

# Cosmological inhomogeneities and relational perturbations of QG condensates

(based on 2112.12677, in collaboration with D. Oriti)

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**Luca Marchetti**

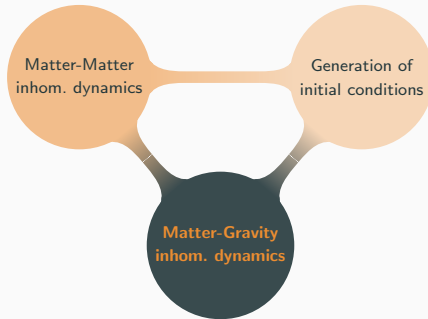
LOOPS '22, 19 July 2022

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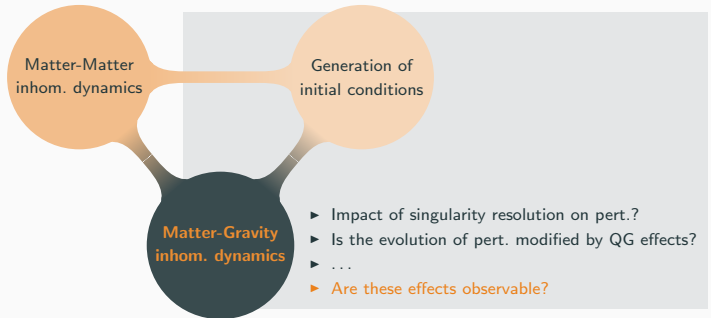


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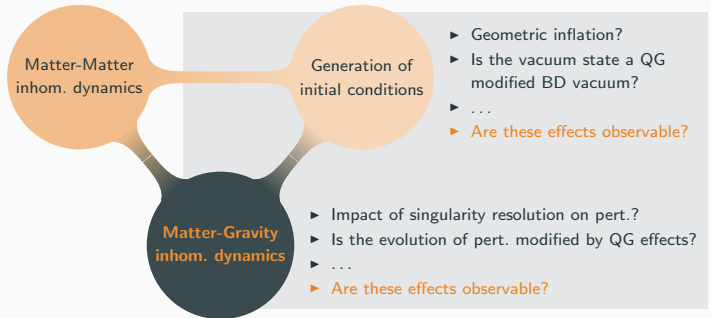
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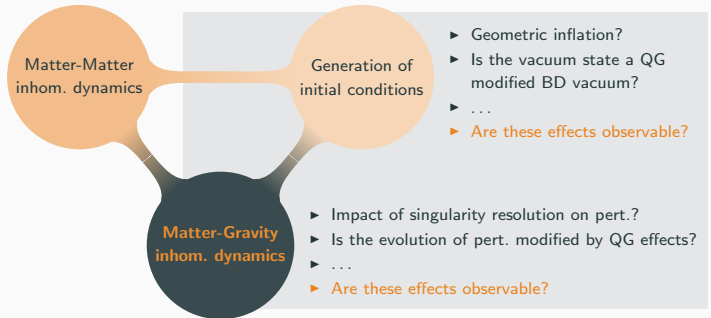
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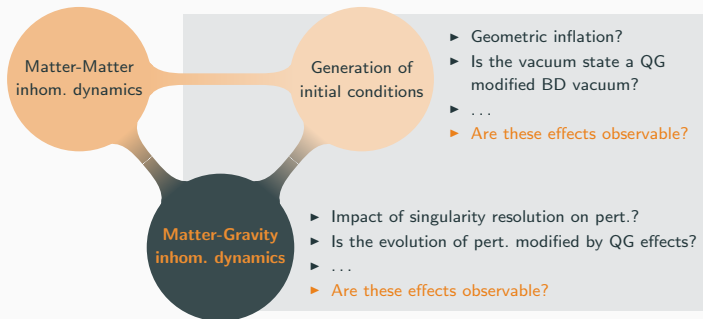
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## Challenges from the QG perspective:

- ▶ How to define inhomogeneities?
- ▶ How to extract macroscopic dynamics?
- ▶ How to construct cosmological geometries?
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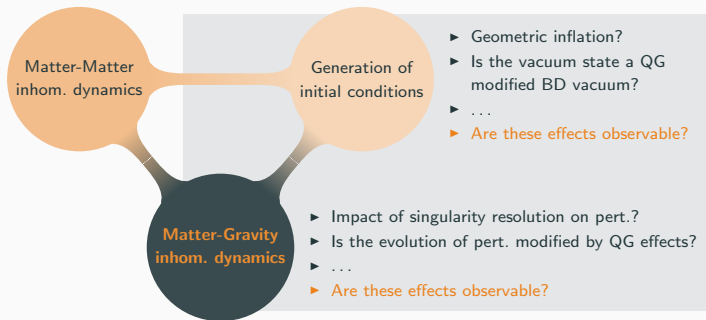


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Relational description

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Relational description

Coarse-graining/  
collective behavior

# (T)GFT condensate cosmology

## (T)GFT condensates

Simplest collective behavior: macroscopic  $\sigma$  dynamics well described in the mean-field approx.

$$|\sigma\rangle = \mathcal{N}_\sigma \exp \left[ \int d^d \chi \int dg_I \sigma(g_I, \chi^\mu) \hat{\varphi}^\dagger(g_I, \chi^\mu) \right] |0\rangle$$



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- ▶ If  $\chi^\mu$  constitute a matter ref. frame:

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Relationality

## Condensate Peaked States

- ▶ If  $\sigma$  is peaked on  $\chi^\mu \simeq x^\mu$ ,  $|\sigma\rangle_x$  encodes relational information about the spatial geometry at  $x^\mu$ .  
 $\sigma = (\text{fixed peaking function } \eta) \times (\text{dynamically determined reduced wavefunction } \tilde{\sigma})$
- ▶  $\langle \hat{\chi}^\mu \rangle_{\sigma_x} \simeq x^\mu$ ,  $\langle \hat{O} \rangle_{\sigma_x} \simeq O[\tilde{\sigma}](x)$ : evolution with respect to  $x^\mu$  is **effectively** relational.

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Eff. dynamics

## Mean-field approximation

- ▶ Mesoscopic regime: large  $N$  (emergence) but negligible interactions.
- ▶ Hydrodynamic approx. of kinetic kernel due to peaking properties.
- ▶ E.o.m. for reduced wavefunction  $\longrightarrow$  e.o.m. for operator averages.

$$\left\langle \frac{\delta S[\hat{\varphi}, \hat{\varphi}^\dagger]}{\delta \hat{\varphi}(g_l, x^\mu, \cdot)} \right\rangle_{\sigma_x} = 0.$$

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Simplest (slightly) relationally inhomogeneous system

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## Classical

- ▶ 4 MCM **reference** fields  $(\chi^0, \chi^i)$ , with Lorentz/Euclidean invariant  $S_\chi$  in field space.
- ▶ 1 MCM **matter** field  $\phi$  dominating the e.m. budget and **relationally inhomog.** wrt.  $\chi^i$ .

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- ▶ (T)GFT field:  $\varphi(g_I, \chi^\mu, \phi)$ , depends on 5 discretized scalar variables.
- ▶ EPRL-like or extended BC model with  $S_{\text{GFT}}$  respecting the classical matter symmetries.

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## Observables

notation:  $(\cdot, \cdot) = \int d^4\chi d\phi d g_I$

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Only isotropic info:  $\hat{V} = (\hat{\varphi}^\dagger, V[\hat{\varphi}])$

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- ▶ CPSs around  $\chi^\mu = x^\mu$ , with
  - $\eta$ : **Isotropic** peaking on rods;
  - $\tilde{\sigma}$ : **Isotropic** distribution of geometric data.
- ▶ Small relational  $\tilde{\sigma}$ -inhomogeneities ( $\tilde{\sigma} = \rho e^{i\theta}$ ):  
 $\rho = \bar{\rho}(\cdot, \chi^0) + \delta\rho(\cdot, \chi^\mu)$ ,  $\theta = \bar{\theta}(\cdot, \chi^0) + \delta\theta(\cdot, \chi^\mu)$ .



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- ▶ Averaged q.e.o.m.  $\longrightarrow$  coupled differential equations for  $\rho$  and  $\theta$ .
- ▶ Decoupling for a range of values of CPSs and large  $N$  (late times).

Dynamic equations  
for  $\langle \hat{V} \rangle_\sigma$ ,  $\langle \hat{\Phi} \rangle_\sigma$

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- ▶ Matching with GR (assuming peaking on matter momenta).
- ▶ Emergent matter and  $G$  defined in terms of microscopic parameters.

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### Perturbations

- ▶ Super-horizon GR matching.
- ▶ **No matching** for intermediate modes (because of different coupling with bkg effective metric)!
- ▶ Effective metric signature determined by CPSs.

# Conclusions





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# Results and perspectives

## Results

- ✓ **Bkg:** Effective bkg dynamics: classical limit & possible singularity resolution.
- ✓ **Bkg:** Emergent matter/gravity constants in terms of microscopic ones.
- ✓ **Pert:** Effective relational localization of (scalar) perturbations.
- ✓ **Pert:** Derivation from full theory of scalar isotr. pert. effective relational dynamics:
  - ✓ Super-horizon GR matching.
  - ✗ No matching with GR at intermediate scales.

## Perspectives (short term)

- ▶ **Bkg:** Inclusion of different matter fields, e.g. scalar field with potential. 
- ▶ **Pert:** Bounce impact on perturbations? 
- ▶ **Pert:** Why sub-horizon GR mismatch? 
  - What (modified) gravity models do match?
  - QG model building issues?
  - Breakdown of some approximations?
- ▶ **Pert:** Out-of-condensate perturbations? 

## Perspectives (long term)

- ▶ Extend the analysis to geometric operators other than the volume (relax isotropy).
- ▶ Study more realistic types of matter.
- ▶ Construct a pert. effective metric and perform a proper SVT decomposition.
- ▶ QG effects on the CMB power spectrum?

# Backup

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# The (T)GFT approach to QG

(Tensorial) Group Field Theories:  
theories of a field  $\varphi : G^d \rightarrow \mathbb{C}$  defined  
on  $d$  copies of a group manifold  $G$ .

$d$  is the dimension of the “spacetime to be” ( $d = 4$ )  
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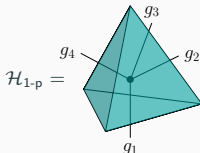
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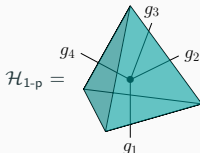
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$S_{\text{GFT}}$  obtained by comparing  $Z_{\text{GFT}}$  with simplicial gravity path integral.

$$Z_{\text{GFT}} = \sum_{\Gamma} \frac{\prod_i \lambda_i^{n_i(\Gamma)}}{\text{sym}(\Gamma)} Z_{\text{GFT}}(\Gamma)$$

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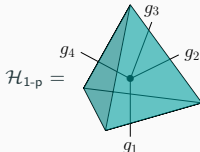
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- ▶ Non-local and combinatorial interactions guarantee the gluing of  $d - 1$ -simplices into  $d$ -simplices.
- ▶  $\Gamma$  are dual to spacetime triangulations.

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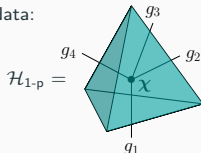
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## Kinematics

Boundary states are  $d - 1$ -simplices decorated with quantum geometric and scalar data:

- ▶ Appropriate (**geometricity**) constraints allow the simplicial interpretation.
- ▶ Group (Lie algebra) variables associated to discretized gravitational quantities.
- ▶ Scalar field discretized on each  $d$ -simplex: each  $d - 1$ -simplex composing it carries values  $\chi \in \mathbb{R}^d$ .



## Dynamics

$Z_{\text{GFT}}$  obtained by comparing  $Z_{\text{GFT}}$  with simplicial gravity + matter path integral.

- ▶ **Non-local and combinatorial** interactions guarantee the gluing of  $d - 1$ -simplices into  $d$ -simplices.
- ▶  $\Gamma$  are **dual to spacetime triangulations**.
- ▶ Scalar field data are **local** in interactions.

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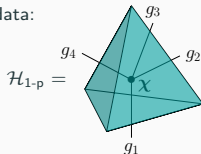
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and  $G$  is the local gauge group of gravity,  
 $G = \text{SL}(2, \mathbb{C})$  or, in many applications,  $G = \text{SU}(2)$ .

## Kinematics

Boundary states are  $d - 1$ -simplices decorated with quantum geometric and scalar data:

- ▶ Appropriate (**geometricity**) constraints allow the simplicial interpretation.
- ▶ Group (Lie algebra) variables associated to discretized gravitational quantities.
- ▶ Scalar field discretized on each  $d$ -simplex: each  $d - 1$ -simplex composing it carries values  $\chi \in \mathbb{R}^d$ .



## Dynamics

$S_{\text{GFT}}$  obtained by comparing  $Z_{\text{GFT}}$  with simplicial gravity + matter path integral.

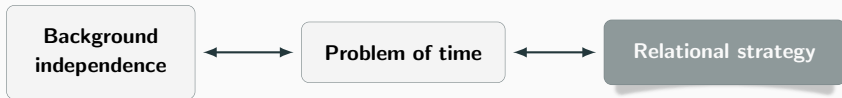
- ▶ **Non-local and combinatorial** interactions guarantee the gluing of  $d - 1$ -simplices into  $d$ -simplices.
- ▶  $\Gamma$  are **dual to spacetime triangulations**.
- ▶ Scalar field data are **local** in interactions.

$$Z_{\text{GFT}} = \sum_{\Gamma} \frac{\prod_i \lambda_i^{n_i(\Gamma)}}{\text{sym}(\Gamma)} Z_{\text{GFT}}(\Gamma)$$

GFTs are QFTs of atoms of spacetime.

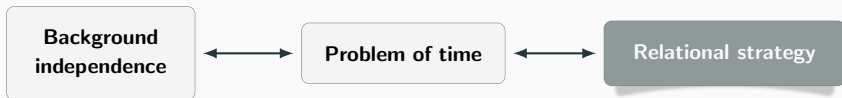


# Relational strategy: the classical and quantum GR perspective



Quite well understood from a classical perspective, less from a quantum perspective.

# Relational strategy: the classical and quantum GR perspective



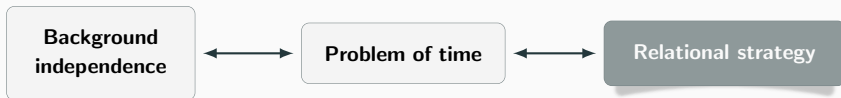
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## Classical

Notion of relationality can be classically encoded in **relational observables**:

- ▶ Take two phase space functions,  $f$  and  $T$  with  $\{T, C_H\} \neq 0$  ( $T$  relational clock).
- ▶ The relational extension  $F_{f,T}(\tau)$  of  $f$  encodes the value of  $f$  when  $T$  reads  $\tau$ .
- ▶ Evolution in  $\tau$  is relational.
- ▶  $F_{f,T}(\tau)$  is a very complicated function, often written in series form.
- ▶ Applications only for (almost) deparametrizable systems, such as GR plus pressureless dust or massless scalar fields.

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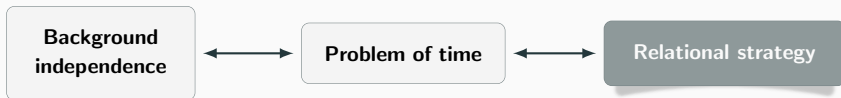
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**Dirac approach**: first quantize, then implement relationality

- ▶ Clock neutral approach: all variables are treated on the same footing.
- ▶ Poor control of the physical Hilbert space.

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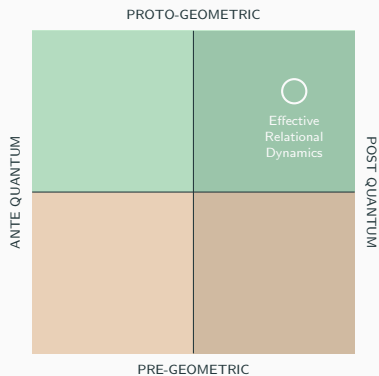
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- ▶ Clock neutral approach: all variables are treated on the same footing.
- ▶ Poor control of the physical Hilbert space.

**Reduced phase space approach**: first implmt relationality, then quantize

- ▶ No quantum constraint to solve.
- ▶ Led to quantization of simple deparametrizable models.
- ▶ Not clock neutral. Too complicated to implement for most of the cases.

# Emergent effective relational dynamics

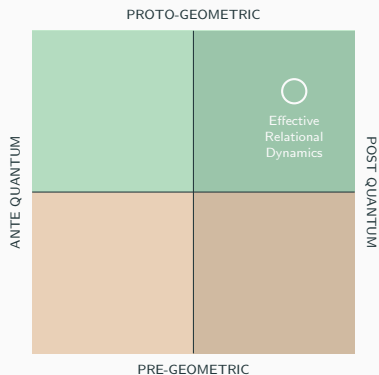


## Basic principles

**Emergence** Rel. dynamics formulated in terms of collective observables and states defined in the microscopic theory.

**Effectiveness** Rel. evolution intended to hold on average. Internal clock not too quantum.

# Emergent effective relational dynamics



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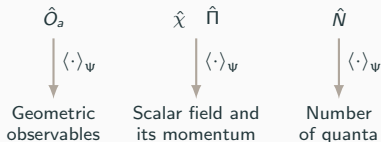
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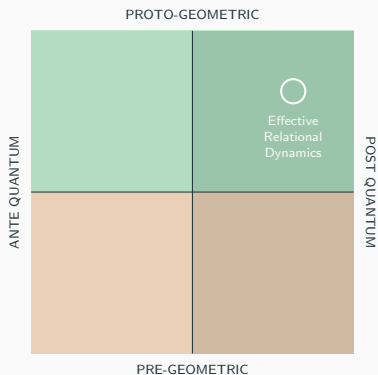
## Concrete example: scalar field clock

### Emergence

- Identify a class of states  $|\Psi\rangle$  which encode **collective behavior** and admit a **continuum** proto-geometric **interpretation**.
- Identify a set of collective observables:



# Emergent effective relational dynamics



## Basic principles

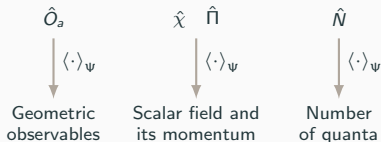
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### Emergence

- ▶ Identify a class of states  $|\Psi\rangle$  which encode **collective behavior** and admit a **continuum** proto-geometric **interpretation**.
- ▶ Identify a set of collective observables:



### Effectiveness

- ▶ It exists a "Hamiltonian"  $\hat{H}$  such that

$$i \frac{d}{d \langle \hat{\chi} \rangle_\Psi} \langle \hat{O}_a \rangle_\Psi = \langle [\hat{H}, \hat{O}_a] \rangle_\Psi,$$

and whose moments coincide with those of  $\hat{\Pi}$ .

- ▶ Relative variance of  $\hat{\chi}$  on  $|\Psi\rangle$  should be  $\ll 1$  and have the characteristic  $\langle \hat{N} \rangle_\Psi^{-1}$  behavior:

$$\sigma_{\hat{\chi}}^2 \ll 1, \quad \sigma_{\hat{\chi}}^2 \sim \langle \hat{N} \rangle_\Psi^{-1}.$$

# Volume at late times

## Classical

---

- ▶ Harmonic gauge:  $N = a^3$ .
- ▶ Negligible contribution of reference matter.

$$(\bar{V}'/\bar{V})^2 = 12\pi G\pi_\phi^{(c)}$$

$$(\bar{V}'/\bar{V})' = 0$$

## Quantum

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- ▶ Wavefunction peaked on  $\pi_\phi = \tilde{\pi}_\phi$ .
- ▶ Domination of single spin  $v_o$ .
- ▶  $\mu_{v_o}(\pi_\phi) \simeq c_{v_o}\pi_\phi$ , with  $4c_{v_o}^2 = 12\pi G$ .

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## Perturbations

### Classical

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- ▶ First order harmonic gauge.
- ▶ Negligible contribution of reference matter.
- ▶ Define  $V(x) = \sqrt{\det q_{ij}} \equiv \bar{V} + \delta V$ .

$$\delta V'' - 6\mathcal{H}\delta V' + 9\mathcal{H}^2\delta V - \bar{V}^{4/3}\nabla^2\delta V = 0.$$

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### Super-horizon

- ▶ Matches the classical solution  $\delta V \propto \bar{V}$ .

### Sub-horizon

- ▶ Same diff. structure but different powers of  $\bar{V}$ .

No matching with GR for arbitrary modes.

# Matter at late times

## Background

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$$\bar{\phi}'' = 0,$$

$$\pi_{\phi}^{(c)} = \text{const.}$$

### Quantum

- ▶ Wavefunction peaked on  $\pi_{\phi} = \tilde{\pi}_{\phi}$ .
- ▶ Domination of single  $\nu_o$ .

$$\langle \hat{\Pi}_{\phi} \rangle_{\bar{\sigma}} = \tilde{\pi}_{\phi} \bar{N},$$

$$\langle \Phi \rangle_{\bar{\sigma}} = \left[ -\partial_{\pi_{\phi}} \left[ \frac{Q_{\nu_o}}{\mu_{\nu_o}} \right] + Q_{\nu_o} \frac{\partial_{\pi_{\phi}} \mu_{\nu_o}}{\mu_{\nu_o}} X^0 \right]_{\pi_{\phi} = \tilde{\pi}_{\phi}}$$

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- ▶  $\pi_{\phi}^{(c)} \equiv \langle \hat{\Pi}_{\phi} \rangle_{\bar{\sigma}} / \bar{N} = \tilde{\pi}_{\phi}$ .
- ▶ Peaking in  $\pi_{\phi} \rightarrow$  peaking in matter field momenta.
- ▶  $\phi \equiv \langle \hat{\Phi} \rangle_{\bar{\sigma}} = -c_{\nu_o}^{-1} + \tilde{\pi}_{\phi} x^0$ ,  $Q_{\nu_o} \simeq \pi_{\phi}^2$ !
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- ▶ Matching at super-horizon scales
- ▶ No matching for intermediate scales.