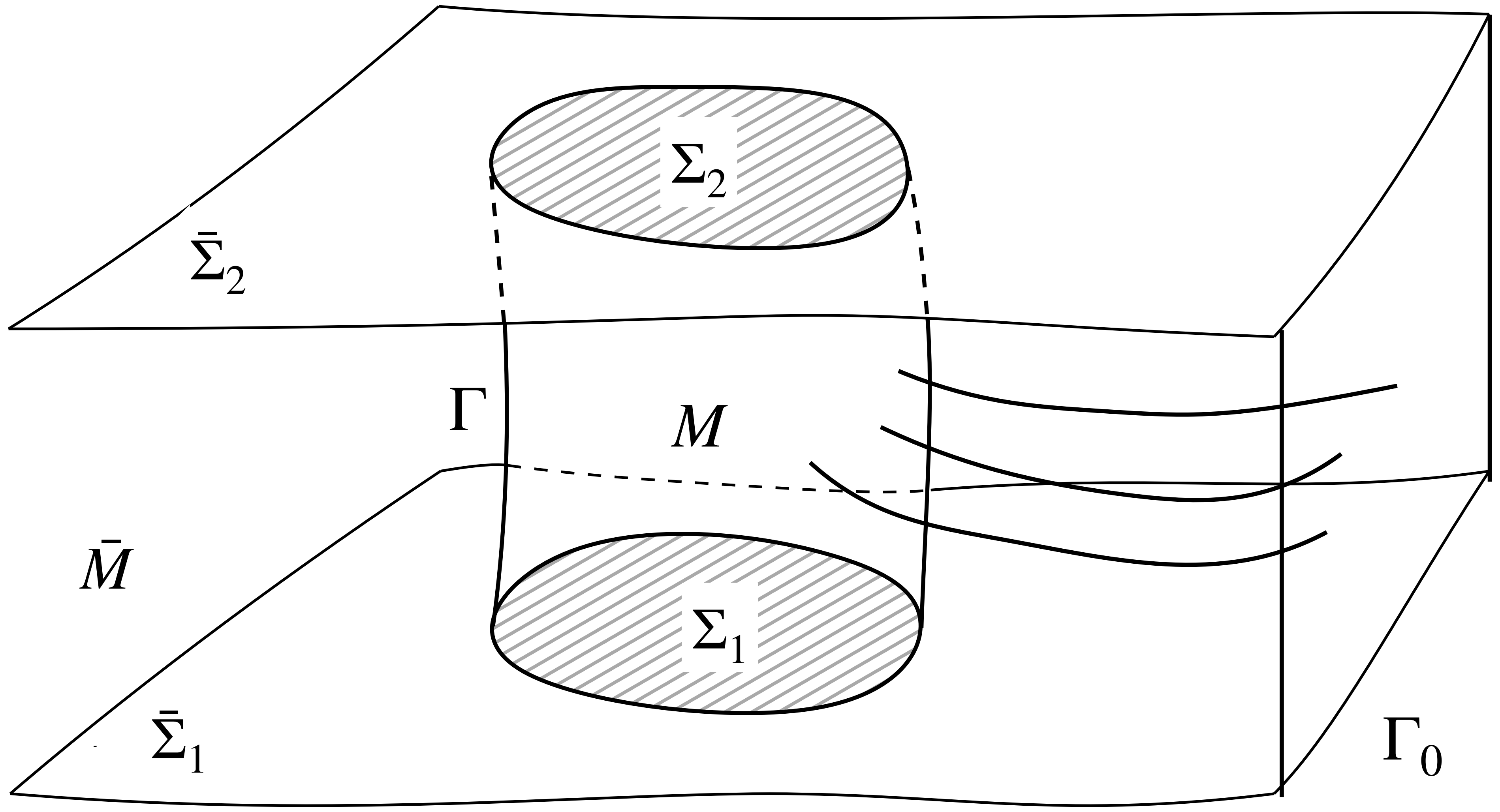
⇒ expect to be gauge (acts on all DoFs) and obey constraint algebra (no gauge broken)

Local subsystems relative to a frame

[Carrozza, Eccles, PH 2205.00913]

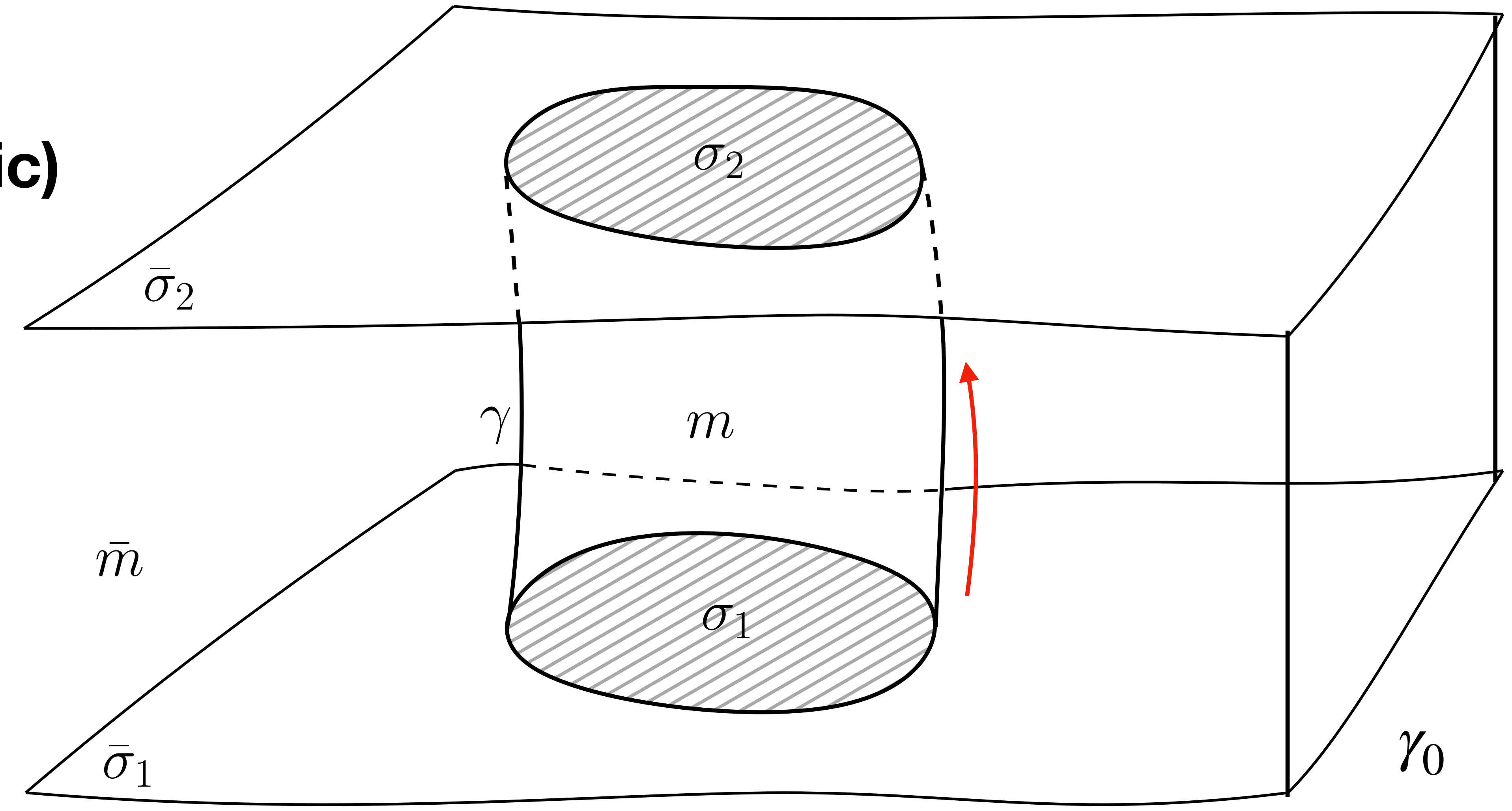
gauge-cov. definition of subregion rel. to frame

gauge-inv. definition of subregion rel. to frame



gauge-cov. (e.g. geodesic) frame

$$\xrightarrow{\mathscr{R}^{-1}[\phi]}$$



spacetime $\mathscr{M}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

orientation space $\mathscr{O}$

“frame reorientation”

diffeos act from the right $\mathscr{R}^{-1} \mapsto \mathscr{R}^{-1} \circ f^{-1}$

$\Rightarrow$ expect to be gauge (acts on all DoFs) and obey constraint algebra (no gauge broken)

diffeos act from the left $\mathscr{R}^{-1} \mapsto \tilde{f} \circ \mathscr{R}^{-1}$

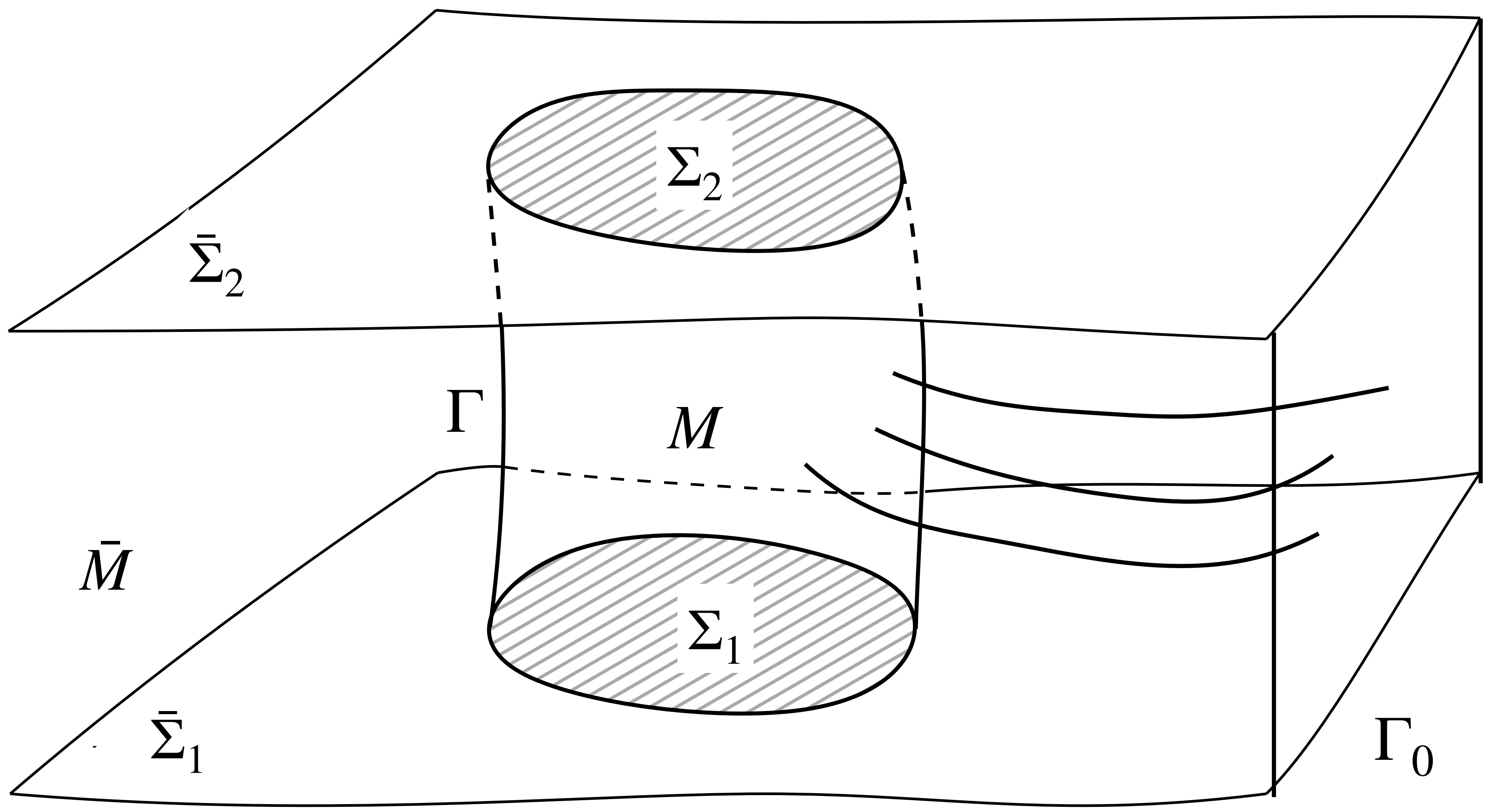
$\Rightarrow$ expect to be physical (changes relation between frame and rest), but non-trivial charge algebra only for diffeos preserving γ

Local subsystems relative to a frame

[Carrozza, Eccles, PH 2205.00913]

gauge-cov. definition of subregion rel. to frame

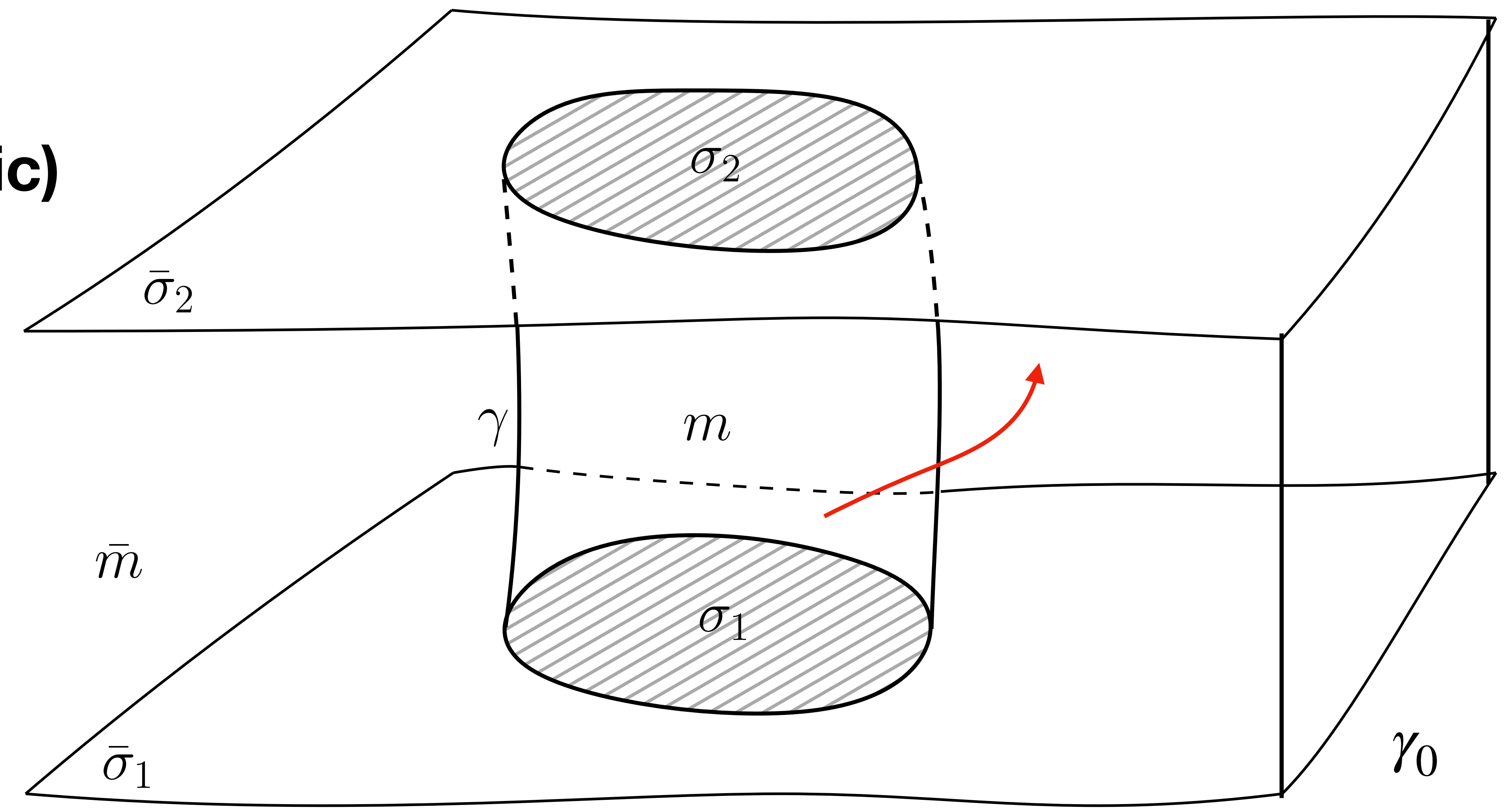
gauge-inv. definition of subregion rel. to frame



spacetime $\mathscr{M}$

gauge-cov. (e.g. geodesic) frame

$\mathscr{R}^{-1}[\phi]$



orientation space $\mathscr{O}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

“frame reorientation”

diffeos act from the right $\mathscr{R}^{-1} \mapsto \mathscr{R}^{-1} \circ f^{-1}$

$\Rightarrow$ expect to be gauge (acts on all DoFs) and obey constraint algebra (no gauge broken)

diffeos act from the left $\mathscr{R}^{-1} \mapsto \tilde{f} \circ \mathscr{R}^{-1}$

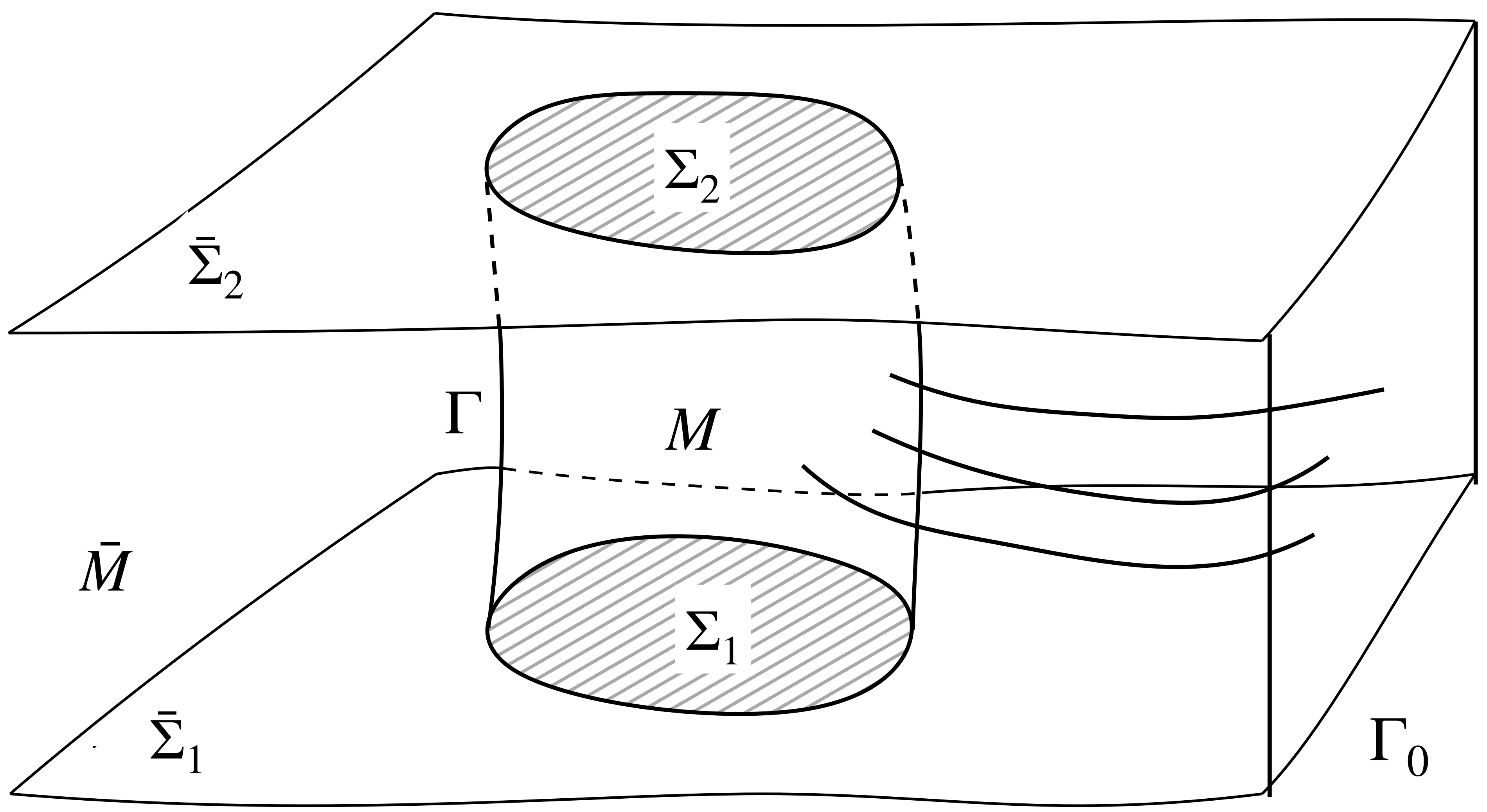
$\Rightarrow$ expect to be physical (changes relation between frame and rest), but non-trivial charge algebra only for diffeos preserving γ (otherwise open system transf.)

Local subsystems relative to a frame

[Carrozza, Eccles, PH 2205.00913]

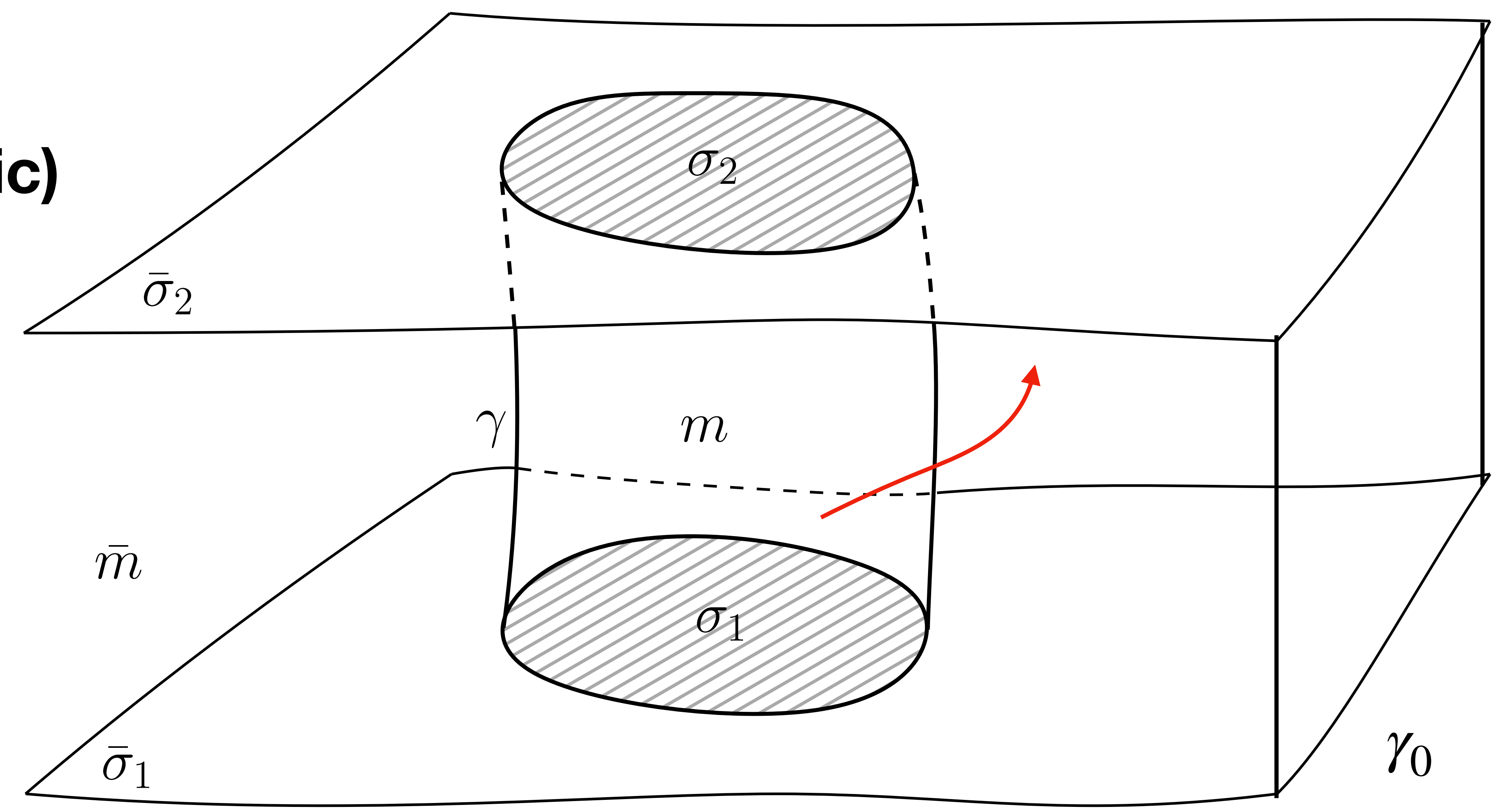
gauge-cov. definition of subregion rel. to frame

gauge-inv. definition of subregion rel. to frame



gauge-cov. (e.g. geodesic) frame

$\mathscr{R}^{-1}[\phi]$



spacetime $\mathscr{M}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

orientation space $\mathscr{O}$

diffeos act from the right $\mathscr{R}^{-1} \mapsto \mathscr{R}^{-1} \circ f^{-1}$

$\Rightarrow$ expect to be gauge (acts on all DoFs) and obey constraint algebra (no gauge broken)

"frame reorientation"

diffeos act from the left $\mathscr{R}^{-1} \mapsto \tilde{f} \circ \mathscr{R}^{-1}$

$\Rightarrow$ expect to be physical (changes relation between frame and rest), but non-trivial charge algebra only for diffeos preserving γ (otherwise open system transf.)

$\Rightarrow$ get unambiguous gauge-inv. subregion presymplectic structure (from post-selection) by requiring preservation under dynamics

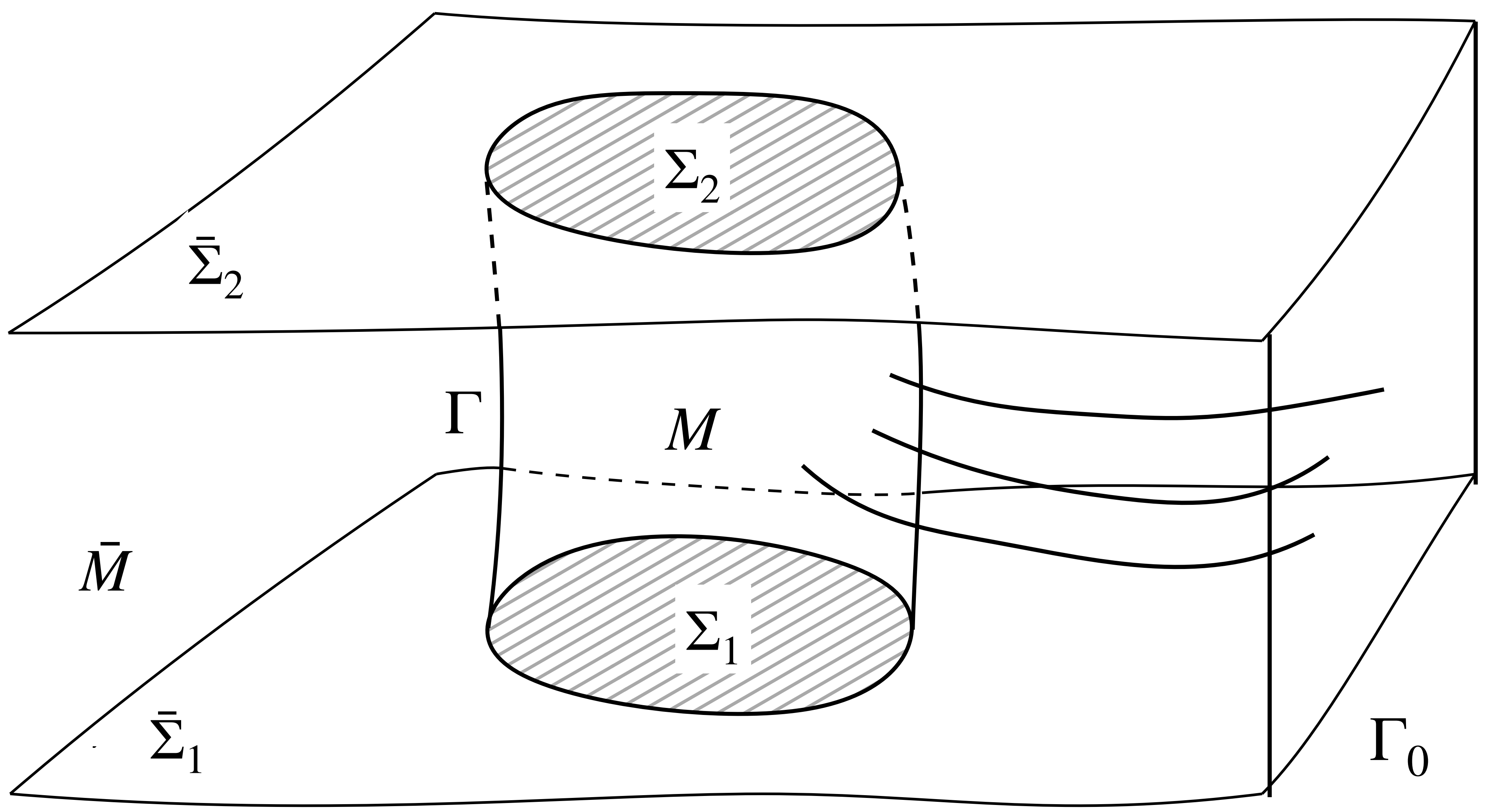
[diff. presymp. structure than Donnelly-Freidel; Speranza; Ciambelli-Leigh-Pai; Freidel, ...]

Local subsystems relative to a frame

[Carrozza, Eccles, PH 2205.00913]

gauge-cov. definition of subregion rel. to frame

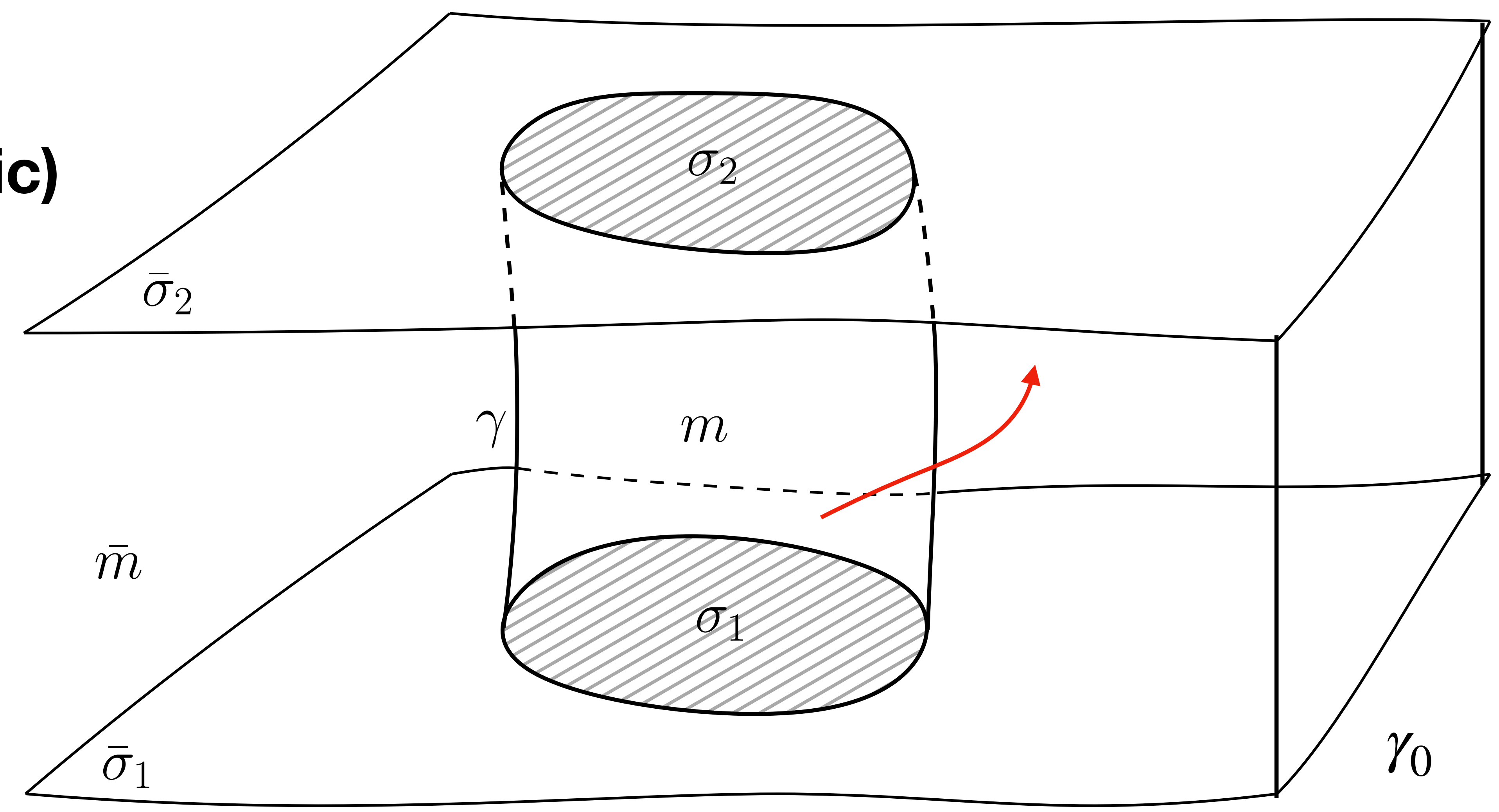
gauge-inv. definition of subregion rel. to frame



spacetime $\mathscr{M}$

gauge-cov. (e.g. geodesic) frame

$\mathscr{R}^{-1}[\phi]$



orientation space $\mathscr{O}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

diffeos act from the right $\mathscr{R}^{-1} \mapsto \mathscr{R}^{-1} \circ f^{-1}$

⇒ expect to be gauge (acts on all DoFs) and obey constraint algebra (no gauge broken)

find full diffeo constraint algebra:

$$-I_{V_\xi}\Omega = \delta C[\xi] \approx 0$$

$$\{C[\xi], C[\zeta]\} = -C\big[[\xi,\zeta]\big]$$

[cov. version of Isham, Kuchar '85]

"frame reorientation"

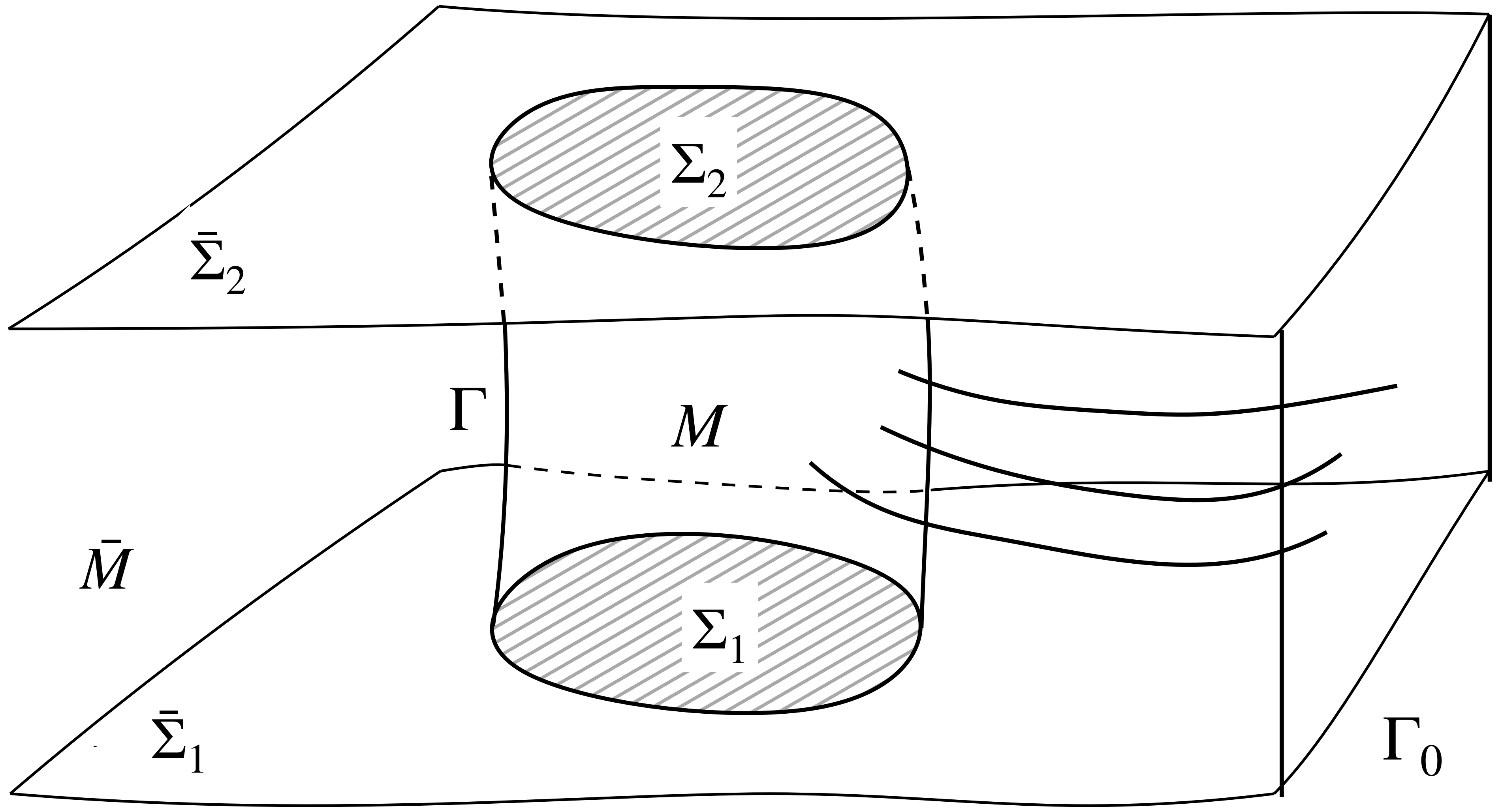
diffeos act from the left $\mathscr{R}^{-1} \mapsto \tilde{f} \circ \mathscr{R}^{-1}$

⇒ expect to be physical (changes relation between frame and rest), but non-trivial charge algebra only for diffeos preserving γ (otherwise open system transf.)

Local subsystems relative to a frame

[Carrozza, Eccles, PH 2205.00913]

gauge-cov. definition of subregion rel. to frame

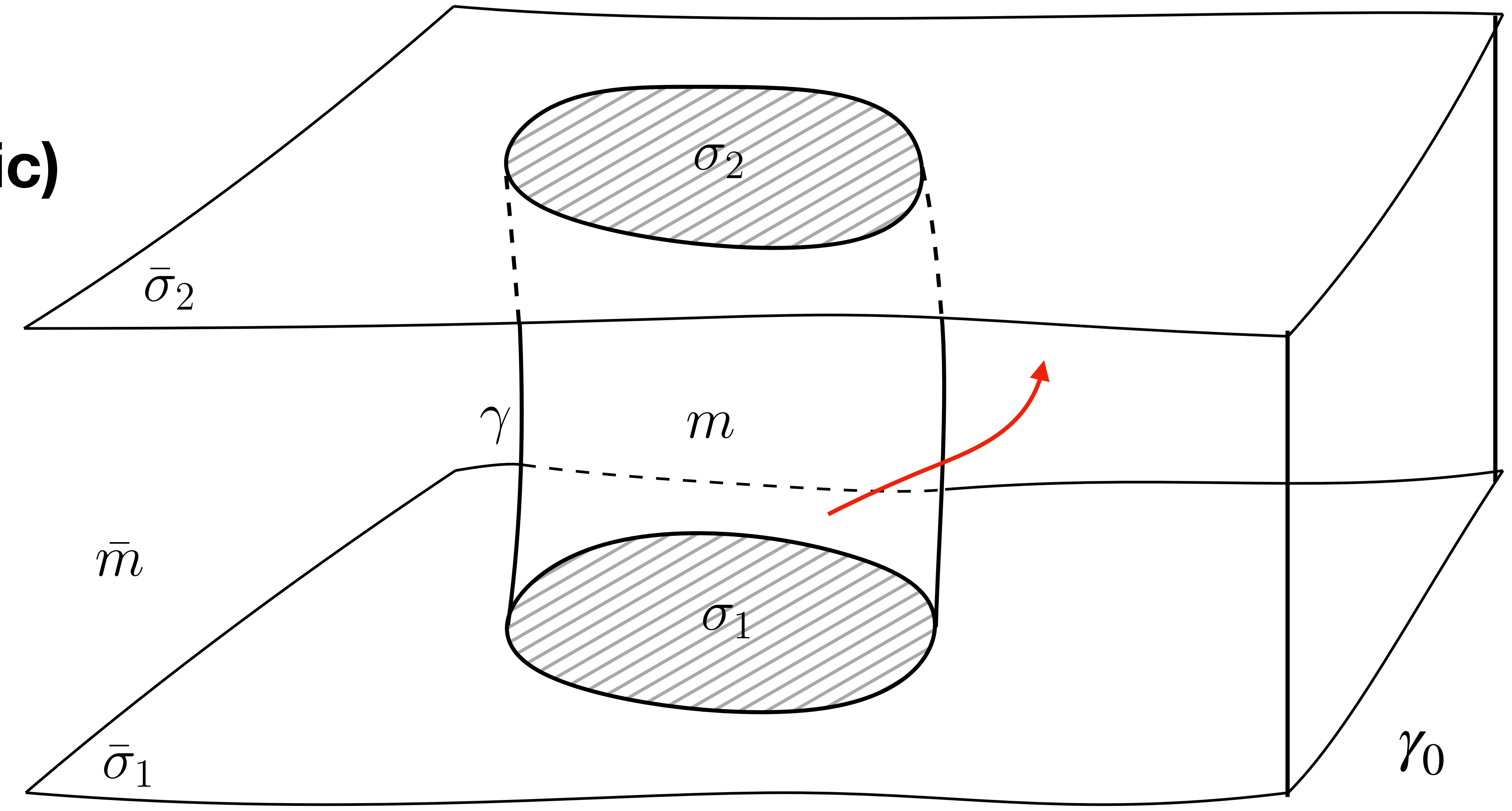


spacetime $\mathscr{M}$

gauge-cov. (e.g. geodesic) frame

$$\mathscr{R}^{-1}[\phi]$$

gauge-inv. definition of subregion rel. to frame



orientation space $\mathscr{O}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

diffeos act from the right $\mathscr{R}^{-1} \mapsto \mathscr{R}^{-1} \circ f^{-1}$

$\Rightarrow$ expect to be gauge (acts on all DoFs) and obey constraint algebra (no gauge broken)

find full diffeo constraint algebra:

$$-I_{V_\xi}\Omega = \delta C[\xi] \approx 0$$

$$\{C[\xi], C[\zeta]\} = -C\big[[\xi, \zeta]\big]$$

[cov. version of Isham, Kuchar '85]

"frame reorientation"

diffeos act from the left $\mathscr{R}^{-1} \mapsto \tilde{f} \circ \mathscr{R}^{-1}$

$\Rightarrow$ expect to be physical (changes relation between frame and rest), but non-trivial charge algebra only for diffeos preserving γ (otherwise open system transf.)

charges not in general integrable:

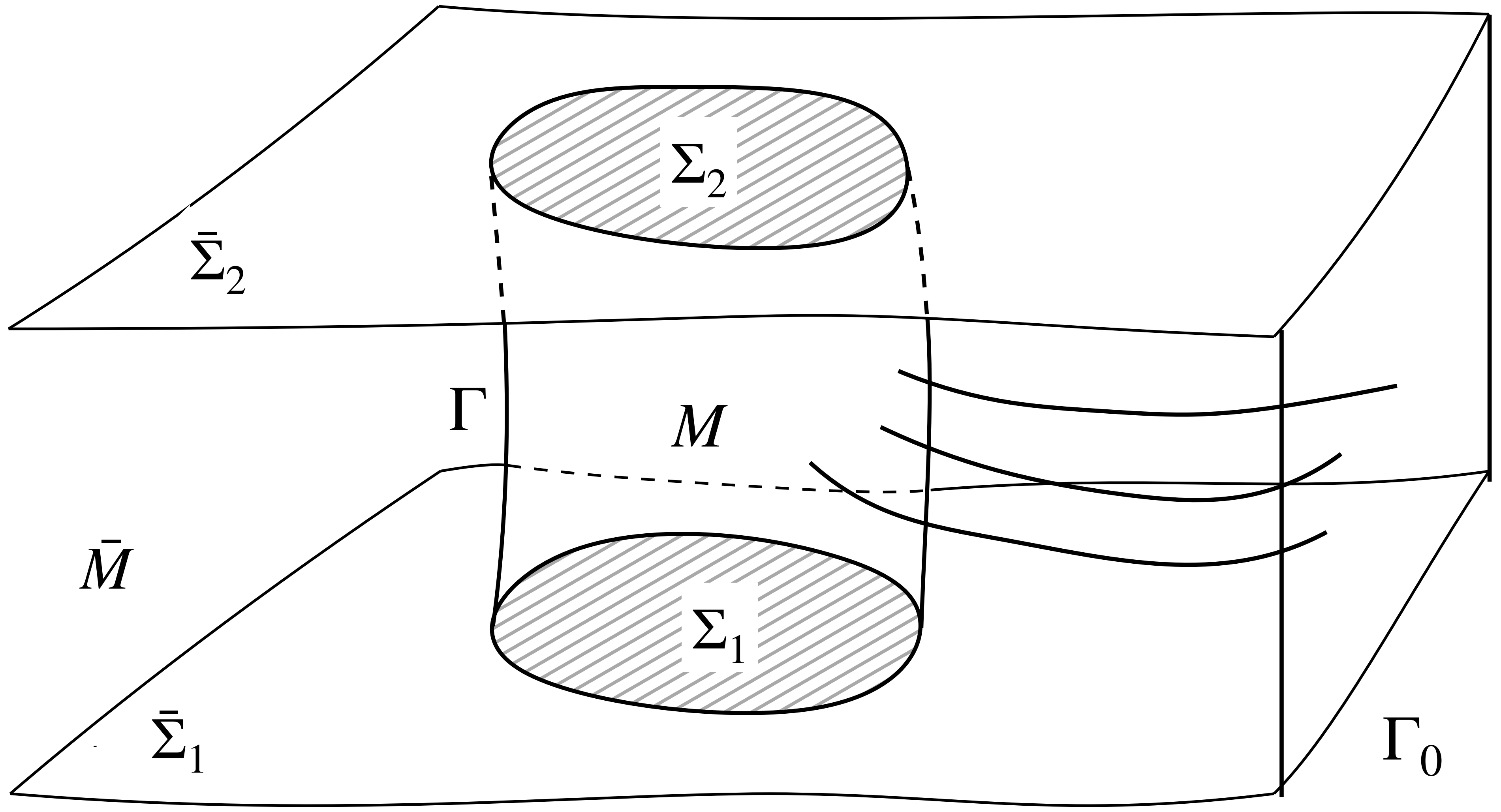
$$-I_{W_\rho}\Omega = \delta Q[\rho] + \text{flux}$$

Local subsystems relative to a frame

[Carrozza, Eccles, PH 2205.00913]

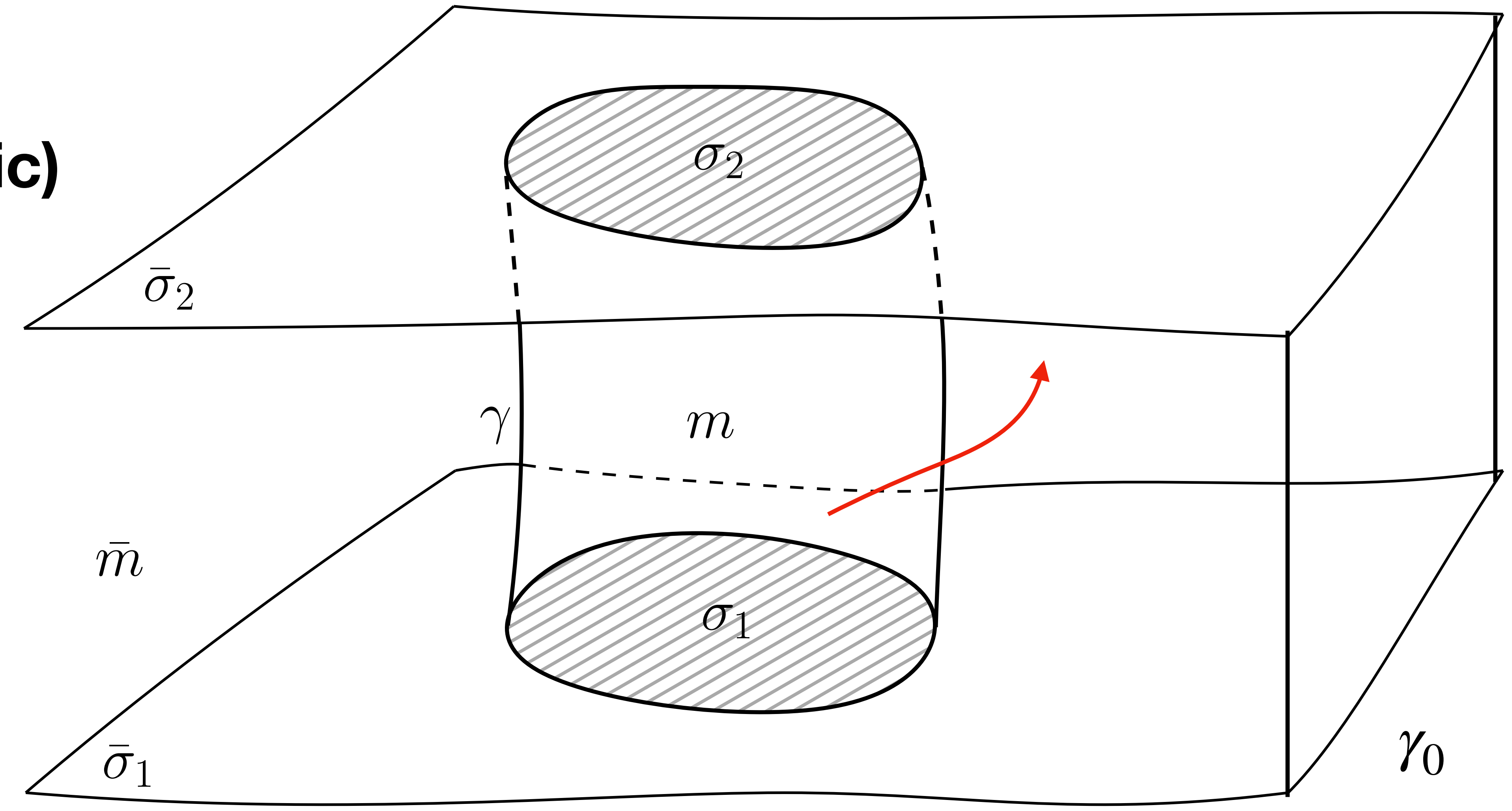
gauge-cov. definition of subregion rel. to frame

gauge-inv. definition of subregion rel. to frame



gauge-cov. (e.g. geodesic) frame

$\mathscr{R}^{-1}[\phi]$



spacetime $\mathscr{M}$

realisation of grav. edge modes as frame for subregion originating in complement (no need to postulate!)

orientation space $\mathscr{O}$

"frame reorientation"

diffeos act from the right $\mathscr{R}^{-1} \mapsto \mathscr{R}^{-1} \circ f^{-1}$

$\Rightarrow$ expect to be gauge (acts on all DoFs) and obey constraint algebra (no gauge broken)

find full diffeo constraint algebra:

$$-I_{V_\xi}\Omega = \delta C[\xi] \approx 0$$

$$\{C[\xi], C[\zeta]\} = -C\big[[\xi,\zeta]\big]$$

[cov. version of Isham, Kuchar '85]

diffeos act from the left $\mathscr{R}^{-1} \mapsto \tilde{f} \circ \mathscr{R}^{-1}$

$\Rightarrow$ expect to be physical (changes relation between frame and rest), but non-trivial charge algebra only for diffeos

integrable for $\rho \parallel \gamma$ and bdry conds.:

$-I_{W_\rho}\Omega = \delta Q_H[\rho] \not\approx 0$ $\Rightarrow$ generate centrally extended corner algebra

$$\{Q_H[\rho], Q_H[\kappa]\} = -Q\big[[\rho,\kappa]\big] - K_{\rho,\kappa} \qquad \{Q[\rho], K_{\kappa,\kappa'}\} = \{K_{\rho,\rho'}, K_{\kappa,\kappa'}\} = 0$$

[consistent with Chandrasekaran, Speranza '20]

Unifying picture of dynamical frame covariance

	Special relativity	Quantum relativity	Gauge relativity	General relativity
frame orientations (group valued frame)	tetrad $e^\mu_a \in \mathrm{SO}_+(3,1)$	coherent state $\lvert\phi(g)\rangle \in \mathscr{H}_R \quad g \in G$	field-dep. gauge transf. $U[\phi] : \mathscr{M} \to G$	field-dep. diffeo $\mathscr{R}^{-1}[\phi] : \mathscr{M} \to \mathscr{O}$
gauge transf./ gauge covariance	spacetime Lorentz tr. $\Lambda^\mu{}_\nu\, e^\nu_a \quad \Lambda^\mu{}_\nu \in \mathrm{SO}_+(3,1)$	left action: $U(g')\lvert\phi(g)\rangle = \lvert\phi(g'g)\rangle$	left action $g \triangleright U = gU$	spacetime diffeo $\mathscr{R}^{-1}[\phi] \circ f^{-1} \quad f \in \mathrm{Diff}(\mathscr{M})$
frame reorientations/ "symmetries" (act only on the frame)	frame Lorentz tr. $\Lambda_a{}^b\, e^\mu_b \quad \Lambda_a{}^b \in \mathrm{SO}_+(3,1)$	action from the right $V_R(g')\lvert\phi(g)\rangle = \lvert\phi(gg'^{-1})\rangle$	action from the right $g \odot U = Ug^{-1}$	orientation space diffeo $\tilde{f} \circ \mathscr{R}^{-1}[\phi] \quad \tilde{f} \in \mathrm{Diff}(\mathscr{O})$
relational observables $O_{F,R}(g)$ "what's value of F when RF is in orientation g?"	$F_{\mu\ldots} e^\mu_a \Lambda_b{}^a(g^{-1})\cdots$	$\int_G dg' \hat{U}_{RS}(g')(\lvert\phi(g)\rangle\langle\phi(g)\rvert \otimes F_S)$	$(U(x)g(x)^{-1})^{-1} \triangleright F(x)$	$(\mathscr{R}[\phi]\,\tilde{f}(g))^* F[\phi]$
"jumping into RF perspective" (gauge fixing)	$e^\mu_a = \Lambda^\mu{}_a(g)$	conditioning on frame or. $\varphi(g) = \langle\phi(g)\rvert \otimes \mathbf{1}_S$	$U[\phi_0](x) = g(x)$	$\mathscr{R}^{-1}[\phi_0](x) = g^{-1}(x)$
RF change 1 (rel. cond. gauge transf.)	$\Lambda^\mu{}_\nu(e,e') = \Lambda^\mu{}_{a'}(g') e'^{a'}_\kappa\, e^\kappa_b \Lambda_\nu{}^b(g)$	$\varphi_2(g') \circ \varphi_1^{-1}(g)$	$g'\, U_2^{-1}[\phi]\, U_1[\phi]\, g^{-1}$	$g' \circ \mathscr{R}_2^{-1}[\phi] \circ \mathscr{R}_1[\phi] \circ g^{-1}$
RF change 2 (rel. cond. symmetry) (1st frame relative to 2nd)	$\Lambda^{a'}{}_b(e,e') = e'^{a'}_\mu\, e^\mu_b$	long expression (see 2110.13824)	$U_2^{-1}[\phi] U_1[\phi]$	$\mathscr{R}_2^{-1}[\phi] \circ \mathscr{R}_1[\phi]$

2 commuting group actions

Conclusions

- **Unifying picture of special, quantum, gauge and general relativity**
  in terms of dynamical frames ⇒ quantum relativity as natural extension of relativity principle into quantum realm
- **Systematic method for changing QRF perspectives**
  accommodates RFs in relative superposition

  see also F. Mele's talk today!
- **Gauge-inv. subsystems depend on choice of dyn. frame**
  ⇒ correlations, thermal properties, dynamics, …. depend on frame
- **Edge modes as dyn. frames**
  corner symmetries as frame reorientations
- **General framework for dyn. frames and relational observables in gravity**
  generalises and unifies previous approaches, relational update of general covariance

  see also C. Goeller's talk today!

Appendix

Relational bulk dynamics

- **assume frame $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \to \mathscr{O}$ sufficiently differentiable $\Rightarrow$ orientation space $\mathscr{O}$ a spacetime with causal structure**

Relational bulk dynamics

- **assume frame $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \to \mathscr{O}$ sufficiently differentiable $\Rightarrow$ orientation space $\mathscr{O}$ a spacetime with causal structure**

  timelike vector field $\xi[\phi]$ on $\mathscr{O}$ s.t. $$0 \neq \mathscr{L}_\xi O_{A,\mathscr{R}}[\phi]$$

Relational bulk dynamics

- **assume frame $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \to \mathscr{O}$ sufficiently differentiable $\Rightarrow$ orientation space $\mathscr{O}$ a spacetime with causal structure**

timelike vector field $\xi[\phi]$ on $\mathscr{O}$ s.t. $$0 \neq \mathscr{L}_\xi O_{A,\mathscr{R}}[\phi] = \{O_{A,\mathscr{R}}, H_\xi\}[\phi]$$

to interpret as time evolution, need non-trivial Hamiltonian

$\Rightarrow$ **could then find time fct on $\mathscr{O}$ s.t.** $$\mathscr{L}_\xi \tilde{t}[\phi] = 1$$

$\Rightarrow$ **relational time evolution would correspond to A evolving rel. to $t[\phi] = \tilde{t}[\phi] \circ \mathscr{R}^{-1}[\phi]$ in spacetime $\mathscr{M}$**

Relational bulk dynamics

- assume frame $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \to \mathscr{O}$ sufficiently differentiable $\Rightarrow$ orientation space $\mathscr{O}$ a spacetime with causal structure

timelike vector field $\xi[\phi]$ on $\mathscr{O}$ s.t. $$0 \neq \mathscr{L}_\xi O_{A,\mathscr{R}}[\phi] = \{O_{A,\mathscr{R}}, H_\xi\}[\phi]$$

to interpret as time evolution, need non-trivial Hamiltonian

$\Rightarrow$ could then find time fct on $\mathscr{O}$ s.t. $\mathscr{L}_\xi \tilde{t}[\phi] = 1$

$\Rightarrow$ relational time evolution would correspond to A evolving rel. to $t[\phi] = \tilde{t}[\phi] \circ \mathscr{R}^{-1}[\phi]$ in spacetime $\mathscr{M}$

how can we get such a Hamiltonian?

Relational bulk dynamics

- **assume frame $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \to \mathscr{O}$ sufficiently differentiable $\Rightarrow$ orientation space $\mathscr{O}$ a spacetime with causal structure**

  timelike vector field $\xi[\phi]$ on $\mathscr{O}$ s.t. $$0 \neq \mathscr{L}_\xi O_{A,\mathscr{R}}[\phi] = \{O_{A,\mathscr{R}}, H_\xi\}[\phi]$$

  to interpret as time evolution, need non-trivial Hamiltonian

  $\Rightarrow$ could then find time fct on $\mathscr{O}$ s.t. $$\mathscr{L}_\xi \tilde{t}[\phi] = 1$$

  $\Rightarrow$ relational time evolution would correspond to A evolving rel. to $t[\phi] = \tilde{t}[\phi] \circ \mathscr{R}^{-1}[\phi]$ in spacetime $\mathscr{M}$

- **covenient to use covariant phase space formalism** [Wald, Crnkovic, Witten, Iyer, Lee, Harlow, Wu,…]

$$\delta L[\phi] = E[\phi] + d\theta[\phi]$$

EoM term: $E \approx 0$

bdry term

Relational bulk dynamics

- **assume frame $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \to \mathscr{O}$ sufficiently differentiable $\Rightarrow$ orientation space $\mathscr{O}$ a spacetime with causal structure**

timelike vector field $\xi[\phi]$ on $\mathscr{O}$ s.t. $$0 \neq \mathscr{L}_\xi O_{A,\mathscr{R}}[\phi] = \{O_{A,\mathscr{R}}, H_\xi\}[\phi]$$

to interpret as time evolution, need non-trivial Hamiltonian

$\Rightarrow$ **could then find time fct on $\mathscr{O}$ s.t.** $\mathscr{L}_\xi \tilde{t}[\phi] = 1$

$\Rightarrow$ **relational time evolution would correspond to A evolving rel. to $t[\phi] = \tilde{t}[\phi] \circ \mathscr{R}^{-1}[\phi]$ in spacetime $\mathscr{M}$**

- **covenient to use covariant phase space formalism** [Wald, Crnkovic, Witten, Iyer, Lee, Harlow, Wu,…]

$$\delta L[\phi] = E[\phi] + d\theta[\phi]$$

EoM term: $E \approx 0$

presymplectic potential

Relational bulk dynamics

- assume frame $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \to \mathscr{O}$ sufficiently differentiable $\Rightarrow$ orientation space $\mathscr{O}$ a spacetime with causal structure

timelike vector field $\xi[\phi]$ on $\mathscr{O}$ s.t. $$0 \neq \mathscr{L}_\xi O_{A,\mathscr{R}}[\phi] = \{O_{A,\mathscr{R}}, H_\xi\}[\phi]$$

to interpret as time evolution, need non-trivial Hamiltonian

$\Rightarrow$ could then find time fct on $\mathscr{O}$ s.t. $$\mathscr{L}_\xi \tilde{t}[\phi] = 1$$

$\Rightarrow$ relational time evolution would correspond to A evolving rel. to $t[\phi] = \tilde{t}[\phi] \circ \mathscr{R}^{-1}[\phi]$ in spacetime $\mathscr{M}$

- covenient to use covariant phase space formalism [Wald, Crnkovic, Witten, Iyer, Lee, Harlow, Wu,…]

$$\delta L[\phi] = E[\phi] + d\theta[\phi]$$

EoM term: $E \approx 0$

presymplectic potential

$\Rightarrow$ presymplectic 2-form on space of solutions $\mathscr{S}$: $$\Omega = \int_\Sigma \delta\theta[\phi]$$

Relational bulk dynamics

- **assume frame $\mathscr{R}^{-1}[\phi] : \mathscr{N}[\phi] \to \mathscr{O}$ sufficiently differentiable $\Rightarrow$ orientation space $\mathscr{O}$ a spacetime with causal structure**

timelike vector field $\xi[\phi]$ on $\mathscr{O}$ s.t. $$0 \neq \mathscr{L}_\xi O_{A,\mathscr{R}}[\phi] = \{O_{A,\mathscr{R}}, H_\xi\}[\phi] \qquad (*)$$

to interpret as time evolution, need non-trivial Hamiltonian

$\Rightarrow$ could then find time fct on $\mathscr{O}$ s.t. $\mathscr{L}_\xi \tilde{t}[\phi] = 1$

$\Rightarrow$ relational time evolution would correspond to A evolving rel. to $t[\phi] = \tilde{t}[\phi] \circ \mathscr{R}^{-1}[\phi]$ in spacetime $\mathscr{M}$

- **covenient to use covariant phase space formalism** [Wald, Crnkovic, Witten, Iyer, Lee, Harlow, Wu,…]

$$\delta L[\phi] = E[\phi] + d\theta[\phi]$$

EoM term: $E \approx 0$

presymplectic potential

$\Rightarrow$ presymplectic 2-form on space of solutions $\mathscr{S}$: $$\Omega = \int_\Sigma \delta\theta[\phi]$$

want vector field V_ξ on $\mathscr{S}$ s.t. $\begin{cases} I_{V_\xi}\Omega = \delta H_\xi & \textbf{(integrability) and} \\ I_{V_\xi}\delta O_{A,\mathscr{R}} = \mathscr{L}_\xi O_{A,\mathscr{R}} & \end{cases}$

$\Rightarrow$ if $H_\xi \not\approx 0$ (so not a constraint), get (*)

Relational bulk dynamics

- **assume frame $\mathscr{R}^{-1}[\phi]:\mathscr{N}[\phi]\to\mathscr{O}$ sufficiently differentiable $\Rightarrow$ orientation space $\mathscr{O}$ a spacetime with causal structure**

timelike vector field $\xi[\phi]$ on $\mathscr{O}$ s.t. $\quad 0\neq\mathscr{L}_\xi O_{A,\mathscr{R}}[\phi]=\{O_{A,\mathscr{R}},H_\xi\}[\phi]\quad$ (*)

to interpret as time evolution, need non-trivial Hamiltonian

$\Rightarrow$ **could then find time fct on $\mathscr{O}$ s.t.** $\quad \mathscr{L}_\xi\tilde{t}[\phi]=1$

$\Rightarrow$ **relational time evolution would correspond to A evolving rel. to $t[\phi]=\tilde{t}[\phi]\circ\mathscr{R}^{-1}[\phi]$ in spacetime $\mathscr{M}$**

- **covenient to use covariant phase space formalism** [Wald, Crnkovic, Witten, Iyer, Lee, Harlow, Wu,…]

$$\delta L[\phi]=E[\phi]+d\theta[\phi]$$

EoM term: $E\approx 0$

presymplectic potential

$\Rightarrow$ **presymplectic 2-form on space of solutions $\mathscr{S}$:** $\quad \Omega=\int_\Sigma\delta\theta[\phi]$

want vector field V_ξ on $\mathscr{S}$ s.t.
$$\begin{cases} I_{V_\xi}\Omega=\delta H_\xi & \text{(integrability) and}\\ I_{V_\xi}\delta O_{A,\mathscr{R}}=\mathscr{L}_\xi O_{A,\mathscr{R}} \end{cases}$$

$\Rightarrow$ **if $H_\xi\not\approx 0$ (so not a constraint), get (*)**

solves "problem of time" in bulk

$\Rightarrow$ **e.g. for grav. charged observables, H_ξ can be bdry Hamiltonian, but in many cases can also find Hamiltonian in cases without bdry**

Relational microcausality

Given two relational observables O_i associated with frame fields $\mathscr{R}_i^{-1}$ that transform trivially under large diffeos, then

$$\{O_1, O_2\}[\phi] = 0$$

provided that $\mathscr{R}_i[\phi](\mathrm{supp}_{\mathscr{O}_\mathrm{i}}(\mathrm{O_i})) \subset \mathscr{M} \backslash \partial \mathscr{M}$ (supports of undressed observables) are spacelike separated

Relational microcausality

Given two relational observables O_i associated with frame fields $\mathscr{R}_i^{-1}$ that transform trivially under large diffeos, then

$$\{O_1, O_2\}[\phi] = 0$$

provided that $\mathscr{R}_i[\phi](\mathrm{supp}_{\mathscr{O}_\mathrm{i}}(\mathrm{O_i})) \subset \mathscr{M} \backslash \partial \mathscr{M}$ (supports of undressed observables) are spacelike separated

in a nutshell: $\{O_1, O_2\}[\phi] = I_{V_2} I_{V_1} \Omega = \int_\Sigma \delta\theta[\phi, \delta\phi_1, \delta_2\phi]$**,** **where** $I_{V_i}\Omega = \delta O_i$ **and** $\delta_i\phi = I_{V_i}\delta\phi$

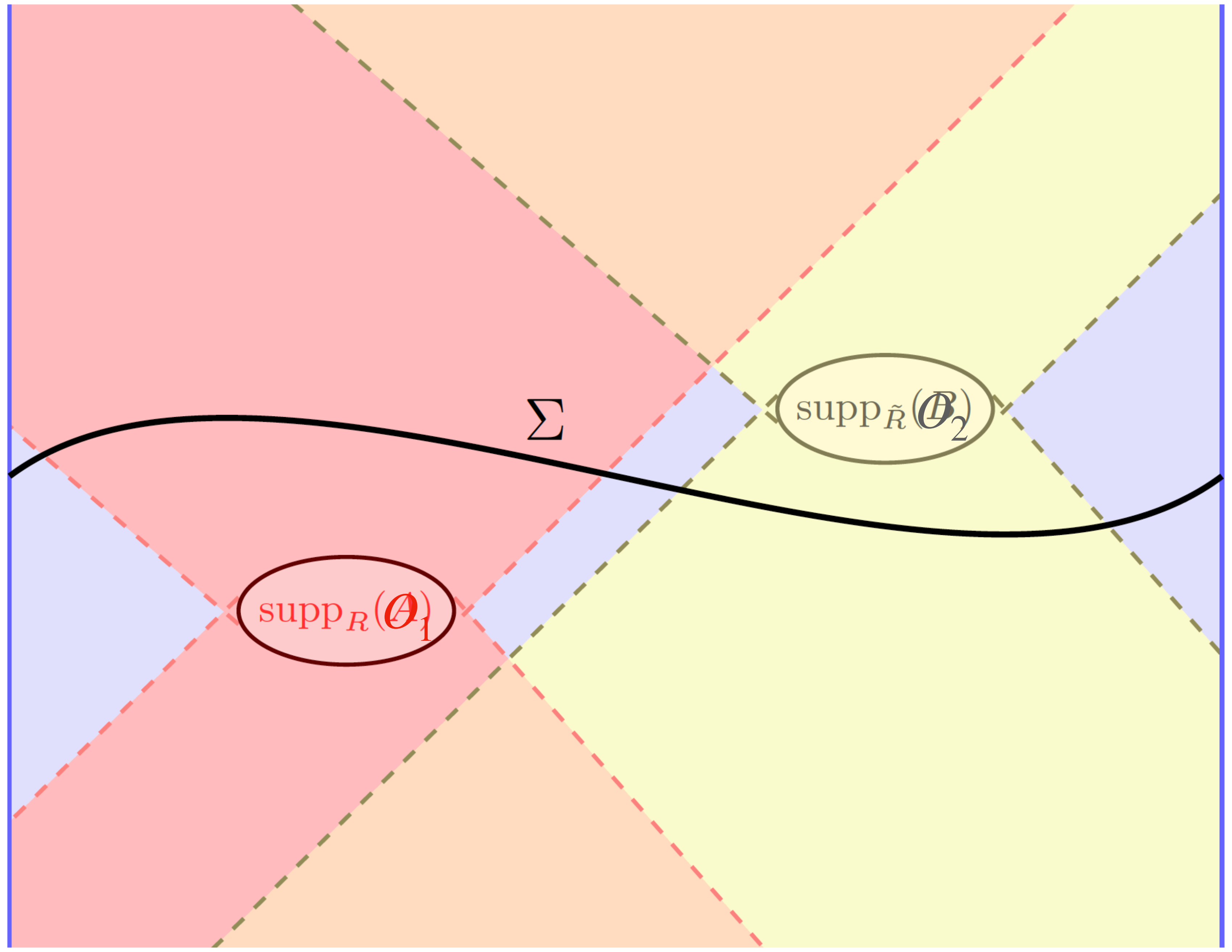
Relational microcausality

Given two relational observables O_i associated with frame fields $\mathscr{R}_i^{-1}$ that transform trivially under large diffeos, then

$$\{O_1, O_2\}[\phi] = 0 \qquad \textbf{(*)}$$

provided that $\mathscr{R}_i[\phi](\mathrm{supp}_{\mathscr{O}_\mathrm{i}}(\mathrm{O_i})) \subset \mathscr{M}\backslash\partial\mathscr{M}$ (supports of undressed observables) are spacelike separated

in a nutshell: $\{O_1, O_2\}[\phi] = I_{V_2} I_{V_1}\Omega = \int_\Sigma \omega[\phi, \delta\phi_1, \delta_2\phi]$, **where** $I_{V_i}\Omega = \delta O_i$ **and** $\delta_i\phi = I_{V_i}\delta\phi$



⇒ using Peierls bracket can show that, on space of solutions $\mathscr{S}$, can always choose gauge s.t. $\delta_i\phi$ vanishes outside domain of influence of $\mathscr{R}_i[\phi](\mathrm{supp}_{\mathscr{O}_\mathrm{i}}(\mathrm{O_i}))$

⇒ can choose Cauchy slice Σ s.t. $\delta_i\phi$ are nowhere simultaneously non-vanishing ⇒ (*)

[generalises Marolf '15]

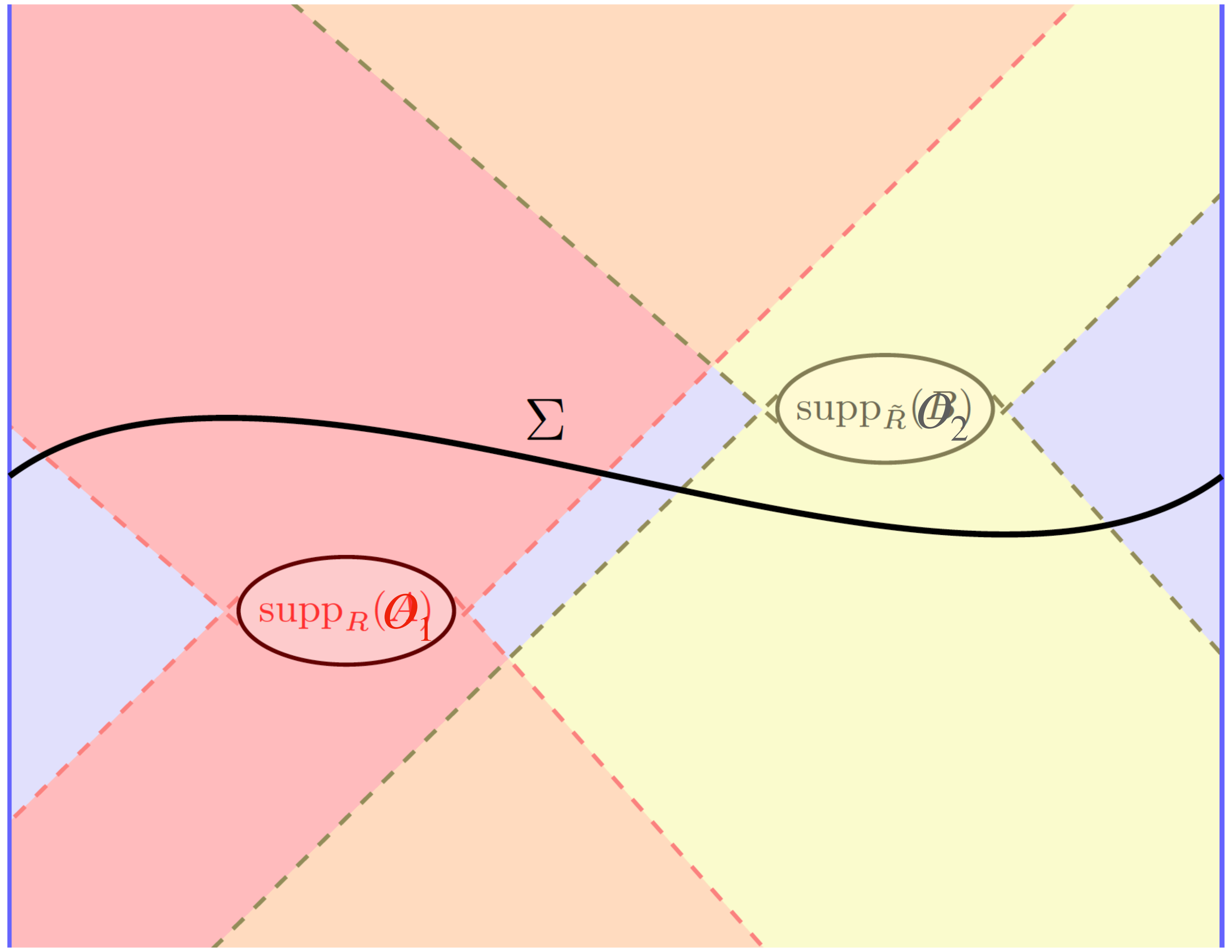
Relational microcausality

Given two relational observables O_i associated with frame fields $\mathscr{R}_i^{-1}$ that transform trivially under large diffeos, then

$$\{O_1, O_2\}[\phi] = 0 \qquad \textbf{(*)}$$

provided that $\mathscr{R}_i[\phi](\mathrm{supp}_{\mathscr{O}_i}(\mathrm{O_i})) \subset \mathscr{M} \backslash \partial \mathscr{M}$ (supports of undressed observables) are spacelike separated

in a nutshell: $\{O_1, O_2\}[\phi] = I_{V_2} I_{V_1} \Omega = \int_\Sigma \omega[\phi, \delta\phi_1, \delta_2\phi],$ **where** $I_{V_i}\Omega = \delta O_i$ **and** $\delta_i \phi = I_{V_i} \delta\phi$



⇒ using Peierls bracket can show that, on space of solutions $\mathscr{S}$, can always choose gauge s.t. $\delta_i\phi$ vanishes outside domain of influence of $\mathscr{R}_i[\phi](\mathrm{supp}_{\mathscr{O}_i}(\mathrm{O_i}))$

⇒ can choose Cauchy slice Σ s.t. $\delta_i\phi$ are nowhere simultaneously non-vanishing ⇒ (*)

[generalises Marolf ’15]

⇒ challenges with bdry conditions for frames that transform non-trivially under large diffeos

[partly connects with perturb. treatment of Donnelly, Giddings ’15]