Quantum aspects of stimulated Hawking radiation in an analog white-black hole pair

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This talk is based on:

Ivan Agullo, Anthony J. Brady, and Dimitrios Kranas. “Quantum Aspects of Stimulated Hawking Radiation in an Optical Analog White-Black Hole Pair”. In: Phys. Rev. Lett. 128 (9 Feb. 2022), p. 091301.

Context

- **Hawking effect**: Creation of entangled pairs of particles by black hole event horizons. S. W. Hawking, Black hole explosions, Nature 248, 30 .

- **Direct observation?** For typical black hole masses: $T_H/T_{CMB} \ll 1 \rightarrow$ signal burried under CMB.

- **Analog black holes**: A viable alternative for the experimental observation of the Hawking effect.

Optical black holes: The Kerr effect

- Ingredients:
  1. Dielectric medium.
  2. Strong EM pulse (pump).
  3. Weak EM pulse (probe).

- The mechanism: Kerr effect, i.e. non-linear interaction between the strong pulse and the medium.

$$\boxed{n_{eff}(\omega, x, t) = n(\omega) + \delta n(x, t), \quad \delta n(x, t) = \alpha E_s^2(x, t)}$$

Modes and emergent causal structure

- In the comoving frame (χ, τ) w.r.t to the strong pulse, the system has time symmetry $\rightarrow$ Conservation of frequency.
- For $\chi < -\chi_\mathsf{h}$, $\chi > +\chi_\mathsf{h}$: 4 modes (1 right-moving and 3 left-moving).
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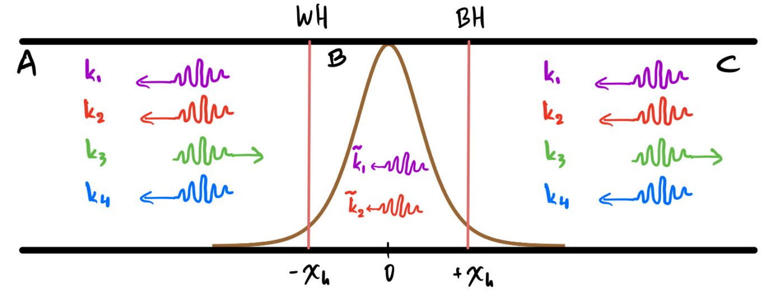

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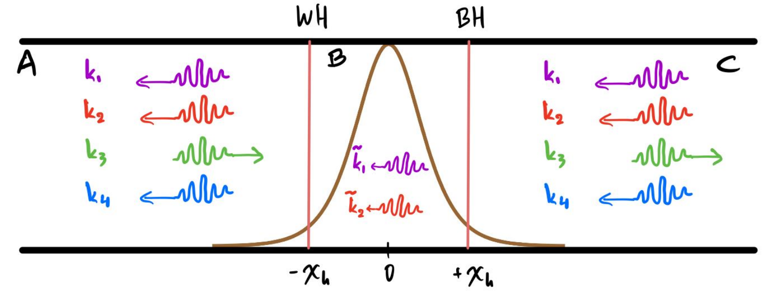

- Hawking particle creation at the WH and BH horizons out of vacuum fluctuations!

Observations?

- T. G. Philbin et al, (2008), Science 319, 1367.
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Successes

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Open questions

- Observation of a blackbody spectrum (dispersion the main obstacle and absence of perfect horizons).
- Generation of entanglement (fragile to background thermal noise).

The effect of background noise

- As input states, we restrict ourselves to the family of Gaussian quantum states and we quantify entanglement by means of Logarithmic Negativity (LN).
- We assume that the input modes are in thermal equilibrium with a blackbody bath of "environment" photons.

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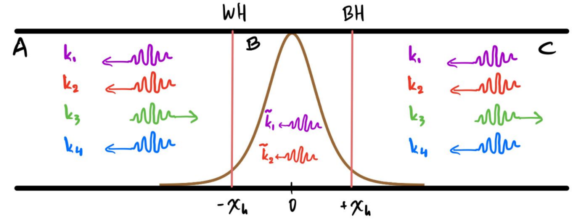

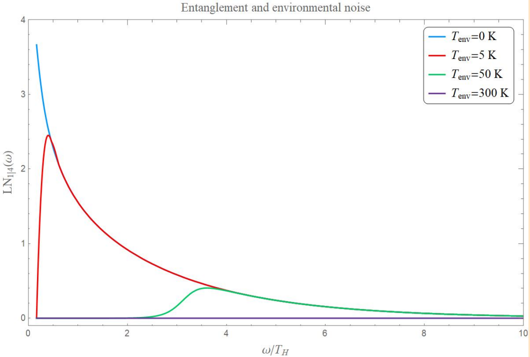

- We find that the presence of the thermal background degrades the entanglement generated in the Hawking process.

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Stimulated Hawking emission: Input non-vacuum initial state (on top of the environmental noise).

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- Single-mode squeezed state:

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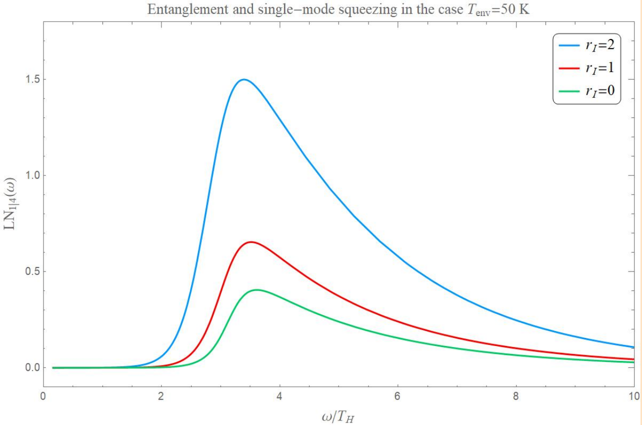

- Single-mode squeezed inputs **enhance** the **entanglement** generated by the horizons.

The protocol to extract T_H from observations

Intensities (classical signal)

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Entanglement (quantum signal)

- Theoretically, we compute the function $LN_{1|4}(\omega) = f(\omega; T_H, T_{env}, r_I)$.
- Measure all mode correlations $\left\langle \{\hat{a}_i^{out}(\omega), \hat{a}_j^{out}(\omega)\} \right\rangle$, $\left\langle \{\hat{a}_i^{out}(\omega), \hat{a}_j^{out\,\dagger}(\omega)\} \right\rangle$, $\left\langle \{\hat{a}_i^{out\,\dagger}(\omega), \hat{a}_j^{out\,\dagger}(\omega)\} \right\rangle$.
- Construct $LN_{1|4}(\omega)$.
- Obtain T_H from $LN_{1|4}(\omega)$.

Take home messages

- The black hole causal structure can be reproduced in dispersive, inhomogeneous media.
- Stimulating the process with a single-mode squeeze state is a promising strategy for the detection of the Hawking effect as it enhances both the Hawking intensity and the entanglement generated by the Hawking process.
- Connecting observations (intensities and entanglement) with the causal horizon is essential for the experimental confirmation of the Hawking effect.

Additional Slides

Context

- **Hawking effect**: Creation of entangled pairs of particles by black hole event horizons. S. W. Hawking, Black hole explosions, Nature 248, 30 .
- **Direct observation?** For typical black hole masses $T_H/T_{CMB} \ll 1 \rightarrow$ signal burried under CMB.
- **Analog black holes**: Dispersive media where propagating perturbations experience causal horizons mimicking the structure of black hole/white hole spacetimes. Popular analog models include: 1) Hydrodynamic systems, Bose Einstein Condensates, Optical media, etc.

River-analog of black holes

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River metric in 1+1D:

$$ds^2 = -u^2 dt^2 + (dx - V(x)dt)^2$$

Acoustic horizon condition $|V(x)| = u$.

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Acoustic horizon condition $|V(x)| = u$.

$$ds^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2$$

$$ds^2 = -c^2 d\tilde{t}^2 + \left(dr + c\sqrt{\frac{r_S}{r}} d\tilde{t}\right)^2$$

Flow velocity: $V(r) = -c\sqrt{\frac{r_s}{r}}$.

Optical black holes: The (microscopic) model

$$\mathcal{L} = \underbrace{\frac{1}{2}\left[|\partial_t A|^2 - |\partial_x A|^2\right]}_{\text{EM field}} + \underbrace{\frac{1}{2}\left[|\partial_t \psi|^2 - \Omega^2 \psi\right]}_{\text{medium}} + \underbrace{gRe\left[\psi \partial_t A^*\right]}_{\text{linear interaction}}$$

$$\Omega(x,t) = \Omega_o + \underbrace{\alpha_{ST}|\mathcal{E}_s(x,t)|^2}_{\text{nonlinear interaction}}$$

$$n_{eff}(\omega_{lab}, x, t) = \sqrt{1 + \frac{g^2}{\Omega^2(x,t) - \omega_{lab}^2}}$$

Wave equation

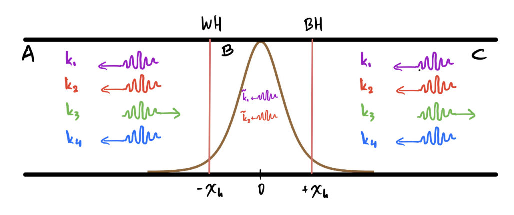
$$\partial_t^2 A(x,t) - \partial_x^2 A(x,t) = -g\partial_t \psi(x,t)$$
$$\partial_t^2 \psi(x,t) + \Omega^2(x,t)\psi(x,t) = g\partial_t A(x,t)$$

$$\downarrow$$

$$\left\{\left(\partial_\chi^2 + \omega^2\right)\left[\gamma^2\left(u\partial_\chi + i\omega\right)^2 + \Omega^2(\chi)\right] - \gamma^2 g^2\left(u\partial_\chi^2 + i\omega\right)^2\right\}\psi_\omega(\chi) = 0$$

Dispersion relation in the comoving frame

$$\gamma(\omega + uk) = \pm\Omega(\chi)\sqrt{1 + \frac{g^2}{\omega^2 - k^2 - g^2}}$$

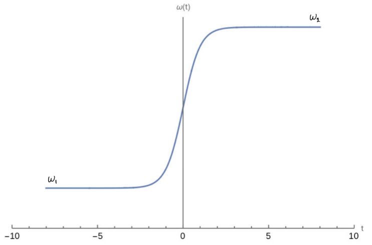


Particle creation: a simple example

Consider a time dependent quantum harmonic oscillator.

$$\hat{H} = \frac{1}{2}\hat{p} + \frac{1}{2}\omega^2(t)\hat{x}^2 \to \frac{d^2\hat{x}}{dt^2} + \omega(t)\hat{x} = 0$$

$$[\hat{x}, \hat{p}] = i, \quad [\hat{a}, \hat{a^\dagger}] = 1$$



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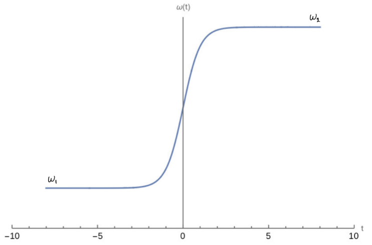

For $t \ll 0$:

- $\omega(t) = \omega_1 \rightarrow \hat{x}(t) = \frac{1}{\sqrt{2\omega_1}}\left(\hat{a}_1 e^{-i\,\omega_1\,t} + \hat{a^\dagger}_1 e^{+i\,\omega_1\,t}\right)$
- $\hat{a}_1 \left|0\right\rangle_1 = 0,\ \left\langle 0\right| \hat{a}_1^\dagger \hat{a}_1 \left|0\right\rangle_1 = 0$

For $t \gg 0$:

- $\omega(t) = \omega_2 \rightarrow \hat{x}(t) = \frac{1}{\sqrt{2\omega_2}}\left(\hat{a}_2 e^{-i\,\omega_2\,t} + \hat{a^\dagger}_2 e^{+i\,\omega_2\,t}\right)$
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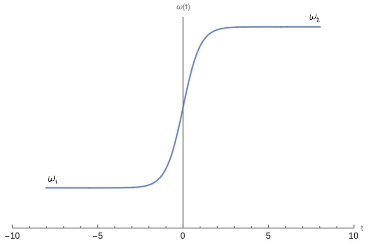

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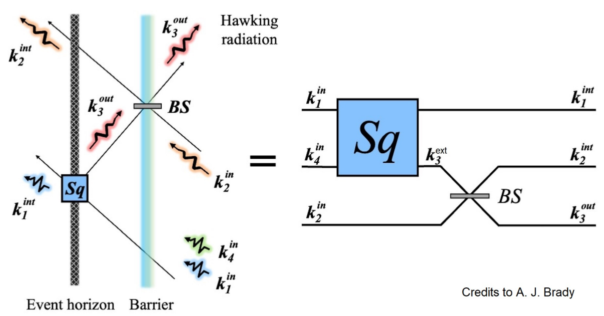
Time evolution:

$$e^{-i\,\omega_1\,t} \rightarrow \alpha e^{-i\,\omega_2\,t} + \beta e^{+i\,\omega_2\,t}, \quad \hat{a}_1 \rightarrow \alpha^* \hat{a}_2 - \beta^* \hat{a}_2^\dagger \quad \rightarrow \quad \left\langle 0\right| \hat{a}_2 \hat{a}_2^\dagger \left|0\right\rangle_2 = |\beta|^2$$

WH-BH scattering via a quantum circuit



The scattering process at the black hole can be modeled via a **two-mode-squeezer** followed by a **beam splitter**.

Squeezer

$$\hat{a}_3^{ext} = \cosh r_H \; \hat{a}_4^{in} + e^{i\phi} \sinh r_H \; (\hat{a}_1^{in})^\dagger$$

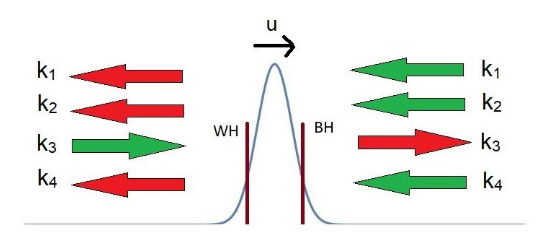
$$\hat{a}_1^{int} = \cosh r_H \; \hat{a}_1^{in} + e^{i\phi} \sinh r_H \; (\hat{a}_4^{in})^\dagger$$

Beam splitter

$$\hat{a}_3^{out} = \cos\theta \; \hat{a}_3^{ext} - \sin\theta \; \hat{a}_2^{in}$$

$$\hat{a}_2^{int} = \cos\theta \; \hat{a}_2^{in} + \sin\theta \; \hat{a}_3^{ext}$$

White hole-Black hole scattering



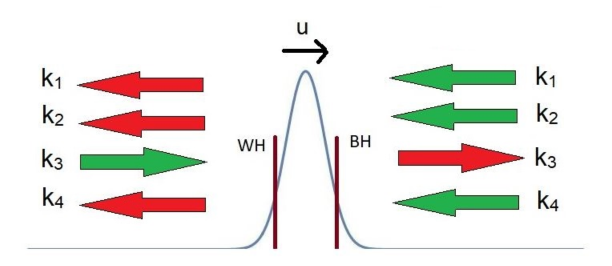
In the asymptotic regions ($\chi \to \pm\infty$):

$$\hat{A}(\chi,\tau) = \int d\omega \sum_{j=1}^{4} \left(\hat{a}_{j,\omega} e^{ik_j(\omega)\chi} e^{-i\omega\tau} + \hat{a}^{\dagger}_{j,\omega} e^{-ik_j(\omega)\chi} e^{+i\omega\tau} \right)$$

$$\hat{a}_i^{out} = \sum_{j}^{4} \left(\alpha_{ij} \hat{a}_i^{\,in} + \beta_{ij} \hat{a}_j^{\,in\,\dagger} \right)$$

Hawking radiation

Hawking radiation emitted via the particle creation process at the analog BH horizon.



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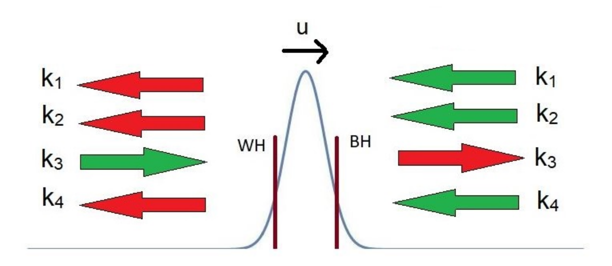

$$\hat{a}_3^{out} = \beta_{31} \left(\hat{a}_1^{in}\right)^\dagger + \alpha_{32}\, \hat{a}_2^{in} + \alpha_{33}\, \hat{a}_3^{in} + \alpha_{34}\, \hat{a}_4^{in}$$

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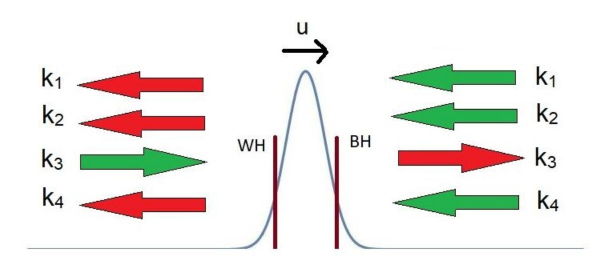

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Particle creation from quantum nothing!

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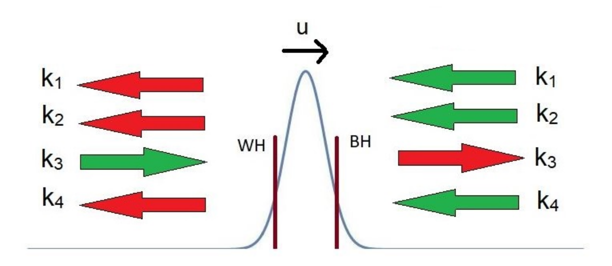

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$$\boxed{|\beta_{31}|^2 = \frac{1 - f(\omega)}{e^{\omega/T_H} - 1}, \qquad T_H = \frac{u\xi}{2\pi}}$$

Linder, Schültzhold, Unruh, 2018

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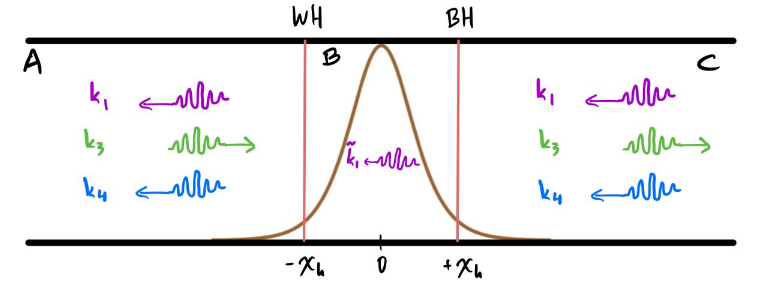

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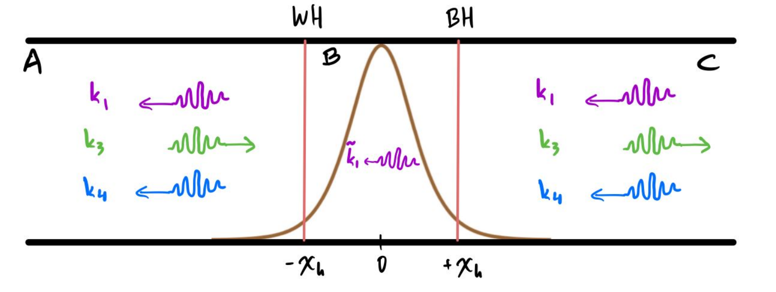

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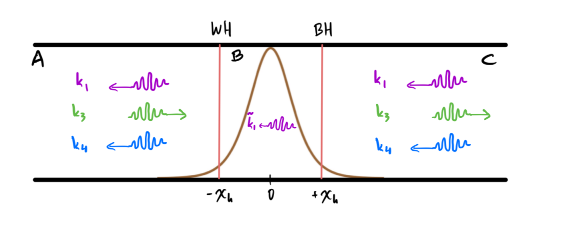
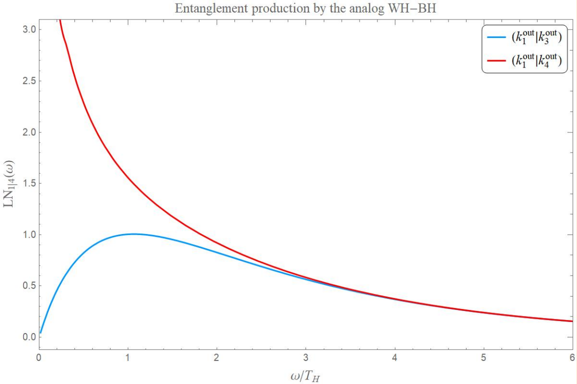
Gaussian states and Logarithmic Negativity

- We work with the family Gaussian states, where all the information about the quantum state is encoded in the first moments $\vec{\mu} = \left\langle \vec{A} \right\rangle$ and the covariance matrix $\sigma = \left\langle \{\vec{A} - \mu, \vec{A} - \mu\} \right\rangle$, where $\vec{A} = (\hat{a}_1, \hat{a}_1^\dagger, \hat{a}_2, \hat{a}_2^\dagger, \hat{a}_3, \hat{a}_3^\dagger, \hat{a}_4, \hat{a}_4^\dagger)^T$.
- The evolution of the state is given by: $\vec{A}_{out} = S \cdot \vec{A}_{in}$, $\vec{\mu}_{out} = S \cdot \vec{\mu}_{in}$, $\sigma_{out} = S \cdot \sigma_{in} \cdot S^T$.
- We use Logarithmic negativity (LN), a well-known entanglement monotone in quantum information theory, to study and quantify the entanglement produced in the Hawking process. LN can be easily computed from the first and second moments.

Entanglement structure of the analog WH-BH





- The BH and WH horizons behave as frequency-dependent two-mode squeezers generating entanglement in the modes $(k_1^{out}|k_3^{out})$ and $(k_1^{out}|k_4^{out})$, respectively.
- At low frequencies: $LN_{1|4} > LN_{1|3}$.
- At larger frequencies: $LN_{1|4} \approx LN_{1|3}$.

Generation of entanglement from vacuum fluctuations

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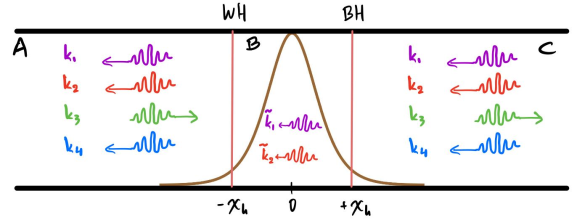
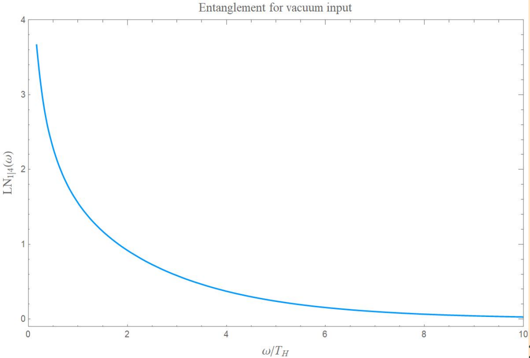
- The strongest correlated couple is the WH Hawking pair of modes $(k_1^{out}|k_4^{out})$.





Observations?

- T. G. Philbin et al, (2008), Science 319, 1367.
- S. Weinfurtner et al, (2011), Phys. Rev. Lett. 106, 021302
- J. Steinhauer, (2016), Nature Phys. 12, 959-965.
- J. Drori et al, (2019), Phys. Rev. Lett. 122, 010404.

Successes

- Generation of horizons.
- Particle production via the stimulated process.

Open questions

- Observation of a blackbody spectrum (dispersion the main obstacle and absence of perfect horizons).
- Generation of entanglement (fragile to background thermal noise).

The protocol to extract T_H from observations

Intensities (classical signal)

- $\langle \hat{n}_3(\omega) \rangle = A(\omega)(e^{\omega/T_H} - 1)^{-1}$

Entanglement (quantum signal)

$$LN_{1|4}(\omega) = -\log_2 \left\{ \frac{1}{4} \left[7 - 4\cosh 2r(\omega) + 8\cosh 4r(\omega) + 4\cosh 6r(\omega) + \cosh 8r(\omega) \right. \right.$$
$$\left. \left. - 16\cosh^2 r(\omega) \cosh 2r(\omega)^{3/2} \left(9 + 6\cosh 2r(\omega) + \cosh 4r(\omega)\right)^{1/2} \sinh r(\omega) \right]^{1/2} \right\}$$

- Measure all mode correlations $\langle \{\hat{a}_i(\omega)\hat{a}_j(\omega)\} \rangle$
- Construct $LN_{1|4}(\omega)$
- Obtain T_H from $LN_{1|4}(\omega)$

Take home messages

- The black hole causal structure can be reproduced in dispersive, inhomogeneous media.
- Frequency conservation and dispersion single out a finite number of degrees of freedom interacting with each other, allowing us to import theoretically rigorous and experimentally accessible tools from quantum information theory to study the entanglement in the Hawking process.
- Stimulating the process with a single-mode squeeze state is a promising strategy for the detection of the Hawking effect as it enhances both the Hawking intensity and the entanglement generated by the Hawking process.