Modified black hole spacetime & upcoming quasinormal mode predictions

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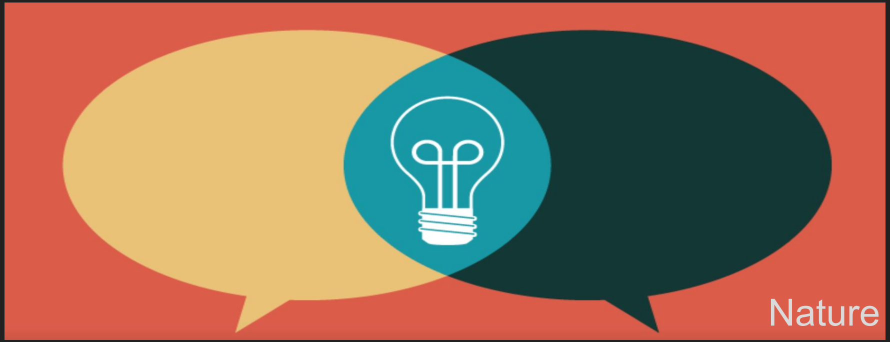
LOOPS 22

Big questions



How can we reconcile general relativity with quantum mechanics? What are the necessary attributes of a superseding theory of Quantum Gravity?

What is the nature of spacetime and black holes?

How can we use modified black hole spacetimes to probe these questions?

Canonical Quantization

What observable consequences would distinguish these possibilities? QNM's

Quasiclassical solutions for static quantum black holes

Coauthors: Gianni Sims, Manuel Díaz, and Martin Bojowald

https://arxiv.org/abs/2012.07649

1. Start with a spherically symmetric black hole
2. Add a quantum correction in the Hamiltonian constraint
3. Preserve covariance
4. Calculate the consequences in the static case and beyond
5. Next: predict gravitational wave signals

Goal: calculate quasiclassical space-time dynamics with non-local quantum corrections, using canonical methods of non-adiabatic quantum dynamics, and extend this to an effective quantum field theory

Modification

Most non-local effects in black hole models assume a specific non-local action. We used a new systematic quasiclassical formulation with non-local corrections derived in a canonical quantization.

We introduce an additional field in the Hamiltonian constraint (phi3) and treat it as a quantum correction on the existing metric field (phi2)

Static gauge choice for the other metric field (phi1) equal to x^2

with the classical $H[N]$ and

$$H[N] = -\int \mathrm{d}x N(x) \left(\frac{\phi_2 p_2^2}{2\sqrt{\phi_1}} + 2\sqrt{\phi_1} p_1 p_2 + \left(1 - \left(\frac{\phi_1'}{\phi_2} \right)^2 \right) \frac{\phi_2}{2\sqrt{\phi_1}} - 2 \left(\frac{\phi_1'}{\phi_2} \right)' \sqrt{\phi_1} \right) \tag{27}$$

as before, and a correction

$$\begin{aligned} H_2[N] &= \int \mathrm{d}x N(x) \left(\frac{1}{2} \frac{\partial^2 H}{\partial p_2^2} \left(p_3^2 + \frac{U}{\phi_3^2} \right) + \frac{\partial^2 H}{\partial \phi_2 \partial p_2} \phi_3 p_3 + \frac{1}{2} \frac{\partial^2 H}{\partial \phi_2^2} \phi_3^2 + \frac{\partial^2 H}{\partial \phi_2 \partial \phi_2'} \phi_3 \phi_3' \right) \\ &= -\int \mathrm{d}x N(x) \left(\frac{\phi_2 p_3^2}{2\sqrt{\phi_1}} + \frac{\phi_3 p_2 p_3}{\sqrt{\phi_1}} + \left(6 \frac{\sqrt{\phi_1} \phi_1' \phi_2'}{\phi_2^4} - \frac{1}{2} \frac{(\phi_1')^2}{\sqrt{\phi_1} \phi_2^3} - 2 \frac{\phi_1'' \sqrt{\phi_1}}{\phi_2^3} \right) \phi_3^2 \right. \\ &\quad \left. -4 \frac{\sqrt{\phi_1} \phi_1' \phi_3 \phi_3'}{\phi_2^3} + \frac{U \phi_2}{2\sqrt{\phi_1} \phi_3^2} \right) . \end{aligned} \tag{28}$$

Note: x is the spatial, radial coordinate. The phi’s and p’s are degrees of freedom in the metric. They are functions of the tetrad variables relating to extrinsic curvature, etc.

Preserving covariance

Requiring the quantum-corrected constraints to preserve Poisson brackets, determined from hypersurface deformations, enforces covariance.

Note: (21) modified with Ex/Ephi^2. Diffeomorphism Constraint modification simplifies in static case.

$$\{D[M_1], D[M_2]\} = D[M_1 M_2' - M_2 M_1'] \tag{19}$$

$$\{H[N], D[M]\} = -H[M N'] \tag{20}$$

$$\{H[N_1], H[N_2]\} = -D[E^x (E^\varphi)^{-2} (N_1 N_2' - N_2 N_1')] \,. \tag{21}$$

Higher order constraints

Preserving covariance required the introduction of a higher order correction to the Hamiltonian constraint:

$$
\begin{aligned}
H_{\phi_2}[L] &= \int \mathrm{d}x L(x)\left(\frac{\partial H}{\partial \phi_2}\phi_3^2 + \frac{\partial H}{\partial \phi_2'}\phi_3\phi_3' + \frac{\partial H}{\partial p_2}\phi_3 p_3\right) \\
&= -\int \mathrm{d}x L(x)\Bigg(\left(\frac{p_2^2}{2\sqrt{\phi_1}} + \frac{1}{2\sqrt{\phi_1}} + \frac{(\phi_1')^2 + 4\phi_1\phi_1''}{2\sqrt{\phi_1}\phi_2^2} - 4\frac{\sqrt{\phi_1}\phi_1'\phi_2'}{\phi_2^3}\right)\phi_3^2 \\
&\quad + \frac{2\sqrt{\phi_1}\phi_1'}{\phi_2^2}\phi_3\phi_3' + \left(\frac{\phi_2 p_2}{\sqrt{\phi_1}} + 2\sqrt{\phi_1}p_1\right)\phi_3 p_3\Bigg) . \qquad (37)
\end{aligned}
$$

Calculation Outline

Calculated equations of motion: poisson bracket of the unfixed fields and their momenta with the sum of modified Hamiltonian constraints.

Proceed Perturbatively: Perturb around the classical solutions for N (lapse function) and phi2, with small corrections.

Simplify: Impose statistic conditions

Solve the equations of motion and constraints for these corrections, introducing an integration constant for each. The new degrees of freedom imply that quantum extended theories like this are more complex than classical ones.

Analyze asymptotic behavior around the horizon and infinity

Static analysis

We are identifying quantum effects from canonical quantization of this spherically symmetric constrained system

Solutions in almost-closed form

The Planck-scale corrections have more effect than you might expect! Potential new effects near the horizon and asymptotically

Our model is sensitive to new, possibly non-local corrections while maintaining general covariance

Conclusions

Obtained a theory for two independent fields, representing a single classical metric component (phi2) and its quantum fluctuation (phi3).

Quasiclassical approximation breaks down before horizon, meaning that non-local effects may be crucial for horizon dynamics of quantum black holes (need higher-order quasiclassical approximations to confirm)

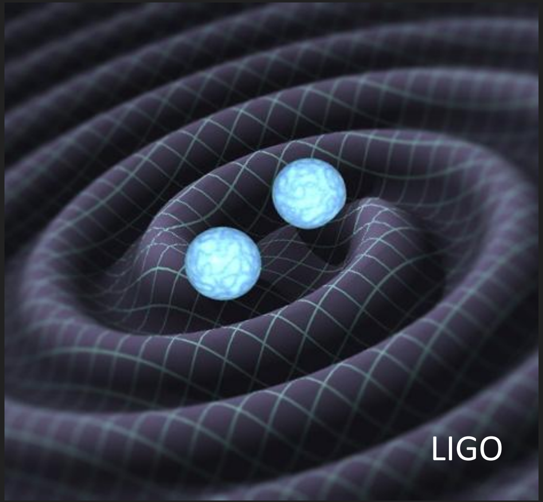
Quasiclassical methods are promising for inhomogeneous models of QG, allowing explicit derivations of quantum corrections with only canonical quantization requirements.

Current Work: Quasinormal Mode Collaboration

Collaborating with Karim Noui at Université Paris-Saclay

Applying his novel derivation method for quasinormal mode equations to this modified black hole model

In the process, generalizing his method to apply to other modified spacetimes

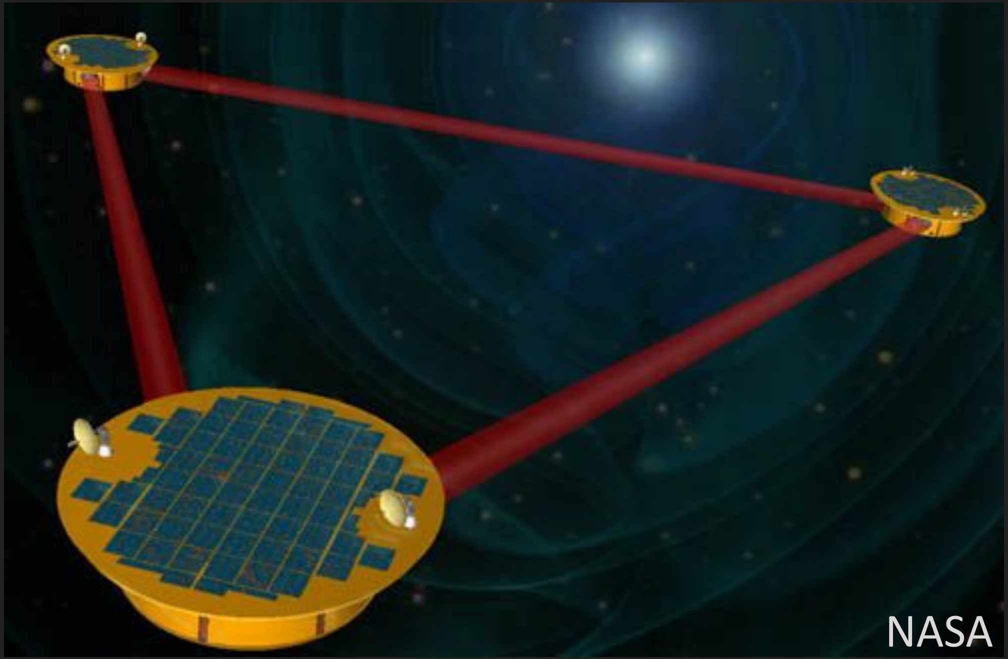

LIGO

Goal:

Calculate gravitational wave quasinormal mode signals detectable by LISA.

These measurements could either rule out or support certain modifications.

These novel results would limit the scope of possible theories of quantum gravity.



Email for follow-up discussions

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Equations of motion

Calculated by taking the poisson bracket of the unfixed fields and their momenta with the sum of modified Hamiltonian constraints:

$$\begin{aligned}\dot{\phi}_2 &= \{\phi_2, \bar{H}[N] + H_{\phi_2}[L]\} = \frac{\delta \bar{H}[N]}{\delta p_2} + \frac{\delta H_{\phi_2}[L]}{\delta p_2} \\ &= \left(2xp_1 + \frac{\phi_2 p_2}{x} + \frac{\phi_3 p_3}{x}\right) + \frac{\phi_3}{x}\left(\phi_3 p_2 + \phi_2 p_3\right) L\end{aligned}$$

$$\begin{aligned}\dot{\phi}_3 &= \{\phi_3, \bar{H}[N] + H_{\phi_2}[L]\} = \frac{\delta \bar{H}[N]}{\delta p_3} + \frac{\delta H_{\phi_2}[L]}{\delta p_3} \\ &= \frac{1}{x}(\phi_2 p_3 + \phi_3 p_2)N + \left(\frac{\phi_2 p_2}{\sqrt{\phi_1}} + 2\sqrt{\phi_1} p_1\right)\phi_3 L\,,\end{aligned}$$

$$\begin{aligned}0 &= \dot{p}_2 = \{p_2, \bar{H}[N] + H_{\phi_2}[L]\} = -\frac{\delta \bar{H}[N]}{\delta \phi_2} - \frac{\delta H_{\phi_2}[L]}{\delta \phi_2} \\ 0 &= \dot{p}_3 = \{p_3, \bar{H}[N] + H_{\phi_2}[L]\} = -\frac{\delta \bar{H}[N]}{\delta \phi_3} - \frac{\delta H_{\phi_2}[L]}{\delta \phi_3}\end{aligned}$$

Simplify constraints with static condition

With these conditions, we obtain the Hamiltonian constraint

$$\bar{H}[N] = -\int \mathrm{d}x N(x) \left(\frac{\phi_2}{2x} - \frac{2x}{\phi_2} - 4x \left(\frac{x}{\phi_2} \right)' + \left(12 \frac{x^2 \phi_2'}{\phi_2^4} - \frac{6x}{\phi_2^3} \right) \phi_3^2 \right. \tag{45}$$
$$\left. - 8 \frac{x^2 \phi_3 \phi_3'}{\phi_2^3} + \frac{U \phi_2}{2x\phi_3^2} \right) ,$$

the higher-order constraint

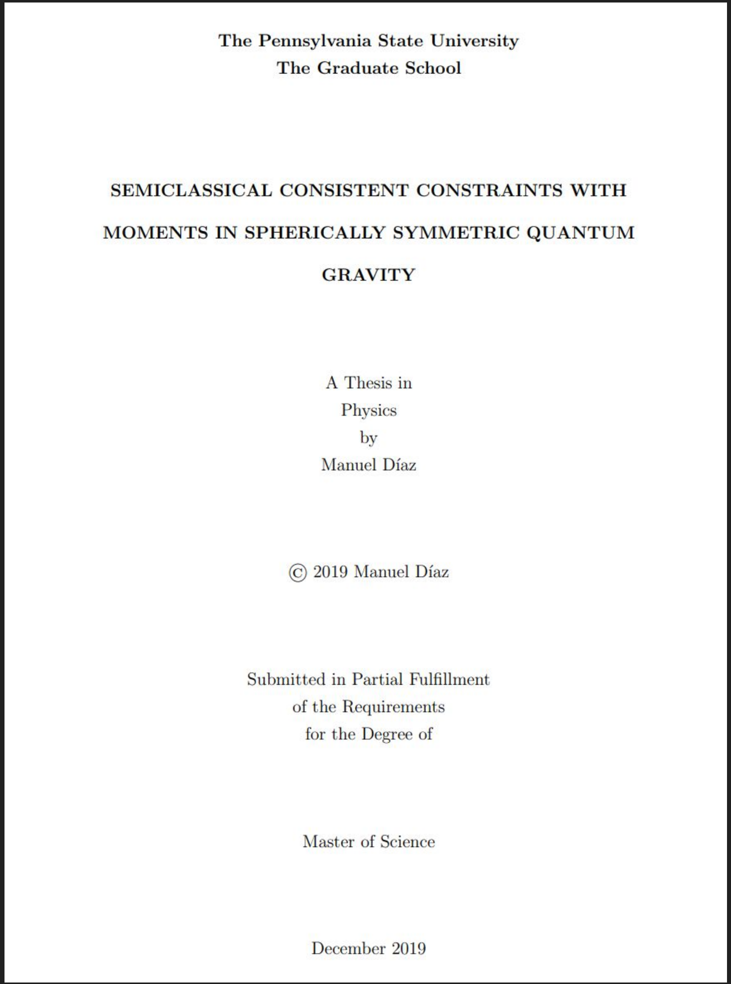
$$H_{\phi_2}[L] = -\int \mathrm{d}x L(x) \left(\left(\frac{1}{2x} + \frac{6x}{\phi_2^2} - 8 \frac{x^2 \phi_2'}{\phi_2^3} \right) \phi_3^2 + \frac{4x^2}{\phi_2^2} \phi_3 \phi_3' \right) \tag{46}$$

Demonstrated extendability

2019 Masters Thesis by Manuel Díaz:

Semiclassical consistent constraints with moments in spherically symmetric quantum gravity

Proof of concept that this modified black hole model can be extended beyond the static case, with additional corrections (adding another field)



The Pennsylvania State University
The Graduate School

SEMICLASSICAL CONSISTENT CONSTRAINTS WITH MOMENTS IN SPHERICALLY SYMMETRIC QUANTUM GRAVITY

A Thesis in
Physics
by
Manuel Díaz

Submitted in Partial Fulfillment
of the Requirements
for the Degree of

Master of Science

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