Reduced phase space quantisation in LQG and loop quantum cosmology

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joint work with Max Fahn, Michael Kobler; Bao-Fei Li, Parampreet Singh and Stefan Weigl

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Plan of the talk

I. Motivation
II. Review some of existing results on reduced phase space quantisation in loop quantum gravity
III. Applications in loop quantum cosmology
IV. Classical versus quantum clocks
V. Open constrained quantum system and reduced phase phase space quantisation
VI. Summary and conclusions

I. Motivation: Gauge Invariance & Quantisation

Classical Theory: gauge fixing and gauge invariant formulation well established, also relation between different choices

Quantisation

Quantise all kinematical dof or only physical ones

Dirac quantisation versus reduced quantisation

Apply gauge fixing or construction of observables before or after quantisation, quantum versus classical clocks (reference fields)

Clocks in closed and open quantum systems

Physical imprint of chosen reference fields?

II. Reduced Quantisation

1.) Reduce with respect to (part of) the constraints classically

$\longrightarrow$ Construction of observables or gauge fixing

Choice: gauge invariant formulation and construct Dirac observables of at the classical level (classical clocks)

[Rovelli '90, Dittrich '04, Thiemann'04, Pons et al.]

2.) Find representations of the algebra of observables, which support the dynamics

Question: How does a particular choice for 1. influences the resulting quantum model

II. Observables in GR

Dirac observable in GR: (A^j_a, E^a_j)

$$\{\mathcal{O}, C\} = \{\mathcal{O}, \vec{C}\} = \{\mathcal{O}, G\} \approx 0$$

here: Reduction only wrt part of the set of constraints
remaining ones will be solved via Dirac quantisation

Framework: Relational Formalism [Rovelli '90]

Idea: Constraints can be classically reduced by means of reference fields, one for each constraint: $C_I, T_I \quad (A^j_a, E^a_j, T_I, P_I)$

Geometrical clocks [Talks by M.Fahn, S.Weigl] here: matter clocks

Reduction wrt C_I reference fields T_I

$$C_I(A, E, T_J, P_J) = C_I^{\mathrm{geo}} + C_I^{T_J}(A, E, T_J, P_J) = 0$$

II. Observables in GR

Idea: Solve C_I for the momenta P_I

$$\tilde{C}_I(A, E, T_J, P_I) = P_I + h_I(A, E, T_J) = 0$$

$\longrightarrow$ Now abelian: $\{\tilde{C}_I, \tilde{C}_J\} = 0$

We are interested only in those reference fields such that:

1.) $$\tilde{C}_0(A, E, T_0, P_0) = P_0 + h_0(A, E, T_0) = 0$$

with $h_0 = h_0(A, E)$ or $h_0 = h_0(A, E, \partial_a T_0)$

2.) Algebra of observables not too complicated: Quantisation

$\longrightarrow$ in models discussed here isomorphic to a sub algebra of the kinematical algebra

II. Observables in GR

Construction of observables: C_I reference fields T_I $\{T^I, C_J\} = \delta^I_J$

Given gauge variant phase space function f: [Rovelli '90, Dittrich '04, Thiemann'04, Pons et al.]

$$\mathcal{O}_f(\tau^I) = f + (\tau^I - T^I)\{f, C_I\} + \frac{1}{2!}(\tau^I - T^I)(\tau^J - T^J)\{\{f, C_I\}, C_J\} + \cdots$$

$$\mathcal{O}_f(\tau^I) = \left[\xi^I \exp(\{., C_I\}) \cdot f\right]\Big|_{\xi^I := \tau^I - T^I}$$

Observable : Combination of original phase space variables

3.) Dynamics can be quantised in LQG representation

$$\frac{d}{d\tau^0}\mathcal{O}_f(\tau^0) = \{\mathcal{O}_f(\tau^0), H_{\rm phys}\}$$

A naive generalisation of the 1 scalar field model to 4 scalar fields yields a reduced model that cannot be quantised in LQG [K.G., Vetter `19]

⟶ ex: Dirac and reduced quantisation yield very different models

Which Reference Matter?

Introduce additional matter fields coupled to gravity

Dynamically coupled observer, no ideal observer

Consider different dust and scalar field models

[Kuchar, Torre `91] [Rovelli, Smolin `93] [Brown, Kuchar `95] [Kuchar, Romano`95]

[Bicak, Kuchar `97] [Thiemann `06] [K.G., T. Thiemann `10]

[Domagala, K.G., Kaminski, Lewandowski `10] [K.G., T. Thiemann `12,] [Husain, Pawlowski `13]

[K.G., Vetter`19] [Han, Liu '19].....

II. Reference Matter: dust models

Lagrangian can obtain up to 8 scalar fields: $(T_I, \rho, W_j)\, I = 0, \cdots 3, j = 1, 2, 3.$

All models include second class constraints: after reduction T_I

$$\mathcal{L}_D = -\frac{1}{2}\sqrt{|g|}\left(g^{\mu\nu}\left[\rho\nabla_\mu T_0 \nabla_\nu T_0 + \alpha(\rho)V_\mu V_\nu + 2\beta(\rho)(\nabla_\mu T_0)V_\nu\right] + \Lambda(\rho)\right)$$

with $V_\mu := W_j \nabla_\mu T^j$ $\alpha(\rho), \beta(\rho), \Lambda(\rho)$ arbitrary functions of ρ

matter can be interpreted as dust, has same $T_{\mu\nu}$

Particular models considered so far:

$$\mathcal{L}_{TD}: \quad \alpha = \beta = \Lambda = \rho \quad \text{[Brown, Kuchar `95]}$$

$$\mathcal{L}_{ND}: \quad \alpha = 1, \beta = \Lambda = \rho = 0 \quad \text{[Bícak, Kuchar `97]}$$

$$\mathcal{L}_{NRD}: \quad \alpha = \beta = 0, \Lambda = \rho \quad \text{[Brown, Kuchar `95]}$$

$$\mathcal{L}_{GD}: \quad \alpha = 0, \beta = 1, \Lambda = \rho \quad \text{[Kuchar, Torre `91]}$$

II. Reference Matter: Scalar

Lagrangian obtains 1 scalar field: T_0

$$\mathcal{L}_{\rm S} = \sqrt{|g|}L(I) \quad I := -\frac{1}{2}g^{\mu\nu}(\nabla_\mu T_0)(\nabla_\nu T_0)$$

Particular models considered so far:

massless scalar field: $\mathcal{L}_S = \sqrt{|g|}I$ [Rovelli, Smolin `93] [Kuchar, Romano`95]

[Domagala, K.G.,Kaminski, Lewandowski `10]

General case: $\mathcal{L}_S = \sqrt{|g|}L(I)$ [Thiemann `06]

Couple 7 additional fields:

[K.G., Vetter `19]

$$\mathcal{L}^{\rm four}_{\rm scalar} = -\sqrt{-g}\frac{1}{2}g^{\mu\nu}\chi^0_{,\mu}\chi^0_{,\nu} - \sqrt{-g}\frac{1}{2}g^{\mu\nu}M_{jj}\chi^j_{,\mu}\chi^j_{,\nu}$$

II. Reduced Dynamics

One can show that for all considered models:

$$H_{\rm phys} = \int_{\mathcal{S}} d^3\sigma H(\sigma)$$

dust: (i) $H(\sigma) := O_{h_0,T_I}(\sigma)$

dust: (ii) $H(\sigma) := O_{\tilde{h}_0,T_0}(\sigma)$

1 scalar: $H(\sigma) := O_{h_0,T_0}(\sigma)$

$$C^{\rm geo} = F^I_{jk}\frac{\epsilon^{IJK}E^j_J E^k_K}{\sqrt{\det(E)}} \qquad C^{\rm geo}_j = F^K_{jk}E^k_K$$

Particular Models:

$$\mathcal{L}_{\rm TD}: \quad H(\sigma) = \sqrt{(C^{\rm geo})^2 - Q^{jk}C^{\rm geo}_j C^{\rm geo}_k}(\sigma)$$

$$\mathcal{L}_{\rm NRD}: \quad H(\sigma) = |C^{\rm geo}|(\sigma)$$

$$\mathcal{L}_{\rm GD}: \quad H(\sigma) = C^{\rm geo}(\sigma)$$

$$\mathcal{L}_{\rm S}: \quad H(\sigma) = \sqrt{-\sqrt{Q}C^{\rm geo} + \sqrt{Q}\sqrt{(C^{\rm geo})^2 - Q^{jk}C^{\rm geo}_j C^{\rm geo}_k}}(\sigma)$$

II. Reduced Quantisation LQG

Direct access to $\mathcal{H}_{\rm phys}$ Need operators $\hat{C}^{\rm geo}, \quad \widehat{Q^{ij}C_j^{\rm geo}C_k^{\rm geo}}$

Algebra of Dirac observables

$$\{A_j^J(\sigma), E_k^K(\sigma')\} = O_{\{A,E\}^*,T} = O_{\{A,E\},T} = \kappa\delta_K^J\delta_j^k\delta(\sigma,\sigma')$$

Since $\mathrm{H}_{\rm phys}$ spatially diff-inv. if $\mathcal{H}_{\rm phys} = \mathcal{H}^G_{\rm AL}$ then graph-preserving

Notion of a minimal loop

$$\hat{C}_{\mu,\gamma,v} := \frac{1}{\ell_P^2\,|T_v(\gamma)|}\sum_{(e_1,e_2,e_3)\in T_v(\gamma)}\epsilon^{IJK}\frac{1}{|L_{\gamma,v,e_I,e_J}|}\sum_{\alpha\in L_{\gamma,v,e_I,e_J}}\mathrm{Tr}\left(\tau_\mu\hat{h}_\alpha\hat{h}_{e_K}\left[\hat{h}^{-1}_{e_K},\hat{V}_{\gamma,v}\right]\right) \qquad \tau_\mu = (\mathbf{1}, -i\sigma_j)$$

$$\mu = 0,\cdots 3.$$

Ex: BK model

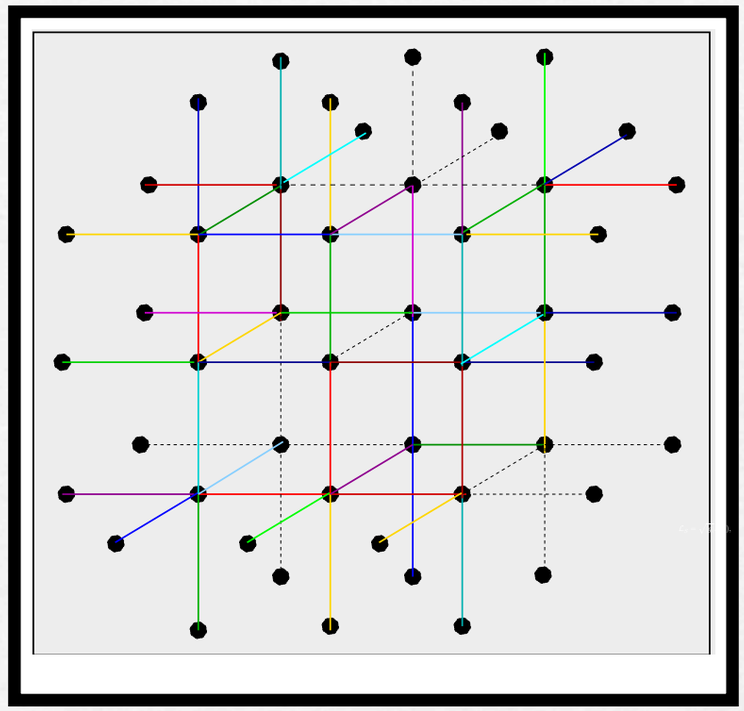
$$\widehat{\boldsymbol{H}}_{\rm phys} = \bigoplus_\gamma \widehat{\boldsymbol{H}}_{{\rm phys},\gamma} \qquad \widehat{\mathbf{H}}_{{\rm phys},\gamma} := \sum_{v\in V(\gamma)}\sqrt{\left|P_\gamma\left[\hat{C}^\dagger_{0,\gamma,v}\hat{C}_{0,\gamma,v} - \frac{1}{4}\hat{C}^\dagger_{I,\gamma,v}\hat{C}_{I,\gamma,v}\right]P_\gamma\right|} \qquad P_\gamma : \mathcal{H}_{\rm phys}\to\mathcal{H}_\gamma$$

II. Reduced Quantisation AQG

Quantise in framework of AQG: 6-valent infinite graph abstract graph

Infinite graph is fundamental

But subgraphs can change, graph-preserving looks different here



Physical Hilbert space: $\mathcal{H}_{\rm phys} = \mathcal{H}_{\rm ITP}$

$$\hat{C}_{\mu,v} := \frac{1}{24\ell_P^2} \sum_{s_1,s_2,s_3=\pm 1} s_1 s_2 s_3 \epsilon^{I_1 I_2 I_3} \, \mathrm{Tr}\left(\tau_\mu A\left(\alpha_{v;I_1 s_1, I_2 s_2}\right) A\left(e_{v;I_3 s_3}\right) \left[A\left(e_{v;I_3 s_3}\right)^{-1}, \hat{V}_v\right]\right)$$

Example BK model
no projection operators needed

$$\widehat{\mathbf{H}}_{\rm phys} := \sum_v \sqrt{\left| \hat{C}^\dagger_{0,v} \hat{C}_{0,v} - \frac{1}{4} \hat{C}^\dagger_{I,v} \hat{C}_{I,v} \right]|}$$

Back to main question of the talk:

Reduced Phase Space Quantisation (RPSQ) in LQG

Lewandowski, Domagala, Kaminski, Pawłowski, Husain, Thiemann, Vetter, K.G .....

Allows access to physical Hilbert space

Can be performed using reference matter Rovelli, Dittrich

Use relational formalism, back reaction reference fields

How do we know quantisation of dynamics „correct"?

1st consistency check semiclassical limit:

Consider graph-preserving operators

$$\langle \Psi_{(A,E)} \,|\, \hat{\mathrm{H}}_{\mathrm{phys}} \,|\, \Psi_{(A,E)} \rangle = \mathrm{H}_{\mathrm{phys}}(A,E) \rangle + O(\hbar)$$

Former result for master constraint + semiclassical perturbation theory
Recent work by Zhang Song and Han to compute corrections

Back to main question of the talk:

Main question:

Physical imprint of chosen reference matter ?

Strategy: Consider simpler systems with different clocks

I. Use LQC as a simplified setup to test this
First step: Apply effective techniques to models

II. Understand choice of classical clock more in detail

III. Consider constrained open quantum systems

III. Symmetry Reduction k=0 FLRW

Consider three models that differ by choice of clock

Difference to APS Ashtekar, Pawlowski, Singh model: 2-fluid models inflaton + clock

- more complicated + inflaton potential causes no problems

Reduced Phase space FLRW k=0

Dirac Observables $\mathcal{O}_b,\ \mathcal{O}_V,\ \mathcal{O}_\varphi,\ \mathcal{O}_{\pi_\varphi}$

Algebra Dirac observables $\{\mathcal{O}_b, \mathcal{O}_V\} = 4\pi G\gamma \quad \{\mathcal{O}_\varphi, \mathcal{O}_{\pi_\varphi}\} = 1$

Physical Hamiltonians

$$\mathbf{H}^{\mathrm{FLRW,dust}} = -\frac{3\mathcal{O}_V}{8\pi G\gamma^2}\mathcal{O}_b^2 + \frac{\mathcal{O}_{\pi_\varphi}^2}{2\mathcal{O}_V} + \mathcal{O}_V U(\mathcal{O}_\varphi)$$

$$\mathbf{H}^{\mathrm{FLRW,scalar}} = \sqrt{-2\mathcal{O}_V\left(-\frac{3\mathcal{O}_V}{8\pi G\gamma^2}\mathcal{O}_b^2 + \frac{\mathcal{O}_{\pi_\varphi}^2}{2\mathcal{O}_V} + \mathcal{O}_V U(\mathcal{O}_\varphi)\right)}$$

III. Reduced Phase Space Quantisation LQC

Dust models: Physical Hilbert space

Physical Hilbert space:

$$\mathcal{H}_{\rm phys} = \mathcal{H}_{\rm phys,grav} \otimes \mathcal{H}_{{\rm phys},\varphi}$$

gravity:

$$\mathcal{H}_{\rm phys,grav} = L_2(\overline{\mathbb{R}}, d\mu_{\rm Bohr})$$

matter:

$$\mathcal{H}_{{\rm phys},\varphi} = L_2(\mathbb{R}, d\mathcal{O}_\varphi)$$

Physical inner product:

$$\langle \Psi, \widetilde{\Psi} \rangle_{\rm phys} := \langle \psi, \widetilde{\psi} \rangle_{\rm grav} \langle \psi, \widetilde{\psi} \rangle_{\varphi} \qquad \langle \psi, \tilde{\psi} \rangle_{\rm grav} := \lim_{D\to\infty} \frac{1}{2D} \int_{-D}^{D} d\mathcal{O}_V \overline{\psi}(\mathcal{O}_V) \widetilde{\psi}(\mathcal{O}_V)$$

Volume representation:

$$\widehat{\mathcal{O}}_V |\nu\rangle = \left(\frac{8\pi G\gamma}{6}\right)^{3/2} \frac{|\nu|}{K} |\nu\rangle, \qquad \widehat{\mathcal{O}}_{N_{\bar\mu}} |\nu\rangle = |\nu+1\rangle, \quad \mathcal{O}_{N_{\bar\mu}} = e^{i\frac{\bar\mu \mathcal{O}_c}{2}}$$

III. Reduced Phase Space Quantisation LQC

Scalar field model: Physical Hilbert space

Physical Hilbert space:

$$\mathcal{H}_{\rm phys} = \mathcal{H}_{\rm phys,grav} \otimes \mathcal{H}_{\rm phys,\varphi}$$

gravity:

$\mathcal{H}_{\rm phys,grav} = L_2(\overline{\mathbb{R}}, d\tilde{\mu}_{\rm Bohr})$

matter:

$\mathcal{H}_{\rm phys,\varphi} = L_2(\mathbb{R}, d\mathcal{O}_\varphi)$

Physical inner product:

$$\langle \Psi, \widetilde{\Psi} \rangle_{\rm phys} := \langle \psi, \widetilde{\psi} \rangle_{\rm grav} \langle \psi, \widetilde{\psi} \rangle_{\varphi}$$

$$\langle \psi, \tilde{\psi} \rangle_{\rm grav} := \lim_{D\to\infty} \frac{1}{2D} \int_{-D}^{D} d\mathcal{O}_V B(\mathcal{O}_V) \overline{\psi}(\mathcal{O}_V) \widetilde{\psi}(\mathcal{O}_V) \qquad B(\mathcal{O}_V) = \left(\frac{3}{2}\right)^3 K |\mathcal{O}_V| |\mathcal{O}_V + 1|^{1/3} - |\mathcal{O}_V - 1|^{1/3}|^3$$

Volume representation: $\widehat{\mathcal{O}}_V |\nu\rangle = \left(\frac{8\pi G\gamma}{6}\right)^{3/2} \frac{|\nu|}{K} |\nu\rangle, \qquad \widehat{\mathcal{O}}_{N_{\bar{\mu}}} |\nu\rangle = |\nu + 1\rangle, \quad \mathcal{O}_{N_{\bar{\mu}}} = e^{i\frac{\bar{\mu}\mathcal{O}_c}{2}}$

III. Quantum Dynamics Quantisation LQC

Quantum dynamics dust and scalar field models

Schrödinger-like equation dust:

$$i\hbar\frac{\partial}{\partial\tau}\Psi(\mathcal{O}_V,\mathcal{O}_\varphi;\tau)=\left(\widehat{\mathbf{H}}^{\rm grav}_{\rm phys}+\widehat{\mathbf{H}}^{\varphi}_{\rm phys}\right)\Psi(\mathcal{O}_V,\mathcal{O}_\varphi;\tau)$$

Schrödinger-like equation scalar field:

$$i\hbar\frac{\partial}{\partial\tau_\chi}\Psi(\mathcal{O}_V,\mathcal{O}_\varphi;\tau)=\sqrt{|-2\hat{B}^{-1}(\mathcal{O}_V)(\widehat{\mathbf{H}}^{\rm grav}_{\rm phys}+\widehat{\mathbf{H}}^{\varphi}_{\rm phys})|}\;\;\Psi(\mathcal{O}_V,\mathcal{O}_\varphi;\tau)$$

Individual Hamiltonian operators

Ashtekar, Pawlowski, Singh '06

$$\widehat{\mathbf{H}}^{\rm grav}_{\rm phys}\Psi(\mathcal{O}_V,\mathcal{O}_\varphi)=$$

$$\mathcal{O}_b=\mathcal{O}_c/\mathcal{O}^{1/2}_{|p|}\qquad\bar{\mu}=\sqrt{\Delta/\mathcal{O}_{|p|}}\quad\Delta=4\sqrt{3}\pi\gamma l^2_{\rm Pl}$$

$$\left(\sin(\bar{\mu}\mathcal{O}_c)\left[\frac{3i{\rm sgn}(\mathcal{O}_V)}{16\pi^2G^2\gamma^3\bar{\mu}^3}\left(\sin\left(\frac{\bar{\mu}\mathcal{O}_c}{2}\right)\widehat{\mathcal{O}}_V\cos\left(\frac{\bar{\mu}\mathcal{O}_c}{2}\right)\;-\cos\left(\frac{\bar{\mu}\mathcal{O}_c}{2}\right)\widehat{\mathcal{O}}_V\sin\left(\frac{\bar{\mu}\mathcal{O}_c}{2}\right)\right)\right]\sin(\bar{\mu}\mathcal{O}_c)\right)\psi(\mathcal{O}_V,O_\varphi)$$

$$\widehat{\mathbf{H}}^{\varphi}_{\rm phys}\Psi(\mathcal{O}_V,\mathcal{O}_\varphi)=\quad\left(-\frac{\hbar^2}{2}\widehat{\mathcal{O}^{-3/4}_{|p|}}\partial^2_{\mathcal{O}_\varphi}\widehat{\mathcal{O}^{-3/4}_{|p|}}+\widehat{\mathcal{O}^{3/4}_{|p|}}U(\widehat{\mathcal{O}}_\varphi)\widehat{\mathcal{O}^{3/4}_{|p|}}\right)\Psi(\mathcal{O}_V,\mathcal{O}_\varphi)$$

Inflaton potential does not yield to time-dependent physical Hamiltonian here, but need 2-fluid model

III. Effective Dynamics

Effective dynamics dust models (scalar field clock works similar)

Effective Hamiltonian $\mathbf{H}^{\mathrm{FLRW,dust}}_{\mathrm{eff}} = -\frac{3\mathcal{O}_V}{8\pi G\lambda^2\gamma^2}\sin^2(\lambda\mathcal{O}_b) + \frac{\mathcal{O}^2_{\pi_\varphi}}{2\mathcal{O}_V} + \mathcal{O}_V U(\mathcal{O}_\varphi) \quad \lambda := \sqrt{\Delta}$

Equations of motion:

$$\frac{d\mathcal{O}_V}{d\tau} = \frac{3\mathcal{O}_V}{2\lambda\gamma}\sin(2\lambda\mathcal{O}_b) \qquad \frac{d\mathcal{O}_b}{d\tau} = -\frac{3\sin^2(\lambda\mathcal{O}_b)}{2\gamma\lambda^2} - 4\pi G\gamma\mathcal{O}_{P_\varphi} \qquad \mathcal{O}_{P_\varphi} \text{ pressure}$$

$$\frac{d\mathcal{O}_\varphi}{d\tau} = \frac{\mathcal{O}_\varphi}{\mathcal{O}_V} \qquad \frac{d\mathcal{O}_{\pi_\varphi}}{d\tau} = -\mathcal{O}_V\frac{dU(\mathcal{O}_\varphi)}{d\mathcal{O}_\varphi}$$

Modified FLRW equation with bounce

$$\mathcal{O}^2_H = \frac{\dot{\mathcal{O}}^2_V}{9\mathcal{O}^2_V} = \frac{8\pi G}{3}\mathcal{O}_\rho\left(1 - \frac{\mathcal{O}_\rho}{\rho_{\max}}\right)$$

Same $\rho_{\max}$ like in Dirac quantised LQC

density $\mathcal{O}_\rho$ involves inflaton + dust clock

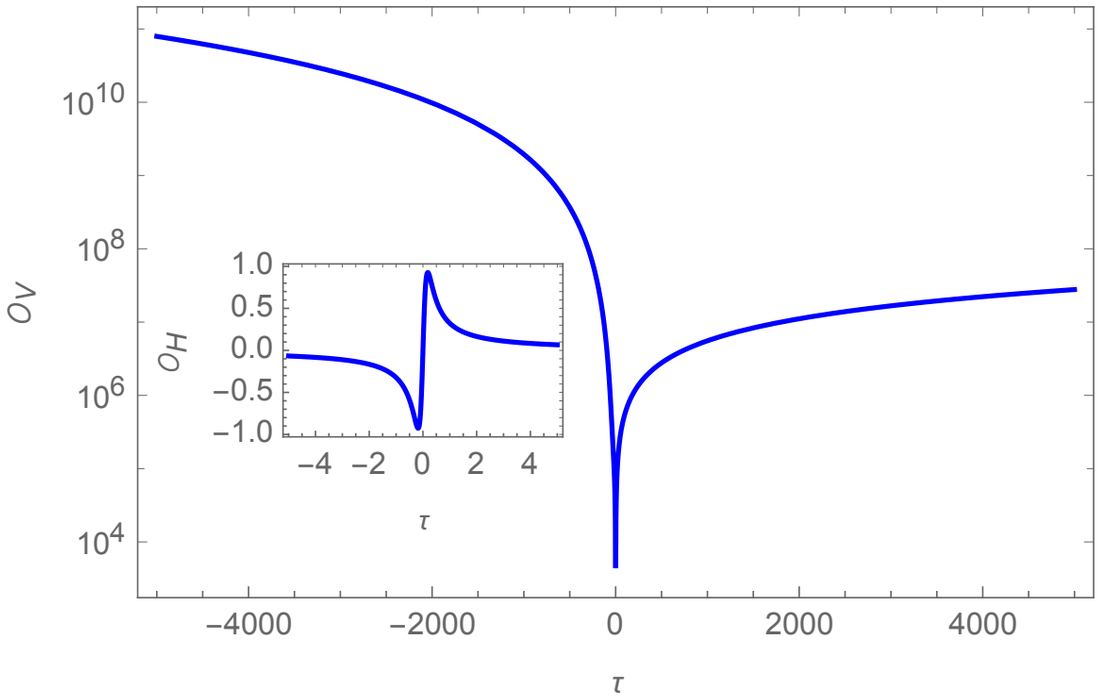
III. More detailed analysis

How does the choice of clocks affect inflation?

We choose a Starobinsky potential $$U = \frac{3m^2}{16\pi G}\left(1 - e^{-\sqrt{\frac{16\pi G}{3}}\mathcal{O}_\varphi}\right)^2 \qquad m = 2.44 \times 10^{-6}$$

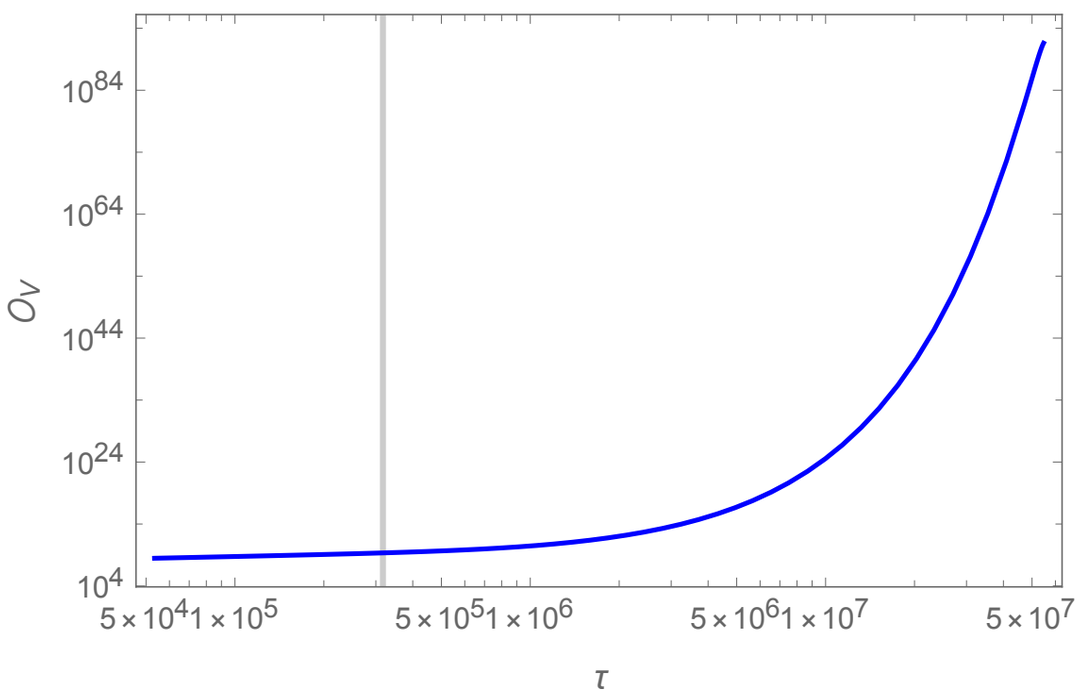
We set the initial conditions at the bounce $\mathcal{O}_{b_i} = \pi/2\lambda \;\; \mathcal{O}_{V_i} = 10^3$ in Planck units

Choosing $\mathcal{O}_{\varphi_i}, \mathcal{O}_{\pi_\varphi}$ determines $\mathbf{H}_{\text{eff}}^{\text{FLRW,dust}}$ and thus $\mathcal{O}_{\rho^{\text{clock}}}$: Parametrise by $\left(\mathcal{O}_\varphi, \mathcal{O}_{\rho^{\text{clock}}}\right)$



$\mathcal{O}_{\varphi_i} = -1.45,\ \mathcal{O}_{\rho^{\text{clock}}} = 10^{-8}$

$\tau = 3.17 \times 10^5$ onset of inflation

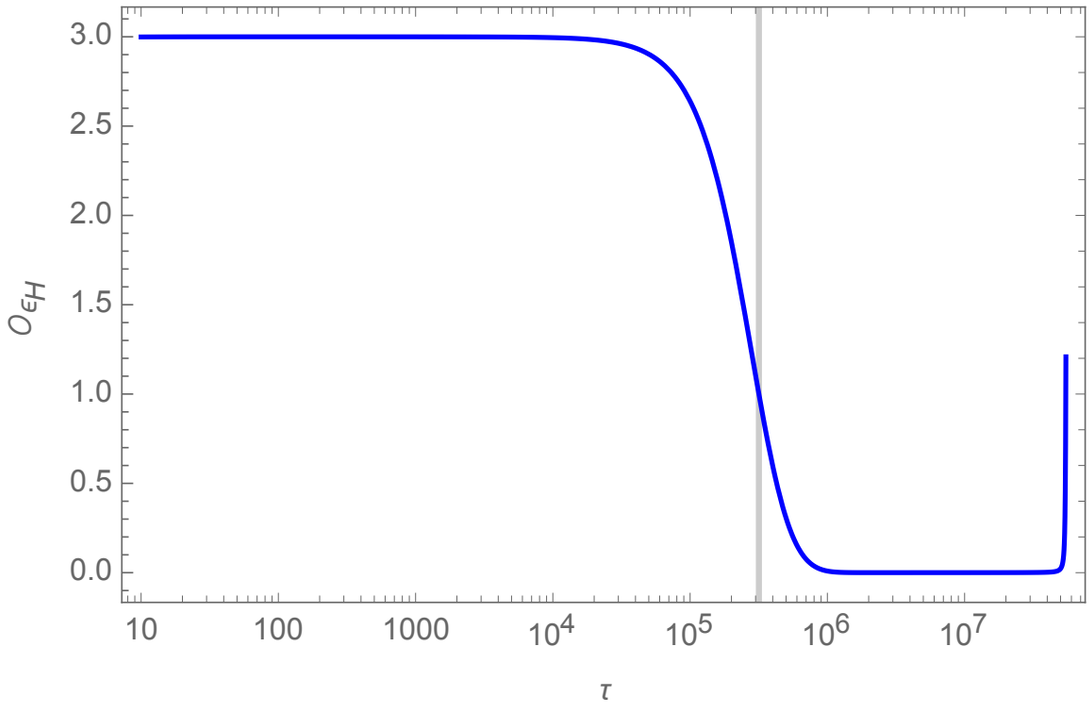


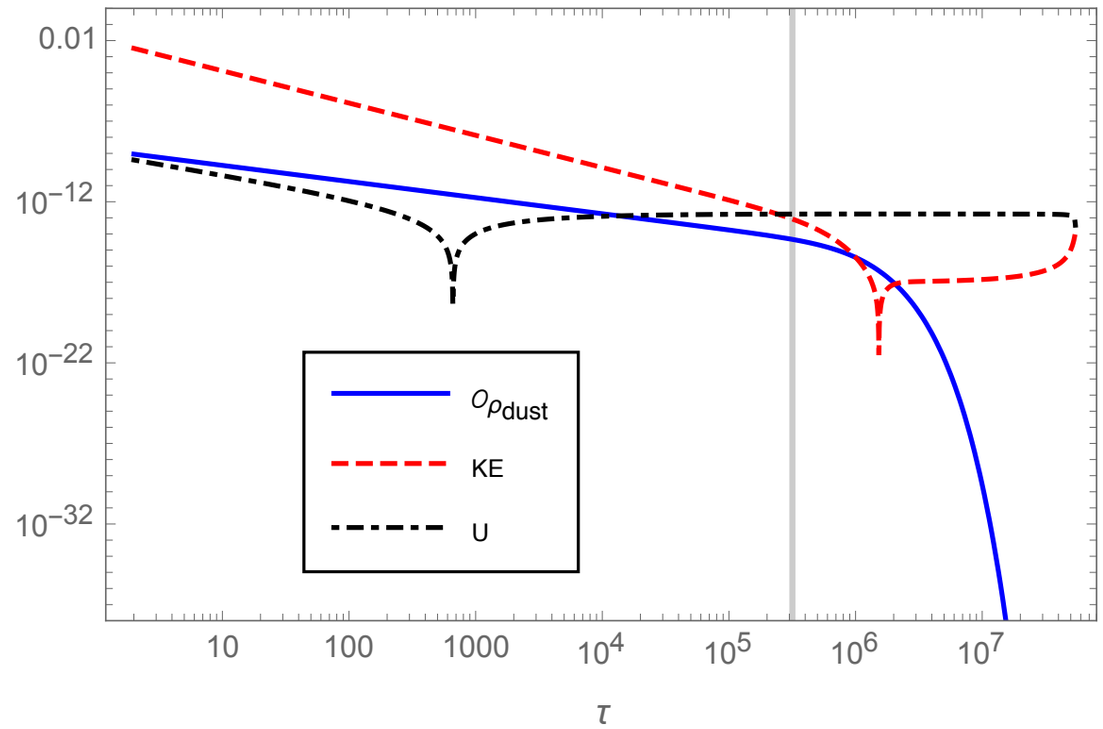
inflation ends at $\tau = 5.46 \times 10^7$

III. More detailed analysis

How does the choice of clocks affect inflation?

Slow roll Parameter





These initial conditions yield 63.1 e-foldings; Dirac quantised LQC without dust yields 63.9 in this case

Li, Wang, Singh

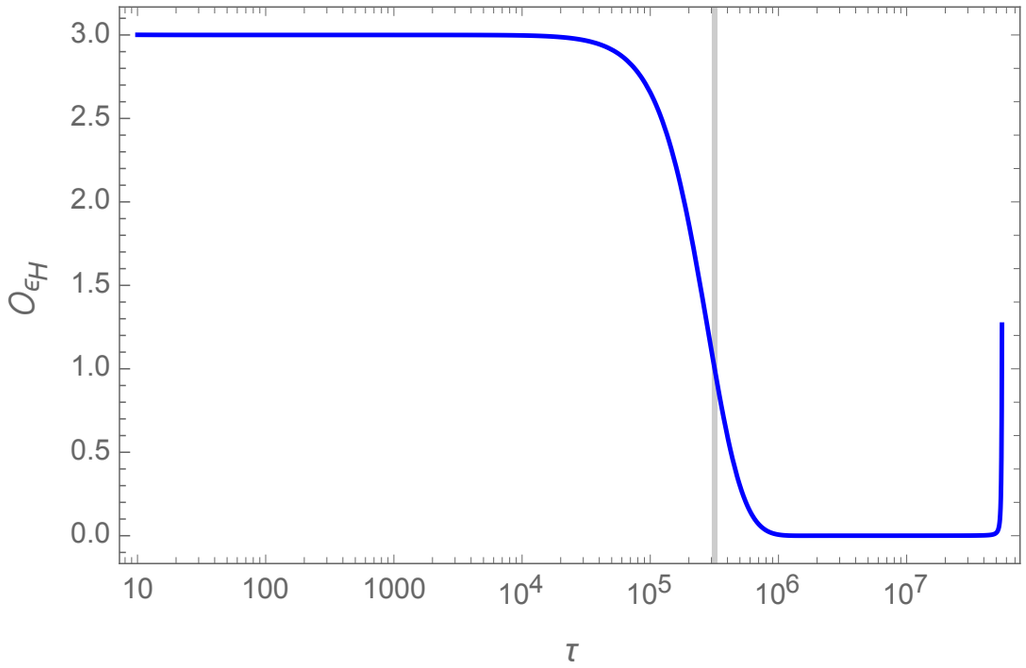
Dust plays a subdominant role from the bounce until the end of inflation

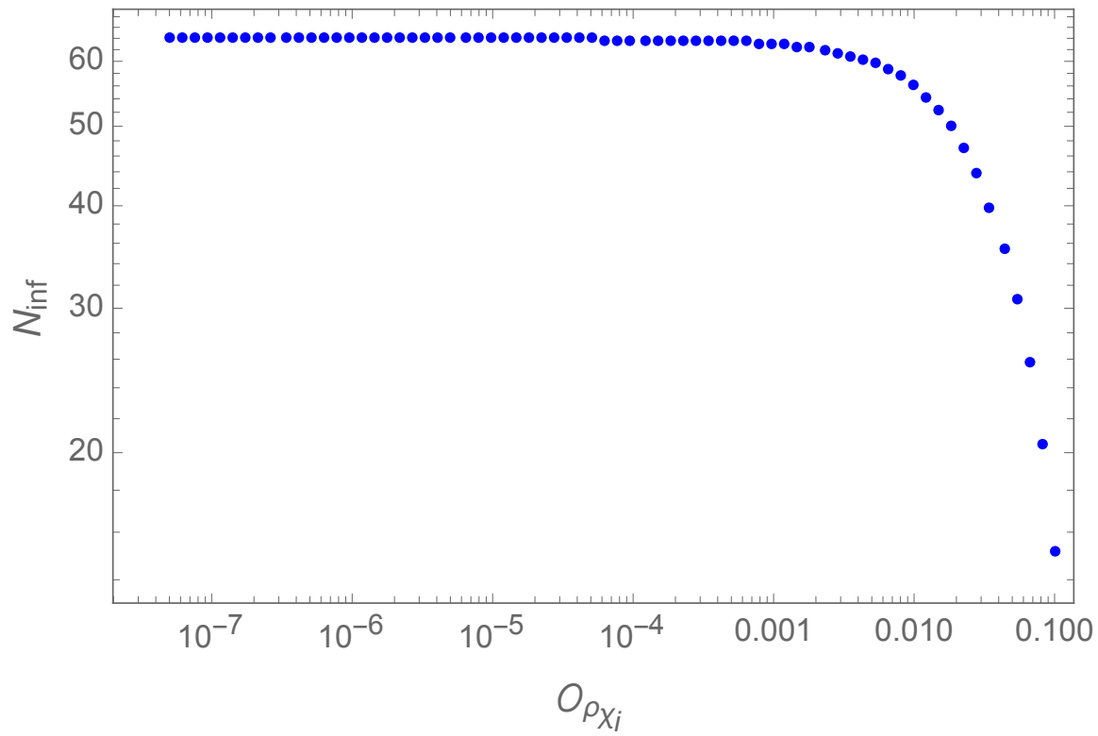
This is what one would expect from a „good" choice of clock

III. More detailed analysis

How does the choice of clocks affect inflation?

Slow roll Parameter $\quad \mathcal{O}_{\varphi_i} = -1.45,\ \mathcal{O}_{\rho^{\rm clock}} = 10^{-8}$





The same initial conditions yield 63.8 e-foldings; closer to value 63.9 (LQC 1-fluid model)

Number of e-foldings decreases with higher clock energy density but effect is less strong than for dust

Here only significant effect if $\mathcal{O}_{\rho^{\rm clock}} \geq 0.001$ in Planck units.

IV. Choosing classical clocks and gauge fixing

We can relate the choice of reference fields to a choice of physical spatial & temp. coord., rel. to gauge fixing [Talk by Philipp Höhn] [Gambini, Pullin]

RPSQ: No quantum properties of reference fields

Stability with respect to (quantum) dynamics [Talk by Stefan Weigl]

[K.G, B.Li, P. Singh, S.A. weigl '21]

Classical clocks: N, N^a

$$\frac{\mathrm{d}}{\mathrm{d}t}G_I^{\mathrm{class}}(t,x) = \{G_I^{\mathrm{class}}(t,x), H_{\mathrm{can}}^{\mathrm{class}}\} + \delta_{I,0}\frac{\partial G_0}{\partial t}(t,x) \approx 0$$

Quantum clocks: $\hat{N}, \hat{N}^a$

$$i\hbar\frac{\mathrm{d}}{\mathrm{d}t}\hat{G}_I(t,x) = \left[\hat{G}_I(t,x), \hat{H}_{\mathrm{can}}\right] + \delta_{I,0}\frac{\partial \hat{G}_0}{\partial t}(t,x) \approx 0$$

For which kind of clocks is $\hat{N}, \hat{N}^a$ just quantisation of N, N^a ?

Gives first idea on possibility to choose consistent classical clocks

IV. Choosing classical clocks and gauge fixing

Considered effective dynamics because good setup to investigate dynamical stability

Q: When does gauge fixing and polymerisation commute ?

More details in talk by Stefan Weigl:

Important aspects:
temporal dependence of gauge fixing conditions
whether dof freedom in gauge fixing condition are polymerised

Matter reference fields:
Assume matter dof are not polymerised then these models fall into a class where gauge fixing and polymerisation commute.

V. Constrained open quantum systems

Idea: Consider a system coupled to an environment [Livine, Feller]

Derive master equation for density matrix of the system

In constrained systems: Need to deal with gauge dof

RPSQ useful because one starts with physical non-van. Hamiltonian

$$\hat{H}_{\rm tot} = \hat{H}_S \otimes \mathbf{1}_\varepsilon + \mathbf{1}_S \otimes \hat{H}_\varepsilon + \hat{H}_{\rm int}$$

$$\frac{\partial}{\partial t}\rho_S(t) = -i\alpha\left[H_S(t) + H_{LS}(t), \rho_S(t)\right] + \mathcal{D}\left(\rho_S(t)\right)$$

Gravitationally induced decoherence: Coupling between environment and system needs not to be put by hand.

V. Constrained open quantum systems

More details in talk by Max Fahn : [M. Fahn, K.G., M. Kobler '22]

Linearised gravity in Ashtekar variables + scalar field

Choose geometrical clocks for Ham., Gauss & diffeo

Slightly different choice than existing ADM clocks Dittrich, Tambornino '07

Obtain Dirac observables and physical Ham. to start with

Derive master equations using Fock quantisation, comparison ADM

[Anastopoulos, Hu '13]; [Blencowe '13]; [Oniga, Wang '16]; [Lagouvardos, Anastopoulos '21]; [Asprea '21]

Master equation with Gibbs state for the environment
not automatically of Lindblad form

Next step: Loop quantisation [K.G., M. Kobler '22]
open scattering model in polymerised quantum mechanics [Talk by Michael Kobler]

VI. Summary and Conclusions

II. Reviewed reduced phase space quantisation for full LQG, different models which one to choose?

III. Reduced phase space quantisation for LQC using two different reference fields (clocks)

Applied effective techniques to analyse the equations of motion

Analysis shows that initial conditions can be chosen such that the clock has no significant effect on dynamics

Both models show qualitatively similar behaviour and can serve as good clocks

IV. Discussed the question what gauge-fixing classical and then quantising means, what do we neglect?

Effective techniques: polymerisation of lapse and shift need to be considered with some care in certain situations to be consistent with effective dynamics

V.: Open quantum systems and choice of clocks: First steps gravitationally induced decoherence in linearised gravity coupled to a scalar field using Ashtekar variables.

Understand more in detail loop inspired open quantum models.

Thank You!

I. Reduced Phase Space General Relativity

Idea: Couple reference matter to gravity+inflaton

$$S = S_{\rm geo} + S_{\rm scalar} + S_{\rm ref}$$

allows construction of Dirac observables

Reduced Phase space for GR+ inflation starting point for quantisation

Dirac Observables dust models $\mathcal{O}_{A^i_a,T}[\tau,\vec{\sigma}],\quad \mathcal{O}_{E^a_i,T}[\tau,\vec{\sigma}],\quad O_{\varphi,T}[\tau,\vec{\sigma}],\quad \mathcal{O}_{\pi_\varphi,T}[\tau,\vec{\sigma}]$

Dirac Observables scalar fields model $\mathcal{O}_{A^i_a,\chi^0}[\tau,\vec{\sigma}],\quad \mathcal{O}_{E^a_i,\chi^0}[\tau,\vec{\sigma}],\quad O_{\varphi,\chi^0}[\tau,\vec{\sigma}],\quad \mathcal{O}_{\pi_\varphi,\chi^0}[\tau,\vec{\sigma}]$

Algebra Dirac observables $\{\mathcal{O}_{A^i_a}(\tau,\vec{\sigma}),\mathcal{O}_{E^b_j}(\tau,\vec{\sigma}')\} = \kappa\delta(\sigma,\sigma')\delta^b_a\delta^i_j$ etc

Classical Dynamics: Physical Hamiltonian $\mathbf{H}^{\rm G}_{\rm phys},\ \mathbf{H}^{\rm BK}_{\rm phys},\ \mathbf{H}^{\rm scalar}_{\rm phys}$

Example for physical Hamiltonians

$$\mathbf{H}^{\rm BK}_{\rm phys} = \frac{1}{2\kappa}\int_{\mathcal{S}} d^3\sigma\,\sqrt{C^2 - Q^{ij}C_iC_j} \qquad \mathbf{H}^{\rm scalar}_{\rm phys} = \frac{1}{2\kappa}\int_{\mathcal{S}} dr^3\sigma\,\sqrt{-2\sqrt{Q}C + 2\sqrt{Q}\sum_{j=1}^{3}\sqrt{Q^{jj}C_jC_j}}$$

IV. Effective Dynamics

Effective dynamics scalar field models

Effective Hamiltonian $$\mathbf{H}_{\text{eff}}^{\text{FLRW,scalar}} = \sqrt{-2\mathcal{O}_V\left(-\frac{3\mathcal{O}_V}{8\pi G\lambda^2\gamma^2}\sin^2(\lambda\mathcal{O}_b)+\frac{\mathcal{O}_{\pi_\varphi}^2}{2\mathcal{O}_V}+\mathcal{O}_V U(\mathcal{O}_\varphi)\right)}$$

Equations of motion:

$$\frac{d\mathcal{O}_V}{d\tau_\chi}=\frac{3\mathcal{O}_N\mathcal{O}_V}{2\lambda\gamma}\sin(2\lambda\mathcal{O}_b) \qquad \frac{d\mathcal{O}_b}{d\tau_\chi}=-4\pi G\gamma\mathcal{O}_N\left(\mathcal{O}_\rho^{\text{scalar}}+\mathcal{O}_P^{\text{scalar}}\right)$$

$$\mathcal{O}_N=-\mathcal{O}_V/\mathbf{H}_{\text{eff}}^{\text{FLRW,scalar}} \qquad \mathcal{O}_\rho^{\text{scalar}}=\frac{\mathcal{O}_{\pi_\varphi}^2}{2\mathcal{O}_V^2}+U(\mathcal{O}_\varphi)+\frac{\mathcal{E}_\chi^2}{2\mathcal{O}_V^2} \qquad \mathcal{O}_P^{\text{scalar}}=\frac{\mathcal{O}_{\pi_\varphi}^2}{2\mathcal{O}_V^2}-U(\mathcal{O}_\varphi)+\frac{\mathcal{E}_\chi^2}{2\mathcal{O}_V^2}$$

$$\frac{d\mathcal{O}_\varphi}{d\tau_\chi}=\frac{\mathcal{O}_N\mathcal{O}_{\pi_\varphi}}{\mathcal{O}_V} \qquad \frac{d\mathcal{O}_{\pi_\varphi}}{d\tau_\chi}=-\mathcal{O}_N\mathcal{O}_V\frac{dU(\mathcal{O}_\varphi)}{d\mathcal{O}_\varphi}$$

Modified Friedmann equation scalar field

$$\tilde{H}^2=\mathcal{O}_N^2\frac{8\pi G}{3}\mathcal{O}_\rho\left(1-\frac{\mathcal{O}_\rho}{\rho_{\max}}\right)$$

density $\mathcal{O}_\rho$ contains inflaton + scalar field

clock, Hubble parameter wrt to τ_χ

Reduced Phase Space General Relativity

Idea: Couple reference matter to gravity+inflaton

$$S = S_{\rm geo} + S_{\rm scalar} + S_{\rm ref}$$

allows construction of Dirac observables

Gaussian and BK dust and scalar fields reference matter

Gaussian

Kuchar, Torre

$$S_{\rm ref} = S^{\rm G}_{\rm dust} = -\int d^3x\sqrt{-g}\left(\frac{\rho}{2}[g^{\mu\nu}T_{,\mu}T_{,\nu}+1] + g^{\mu\nu}T_{,\mu}W_j S^j_\nu\right)$$

Brown-Kuchar

$$S_{\rm ref} = S^{\rm BK}_{\rm dust} = -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}\rho[g^{\mu\nu}\widetilde{U}_\mu\widetilde{U}_\nu + 1]$$

Brown-Kuchar

4-scalar fields

$$S_{\rm ref} = S^{\rm ref}_{\rm scalar} = -\frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}g^{\mu\nu}\chi^0_{,\mu}\chi^0_{,\nu} - \frac{1}{2}\int_{\mathcal{M}} d^4x\sqrt{-g}g^{\mu\nu}M_{jj}\chi^j_{,\mu}\chi^j_{,\nu}$$

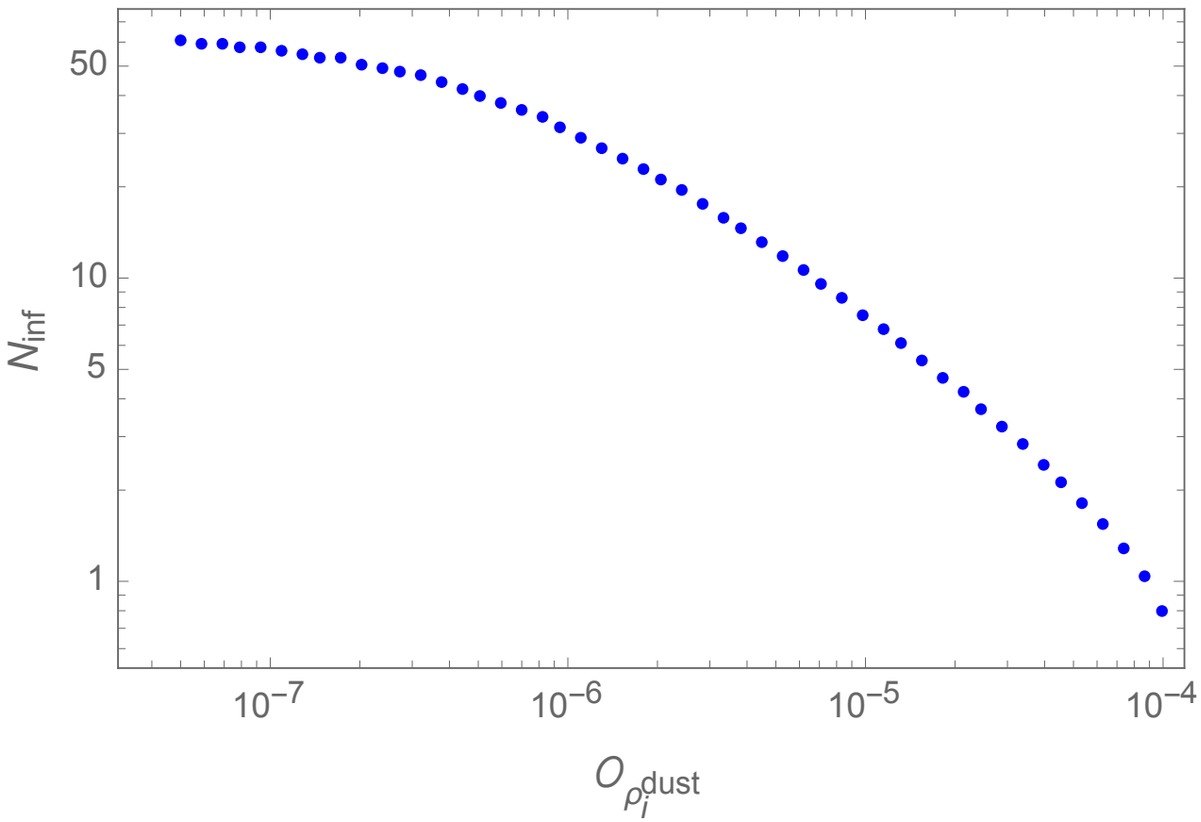
Vetter, K.G.

Full Theory: Choice of reference matter mainly by simplicity

V. More detailed analysis

What happens with increasing dust energy density?

Total number of e-foldings slow roll inflation



Same initial conditions for inflaton

Vary dust energy density from 5×10^{-8} to 10^{-4}

The number of e-foldings decreases with increasing dust energy density

The value of $\mathcal{O}_{\varphi_{on}}$ decreases due to a larger Hubble friction when the dust energy density is increased

How large can the dust energy density be chosen to still allow inflation?

Extreme case: Dust energy density so large that there occurs no inflation, upper bound?

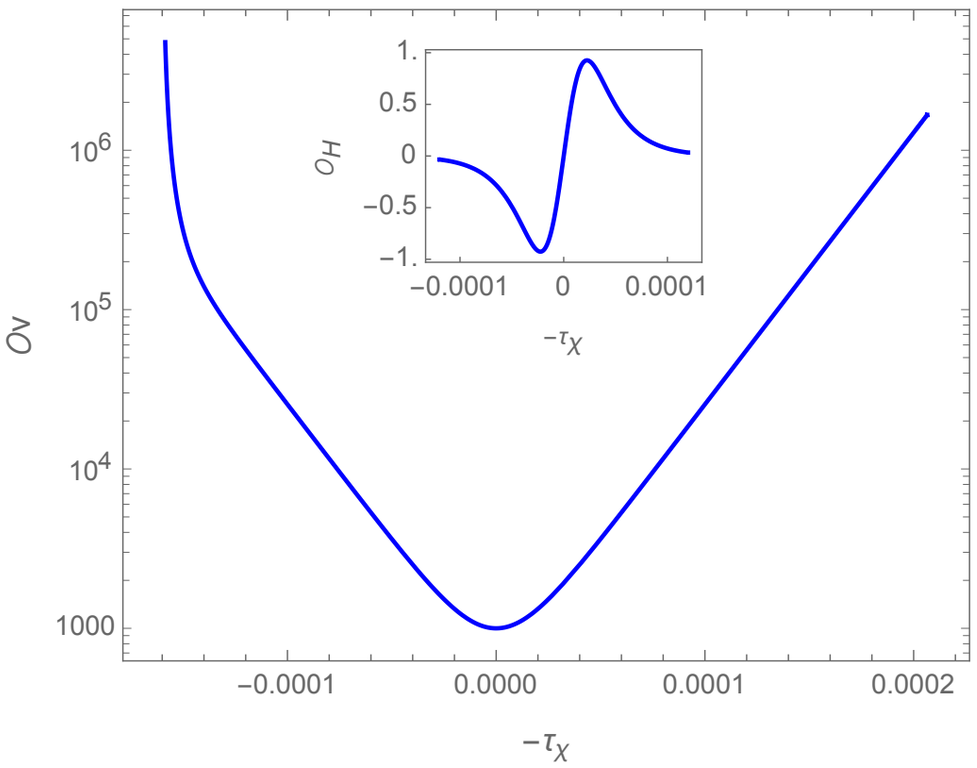
For the initial values chosen here: $\mathcal{O}_{\rho^{clock}} = 1.38 \times 10^{-4}$

More detailed analysis

How does the choice of clocks affect inflation?

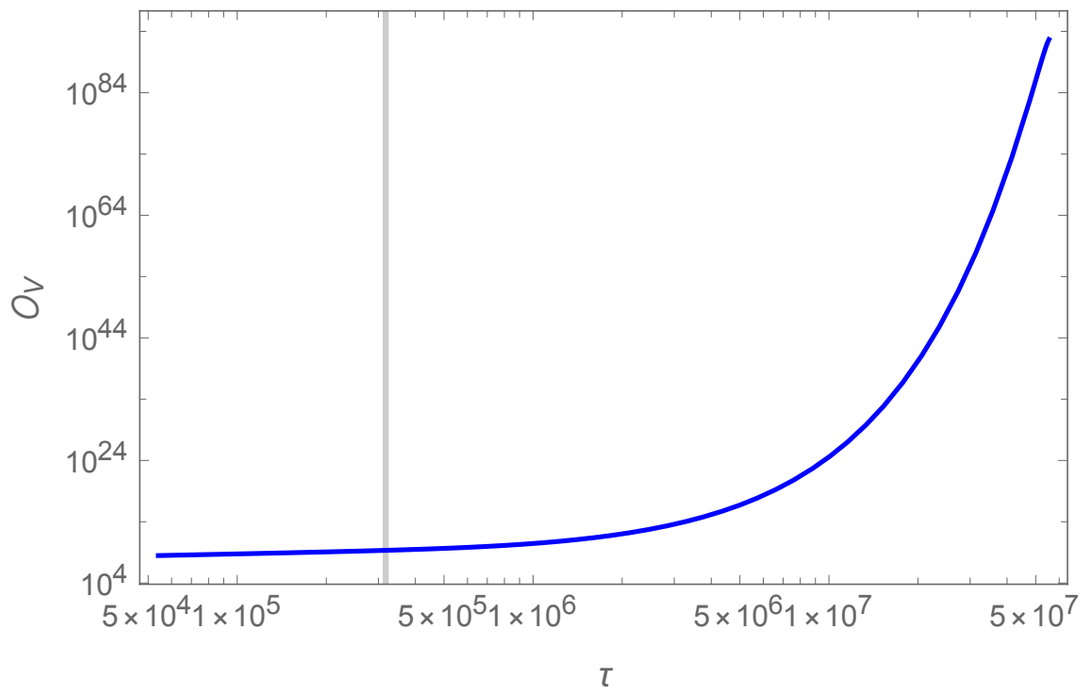
Now we consider the scalar field model wit the same initial conditions $\mathcal{O}_{\varphi_i} = -1.45$, $\mathcal{O}_{\rho^{\text{clock}}} = 10^{-8}$

Evolution of the volume in scalar field time



For better comparison we plot volume wrt to dust time

$\tau = 3.17 \times 10^5$ onset of inflation



inflation ends at $\tau = 5.52 \times 10^7$