Coarse-graining: Holonomy operators and spin network entanglement

Based on the work 2205.11205

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Motivation

(General) Coarse-graining: a kind of compressing information, to study complicated problem with fewer variables.

Why we study coarse-graining?

- To recover classical limit: How to go from LQG to the classical [Borissova,Dittrich,2022].
- To figure out locality: How to localize a subsystem in the presence of diffeomorphism? If we can't, what about relational perspective [Martin-Dussaud,Rovelli,Zalamea,2018]?
- Toward holography: Bulk information is reflected on boundary. What is the role of boundaries [Dittrich,Goeller,Livine,Riello,2018], [Freidel,Geiller,Pranzetti,2020]? How to count the LQG's d.o.fs [Livine,2017]?

Coarse-graining: to simplify graphs

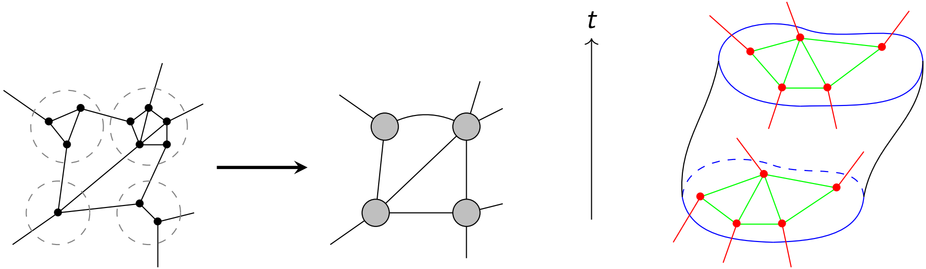
In particular, we are going to coarse-grain spin networks, i.e., to simplify the graphs.

- fined spin networks $\rightarrow$ coarse-grained spin networks.

Objective: In case of absence of locality, we follow the relational perspective and quasi-local boundaries, to investigate

"Geometry emerges from quantum information".

Coarse-graining and open spin networks



(a) An illustration of the partitioning of the graph in order to coarse-grain the spin network state.

(b) Spin sub-network of right-upper Fig.1a evolves along time.

Figure: Coarse-graining emphasizes the relation between coarse-grained vertices instead of fined structure inside sub-graphs.

Bulk-boundary maps

Tensoring boundary representations, notice that

- Coarse-grained vertex carries a non-zero total angular momentum (because gauge symmetry is broken down due to the boundary).
- Spin network is glued from 'open spin networks' with gluers (i) 'bulk holonomies' and (ii) spin-matching, i.e. $|\psi_\Gamma\rangle \neq |\psi_{\Gamma_1}\rangle \otimes |\phi_{\Gamma_2}\rangle$ but

$$|\psi_\Gamma\rangle = \sum \alpha_{12}(g)|\psi_{\Gamma_1}\rangle \otimes |\phi_{\Gamma_2}\rangle$$

We adopt bulk-to-boundary perspective [QC,Livine,Colafranceschi,Chirco,Oriti,2021]

- Spin network: functional from bulk to boundary

$$\psi_\Gamma : \{g_{e\in\Gamma^o}\} \to \mathcal{H}_{\partial\Gamma}\,, \qquad \{g_{e\in\Gamma^o}\} \mapsto |\psi_\Gamma(\{g_{e\in\Gamma^o}\})\rangle\,.$$

Bulk-boundary maps make up a dual Hilbert space $(\mathcal{H}_\partial)^*$, with the scalar product same as spin networks:

$$\langle\phi_{\partial\Gamma}|\psi_{\partial\Gamma}\rangle := \int \prod_{e\in\Gamma^o} \mathrm{d}g_e\,\langle\phi_\Gamma(\{g_{e\in\Gamma^o}\})|\psi_\Gamma(\{g_{e\in\Gamma^o}\})\rangle\,.$$

Let us coarse-grain

There are two manners to do coarse-graining:

- Trace over bulk holonomies (G-twirling operation [Bartlett,Rudolph,Spekkens,2007], Höhn's talk), then obtain a SU(2)-invariant state [QC,Livine,2021],

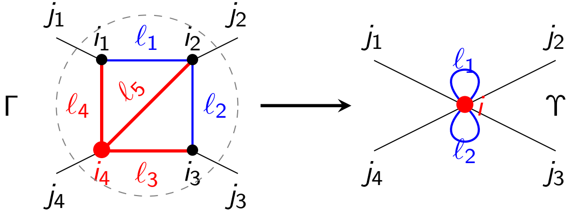
$$\rho_\partial = \int \prod_{e\in\Gamma^\circ} \mathrm{d}g_e \, |\psi_\Gamma(\{g_e\}_{e\in\Gamma^\circ})\rangle\langle\psi_\Gamma(\{g_e\}_{e\in\Gamma^\circ})| \in \mathrm{End}(\mathcal{H}_{\partial\Gamma}) \,, \qquad \rho_\partial = g\,\rho_\partial g^{-1} \,.$$

- Gauge-fix bulk holonomies, then obtain a loopy spin network [Freidel,Livine,2002].

Coarse-graining via gauge-fixing

In this talk, we focus on the 'coarse-graining via gauge-fixing'.
Procedure $\Gamma \to \Upsilon$:

- Choose a 'root vertex' (in red) and a 'maximal tree' in bulk (in red): connect all vertices by a path that doesn't form a loop.
- Implement gauge transformation $g_e \to h_t g_e h_s^{-1}$ along the maximal tree to set these holonomies $g_e \mapsto 1$.
- Contract the maximal tree to the root vertex. Edges out of the maximal tree become self-loops (in blue).
- Studying spin sub-networks via loopy spin networks [Charles,Livine,2016].



Coarse-graining via gauge-fixing

Remarks:

- Gauge-fixing actually defines a unitary for dual boundary Hilbert space $(\mathcal{H}_{\partial\Gamma})^*$.

$$\langle \phi_{\partial\Gamma} | \psi_{\partial\Gamma} \rangle = \langle \phi_{\partial\Upsilon} | \psi_{\partial\Upsilon} \rangle \ .$$

- Density matrix: $\rho_\Gamma = |\psi_\Gamma\rangle\langle\psi_\Gamma| \in \mathrm{End}(\mathcal{H}_\Gamma)$. Reduced density matrix: $\rho_{\Gamma_1} = \mathrm{Tr}_{\mathcal{H}_{\Gamma_2}} \rho_\Gamma = \sum_{\{\phi'\}} \langle \phi'_{\Gamma_2} | \rho_\Gamma | \phi'_{\Gamma_2} \rangle$. It turns out that ρ_{Γ_1} and ρ_{Υ_1} are equivalent up to boundary holonomies — 'local unitary' w.r.t $(\mathcal{H}_{\partial\Gamma_1})^*$.
- Spin network entanglement is preserved under 'coarse-graining via gauge-fixing'.

Entanglement coarse-graining: kinematical level

The spin network entanglement has well coarse-graining behavior.

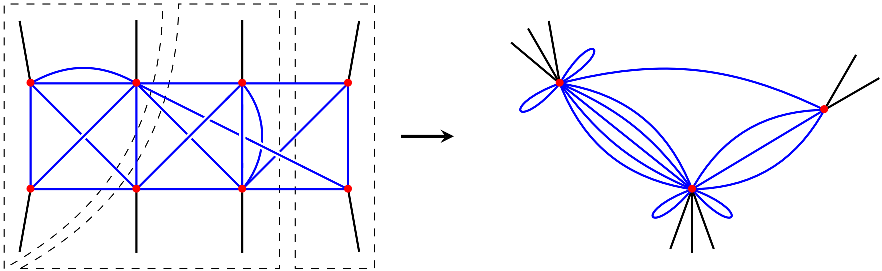


Figure: The illustration for coarse-graining.

What happen if some dynamics come?

Loop holonomy operator and its coarse-graining

We consider loop holonomy operator, because

- It is a non-local operator that can create entanglement [QC,Livine,2022] and corresponds to Ashtekar-Barbero curvature.
- It is a basic block for Euclidean Hamiltonian in the framework of AQG [Giesel,Thiemann,2006], changing the spins associated to intertwiners.

We can derive transformation rules between loop holonomy on fined and coarse-grained graphs which can be proven by Biedenharn-Elliott identity.

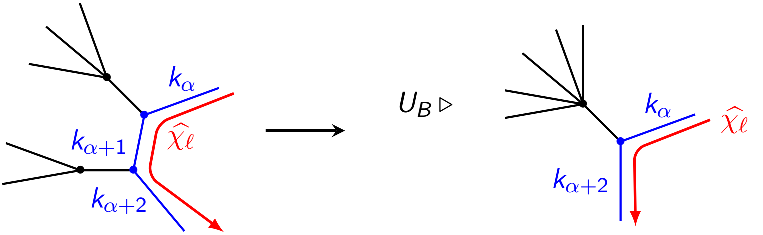


Figure: Gauge-fix the holonomy associated with spin $k_{\alpha+1}$ into $\mathbb{I}$ in the left side, then coarse-grain $k_{\alpha+1}$. What matter to the dynamics are spins $k_{\alpha+2}, k_\alpha$ and recoupled spin J.

Entanglement coarse-graining: dynamical level

The spin network entanglement is still preserved under coarse-graining.

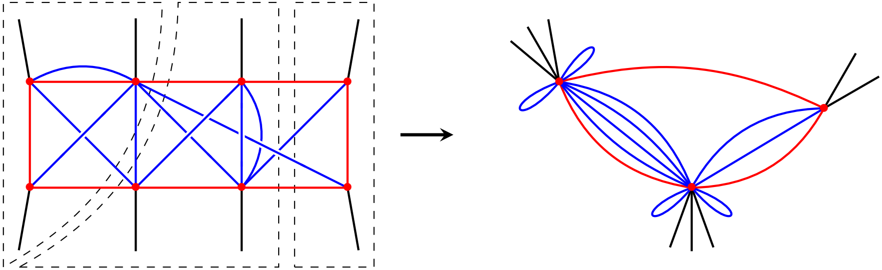


Figure: The illustration for coarse-graining.

Outlook & Perspective

Summary:

- The entanglement between spin sub-networks is preserved under coarse-graining.
- The transformation rules between loop holonomy operators on fined and coarse-grained graph are produced.
- The loop holonomy dynamics of entanglement between spin sub-networks is preserved under coarse-graining.

What are the next?

- What about the full dynamics of LQG? $\rightarrow$ Consider a true LQG Hamiltonian.
- Can we recover entanglement entropic area law? $\rightarrow$ Study 2-vertex model [Borja,Freidel,Diaz-Polo,Garay,Livine,2010].

Thank you for your attention!