

# Semiclassical states, high order quantum corrections and cosmology.

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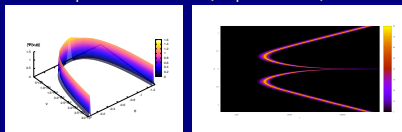
# The problem

- **Determining the evolution of quantum system on the genuine quantum level:** solving partial differential equation(s) or large set of coupled ordinary differential equations

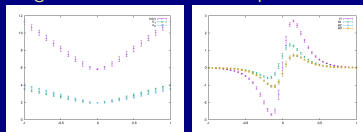
$$i\hbar\partial_t\Psi = \hat{H}\Psi$$

**Example (Loop Quantum Cosmology):**

- Isotropic LQC:  $10^5$  points in  $v$  (equations).



- Anisotropic homogeneous LQC  $10^9$  equations. **Just 3D**



- **More variables: impractical!!!**



## However ...

- **Often interest in specific systems:** behaving classically (small variances)
  - Restriction to semiclassical states very useful.
- **Usually just a couple of DOF** (specific observables) is of interest.
  - No need to track fine details of the wave function shape.
- **Desired:** a method of casting the quantum evolution that:
  - Involves just a couple of relevant degrees of freedom.
  - Allows for control of the effects of neglected DOF with arbitrary precision.
- **The solution:** semiclassical effective dynamics based on Hamburger moments.



# Semiclassical effective dynamics

- **In quantum optics known for a long time**, for cosmology rediscovered by Bojowald, Skirzewski 2006.
- **General construction**: define a set of observables that
  - Encode all the information about the quantum state.
  - Few of them contain relevant information (expectation values and variances of interesting observables) while
  - the rest can be treated as quantum corrections.
  - Set of equations of motion represents the quantum dynamics fully.
- **Closeness and completeness** requires a Poisson algebra of observables.



# Hamburger moments

- **Example:** a 1D quantum system with Heisenberg algebra  $\{x, p, 1\}$  and the hamiltonian  $\hat{H}$

$$[x, p] = i \quad i\hbar \partial_t \Psi(x) = [\hat{H}\Psi](x).$$

- **Observables:** analogs of statistical moments

$$G^{mn} := \int dx dp \, h : (x - \langle x \rangle)^m (p - \langle p \rangle)^n : i \\ := \sum_{i,j=0}^{m,n} \binom{m}{i} \binom{n}{j} (-i\hbar)^{(m+n)-(i+j)} \int dx dp \, x^i p^j : i h x^m p^n : i$$

- Poisson algebra structure **uniquely defined by requirement**

$$\{h : x^m p^n : i, h : x^{m'} p^{n'} : i\} = (i\hbar)^{-1} h [ : x^m p^n : i, : x^{m'} p^{n'} : i ]$$

- $fG^{ab}, G^{cd}g$  polynomial and at most 2nd order in  $G$
- couples  $G^{mn}$  of order  $(m+n)$  up to  $a+b+c+d-1$ .
- **In practice:** found automatically by metalanguage procedures
- **Semiclassicality:** all the  $F^{mn} := h : x^m p^n : i$  must be finite.
  - **In practice:**  $\sum_{m,n \in \mathbb{N}} j G^{mn} j < 1$ .



# The dynamics

- **Taylor-like expansion** of the Hamiltonian

$$H(\hat{x}, \hat{p}) = H(\hbar \hat{x} + i, \hbar \hat{p} + i)$$

- In Weyl ordering ( $x := \hbar \hat{x}, p := \hbar \hat{p}$ ):

$$H(x, p, fG^{mn}g) := \hbar \hat{H}(\hat{x}, \hat{p}) = \sum_{mn} \frac{1}{m!n!} \frac{\partial^{m+n}}{\partial x^m \partial p^n} H(x, p) G^{mn}$$

- In practice the ordering of operators in  $F^{mn}$  is adapted to the factor ordering in  $\hat{H}$ .
- **Any observable** can be decomposed the same way.
- **The equations of motion:** Hamilton-Jacobi equations for  $G^{mn}, x, p,$

$$\dot{G}^{mn} = fH(fG^{mn}g), G^{mn}g$$

- Countable set of equations.
- Polynomial and at most quadratic in  $G^{mn}$  for  $m+n \leq 2$ .  
**Nonlinear!**
- Can be cut-off at arbitrary order  $m+n$ .



# Loop quantized system (LQC)

- **LQC isotropic geometry example**

- **Classical variables:** volume  $V$  and momentum  $b$

$$fV, bg = 4\pi\gamma \frac{P}{\Delta} G = a / \hbar^{1/2}$$

$\gamma$  - Barbero-Immirzi par;  $\Delta$  - LQC area gap

- **Generators:** volume  $V$  and holonomy component  $N = e^{ib}$  (promoted to operators)

- **Hamburger moments**  $G^{ab}$  algebra complex!!

- **Basic observables:**  $V, c = (N + N^{-1})/2, s = (N - N^{-1})/(2i)$

$$fV, cg = as, fV, sg = ac, fs, cg = 0$$

- **Absolute moments:**  $F^{abc} := \hbar \cdot \hat{V}^a \hat{S}^b \hat{C}^c : j ! G^{abc}$

- **Necessary step of transformation**  $G^{abc} \rightarrow \tilde{G}^{ab}$

- Performed via metalanguage



# Simple FRLW inflationary universe

- **The model:** flat isotropic FRLW universe with matter:
  - Dust providing the internal clock (negligible mass to not affect the dynamics).
  - Massive scalar field: quadratic inflationary potential.

- **The metric:**

$$ds^2 = -N^2(t)dt^2 + a^2(t)g_{ij} dx^i dx^j$$

- **Degrees of freedom:**

- **geometry:** canonical pair  $(V = a^3, \pi_V)$  s.t.  $fV, \pi_V g = 1$ ,
- **matter:** canonical pair  $f\phi, p_\phi g$ ,
- **time:** dust field potential  $T$ .

- **The dynamics** generated by the (deparametrized) Hamiltonian

$$H(V, \pi_V, \phi, p_\phi) = 6\pi G V \pi_V^2 + \frac{p_\phi^2}{2V} + \frac{m^2}{2} V \phi^2$$

- Polynomial with exception of  $1/V$  term!





# The quantization

- **Quantization:** Wheeler-deWitt one (following the geometrodynamics program)
  - Both dynamical DOF quantized using **Schrödinger representation**:
    - variables  $(V, \pi_V, \phi, p_\phi)$  promoted to operators,
    - Hilbert space:  $\mathcal{H} = L^2(\mathbb{R}, dV) \otimes L^2(\mathbb{R}, d\phi)$ .
  - **The quantum dynamics** generated by the Schrödinger equation

$$i\hbar\partial_T\Psi_T(V, \phi) = \left[ 6\pi G\hbar\hat{\pi}_V\hat{V}\hat{\pi}_V + \frac{1}{2}\hat{V}^{-1}\hat{p}_\phi^2 + \frac{m^2}{2}\hat{V}\hat{\phi}^2 \right]\Psi_T(V, \phi)$$

- **The semiclassical description:**
  - expectation values  $V, \pi_V, \phi, p_\phi$ ,
  - the moments

$$G^{abcd} := \hbar: (\hat{V} \quad \hbar\hat{V}il)^a (\hat{\pi}_V \quad \hbar\hat{\pi}_Vil)^b : \\ : (\hat{\phi} \quad \hbar\hat{\phi}il)^c (\hat{p}_\phi \quad \hbar\hat{p}_\phi il)^d : i$$

where  $: \mathcal{X}\mathcal{Y} :: = (1/2)(\mathcal{X}\mathcal{Y} + \mathcal{Y}\mathcal{X})$ .



# Framework for numerical analysis

D. Brizuela, TP, 2021

- **The system:**
  - Generated sets of EOM for cut-offs  $n$  from 2 to 6.
    - The respective numbers of EOM: 14, 34, 69, 125, 209.
  - **Initial data:** evaluated moments for Gaussian (and couple other) shapes in basic variables.
  - **Inflaton mass:**  $10^{-6} m_{\text{Pl}}$ .
  - Initial data specified in (high density) **kinetic dominated region** before inflation.
  - Since  $V$  is the infrared regulator we "densitized" the moments

$$G^{abcd} \rightarrow Q^{abcd} := G^{abcd} / V^{a+d}, \quad p_\phi \rightarrow \sigma := p_\phi / V$$

- **The evolution range:** well past inflation end.
- **The data processing:**
  - Compared the **trajectories** of expectation values and higher moments for various order of cut-off.
  - **Convergence:** determined convergence of the trajectories as  $n$  increases.

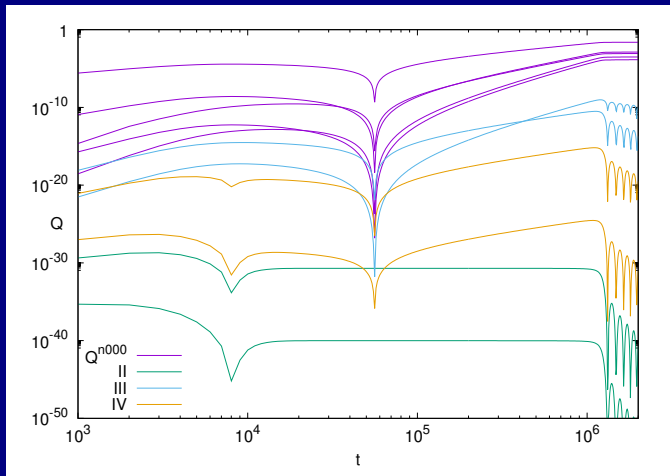


# Regulator removal limit

- **Infrared regulator:**
  - **Action/Hamiltonian/momentum variables** defined by integrating densities over (infinite) Cauchy slice.
  - For finiteness integration restricted to finite comoving region.
  - **Crucial requirement:** model has to admit nontrivial limit as chosen region is expanded to encompass whole slice!
  - **Addressed only partially:** Corichi, Montoya, Singh, ...
- **Crucial properties of EOM:**
  - At 2nd order cut-off  $V$  decouples. **System manifestly invariant** wrt. fiducial cell choice.
  - For higher order  $V$  appears only through terms  $(\sim/V)^n$ .
- **Consequences:**
  - System admits **well defined limit  $V \rightarrow 1$** .
  - Taking that limit is mathematically equivalent to setting  $\sim \rightarrow 0$ : system behaves like **an ensemble of statistical ones**.



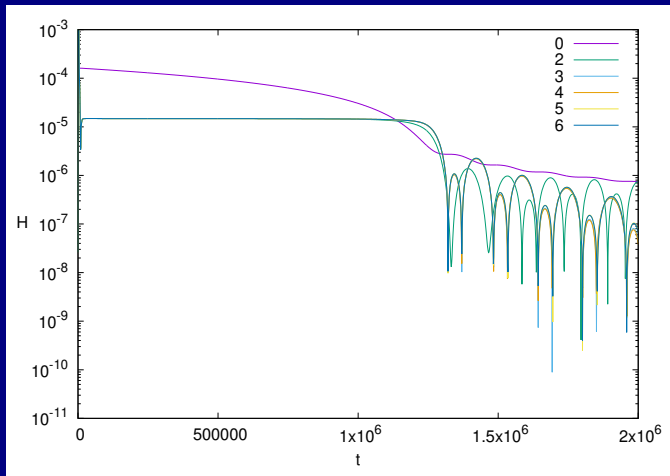
# Densitized central moments



Rysunek: Evolution of sample of the moments for order 6.



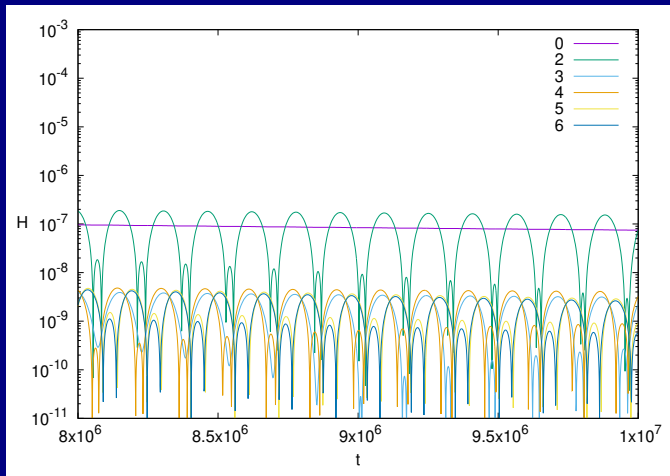
# Hubble parameter corrections



Rysunek: Quantum corrections to  $H_r$ .



# Hubble parameter corrections (2)



Rysunek: Quantum corrections to  $H_r$ : late reheating.



# Distinguished dynamical results

- **Significant dispersion in volume**
  - Moments  $Q^{n000}$  start rapidly increasing mid-inflation, becoming constant upon inflation end.
  - Approximate analysis of 2nd order EOM shows that transition point is distinguished by a certain function of expectation values.
- **Counterintuitive behavior past inflation (classicalization)**
  - After exiting the inflation the quantum corrections to locally measurable observables decrease as the cutoff order increases!
    - **Holds for all evolved data**, though variances in conjugate variables differ by at most 1 level of magnitude.



# General insights

- **General observations inferred:**
  - **Method robustness:** described semiclassical framework is a technically viable and powerful tool of controlling the dynamical evolution of quantum systems up to high order (of quantum corrections).
  - **Consistency check:** it allows to probe the models consistency in the infrared regulator removal limit in a precise and robust manner.
  - **Surprises:** in certain domains the 1st subleading correction can actually be least accurate and including higher order corrections may prove necessary to accurately describe systems past 0th order approximations.
- **Point to remember:**
  - What is the real result is the **limit** to which the trajectories **converge** with increasing order!



Thank you for your attention!

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