Quantum Frame Relativity of Subsystem Correlations and (Thermo)Dynamics

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based on upcoming work

with

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OIST

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The problem: relational definition of subsystems

[Zanardi '01; Zanardi, Lidar, Lloyd '03; Cotler, Penington, Ranard '19]

Tensor Product Structure (TPS)

A TPS $\mathcal{T}$ on $\mathcal{H}$ is an equivalence class of isomorphisms (unitaries) $\mathbf{T} : \mathcal{H} \to \bigotimes_{\alpha=1}^{n} \mathcal{H}_\alpha$ such that

$\mathbf{T}_1 \sim \mathbf{T}_2$ if $\mathbf{T}_2 \circ \mathbf{T}_1^{-1} =$ product of local unitaries $\otimes_\alpha U_\alpha$ and permutations of subsystem factors with equal dimension

same notion of locality

Operationally, subsystems are defined via sets of simultaneously measurable (commuting) subalgebras of observables

$$[\mathcal{A}_\alpha, \mathcal{A}_{\alpha'}] = 0 \quad , \quad \mathcal{A}_\alpha \cap \mathcal{A}_{\alpha'} = \mathbb{C}\mathbf{1} \ , \ \forall\, \alpha \neq \alpha' \qquad \bigvee_\alpha \mathcal{A}_\alpha = \mathcal{L}(\mathcal{H})$$

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How do we define subsystems when external relata are unavailable and there is no preferred factorization (TPS)?

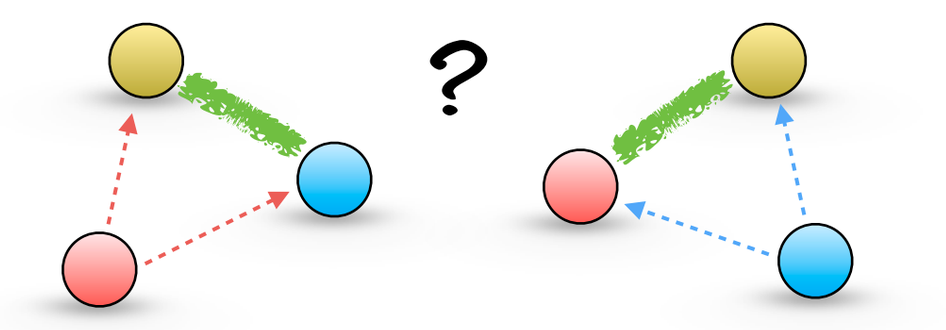
(e.g. gauge theory, cosmology, quantum gravity ⟶ kinematical notion of subsystem not physical)

Use internal RFs
(physical systems, ultimately quantum)

Quantum Reference Frames (QRFs)

[Brukner, Giacomini, Höhn, Müller, Krumm, Smith, Lock, Loveridge, de la Hamette, Galley, Castro-Ruiz, …]

⟶ define gauge-inv. subsystems via relational observables



Different frames identify inequivalent physical notions of subsystems

QRF dependence of physical properties like correlations and (thermo)dynamics

when would two frames agree, or not, on the physical description of a subsystem and its interaction with the remaining DoFs ?

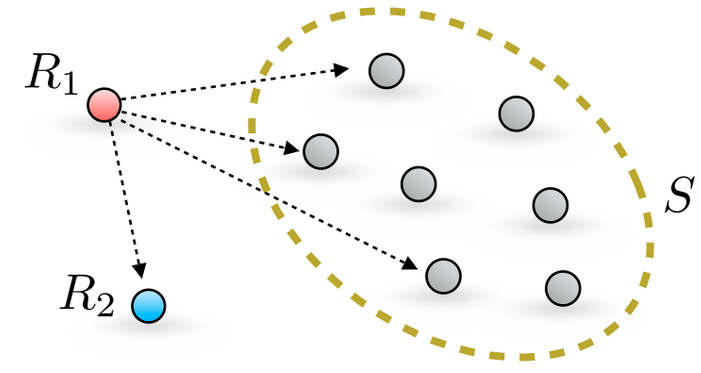
Setup

Krumm, Höhn, Müller '20, '21;
Ahmad, Galley, Höhn, Lock, Smith '21;
de la Hamette, Galley, Höhn, Loveridge, Müller '21

Total system in isolation subject to (finite abelian) gauge group $\mathcal{G}$

e.g. particles with spatial translation symm.
gauge field in some region
dynamical fields subject to spacetime diffeos

$\rho \quad , \quad g \cdot \rho$ indistinguishable $\forall\, g \in \mathcal{G}$



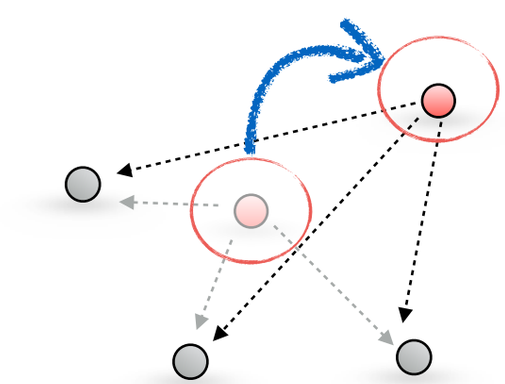
- Kinematical Hilbert space $\mathcal{H}_{\text{kin}} = \mathcal{H}_{R_1} \otimes \mathcal{H}_{R_2} \otimes \mathcal{H}_S$
  externally distinguishable states
  frames system

- gauge redundancy $\longrightarrow$ kinematical subsystems not independent
  gauge-inv. relational info (e.g. relative distances)

(two matter ref. fields, disconnected boundaries, multiple subregions,…)

physical Hilbert space $\mathcal{H}_{\text{phys}} = \left\{ |\psi\rangle_{\text{phys}} \quad \text{s.t.} \quad |\psi\rangle_{\text{phys}} = \; U^g_{R_1} \otimes U^g_{R_2} \otimes U^g_S \;\; |\psi\rangle_{\text{phys}} \right\}$ internally distinguishable gauge-inv. states

relational observables $\mathcal{A}_{\text{phys}} = \mathcal{B}(\mathcal{H}_{\text{phys}}) \;\ni\; O^{g_i}_{f_{\bar{i}}|R_1}$ value of $f_{\bar{i}}$ when R_i is in orientation g_i



- $\mathcal{G}$-frames Quantum version of group-valued RF (cfr. edge modes as RFs [Carrozza, Höhn, '21; Carrozza, Eccles, Höhn, '22]

  Set of $\dim \mathcal{G}$ configuration DoFs $|g_i\rangle_i \;\; (i = 1, 2)$ frame orientations

  $\mathcal{G}$-action on frame config. $U^g_{R_1} \otimes \mathbb{1}_{R_2} \otimes \mathbb{1}_S \;\; , \;\; \mathbb{1}_{R_1} \otimes U^g_{R_2} \otimes \mathbb{1}_S$ frame reorientations

Internally distinguishable
(change relations,
physical transf.)

Jumping into internal perspective and QRF transformations

Krumm, Höhn, Müller '20, '21;
de la Hamette, Galley, Höhn, Loveridge, Müller '21

How to describe S + "other frame" relative to one of the internal frames ?

How to switch between internal frame perspectives ?

Idea: **identify redundant DoFs with those of the frame**

$\mathcal{H}_{\text{phys}}$

Gauge-inv./relational states

redundancy

fix redundancy/condition on orientation of R_1

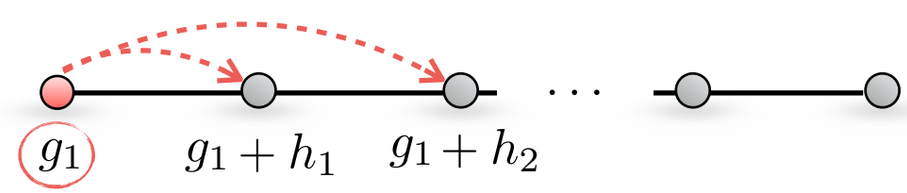
$$\mathcal{R}_1^{g_1} = \langle g_1|_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_S$$

Reduction map
(invertible isomorphism between physical and reduced Hilbert space)

$\mathcal{H}_{R_2|R_1} \otimes \mathcal{H}_{S|R_1}$

relative states to internal frame R_1

no redundancy



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$\mathcal{H}_{\text{phys}}$

perspective-neutral

Different ways of fixing the redundancy

All encoded in physical Hilbert space

reduction to R1 perspective

$$\mathcal{R}_1^{g_1} = \langle g_1|_1 \otimes \mathbb{1}_2 \otimes \mathbb{1}_S$$

reduction to R2 perspective

$$\mathcal{R}_2^{g_2} = \mathbb{1}_1 \otimes \langle g_2|_2 \otimes \mathbb{1}_S$$

different frame perspectives linked through perspective-neutral stage

$\mathcal{H}_{R_2|R_1} \otimes \mathcal{H}_{S|R_1}$

relative states to internal frame R1

$$V_{1\to 2}^{g_1\to g_2} = \mathcal{R}_2^{g_2} \circ (\mathcal{R}_1^{g_1})^{-1}$$

QRF transf. (non-local unitary)

$\mathcal{H}_{R_1|R_2} \otimes \mathcal{H}_{S|R_2}$

relative states to internal frame R2

$\Downarrow$

Different factorizations (TPSs) into physical subsystems relative to the two frames

Commuting local observables relative to R1 are mapped into non-local commuting observables relative to R2

Quantum frame relativity of subsystems

Ahmad, Galley, Höhn, Lock, Smith '21;
de la Hamette, Galley, Höhn, Loveridge, Müller '21
Kotecha, FM, Höhn to appear

⟶ The associated relational observables $O^{g_i}_{f_j \otimes \mathbb{1}_S | R_i}$, $O^{g_i}_{\mathbb{1}_j \otimes f_S | R_i}$ identify inequivalent tensor factorizations of $\mathcal{H}_{\text{phys}}$

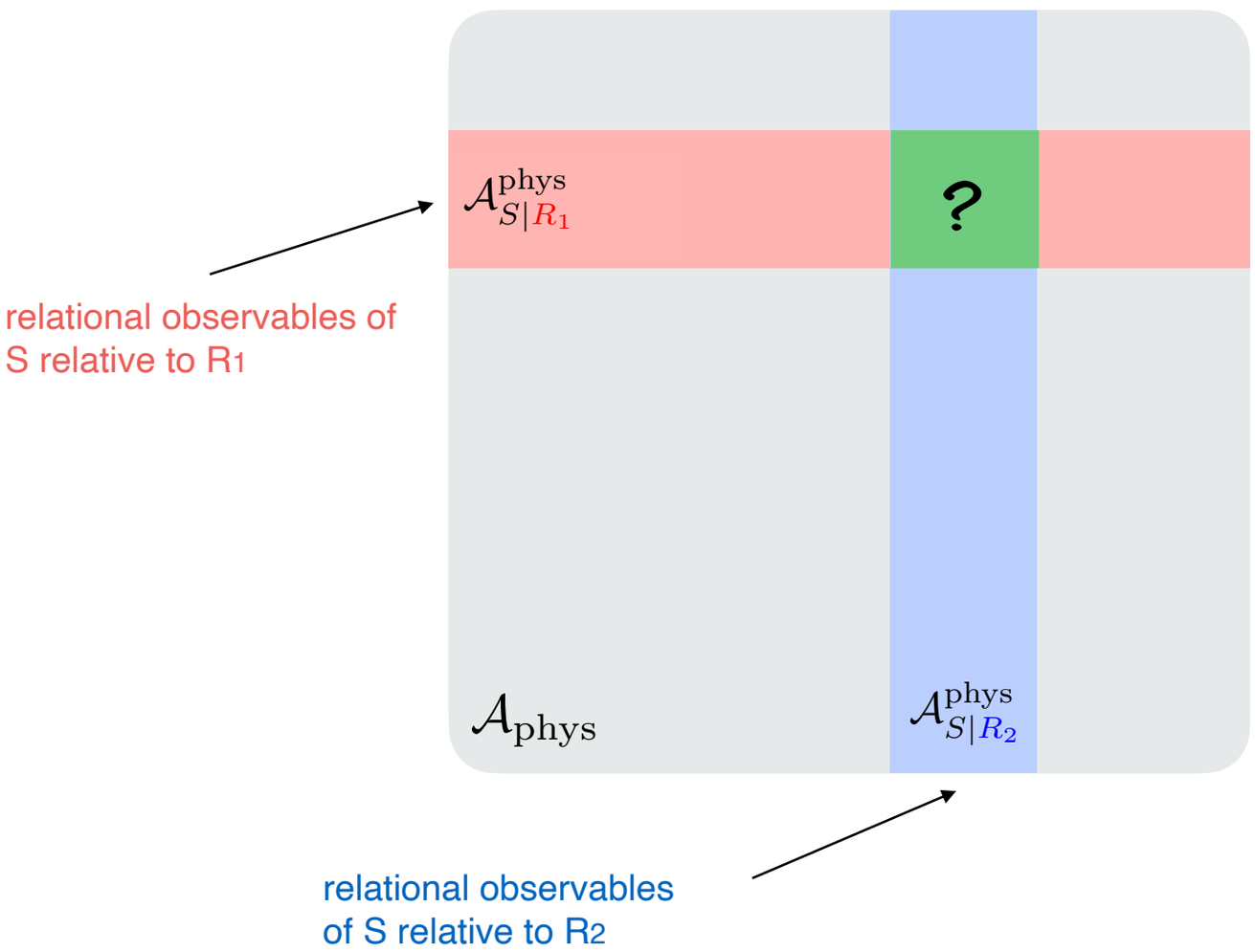
$$\mathcal{A}_{\text{phys}} \simeq \mathcal{A}^{\text{phys}}_{R_2|R_1} \odot \mathcal{A}^{\text{phys}}_{S|R_1} \simeq \mathcal{A}^{\text{phys}}_{R_1|R_2} \odot \mathcal{A}^{\text{phys}}_{S|R_2}$$

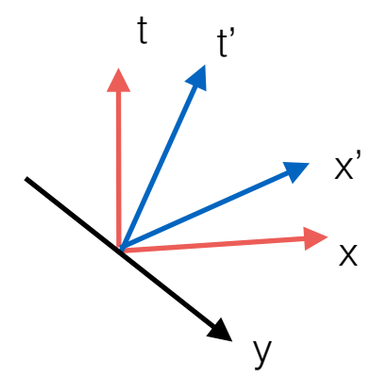
but

$$\mathcal{A}^{\text{phys}}_{S|R_1} \neq \mathcal{A}^{\text{phys}}_{S|R_2}$$

↓

Different gauge-invariant notions of subsystems relative to the two internal QRFs





"different observers have different decompositions of relational length observables into space and time"

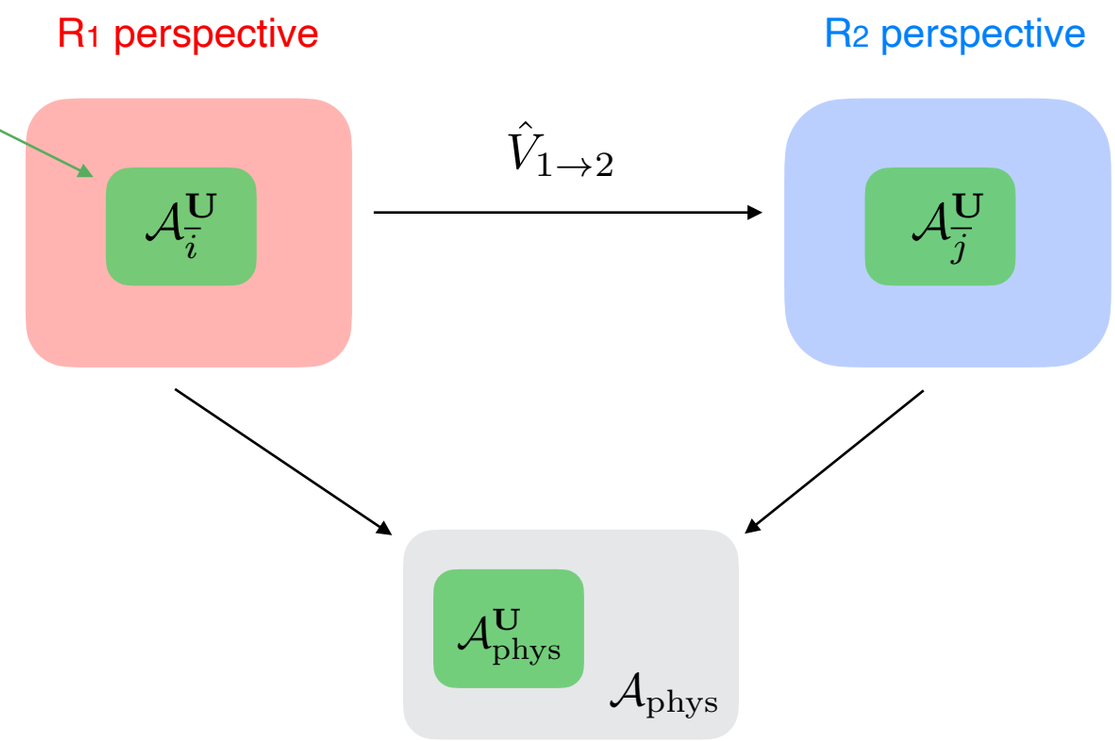
when would two frames agree, or not, on the physical description of a subsystem?

Subalgebras of QRF-invariant operators

Kotecha, FM, Höhn to appear

$$\mathcal{A}_{\bar{i}}^{\mathbf{U}_{Y_j\otimes Z_S}} = \left\{ f_{\bar{i}} \in \mathcal{A}_{R_j|R_i} \otimes \mathcal{A}_{S|R_i} \mid \hat{\mathbf{U}}(f_{\bar{i}}) = (\hat{Y}_j \otimes \hat{Z}_S)(f_{\bar{i}}) \right\}$$

$$\mathbf{U} = \sum_{g\in\mathcal{G}} |g g_i\rangle\langle g_j g^{-1}|_2 \otimes U_S^g$$



Inverse of QRF-transformation $V_{1\to 2}$ within a single perspective

Restores locality altered by the QRF-transformation

$\Downarrow$

operators in the two perspectives differ only by frame exchange and local unitaries

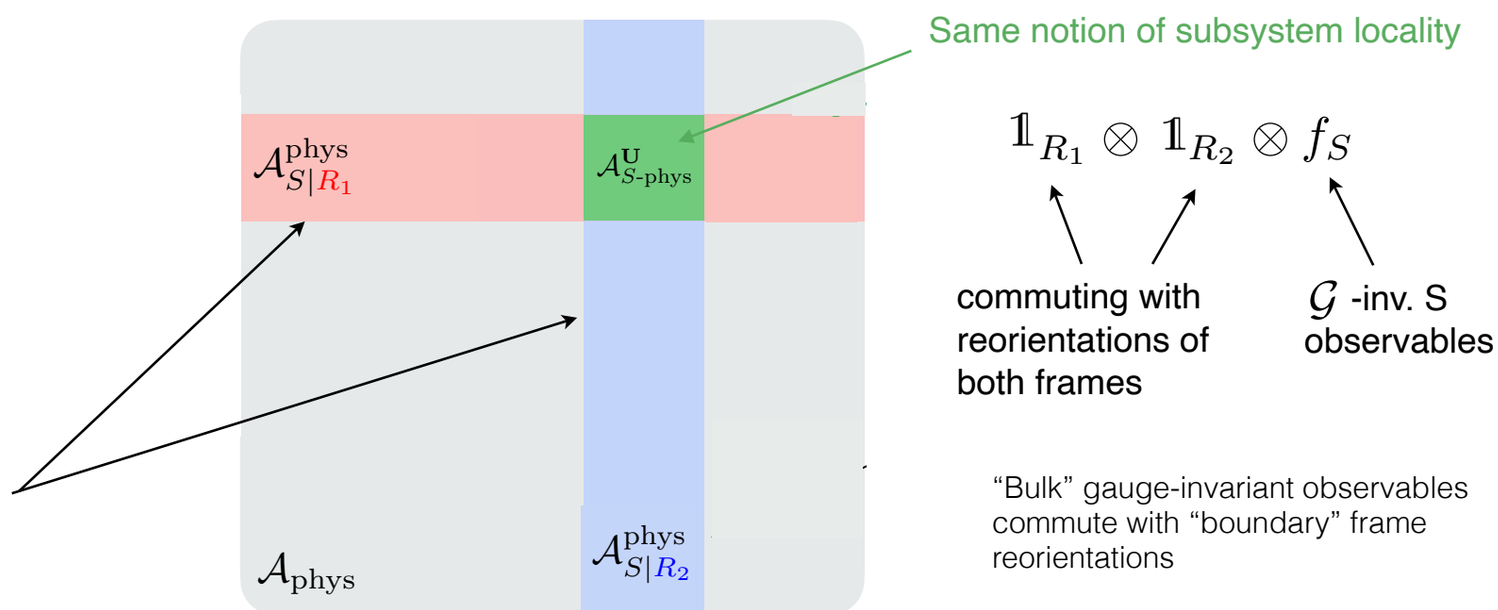
- same degree of locality
- same structure of interactions/correlations

$\Downarrow$

U-inv. observables probe equivalent gauge-inv. descriptions

QRF-relativity of subsystem locality, correlations, and interactions resides in **breaking U-invariance**

Example: local S observables



"Bulk" gauge-invariant observables commute with "boundary" frame reorientations

Subsystem states: correlations and entropies

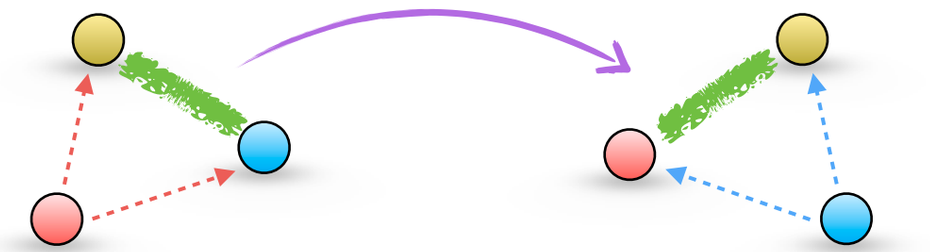
Kotecha, FM, Höhn to appear

QRF-dependence of factorisation $\Longrightarrow$ Correlations/entanglement of S with its complement will in general differ in two perspectives

(See also Giacomini, Castro-Ruiz, Brukner '17;
Ahmad, Galley, Höhn, Lock, Smith '21;
de la Hammette, Galley, Höhn, Loveridge, Müller '21)

Gauge-invariant entanglement entropy in general $\mathcal{S}(\rho_{S|R_1}) \neq \mathcal{S}(\rho_{S|R_2})$
for same global physical state

Unless (mixture of) **U-invariant** global states



Subsystem states & (Rényi) Entropies

For same global physical state, reduced S-state and its correlations/entanglement with other DoFs will in general differ in two perspectives.

When two QRFs agree?

		$\rho_{\bar{i}}$ is **U**-invariant
$\rho_{S\|R_1} = \rho_{S\|R_2}$	✓	✗
$\mathcal{S}_\alpha(\rho_{S\|R_1}) = \mathcal{S}_\alpha(\rho_{S\|R_2}) \;\; \forall\alpha$	✓	✗
$\mathcal{S}(\rho_{S\|R_1}) = \mathcal{S}(\rho_{S\|R_2})$	✓	✗

$\rho_{S|R_2} = Z_S \, \rho_{S|R_1} \, Z_S^\dagger$

Subsystem dynamics and thermodynamics

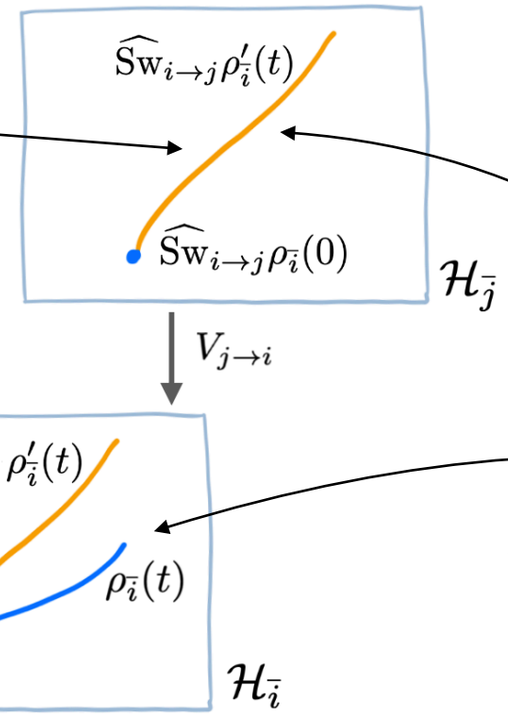
Kotecha, FM, Höhn to appear

For dynamical considerations, it is important whether both the initial states and the Hamiltonian are U-invariant or not

Total system in isolation (unitary dynamics)

Subsystems can interact, exchange heat, work, & energy

$$\rho_{\bar{i}}(0)\ ,\ H_{\bar{i}}\ \in\ \mathcal{A}_{\bar{i}}^{\mathbf{U}_{Y_j}\otimes Z_S}$$



U invariance

$S\|R_1$	$\mapsto$	$S\|R_2$
closed/open dynamics		closed/open dynamics

in the "same" way

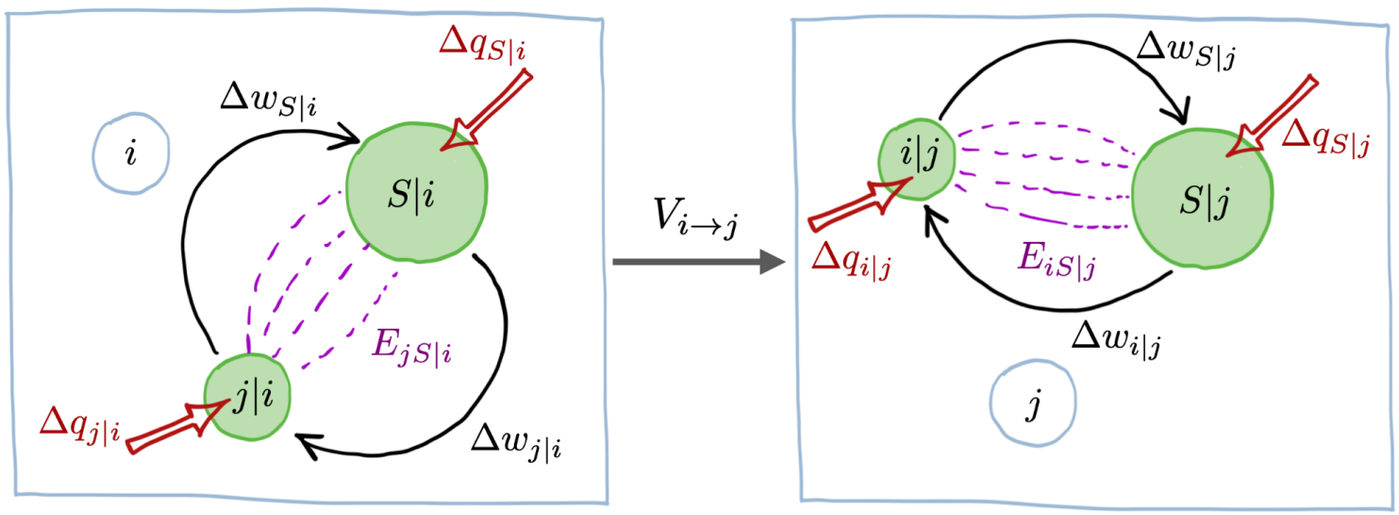
U breaking

At least one between $\rho_{\bar{i}}(0)$ and $H_{\bar{i}}\ \notin\ \mathcal{A}_{\bar{i}}^{\mathbf{U}_{Y_j}\otimes Z_S}$

Different dynamics of S-reduced state in two perspectives

Thermodynamically isolated or closed resp in the "same" way

can map unitary dynamics/zero heat & work exchange into open dynamics/ non-zero heat & work exchange in other perspective



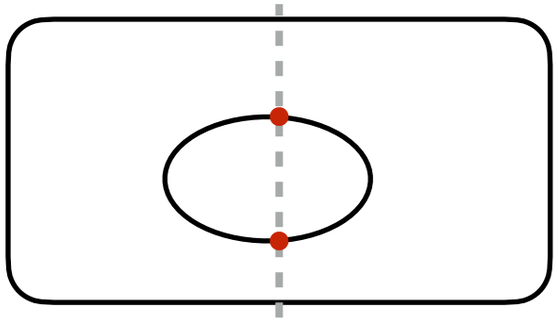
Conclusion and outlook

Take-home message(s)

- Perspective-neutral framework for relational/gauge-invariant description of subsystems and their correlations/interactions
- Different frames mean different physical DoFs when referring to a subsystem
- Frame-dependence or invariance of local & non local observables, subsystem states, correlations, and (quantum) thermodynamics

What next?

- General groups $\mathcal{G}$, spacetime RFs, field theory
- 2nd quantized formulation $\longrightarrow$ changing particle number, phase transitions
- RFs, entanglement, and thermodynamics for subsystems in gauge theories and gravity



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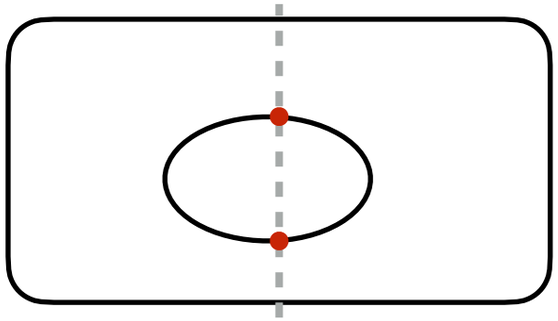

Thank you for your attention !