Isometric boundary-boundary maps in spin networks

based on ArXiv:2207.07625

Simon Langenscheidt, with Eugenia Colafranceschi and Daniele Oriti

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Munich Center for Quantum Science and Technology

Motivation

From AdS/CFT, celestial holography, 3D gravity, corner symmetries $\Longrightarrow$
Holographic correspondences encode parts of gravity

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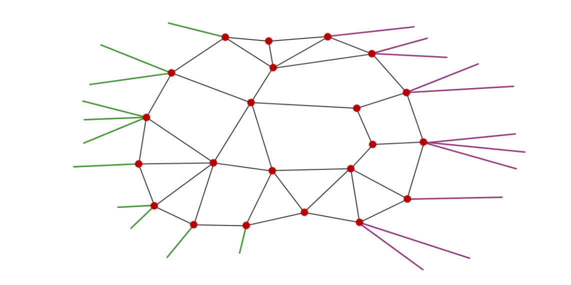

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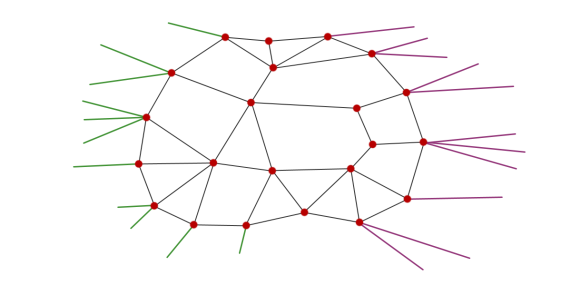
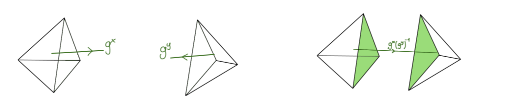


Study restricted class of spin network states using random tensor network techniques:
Fix graph structure and intertwiner data, randomise the rest.

Spin networks as tensor networks

Key strategy: Apply tensor network methods for a typicity statement.



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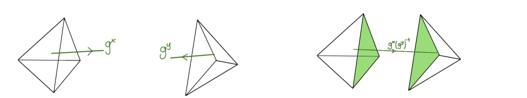

Individual vertex states, apply projection onto maximally entangled link state
$\implies$ Dependence on single group element on glued edge.

$$\Psi(g_1, g_2) = \int dh\, \psi_1(g_1 h)\psi_2(g_2 h) = \Psi(g_1(g_2)^{-1}, e) \tag{1}$$

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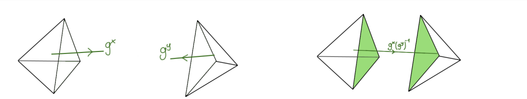

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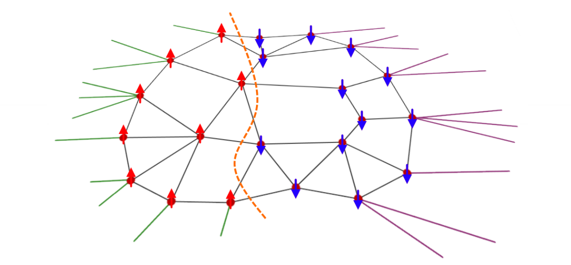
Spin network basis states are naturally tensor network states, with individual tensors: $\Psi \in L^2(G^v/G_{Diag})$

Context and previous work

(Han, Hung '17): Similar method, with focus on boundary entropy.

(Colafranceschi, Oriti, Chirco '21,'22): Isometry degree of maps from intertwiners to boundary edges, with *fixed edge spins*.



Colafranceschi, Oriti, Chirco - 2105.06454 , 2012.12622 , 2110.15166, Han - 1610.02134

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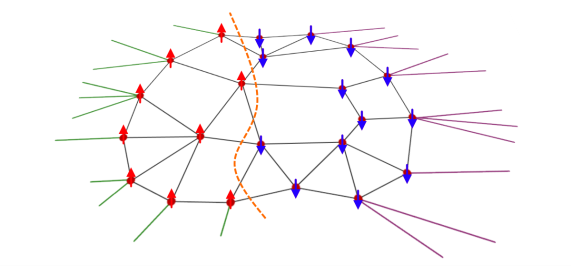

Study entropy and isometry questions through Ising model on graph.

$\implies$ Holographic maps possible only with inhomogeneities in the spins.

$\implies$ Ryu-Takayanagi formula for bulk entropy.

Colafranceschi, Oriti, Chirco - 2105.06454 , 2012.12622 , 2110.15166, Han - 1610.02134

Methods

Extend this to superpositions of different spins. Same method - work in tensor network (PEPS) class of spin network superpositions and randomise over vertex wavefunctions. (Harlow '15, Yang '16, Hayden '16, Qi '17, ...)

Hayden, Harlow, Yang et al - 1601.01694 , 1503.06237 , 1510.03784, JHEP08(2017)060

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Check induced boundary (A)-to-boundary (B) map $\langle\theta_B|\,\Phi\,|\theta_A\rangle = \langle\phi|\,\theta_A, \theta_B\rangle$.

$$|\phi\rangle = \langle\zeta|_{\mathcal{I}} \bigotimes_{e\in E} \langle e| \bigotimes_{x\in V} |\Psi_x\rangle\,, \text{ where } |\Psi_x\rangle = U_x\,|\Psi_0\rangle \text{ random} \tag{2}$$

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- Seperate calculation into sets of spin values $\{j_e : e \in E_\gamma\}$
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$\Longrightarrow$ Equal to partition sum(s) of random Ising model.

$$\langle e^{-S_2}\rangle \hat{=} Z = \sum_{J,K} P(J,K) Z^{J,K} \tag{3}$$

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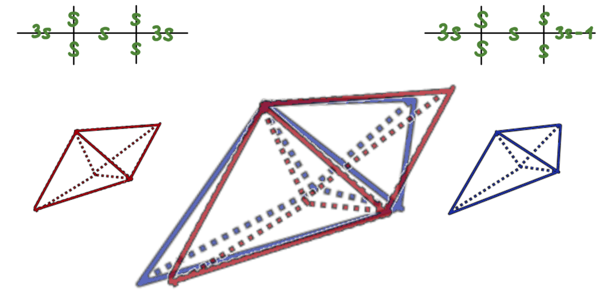
$$\frac{(\Delta A_C)^2}{(A_C)^2} \approx \frac{1}{6} \implies \text{nonvanishing relative uncertainty} \tag{6}$$



Summary

Typical large-spin superpositions of spin networks feature boundary/boundary isometry under simple conditions.
If so, their geometric properties are easily related to the constituents.

Outlook

Possible applications

- Radially symmetric geometries (compare to Yi Li Wang's talk of yesterday)
- Spin network states as channels between boundary models of punctures
- With Dynamics: Causal characterisation of entanglement shadow
- Semiclassical superpositions?

Thank you for your attention!