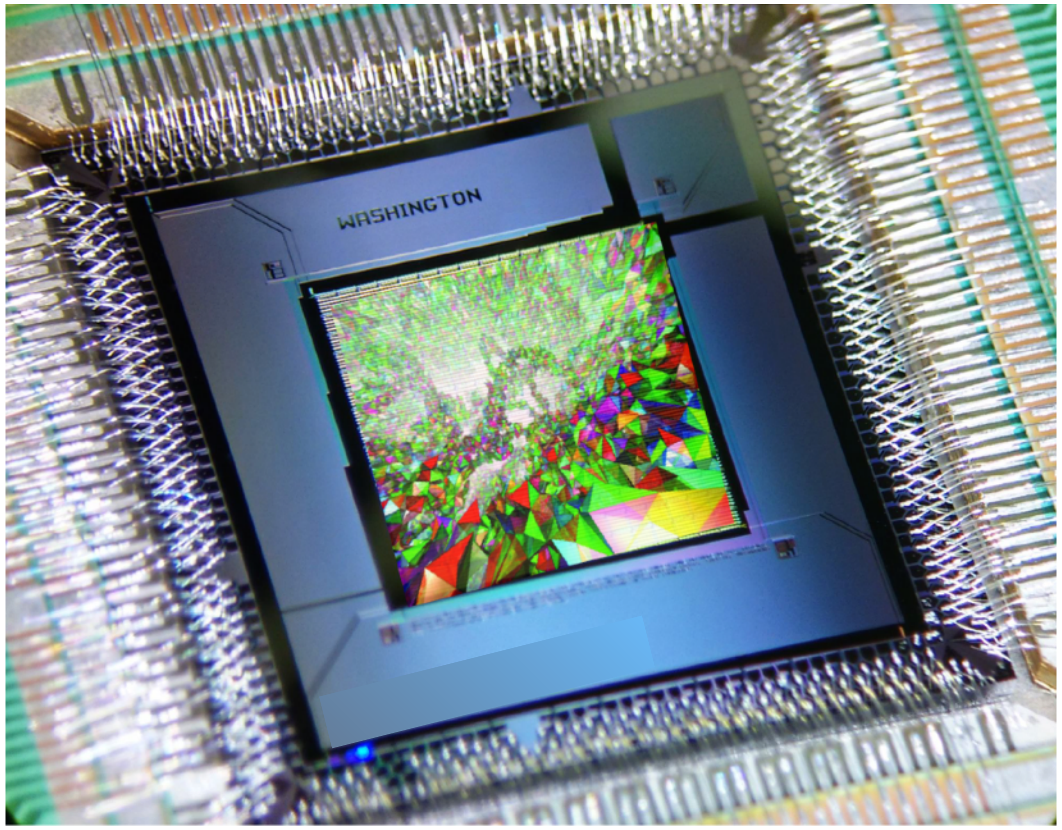
Tying the loops on quantum simulators



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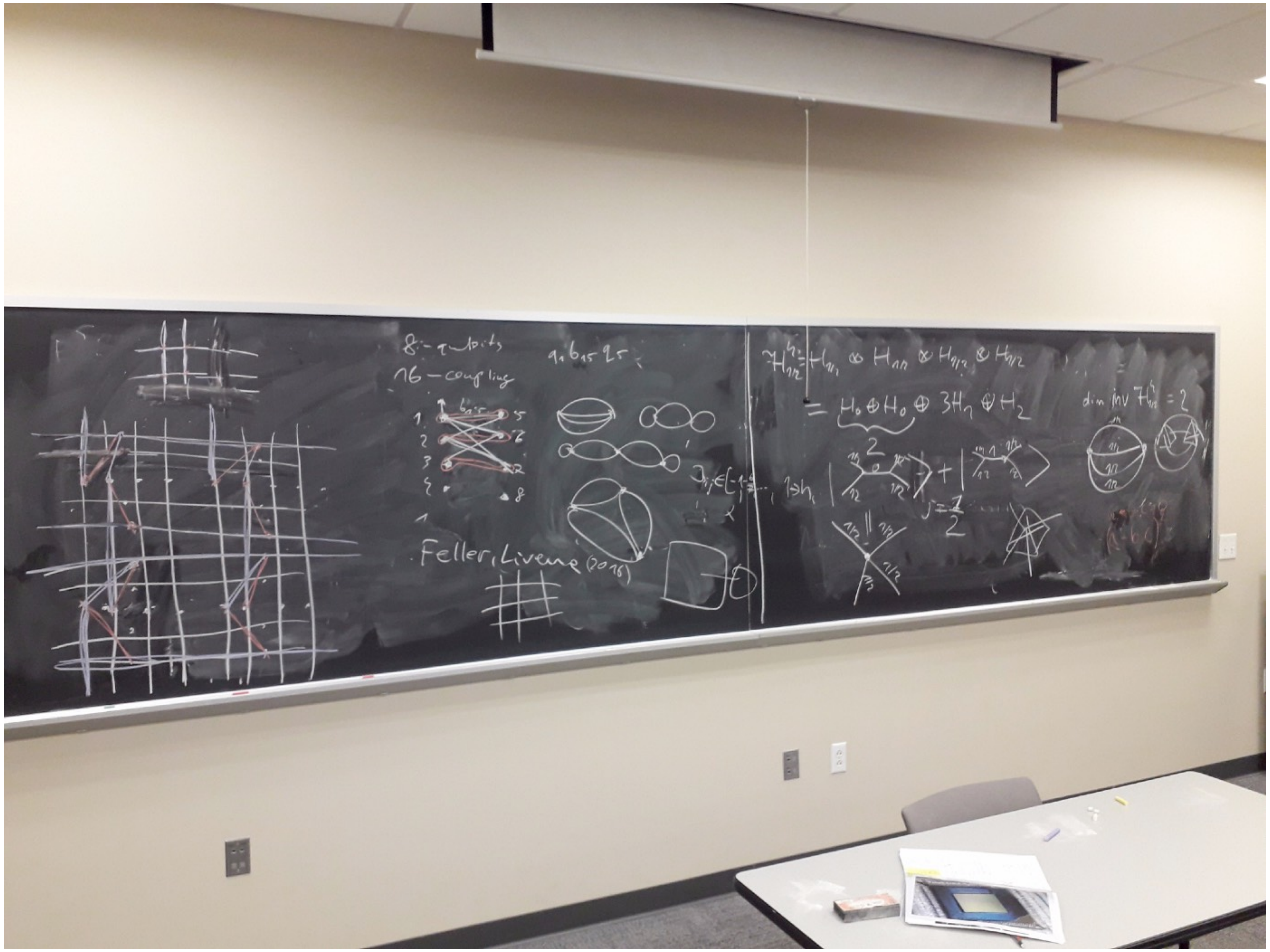
Why simulations of the loops?

- Studies of many-body phenomena in the Planck scale physics, e.g. phase structure, phase transitions
- Analysis of the quantum thermodynamic limit, e.g. quantum phase transitions
- Evaluation of the quantum gravitational amplitudes
- Analysis of the semi-classical limit, e.g. large j limit
- Analysis of the entanglement entropy, e.g area scaling, Page curve
- A framework for simulations of the other gauge field theories
- Complexity of the quantum gravitational processes:
    - Quantum circuit representation provides an upper bound on quantum complexity of quantum gravitational processes
    - Verification of the violation of the extended Church-Turing hypothesis for quantum gravity

A short story of quantum simulating loops



November 2017, JM „Spin Networks on Adiabatic Quantum Computer”,
Penn State

No papers on the subject at that time yet…

2017

- K. Li, *et al.*, *Quantum Spacetime on a Quantum Simulator*, Commun Phys **2,** 122 (2019). [arXiv:1712.08711].

2018

- JM, *Prelude to Simulations of Loop Quantum Gravity on Adiabatic Quantum Computers*, Front. Astron. Space Sci. **8**:571282 (2021), [arXiv:1801.06017]
- JM, *Quantum Gravity on a Quantum Chip*, [arXiv:1803.10592].
- JM, *Spin Foam Vertex Amplitudes on Quantum Computer - Preliminary Results*, Universe **5** (2019) no.8, 179, [arXiv:1810.07100].

2020

- G. Czelusta, JM, *Quantum simulations of a qubit of space*, Phys. Rev. D **103**, 046001 (2021) [arXiv:2003.13124]
- L. Cohen, *et al.*, *Efficient Simulation of Loop Quantum Gravity -- A Scalable Linear-Optical Approach*, Phys. Rev. Lett. **126** (2021) no.2, 020501 [arXiv:2003.03414]
- P. Zhang, *et al., Observation of Two-Vertex Four-Dimensional Spin Foam Amplitudes with a 10-qubit Superconducting Quantum Processor*, [arXiv:2007.13682]

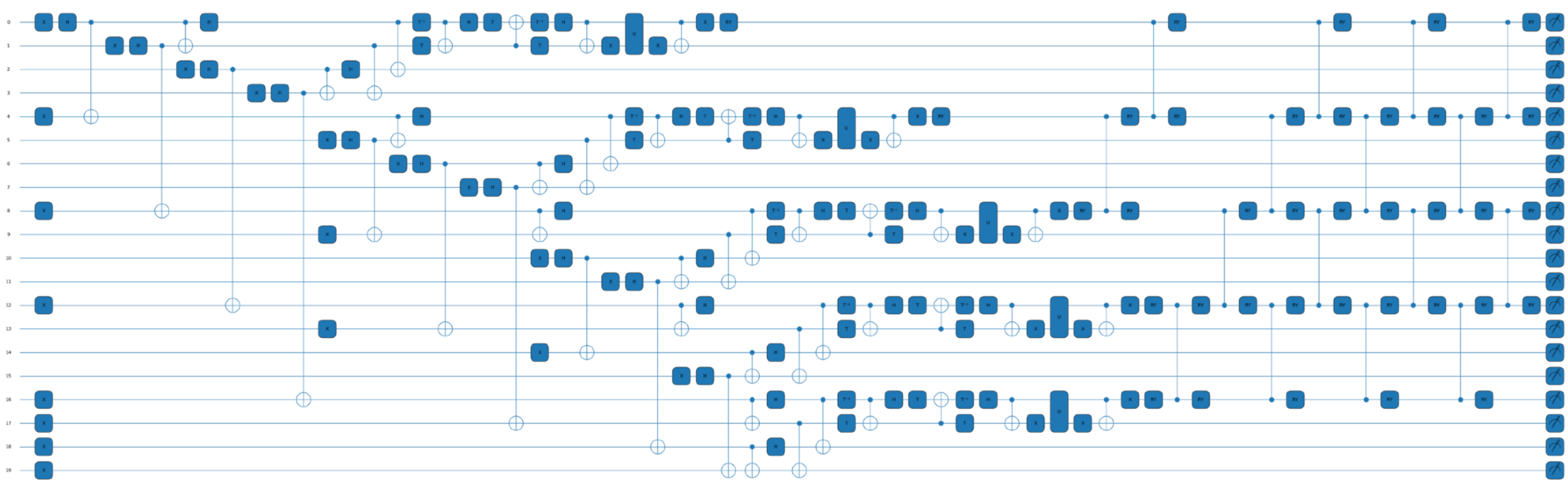
2021

- G. Czelusta, JM, *Quantum variational solving of the Wheeler-DeWitt equation*, Phys. Rev. D **105**, 126005 (2022), [arXiv:2111.03038]

2022

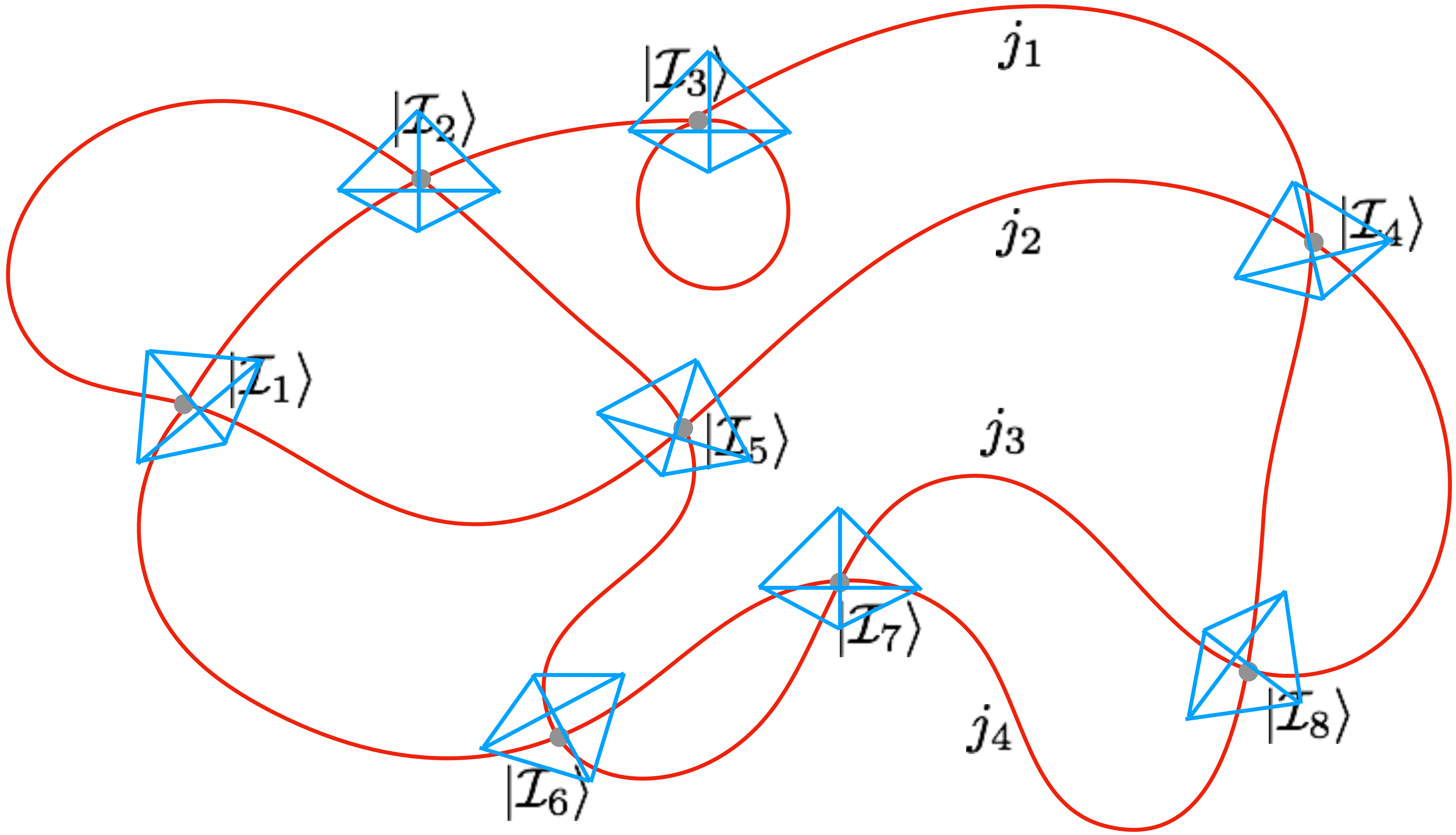
- R. van der Meer, *et al.*, *Experimental Simulation of Loop Quantum Gravity on a Photonic Chip*, [arXiv:2207.00557].

More, in progress…



Kinematics

Spin networks - states of LQG

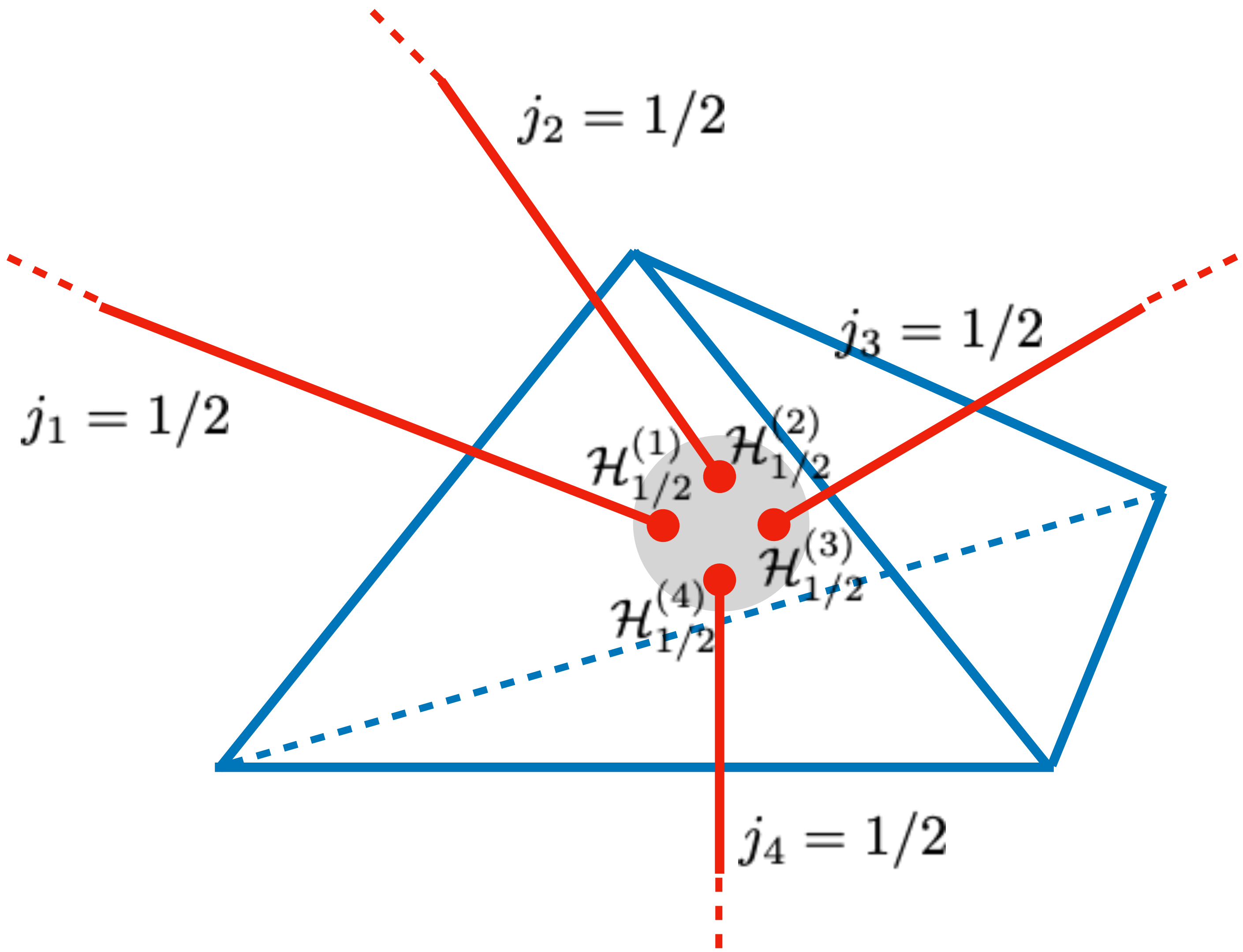


Spin labels - irreducible representations of the SU(2) group: $j_i = 0, \frac{1}{2}, 1, \frac{3}{2}, \ldots$

Local SU(2) gauge invariance (Gauss constraint) implies that spins sum up to zero at the nodes - degeneracy leads to intertwiner spaces.

Ising spin networks

(Feller & Livine, 2016)



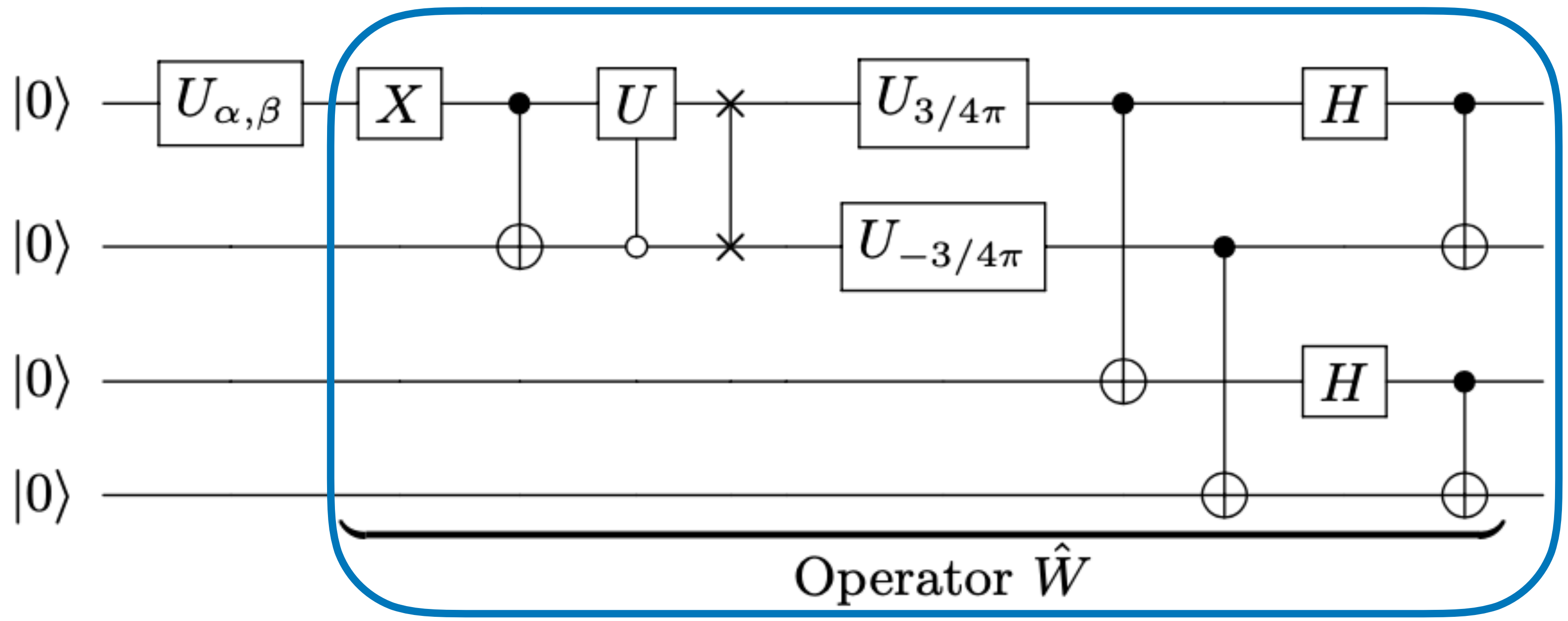
$\hat{P}_{\mathrm{G}}$ - Projection onto spin-0 subspace by the virtue of the Gauss constraint:

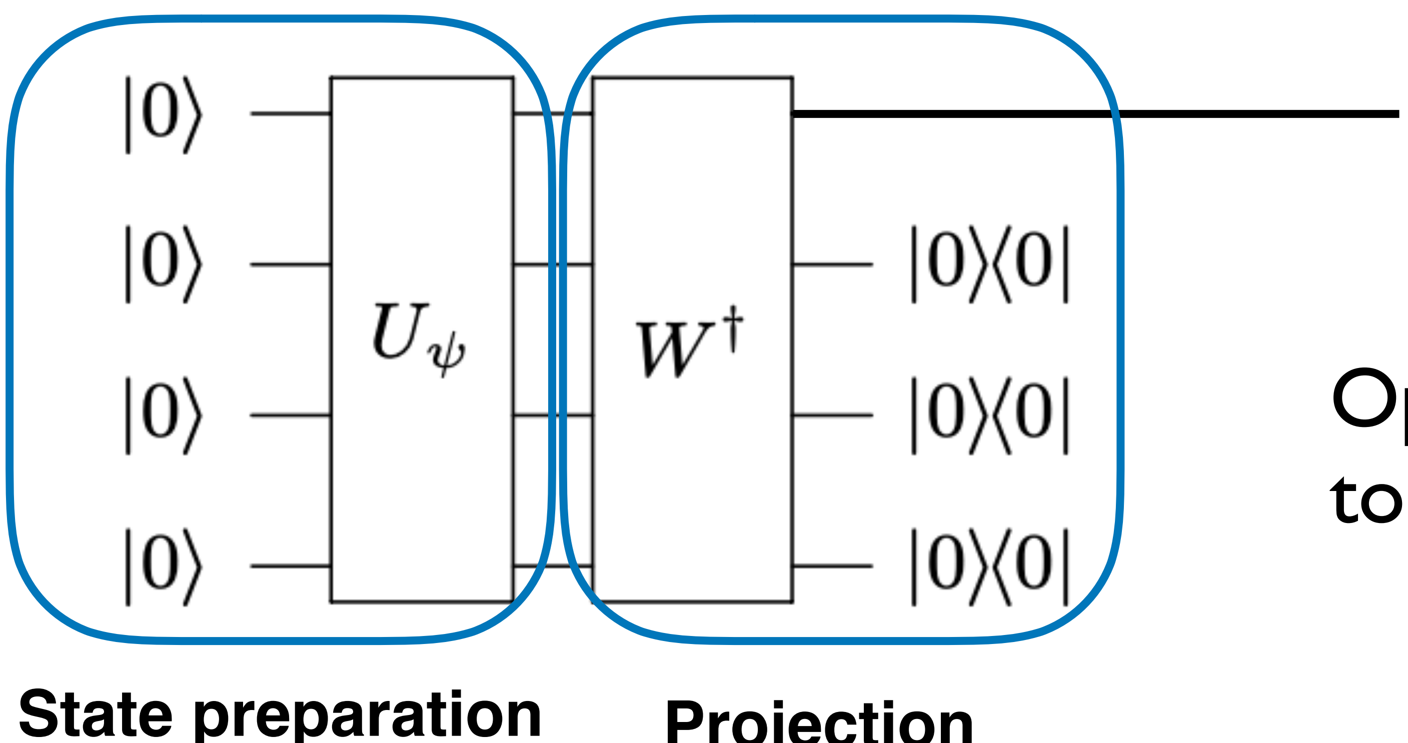
$$\hat{\vec{C}}_G|\psi\rangle = \sum_{i=1}^{4} \hat{\vec{J}}_i|\psi\rangle = 0$$

In consequence: $\dim \mathrm{Inv}(\mathcal{H}^{(1)}_{1/2} \otimes \mathcal{H}^{(2)}_{1/2} \otimes \mathcal{H}^{(3)}_{1/2} \otimes \mathcal{H}^{(4)}_{1/2}) = 2$

New circuit for an Ising node

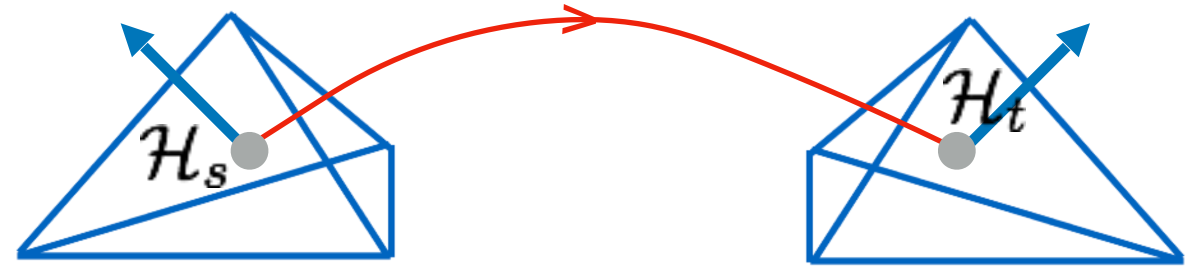
$$\hat{W}\left(\alpha|0\rangle + \beta|1\rangle\right)|000\rangle = |\mathcal{I}(\alpha,\beta)\rangle = \alpha|\iota_0\rangle + \beta|\iota_1\rangle$$





Operator $\hat{W}$ contributes to a „projection" operator

SU(2) holonomies ~ maximal entanglement



Quantum entanglement is „gluing" together faces of tetrahedra.

(Livine 2018; Baytas, Bianchi, Yokomizo 2018; JM, Trześniewski 2020)

The state associated with holonomy can be written as:

$$|\mathcal{E}\rangle = \frac{1}{\sqrt{2}} h^*_{IJ} |I\rangle_s |J\rangle_t \in \mathcal{H}_s \otimes \mathcal{H}_t$$

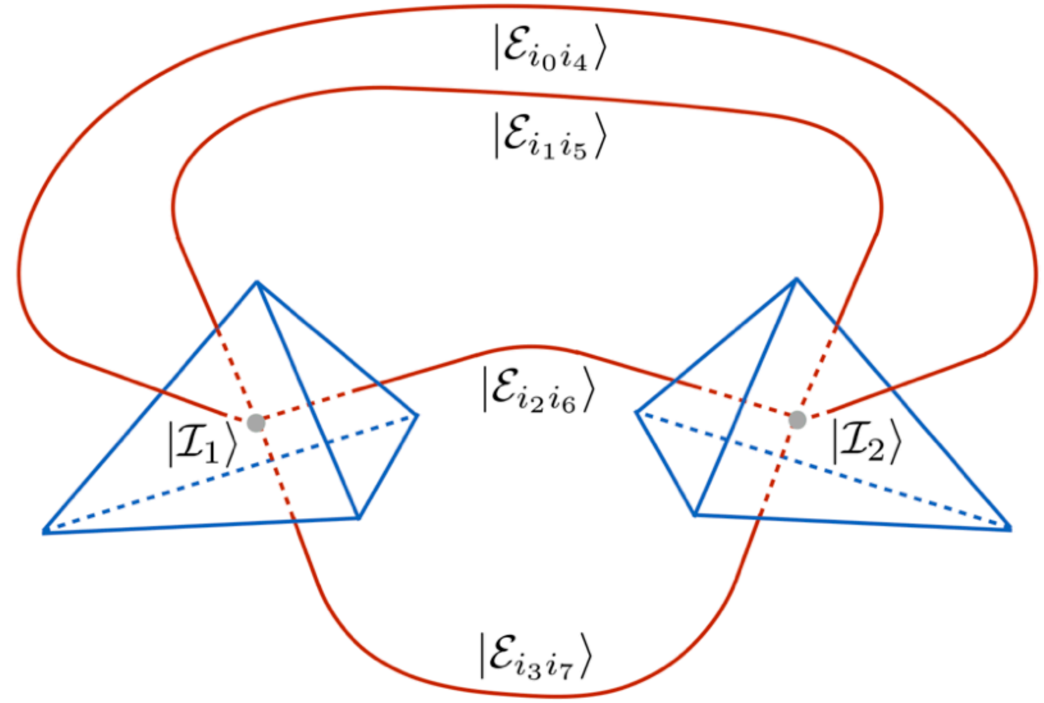
e.g. $|\mathcal{E}_l\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$

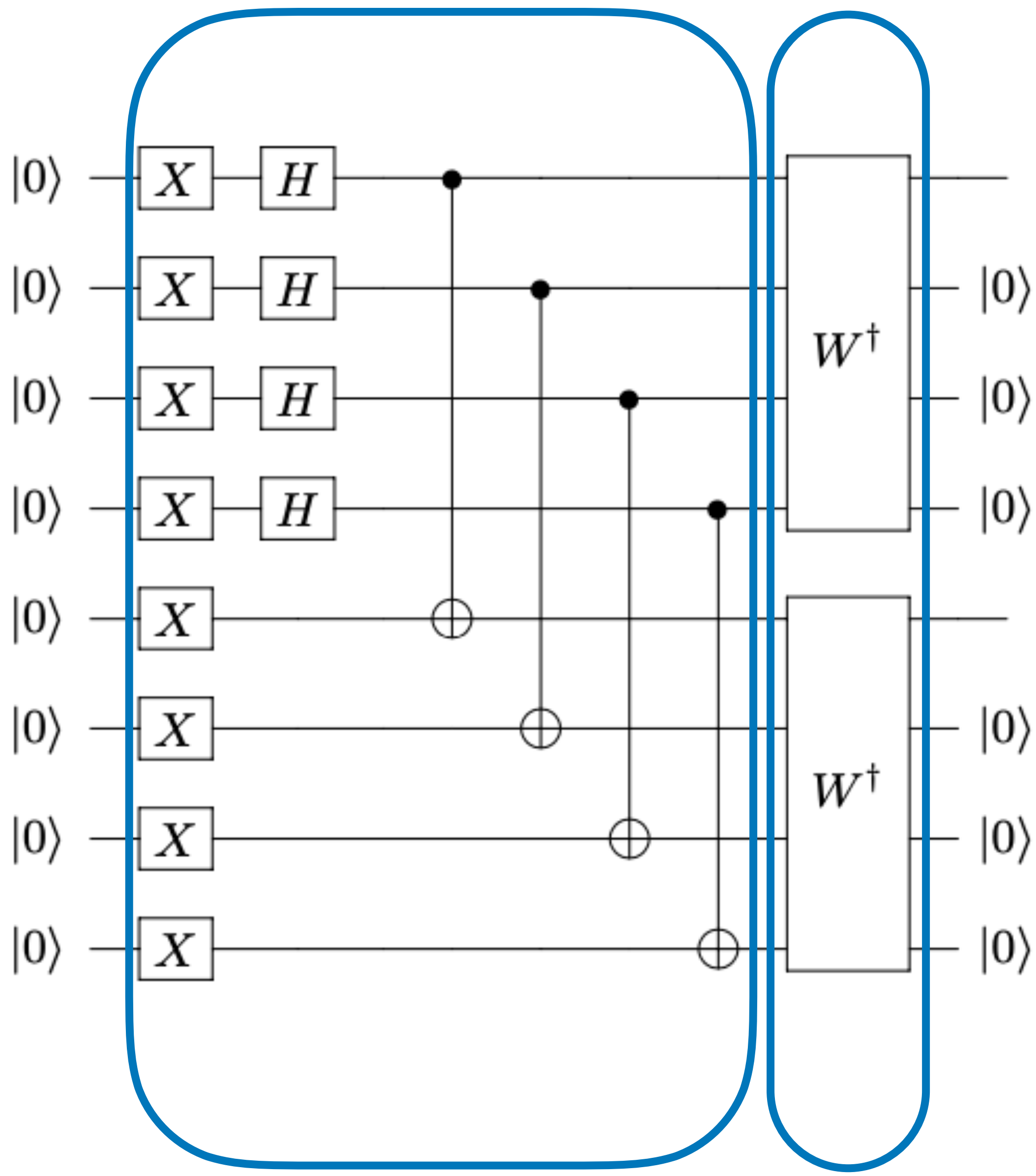
h_{IJ} are matrix components of the SU(2) holonomy.

Based on this Maximally Entangled Spin Network (MESN) states can be introduced:

$$|\mathrm{MESN}\rangle := \hat{P}_G \bigotimes_l |\mathcal{E}_l\rangle$$

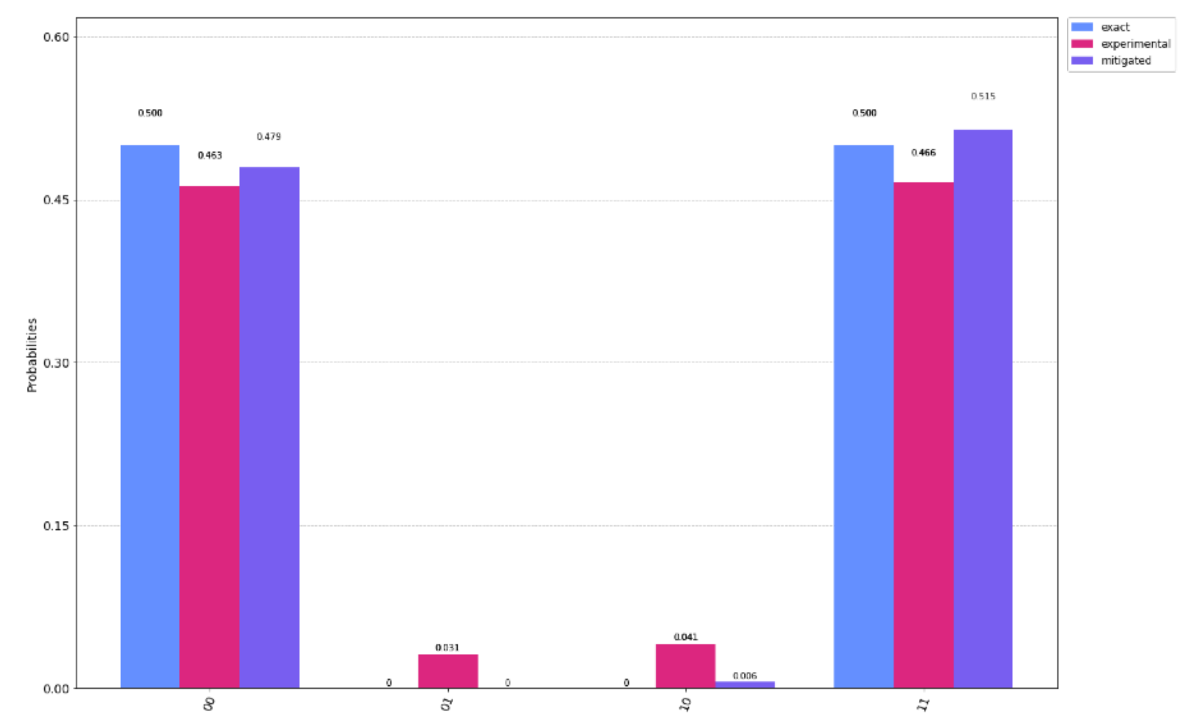
Dipole





Entangled pairs **Projections**

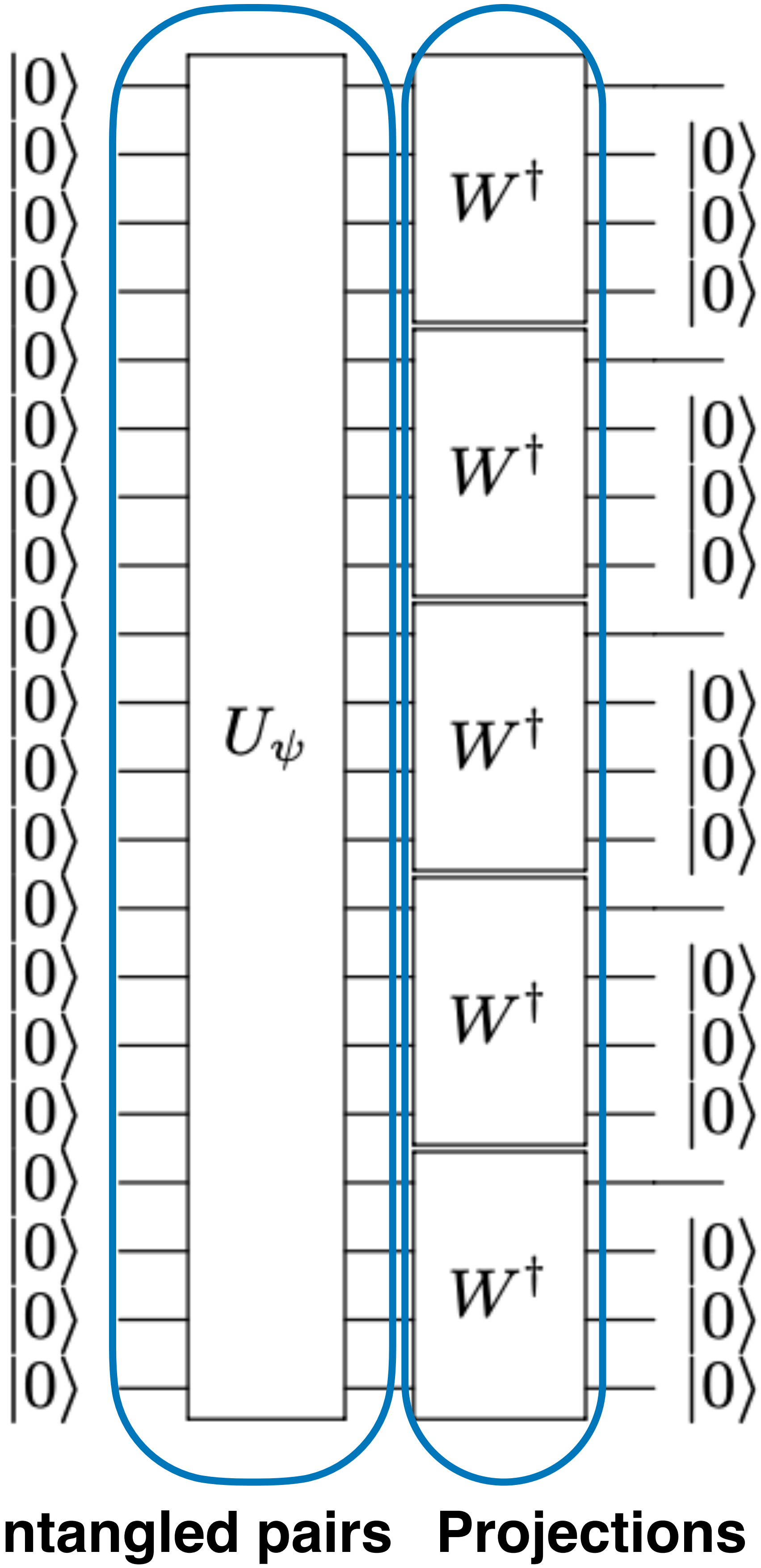
Measured and predicted probabilities:

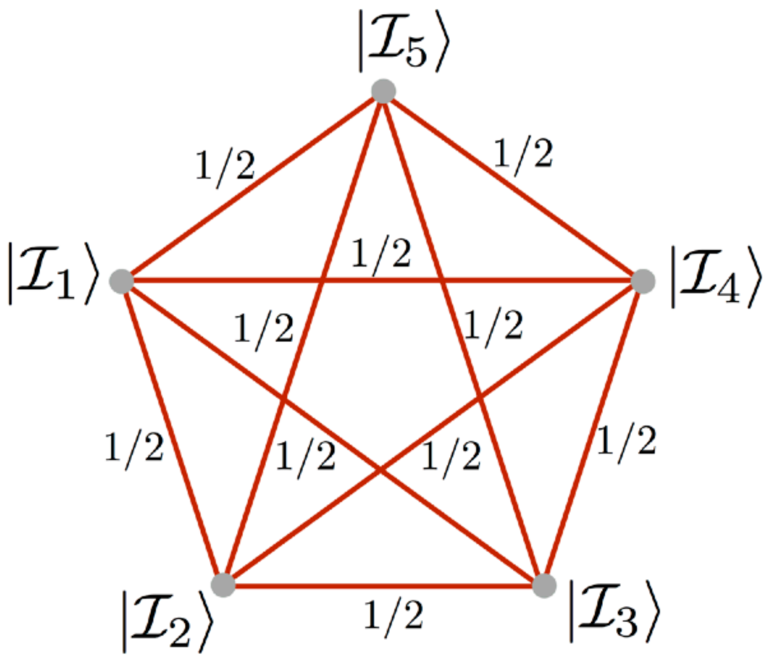


Manila IBM quantum computer

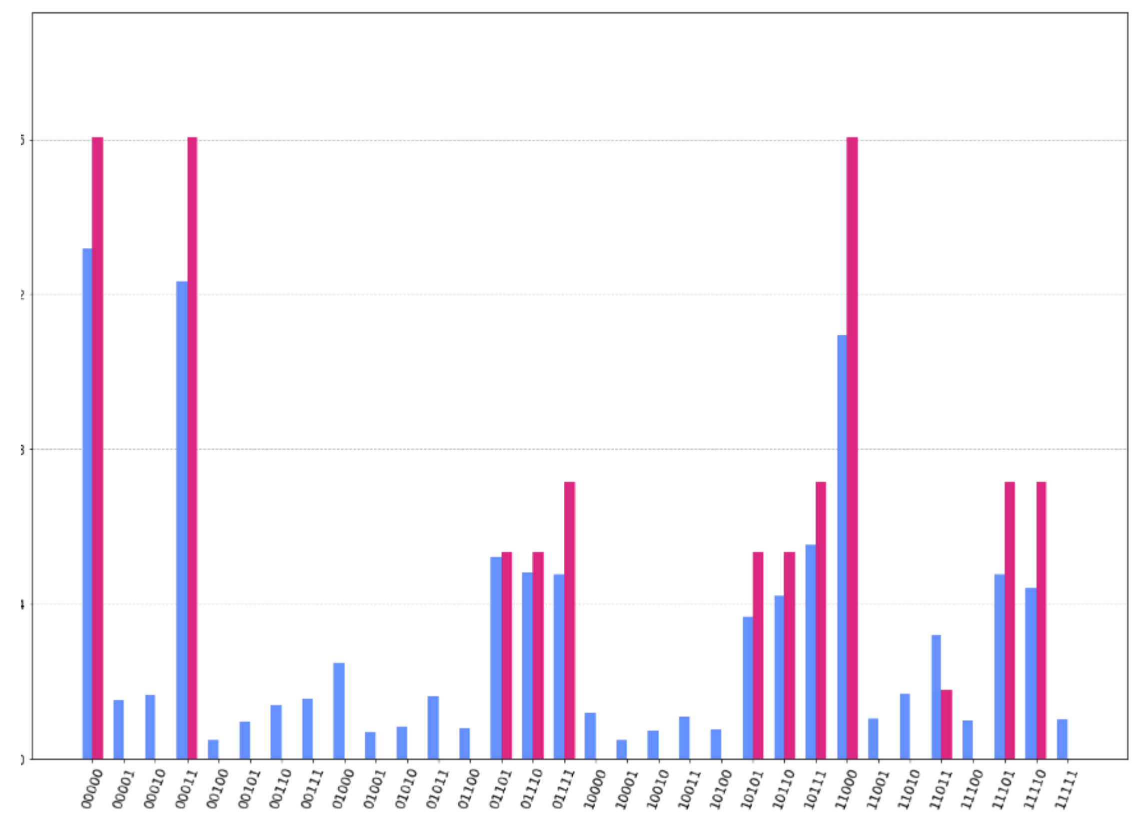
The quantum fidelity of the found state is ≈ 0.99

Pentagram





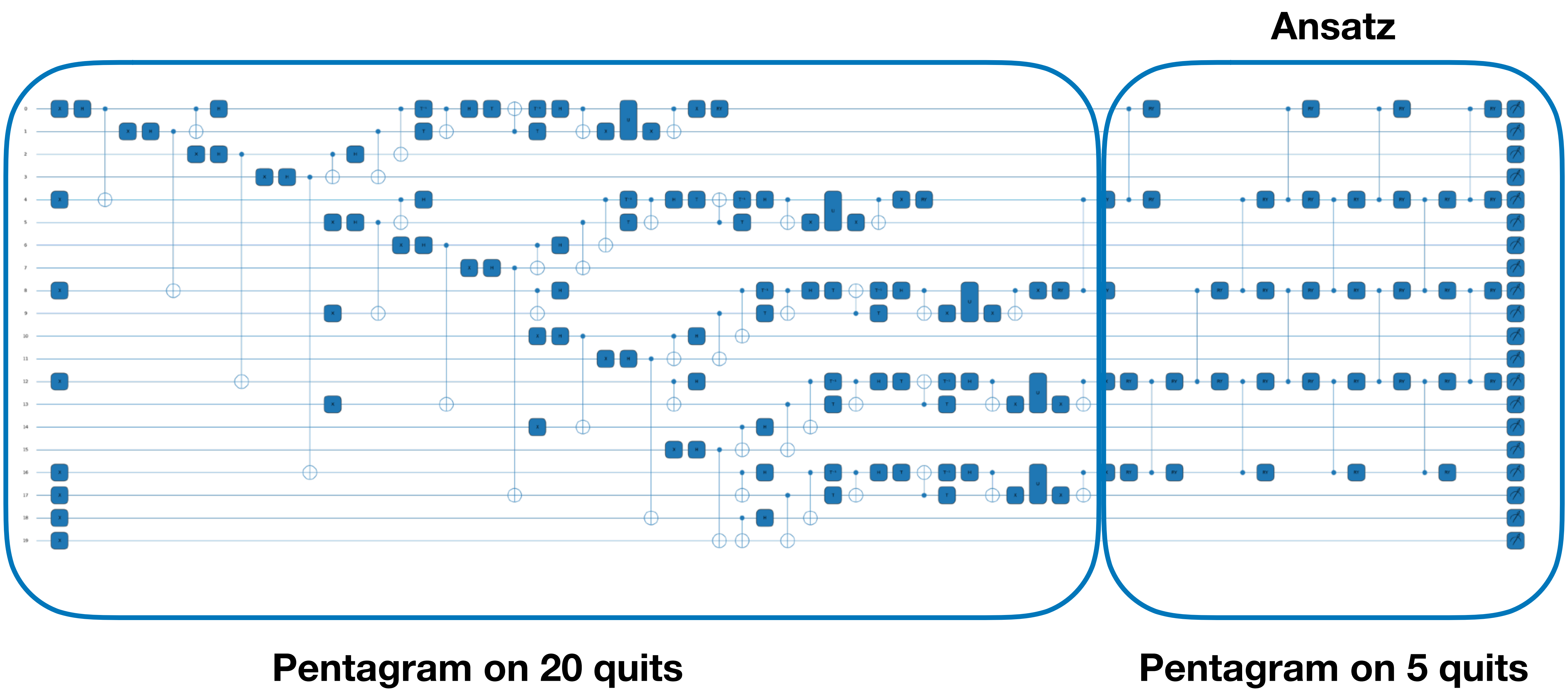
Measured and predicted (from the {15j} symbol) probabilities:



Manila IBM quantum computer

The quantum fidelity of the found state is: ≈ 0.77

Variational extraction of the 5-qubit state of the pentagram.



The probability of the state $|0\rangle^{\otimes 20}$ is maximized.

Kinematics - ongoing and further studies

- Beyond the pentagram - simulations of the Ising spin networks „molecules” with the dozens on nodes (direction of the thermodynamic limit).
- Simulations of dipole spin network in the large j limit (semi-classical limit).
- Understanding the properties of entanglement - measurements of entanglement entropy, mutual informations, Page curves etc.
- Analysis of the fluctuations of the volume.
- Application of the tensor network methods (PEPS).
- Quantum complexity studies (geometric tools).
- Implementations on the new quantum hardware.

Dynamics

Following the Dirac quantization of gravitational system, the physical Hilbertspace $\mathcal{H}_{\mathrm{phys}}$ is constructed by solving the Wheeler-DeWitt (WDW) equation:

$$\hat{C}|\Psi\rangle = 0$$

The WDW is difficult to solve in general case. Solutions are known for certain minisuperspace models.

For $\hat{C}$, being a self-adjoint (but in general non-unitary) operator the physical states correspond to the ground states of the Master Hamiltonian:

$$\hat{H} \propto \hat{C}^2$$

The task is to find the states which minimize

$$\langle\Psi|\hat{C}^2|\Psi\rangle$$

Overview of method of solving the WDW equation for arbitrary #DOF (m) on a quantum computer:

1. Regularize theory to make the Hilbert space finite.

Compact phase space ➔ Finite Hilbert space

Here, the spherical compactification is applied.

2. Apply Variational Quantum Eigensolver (VQE) to find the states minimizing the Master Hamiltonian $\hat{C}^2$:

$$\hat{C}|\psi_0\rangle = 0 \Longleftrightarrow \langle\psi_0|\hat{C}^2|\psi_0\rangle = 0$$

An ansatz on the class of states is needed.

3. Study the large spin limit to recover the affine case.

Loop Quantum Cosmology (LQC)

In the homogeneous and isotropic case, the holonomy-flux algebra reduces to the „cylindrical algebra”:

$$[E, U] = U \qquad [E, U^\dagger] = -U^\dagger \qquad [U, U^\dagger] = 0$$

where $(U, E) \in U(1) \times \mathbb{R}$, which is non-compact.

Employing the fact that $U|\varphi\rangle = e^{i\varphi}|\varphi\rangle$, one can equivalently write:

$$[E, \sin\varphi] = i\cos\varphi \quad [E, \cos\varphi] = -i\sin\varphi \quad [\cos\varphi, \sin\varphi] = 0$$

What we consider in what follows is the SU(2) generalization of the above algebra, so that

$$[-S_z, S_y] = iS_x \qquad [-S_z, S_x] = -iS_y \qquad [S_x, S_y] = iS_z$$

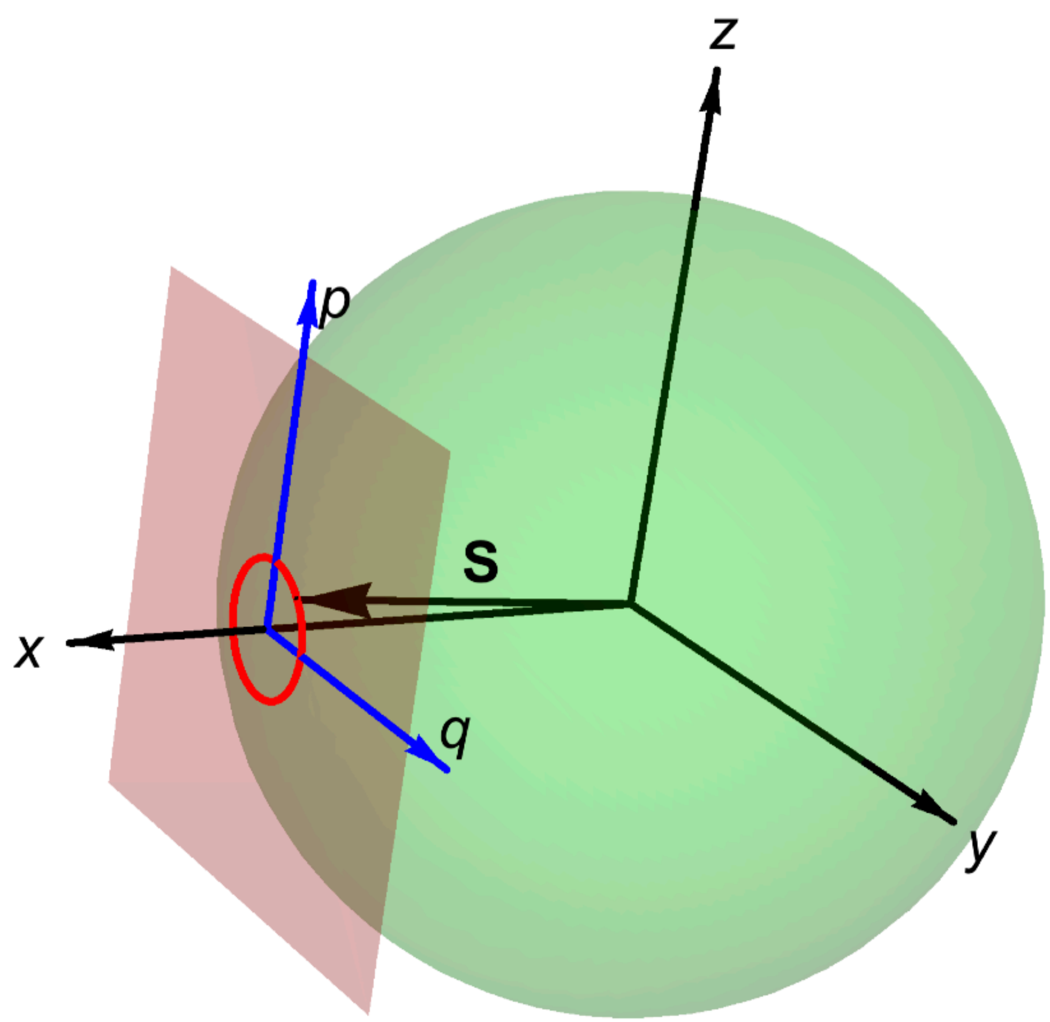
The associated phase space is $\mathbb{S}^2$, so that its volume is finite.

One can introduce the vector

$$\vec{S} = (S_x, S_y, S_z)$$

with the following components:

$$S_x = S\cos\left(\frac{p}{R_1}\right)\cos\left(\frac{q}{R_2}\right),$$
$$S_y = S\sin\left(\frac{p}{R_1}\right)\cos\left(\frac{q}{R_2}\right),$$
$$S_z = -S\sin\left(\frac{q}{R_2}\right).$$



Poisson bracket: $\{f, g\} = \frac{1}{\cos(q/R_2)}\left(\frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial f}{\partial p}\frac{\partial g}{\partial q}\right)$

Employing the above, the su(2) algebra is recovered:

$$\{S_i, S_j\} = \epsilon_{ijk} S_k$$

Compactified flat de Sitter cosmology (Artigas, JM, Rovelli, 2019)

Kinematics (symplectic form)

Affine → 2-sphere

$$\omega = dp \wedge dq \longrightarrow \omega = \cos\left(\frac{q}{R_2}\right) dp \wedge dq,$$

Dynamics (scalar constraint)

$$C = q\left(-\frac{3}{4}\kappa p^2 + \frac{\Lambda}{\kappa}\right) \approx 0 \longrightarrow C = \frac{S_3}{R_1}\left[\frac{3}{4}\kappa\frac{S_2^2}{R_2^2} - \frac{\Lambda}{\kappa}\right] \approx 0$$

$$p \to p_S := \frac{S_y}{R_2} = R_1 \sin\left(\frac{p}{R_1}\right)\cos\left(\frac{q}{R_2}\right),$$

$$q \to q_S := -\frac{S_z}{R_1} = R_2 \sin\left(\frac{q}{R_2}\right),$$

The procedure is ambiguous! We made the simplest choice.

Friedmann equation:

$$H^2 = \frac{\Lambda}{3}\left(\frac{\sin(q/R_2)}{q/R_2}\right)^2\left[\frac{\cos^2(q/R_2) - \delta}{\cos^2(q/R_2)}\right]$$

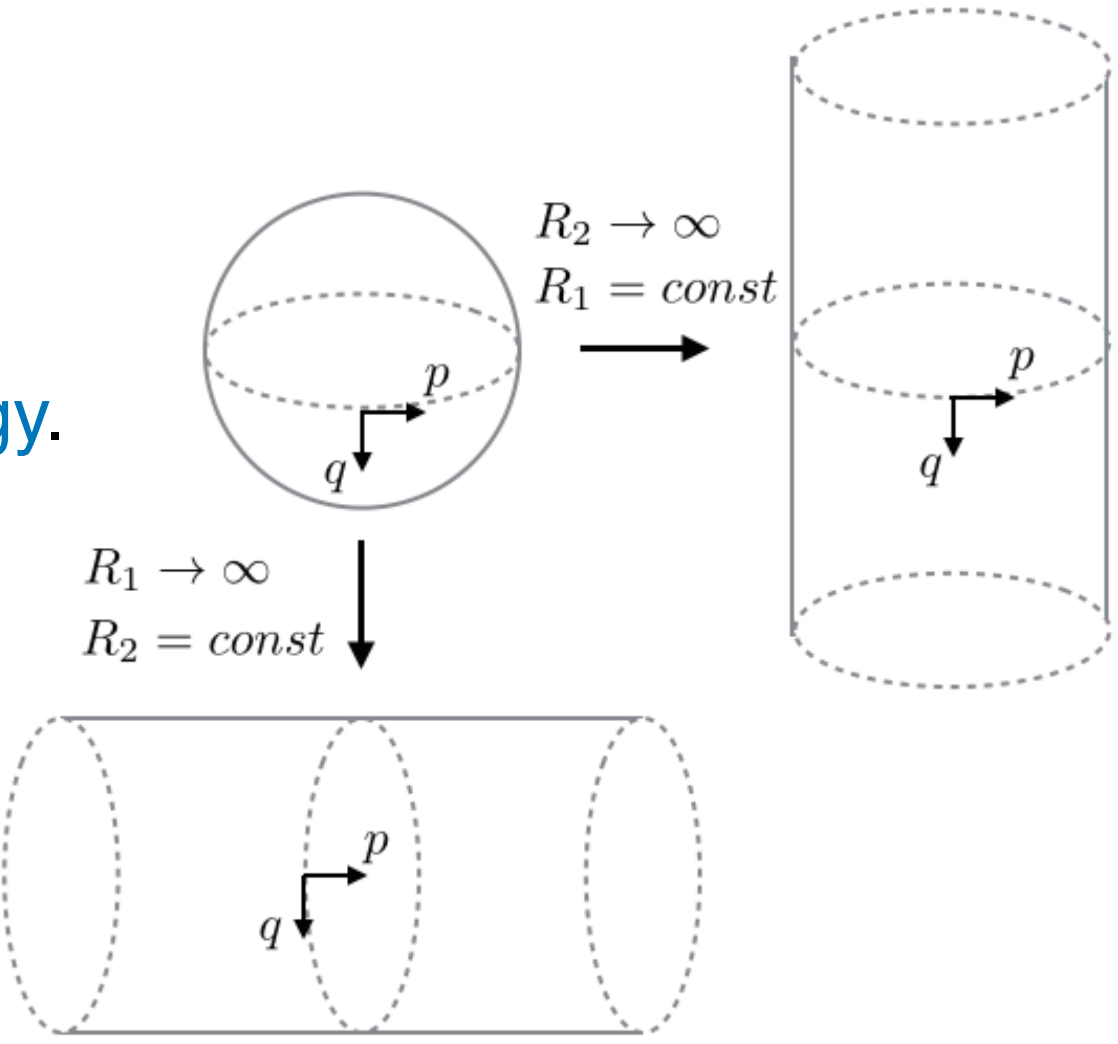
where $\delta := \frac{4}{3}\frac{\Lambda}{R_1^2\kappa^2} \in [0,1]$

The affine case is recovered in the $R_1, R_2 \to \infty$ limit.

Cylindrical limit (LQC)

In the $R_2 \to \infty$ limit the so-called polymerization of momentum is obtained.

This is the case of loop quantum cosmology.



The su(2) algebra reduces to the cylindrical case:

$$\left\{\sin\left(\frac{p}{R_1}\right), \cos\left(\frac{p}{R_1}\right)\right\} = 0,$$
$$\left\{q, R_1 \sin\left(\frac{p}{R_1}\right)\right\} = \cos\left(\frac{p}{R_1}\right),$$
$$\left\{q, \cos\left(\frac{p}{R_1}\right)\right\} = -\frac{1}{R_1}\sin\left(\frac{p}{R_1}\right),$$

The scalar constraint reduces to:

$$C = q\left(-\frac{3\kappa}{4}\frac{\sin^2(\lambda p)}{\lambda^2} + \frac{\Lambda}{\kappa}\right) \approx 0$$

where the polymerization scale is: $\lambda := \frac{1}{R_1}$

The Friedmann equation is:

$$H^2 = \frac{\kappa}{3}\rho_\Lambda\left(1 - \frac{\rho_\Lambda}{\rho_c}\right)$$

where $\rho_c := \frac{3}{4}\frac{\kappa}{\lambda^2}$

Compactified de Sitter in LQC

The symmetrized quantum Hamiltonian constraint:

$$\hat{C} = \frac{1}{3}\left(\hat{S}_z\hat{S}_y\hat{S}_y + \hat{S}_y\hat{S}_z\hat{S}_y + \hat{S}_y\hat{S}_y\hat{S}_z\right) - \delta\hat{S}^2\hat{S}_z \approx 0$$

We have shown that the WdW has always solutions for any δ for the bosonic representations. For the fermionic representations the solutions do not exist, except some particular values of δ.

The simplest non-trivial case is s=1, for which:

$$\hat{C} = 2\left(\frac{1}{6} - \delta\right)\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} = 2\left(\frac{1}{6} - \delta\right)\hat{S}_z$$

The physical state, which satisfy the constraint, is:

$$|\Psi\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = |s = 1, s_z = 0\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

The action of the spin operators on n-qubit (spin-1/2) quantum register is given by:

$$\hat{S}_i = \frac{1}{2}\sum_{j=1}^{n} \mathbb{I}^1 \otimes \ldots \mathbb{I}^{j-1} \otimes \hat{\sigma}_i^j \otimes \mathbb{I}^{j+1} \otimes \ldots \mathbb{I}^n,$$

where n = 2s.

Because $\hat{C}^2$ is **not** a unitary operator, the expectation values cannot be evaluated directly with the use of quantum computing. For this purpose the operator has to be expanded into unitarities (here, the Pauli matrices):

$$\hat{C} = \sum_j c_j \bigotimes_i \hat{\sigma}_{ij}^k$$

so that $\langle \hat{C} \rangle = \sum_j c_j \langle \bigotimes_i \hat{\sigma}_{ij}^k \rangle$

Similarly for $\hat{C}^2$.

Every contributing expectation value has to be evaluated individually. The so-called Hadamard test can be used for this purpose

Variational Quantum Eigensolver

We iteratively search for the minimum of the cost function:

$$c(\alpha) = \frac{a}{\max|\lambda_i|^2}\langle\psi(\alpha)|\hat{C}^\dagger\hat{C}|\psi(\alpha)\rangle + b\left(1 - \frac{\langle\psi(\alpha)|\hat{\vec{S}}^2|\psi(\alpha)\rangle}{s(s+1)}\right)$$

where:

$$a, b \in (0,1)$$

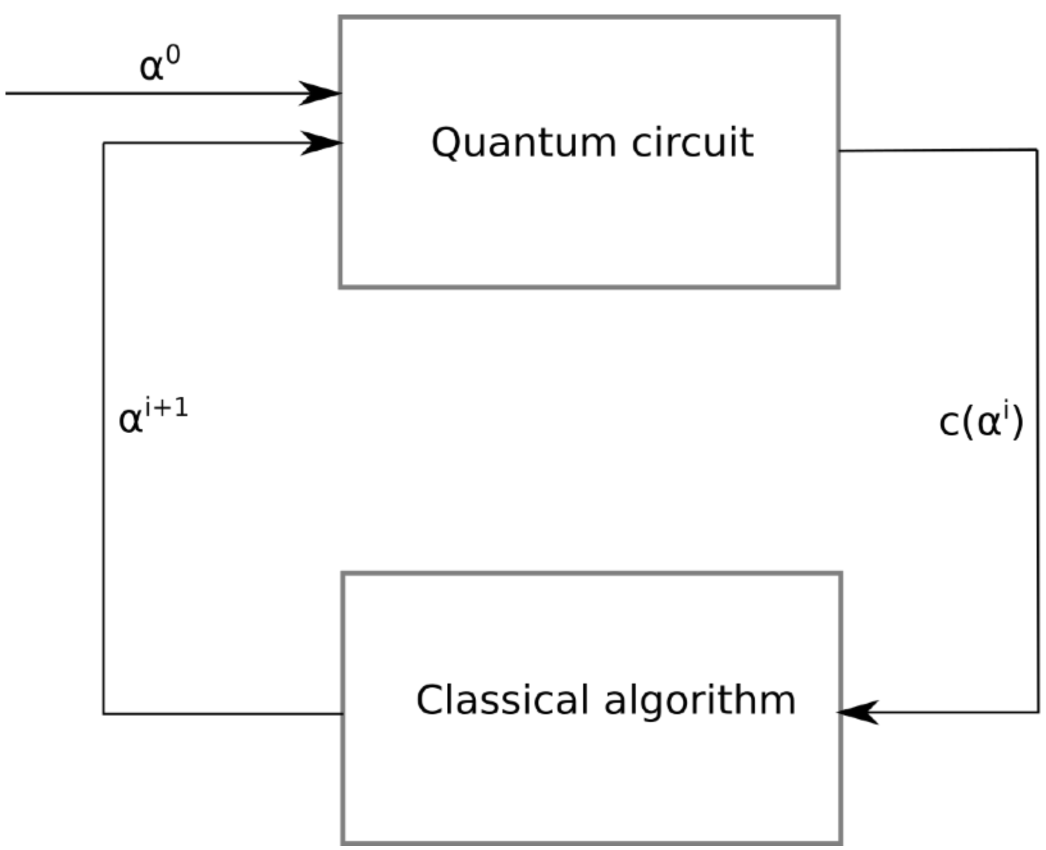
$$a + b = 1$$

The second contribution fixes the spin-s subspace of a quantum register.

Acting iteratively we find:

$$\alpha_{\min} := \mathrm{argmin}_\alpha c(\alpha)$$

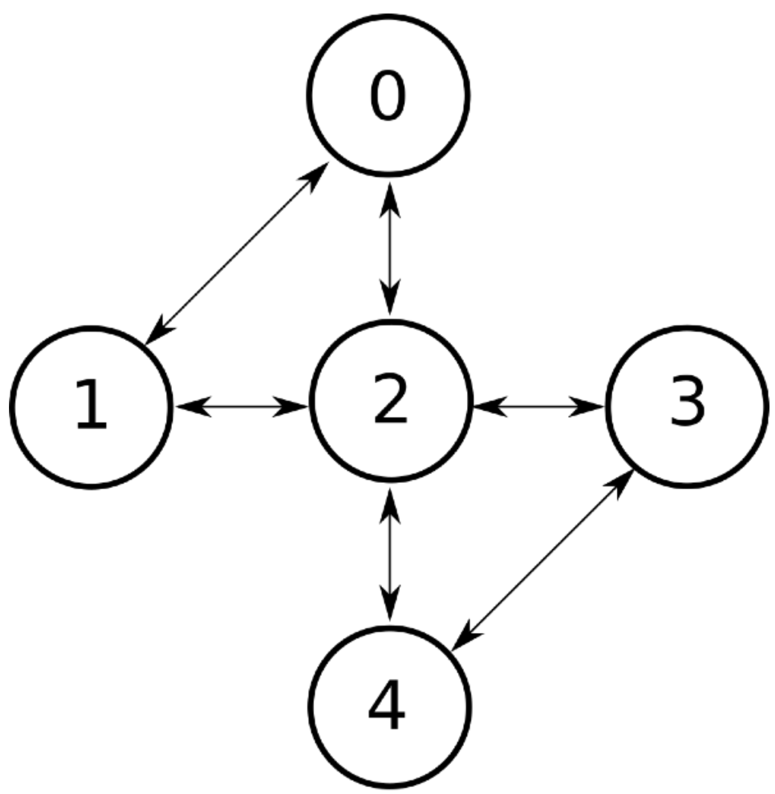
so that $|\psi_0\rangle = |\psi(\alpha_{\min})\rangle$

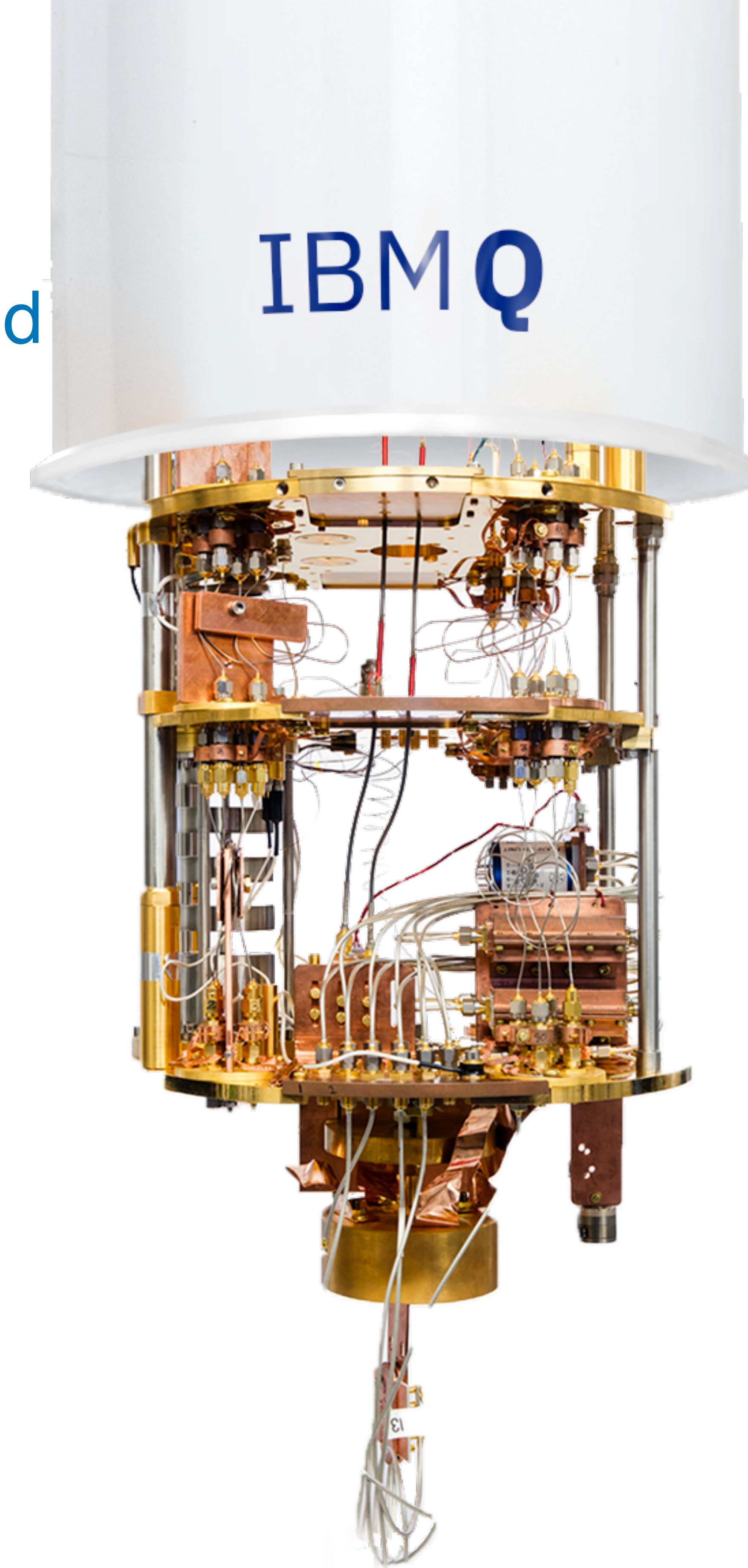


Quantum chip

The expectation values are evaluated on a quantum processor.

Here, the IBM 5-qubit Yorktown superconducting quantum chip has been used. The connectivity of the quantum processor is the following:



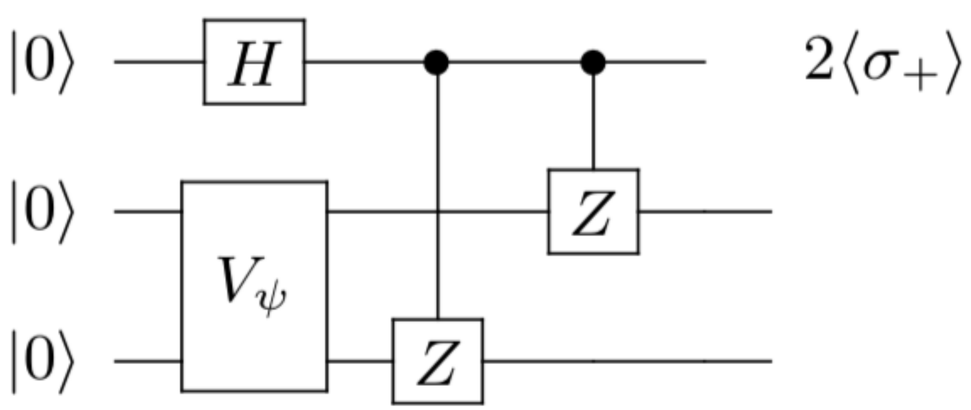


The s=1 case

Here:

$$\langle \hat{C}^2 \rangle = 2\left(\frac{1}{6} - \delta\right)^2 (1 + \langle \sigma_z \otimes \sigma_z \rangle)$$

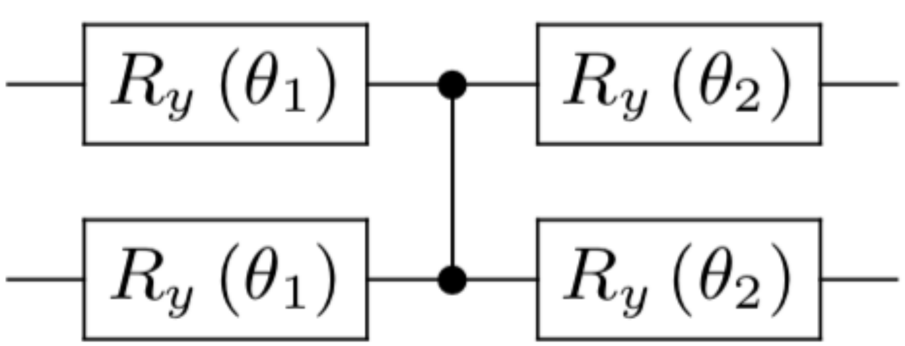
The Hadamard test becomes:



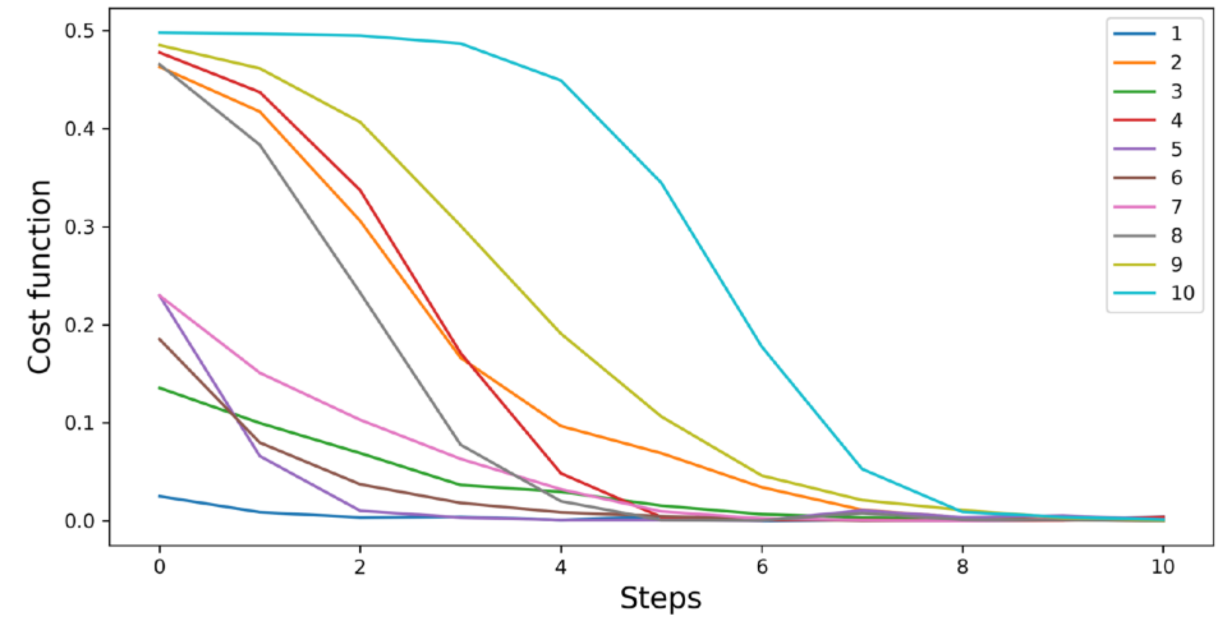
The ansatz for the state

$$|\psi(\alpha)\rangle = |\psi(\theta_1, \theta_2)\rangle = \hat{V}_\psi |00\rangle$$

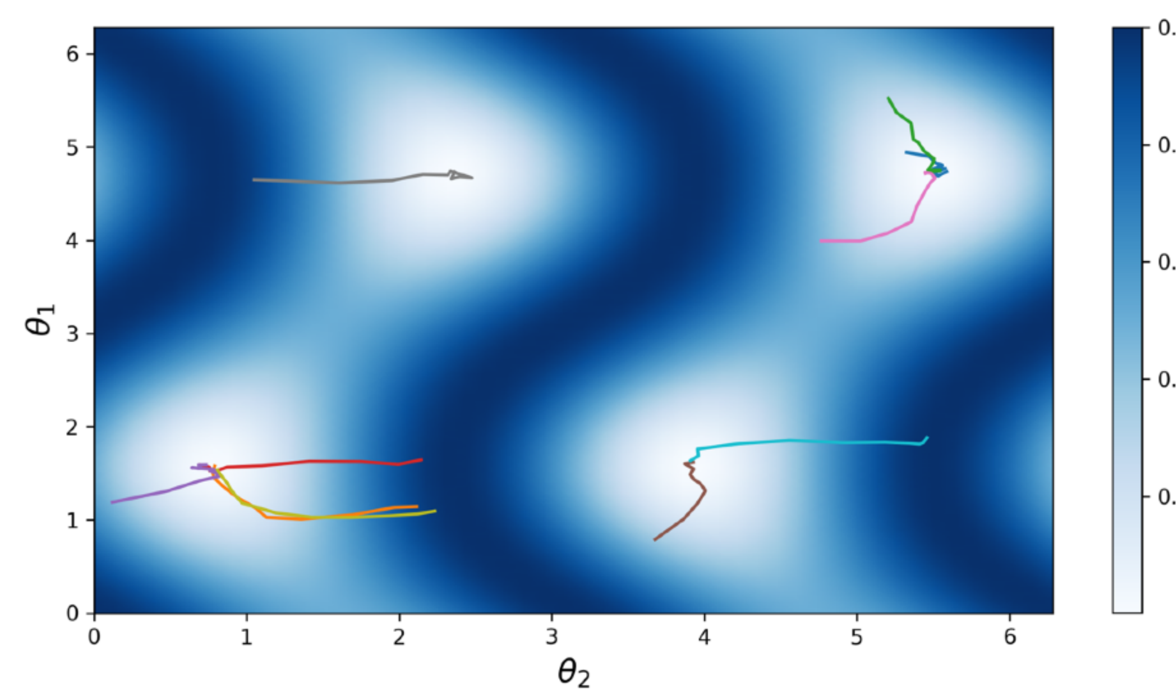
is



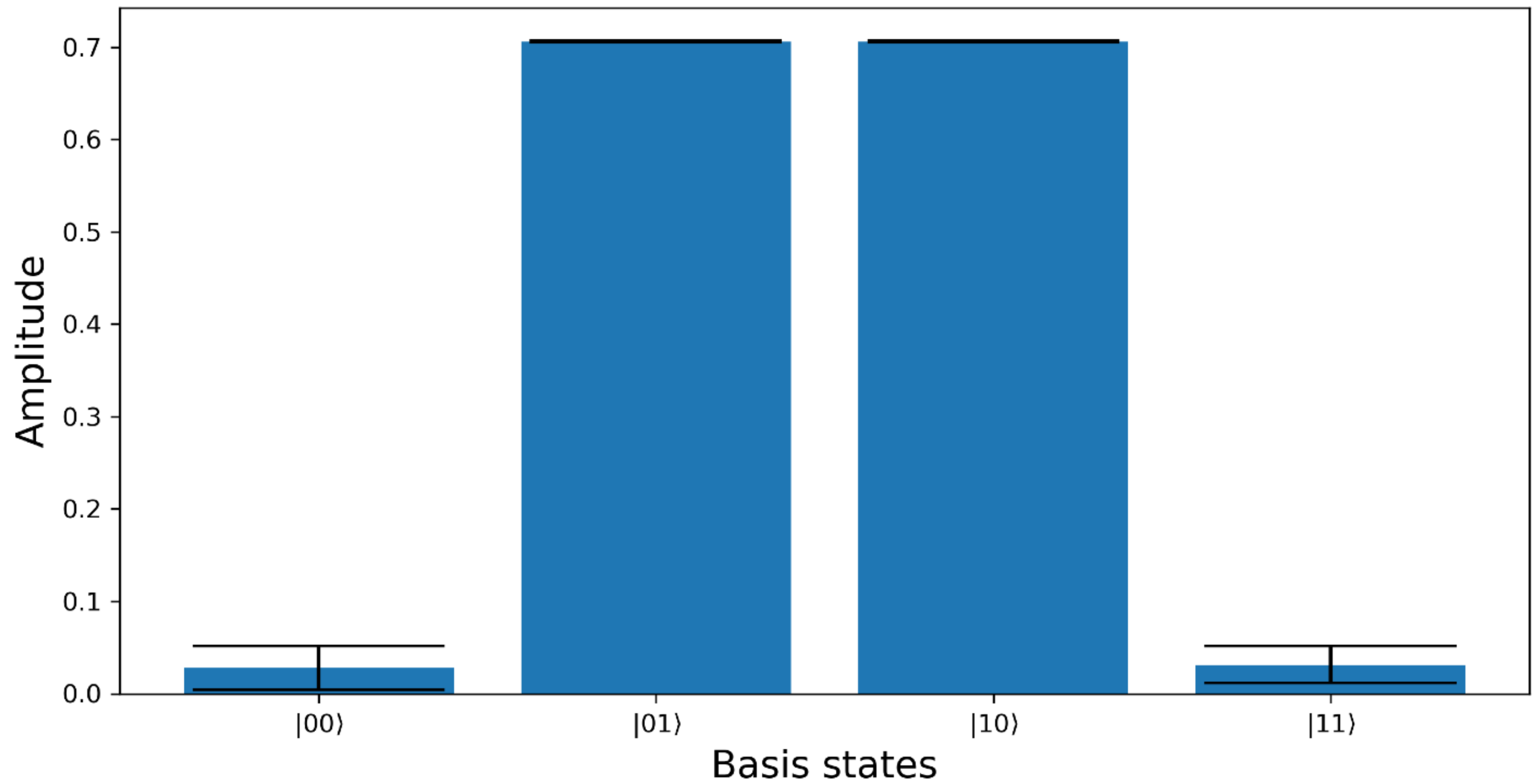
Cost function during minimalization for 10 runs, with randomly initialized parameters:



The cost function landscape:



Averaged (over 10 runs) amplitudes of the final state:



In the simulations, 1024 shots for each circuit have been made.

The theoretically predicted solution to the WDW equation:

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$$

The quantum fidelity of the found state is: 0.997 ± 0.003

Dynamics - ongoing and further studies

- A new representation which utilizes almost all the Hilbert space of the quantum register.
- Increasing the spin s for m=1 (semi-classical limit).
- Inhomogeneous configurations with m>1 interacting copies of the ultralocal patches.
- Implementation of the graph-preserving truncations of the LQG Hamiltonian constraint.
- Computational complexity analysis.
- Implementations on the new quantum hardware.

Thank you!

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