Cosmological Perturbations in Loop Quantum Cosmology

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IPARCOS

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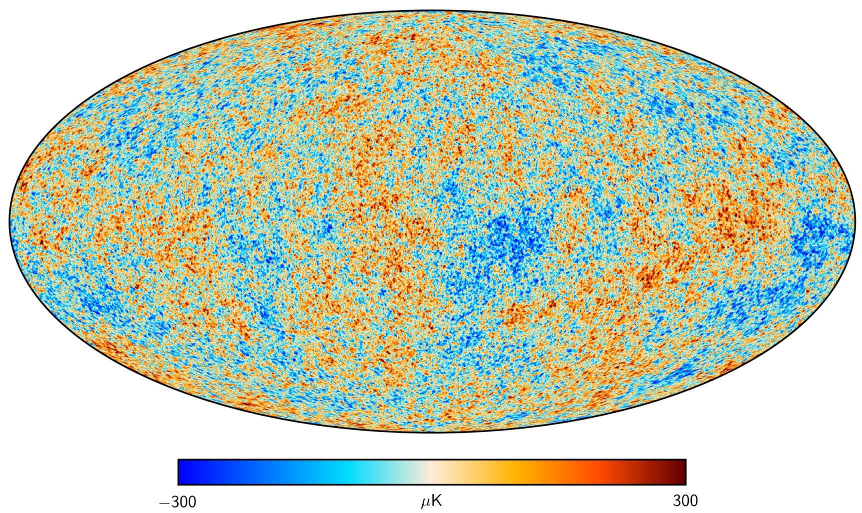
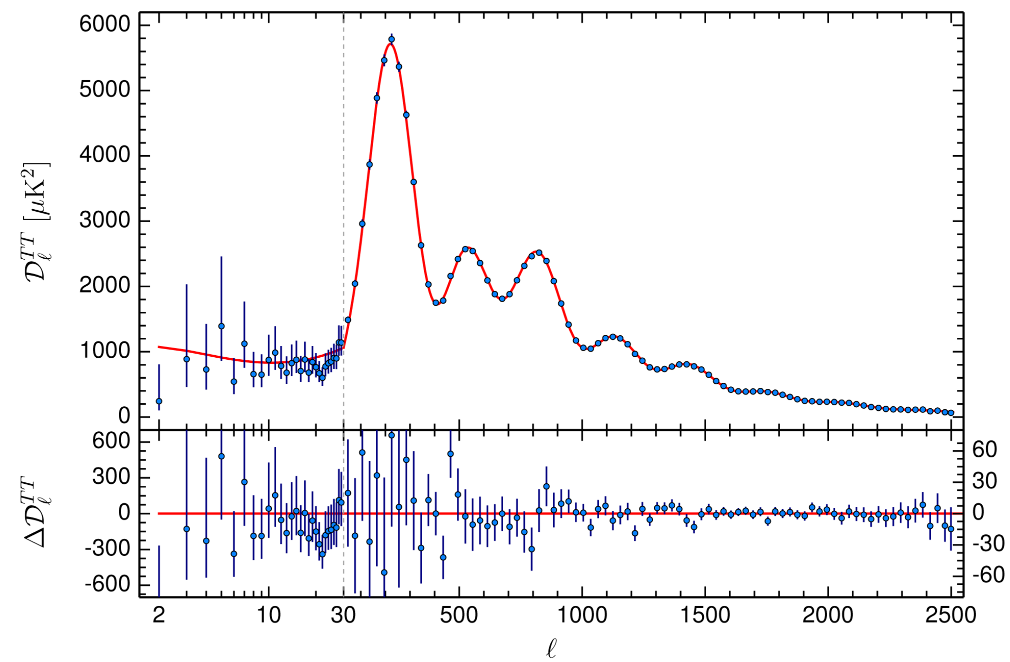
INTRODUCTION

- **Precision Cosmology**: Planck analysis confirms excellent agreement between ΛCDM theoretical predictions and observations



www.cosmos.esa



- The standard cosmological model ignores the pre-inflationary epoch and the initial singularity.

- At Planckian energy densities we expect quantum gravity to play a role.

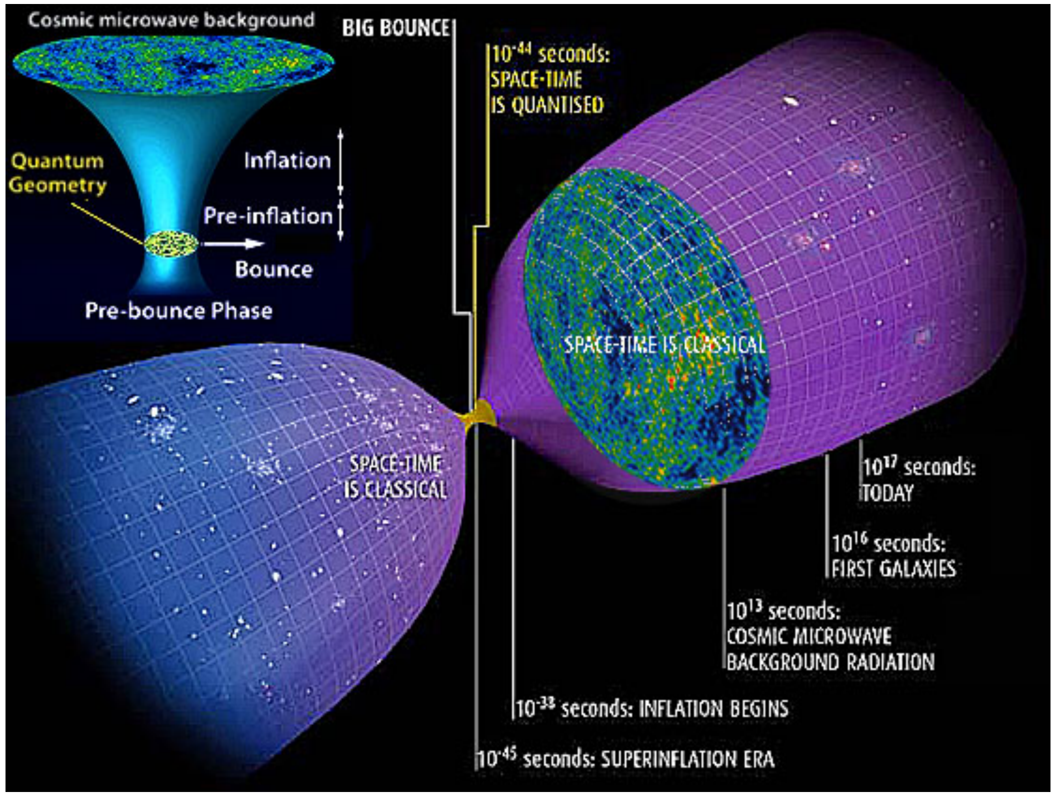
Quantum Cosmology

- Hopes of confronting QC with future observations.

INTRODUCTION

- Framework: **hybrid Loop Quantum Cosmology**
  - LQG inspired treatment for the conformal factor
  - Fock quantization for the perturbations

- Motivation: **singularity-free** description.



http://www.igcscience.org/2316/03/13/loop-quantum-cosmology-and-the-early-universe/

http://gravity.psu.edu/outreach/articles/bigbounce.pdf

- Well-defined pre-inflationary dynamics.

OUTLINE

- Flat FLRW in LQC: quantum bounce and effective dynamics
- Perturbations within LQC: hybrid quantization and effective dynamics
- Observational consequences from LQC
  - Vacuum proposals in LQC: e.g. states of low energy
  - Primordial Power Spectrum
  - Different predictions

FLAT FLRW + SCALAR ϕ in LQC

$$E^a_i \longleftrightarrow p, \qquad A^i_a \longleftrightarrow c \qquad \{c,p\} = \frac{8\pi G\gamma}{3}\,, \quad \{\phi, P_\phi\} = 1$$

- **Improved dynamics:** $(c,p) \rightarrow (b,v), \quad \{b,v\} = 2$

  [Ashtekar, Pawlowski, Singh, 06]

- Kinematical Hilbert space: $\mathcal{H} = \mathcal{H}_g \otimes \mathcal{H}_m$

- Matter: standard representation $\mathcal{H}_m = L^2(\mathbb{R}, d\phi)$, $\hat{P}_\phi = -i\partial_\phi$

- Geometry: **loop representation** $\mathcal{H}_g = \overline{\text{span}\{|v\rangle, v \in \mathbb{R}\}}$, $\langle v|v'\rangle = \delta_{v,v'}$

$$\hat{v}|v\rangle = v|v\rangle \quad , \quad \widehat{e^{\pm ib/2}}|v\rangle = |v \pm 1\rangle$$

"Polymerization": $b \rightarrow \widehat{\sin(b)} = \dfrac{\widehat{e^{ib}} + \widehat{e^{-ib}}}{2i}$

HAMILTONIAN CONSTRAINT OPERATOR

[Ashtekar, Pawlowski, Singh, 06]

$$\hat{C} = -\frac{6}{\gamma^2}\hat{\Omega}^2 + 8\pi G \hat{P}_\phi^2$$

$\hat{\Omega}^2$ symmetric operator representing $(2\pi G\gamma b v)^2$

$$\hat{\Omega}^2|v\rangle = -f_+(v)|v+4\rangle + f_o(v)|v\rangle - f_-(v)|v-4\rangle$$

- **Superselection** sectors: v supported in discrete semilattices
- **Non-degenerate** absolutely continuous spectrum
- (Real) generalized eigenfunctions with eigenvalue k^2 : $e_k^\varepsilon(v)$

they behave as standing waves $e_k^\varepsilon(v) \xrightarrow[v\to\infty]{} r[e^{i\alpha_\varepsilon(k)}\underline{e}_k + e^{-i\alpha_\varepsilon(k)}\underline{e}_{-k}]$

($\underline{e}_k$, $\underline{e}_{-k}$ →) analog WDW eigenfunctions

and are highly suppressed for $v < k^2$

[Martín-Benito, Mena, Olmedo, 09]

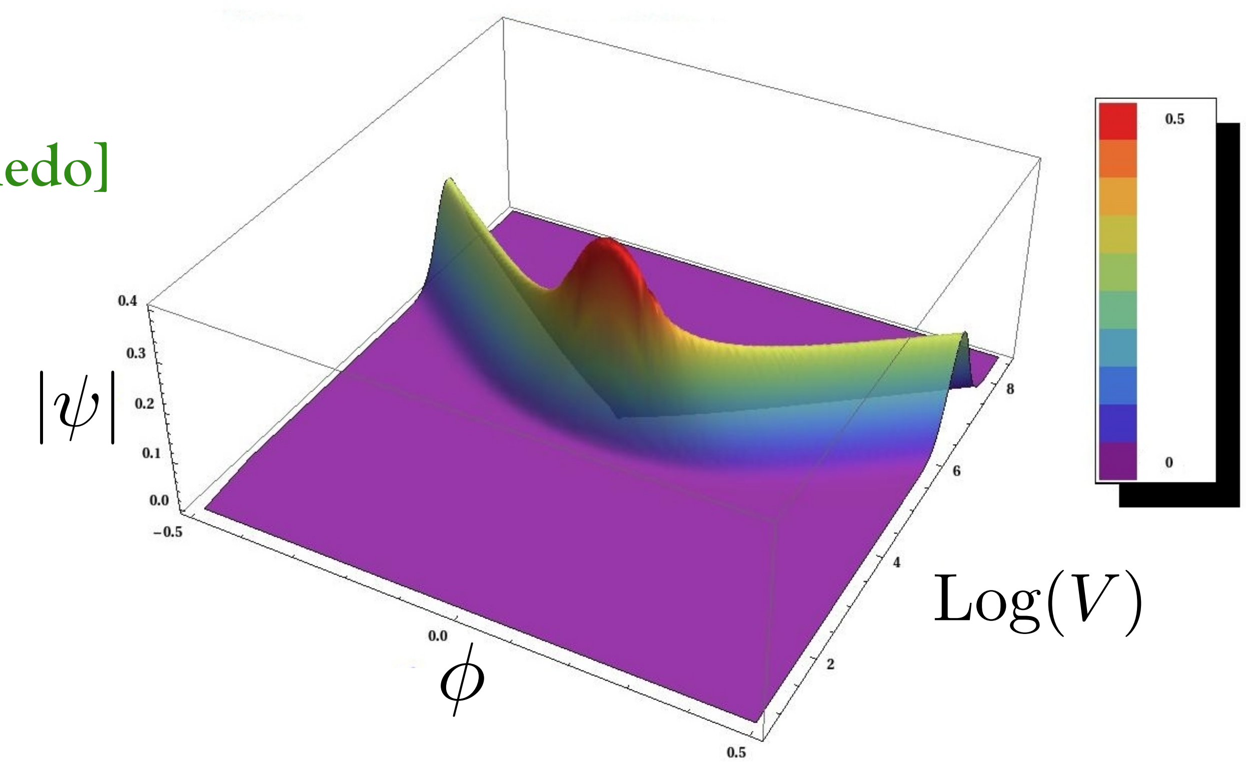
LQC: QUANTUM BOUNCE

[Ashtekar, Pawlowski, Singh, 06]

Big Bang ⟶ **Big Bounce**

- Expectation value of the volume on semiclassical states

[by courtesy of J. Olmedo]



- The spectrum of the energy density is bounded from above: $\rho_c \sim \rho_{Pl}$

[Ashtekar, Corichi, Singh,07]

LQC: EFFECTIVE DYNAMICS

- Background dynamics: flat FLRW + inflaton ϕ

ρ : energy density , P : pressure

☆ GR

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi}{3}a^2\rho, \qquad \frac{a''}{a} = \frac{4\pi}{3}a^2\rho - 4\pi a^2 P,$$

☆ Effective LQC

$$\left(\frac{a'}{a}\right)^2 = \frac{8\pi}{3}a^2\rho\left(1-\frac{\rho}{\rho_c}\right),$$

$$\frac{a''}{a} = \frac{4\pi}{3}a^2\rho\left(1+2\frac{\rho}{\rho_c}\right) - 4\pi a^2 P\left(1-2\frac{\rho}{\rho_c}\right)$$

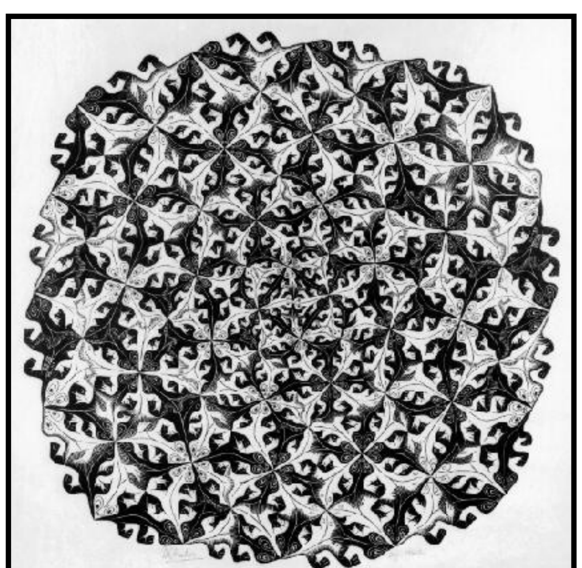
HYBRID QUANTIZATION

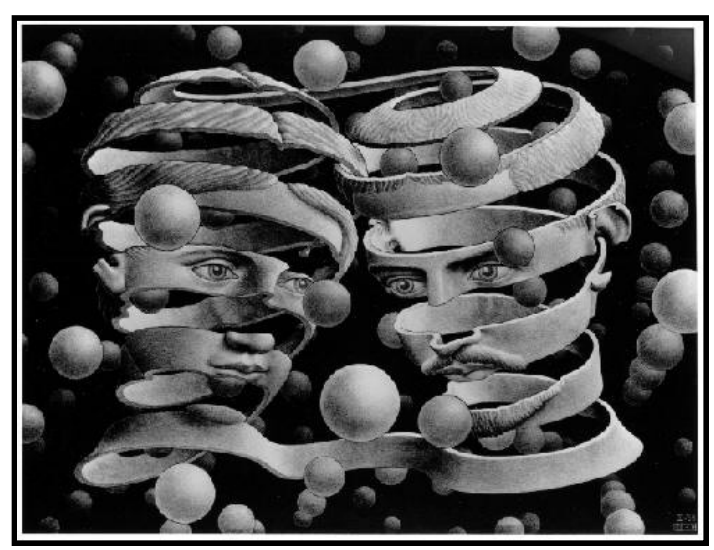
[Garay, Martín-Benito, Mena Marugán,08]

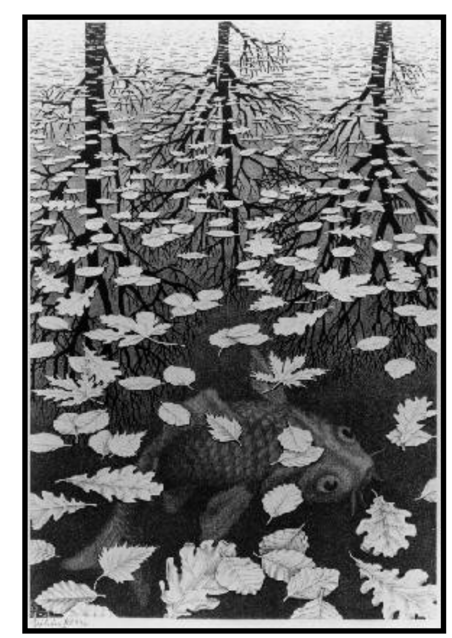
- Inclusion of inhomogeneities in LQC
  - ☆ **Loop quantization** for the zero-mode of the geometry
  - ☆ **Fock quantization** for the inhomogeneities

- Assumption: Main quantum gravity effects are those affecting the global degrees of freedom of the geometry

LQG ⟶ **hybrid LQC** ⟶ **QFT/CS**







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FLRW with cosmological perturbations

- We consider FLRW + inflaton, perturbed at linear order.

- Hybrid quantization of the full symplectic system described by gauge invariant variables [Castelló Gomar, Martín-Benito, Mena Marugán,15]

- Extract effective classical dynamics for the perturbations from the Hamiltonian constraint of the system

PERTURBATIONS: EFFECTIVE DYNAMICS

Tensor modes: $$(\mu_{\vec{k}}^{\pm}(\eta))'' + \left[k^2 + s^{(\mathrm{t})}(\eta)\right] \mu_{\vec{k}}^{\pm}(\eta) = 0$$

Scalar modes: $$v_{\vec{k}}''(\eta) \ + \left[k^2 + s^{(\mathrm{s})}(\eta)\right] v_{\vec{k}}(\eta) = 0$$

Hybrid LQC: [Castelló Gomar, Martín de Blas, Mena Marugán, Olmedo, 17]

$$s^{(\mathrm{t})} = -\frac{4\pi G}{3} a^2 (\rho - 3P) \quad , \quad s^{(\mathrm{s})} = s^{(\mathrm{t})} + \mathcal{U}$$

effective LQC

$$\mathcal{U} = a^2 \left[V_{,\phi\phi} + 48\pi G V(\phi) + 6 \frac{a'\phi'}{a^3 \rho} V_{,\phi} - \frac{48\pi G}{\rho} V^2(\phi) \right]$$

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effective LQC

Dressed metric $$s^{(t)} = -\frac{4\pi}{3}a^2\rho\left(1 + 2\frac{\rho}{\rho_c}\right) + 4\pi a^2 P\left(1 - 2\frac{\rho}{\rho_c}\right)$$

[Agulló, Ashtekar, Nelson,12]

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effective LQC

- Other quantization prescriptions lead to different effective masses

mLQC - I, mLQC - II [Li, Singh, Wang, 20] [Li, Olmedo, Singh, Wang, 20]

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effective LQC

- Other quantization prescriptions lead to different effective masses.
- Few Planck seconds away from the bounce we recover GR.
- Around the bounce (kinematically dominated regime): $s^{(\mathrm{s})} \approx s^{(\mathrm{t})}$

PRIMORDIAL POWER SPECTRUM

- Perturbations are evolved from initial time until the relevant scales have all crossed out the horizon. After that the power spectra of the comoving curvature perturbation and tensor modes remain frozen.

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \frac{|v_{\vec{k}}|^2}{z^2}\Big|_{\eta=\eta_{\rm end}} \qquad \mathcal{P}_{\mathcal{T}}(k) = \frac{32k^3}{\pi} \frac{|\mu^{\pm}_{\vec{k}}|^2}{a^2}\Big|_{\eta=\eta_{\rm end}}$$

$(z = a\dot{\phi}/H)$

- In **standard cosmology (ΛCDM)**, the primordial scalar power spectrum is nearly scale invariant

$$\mathcal{P}_{\mathcal{R}}(k) = A_s \left(\frac{k}{k_\star}\right)^{n_s - 1}$$

spectral index (close but small than 1)

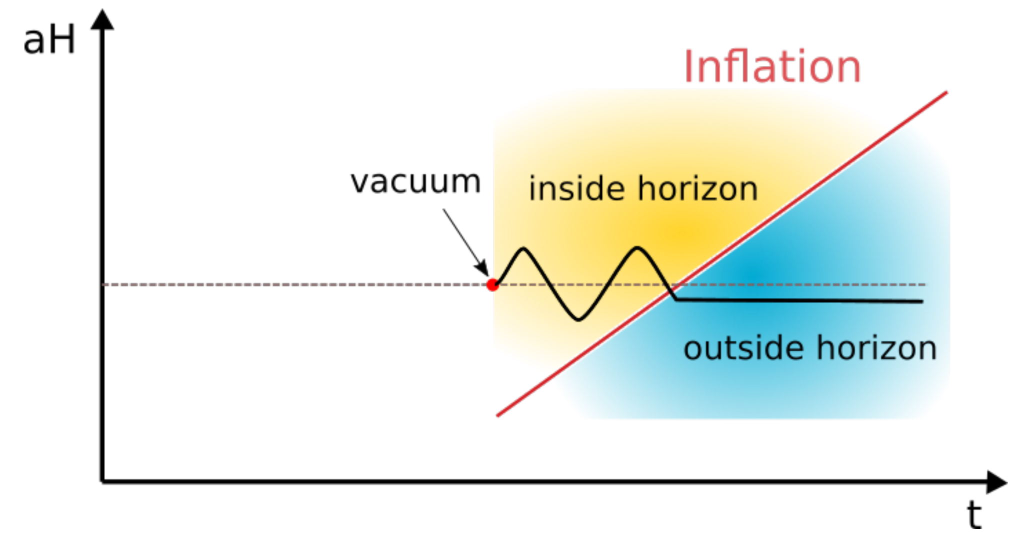
scalar perturbation amplitude

pivot scale (Planck: $0.05\,\mathrm{Mpc}^{-1}$)

LQC OBSERVATIONAL CONSEQUENCES

- In ΛCDM: initial conditions at the onset of inflation ⟶ BD vacuum

It assumes that relevant scales do not feel curvature before.



[by courtesy of R.Neves]

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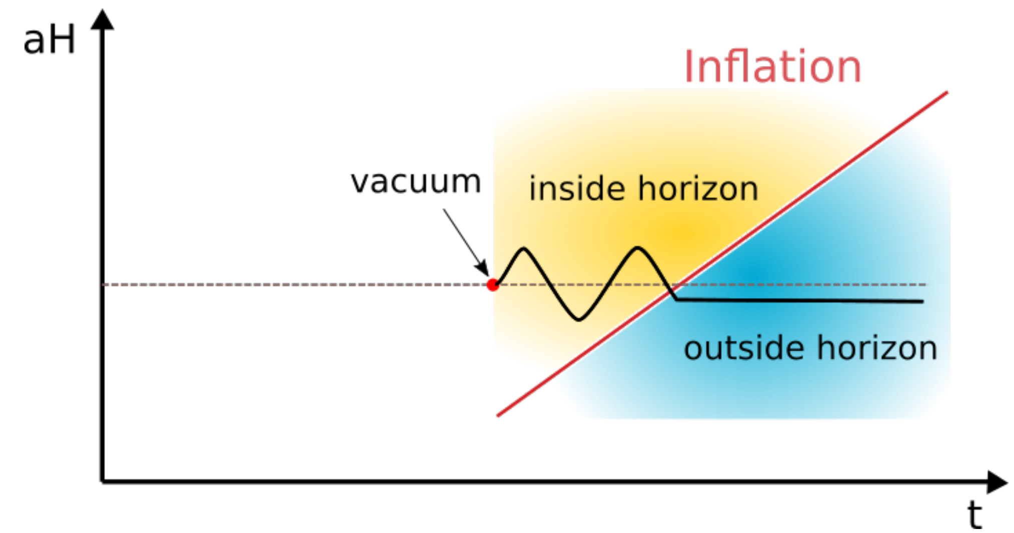
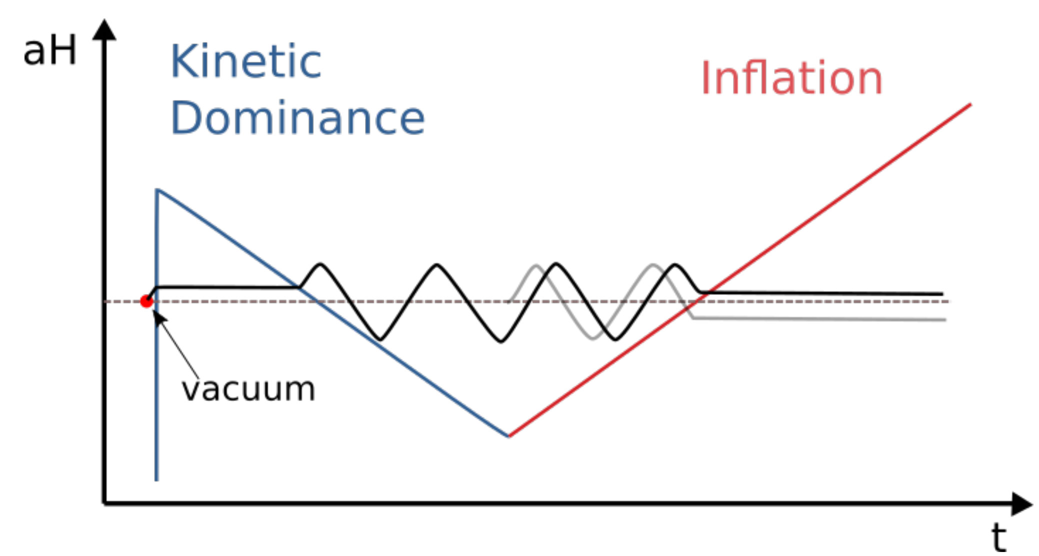


[by courtesy of R.Neves]

- In LQC: pre-inflationary dynamics

  Some (potentially observable) scales experience curvature because of the bounce.



- Observables modes have comoving wave number k in the range $\sim (10^{-4}\,\mathrm{Mpc}^{-1}, 0.5\,\mathrm{Mpc}^{-1})$

LQC OBSERVATIONAL CONSEQUENCES

- While the LQC corrections to the background geometry are significant in the UV, their effect on cosmological perturbations is non-negligible only in the IR.

Cosmic tango between the very small and the very large

[Ashtekar, Gupt, Sreenath, 21]

- Does LQC alleviate current ΛCDM anomalies?:
  power suppression at large angular scales, dipolar asymmetry, tension in the lensing amplitude, preference for odd parity multipoles…

[Ashtekar, Gupt, Sreenath,21] [Agulló, Kranas, Sreenath,20]

PRIMORDIAL POWER SPECTRUM IN LQC

- The predictions obtained from this LQC framework depend mainly on

  ☆ The choice of initial conditions for the background
  (number of e-folds from the bounce to the onset of inflation)

This fixes the scales $k \leq k_{LQC}$ for which the LQC corrections are relevant, and whether they are observable or not. k_{LQC} usually fixed to get agreement with ΛCDM and with observations at small angular scales.

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☆ The choice of initial conditions for the perturbations (Fock vacuum)

⟶ Modified PPS: $$\mathcal{P}_{\mathcal{R}}^{\mathrm{LQC}}(k) = f(k) A_s \left(\frac{k}{k_\star}\right)^{n_s-1}$$; $f(k) \sim 1$ for $k > k_{LQC}$

VACUUM PROPOSALS IN LQC

- When do we set initial conditions?

  - Well before the bounce when all scales of interest are adiabatic? [Agulló, Kranas, Sreenath,20]

  - At the bounce? [Olmedo, Martín de Blas, 16] [Ashtekar, Gupt, Jeong Sreenath,20]

- Several different proposals for the vacuum of the perturbations (some):

[Agulló, Ashtekar, Nelson, 13] 4-th order adiabatic state

[Ashtekar, Gupt, 17] Vacuum motivated by quantum version of the Weyl curvature hypothesis (QHIH)

[Olmedo, Martín de Blas, 16]
[Elizaga de Navascués, Mena-Marugán, Prado, 21] Non-oscillating vacuum

[Neves, Martín-Benito, Olmedo, 21] States of Low Energy

STATES OF LOW ENERGY IN LQC

[Neves, Martín-Benito, Olmedo,21]

- SLEs are Hadamard states that minimize the smeared energy density along $f(t) \in C_0^\infty(\mathbb{R})$ [Olbermann,07] [Banarjee, Niedermaier,20]

$$E(T_k) = \frac{1}{2}\int f(t)\left(|\dot{T}_k|^2 + \omega_k^2 |T_k|^2\right)$$

$$T_k = \frac{v_k}{a} \quad , \quad \ddot{T}_k + 3H(t)\dot{T}_k + \omega_k^2 T_k = 0$$

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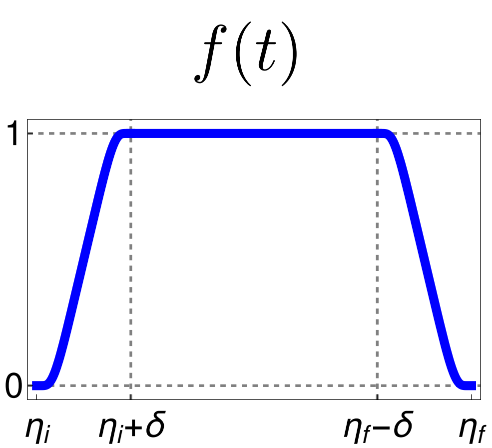
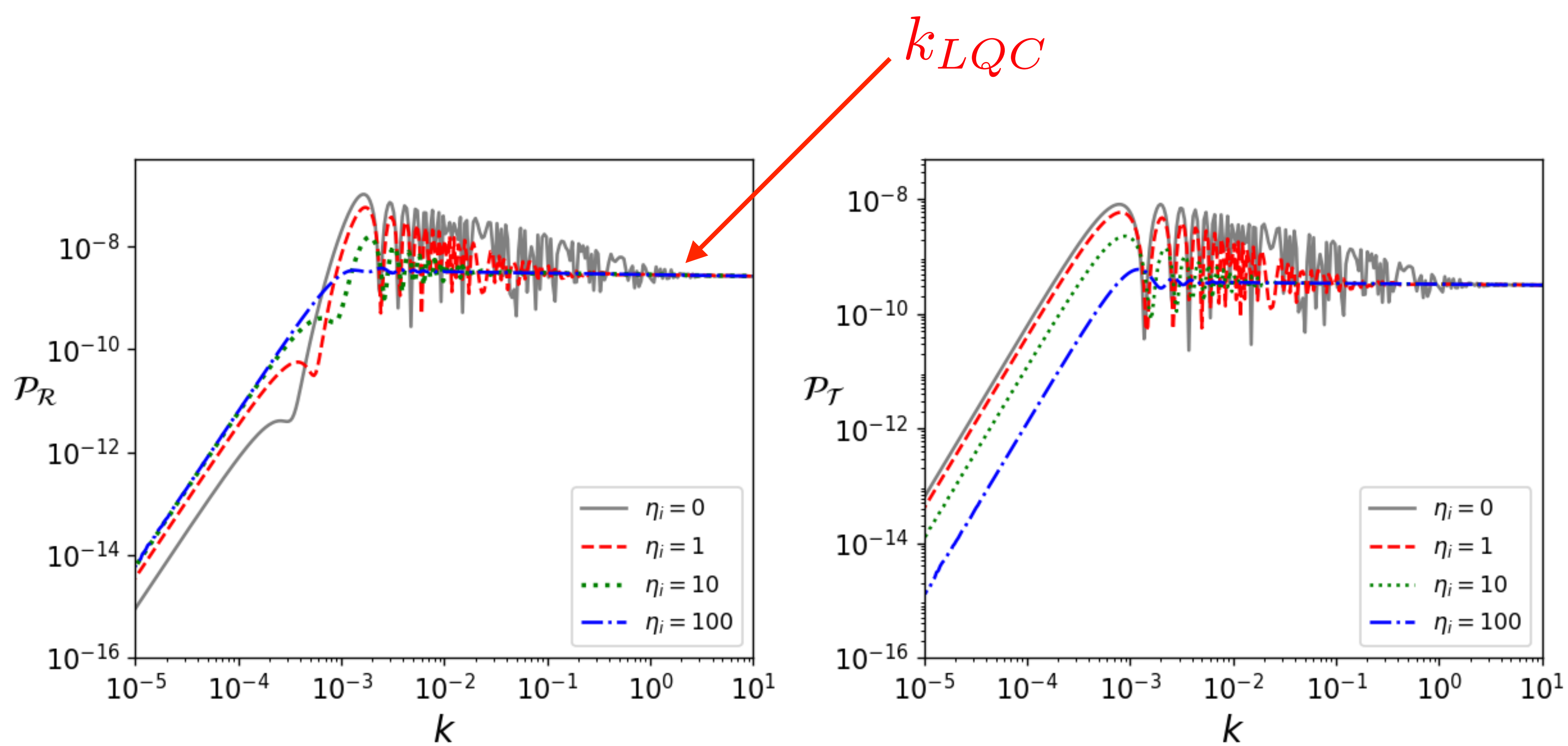
- For functions with support starting at η_i (bounce: $\eta = 0$)





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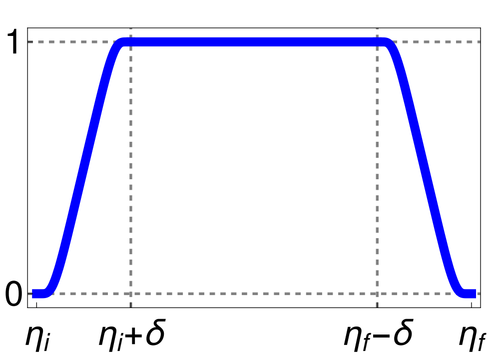
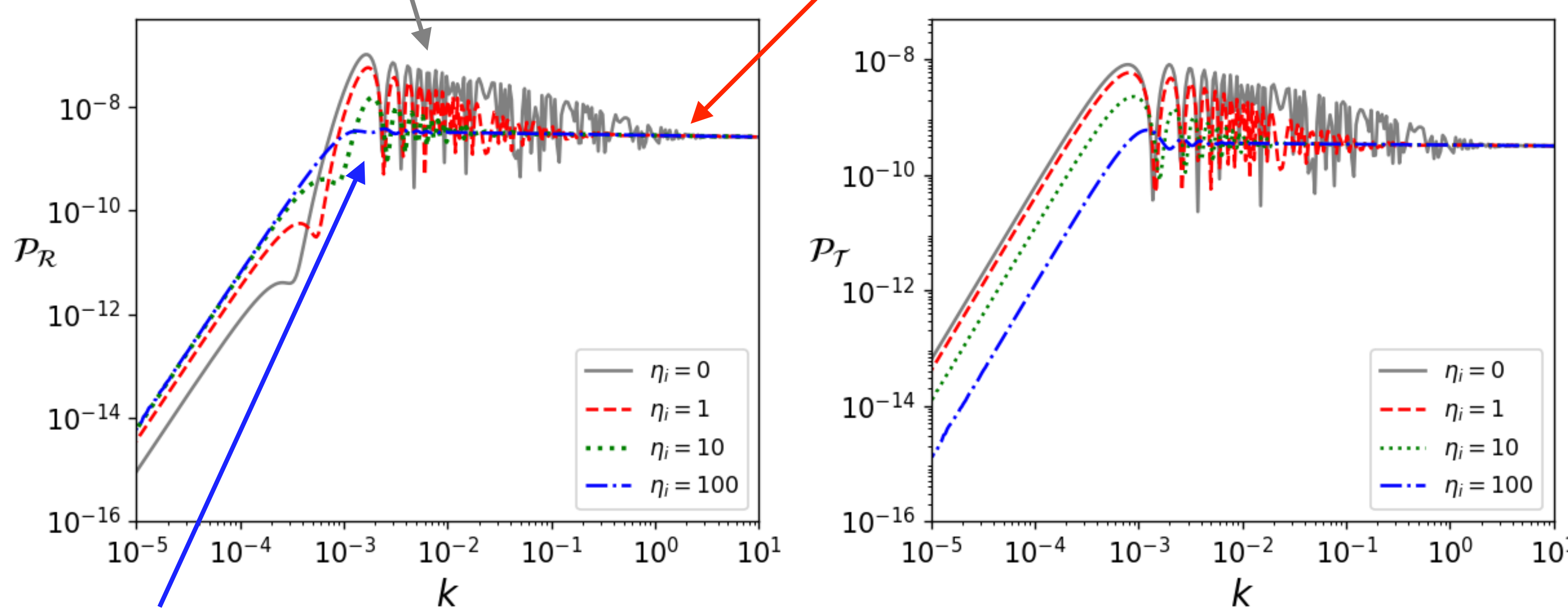
- For functions with support starting at η_i (bounce: $\eta = 0$)

similar to 2nd-order adiabatic vacuum

k_{LQC}

$f(t)$





similar to non-oscillatory vacuum

LQC PREDICTIONS

- Bayesian posterior to cosmological parameters from experimental data using MCMC codes (CosmoMC, Monte Phyton).

- Previous vacuum proposals made within LQC got tension alleviation:

☆ **Non-Gaussianities** due to correlations between IR modes induce monopolar and dipolar modulation of angular PS at large angular scales

⟶ power suppression at large angular scales is not anomalous
⟶ lensing amplitude compatible with 1
⟶ preference for odd parity multipoles
⟶ dipolar asymmetry

[Agulló, Bolliet, Sreenath,18]
[Agulló, Kranas, Sreenath,20]

- 0th-order adiabatic vacuum well before the bounce.

☆ Ashtekar-Gupt vacuum ⟶ PPS with power suppression

⟶ power suppression at large angular scales and A_L compatible with 1

[Ashtekar, Gupt, Jeong Sreenath,20]   [Ashtekar, Gupt, Sreenath,21]

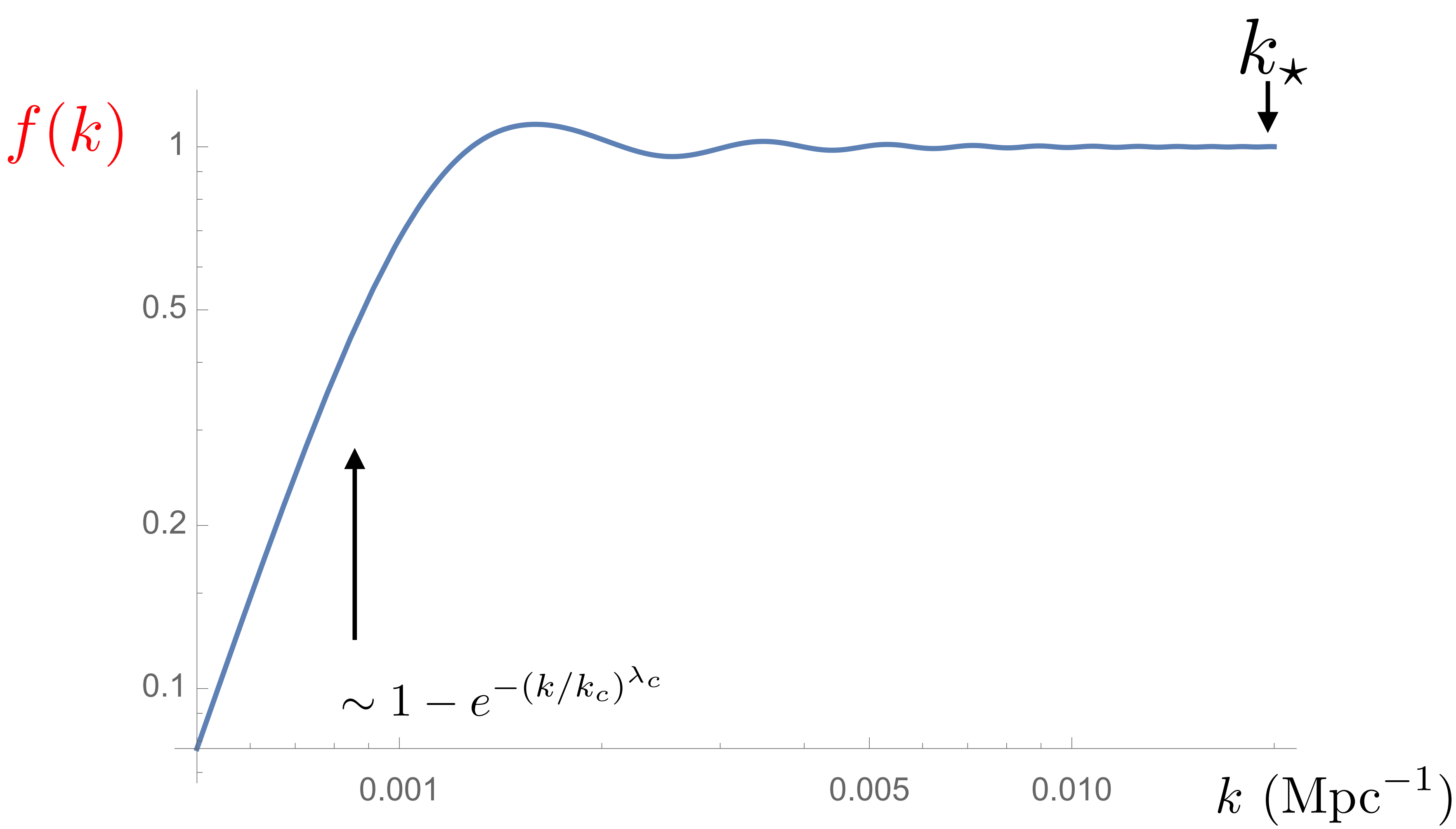
NON-OSCILLATORY VACUUM

[Olmedo, Martín de Blas, 16] [Elizaga de Navascués, Mena-Marugán, Prado, 21]

- Do other vacua lead to similar results? E.g: Non-oscillating vacuum
- Same UV expansion than SLEs $\longrightarrow$ Hadamard state
- PPS:

$$\mathcal{P}_{\mathcal{R}}^{\mathrm{LQC}}(k) = f(k) A_s \left(\frac{k}{k_\star}\right)^{n_s - 1}$$

[Neves, Martín-Benito, Olmedo, w.i.p.]



$\lambda_c \sim 3$

$k_c \sim 0.001$

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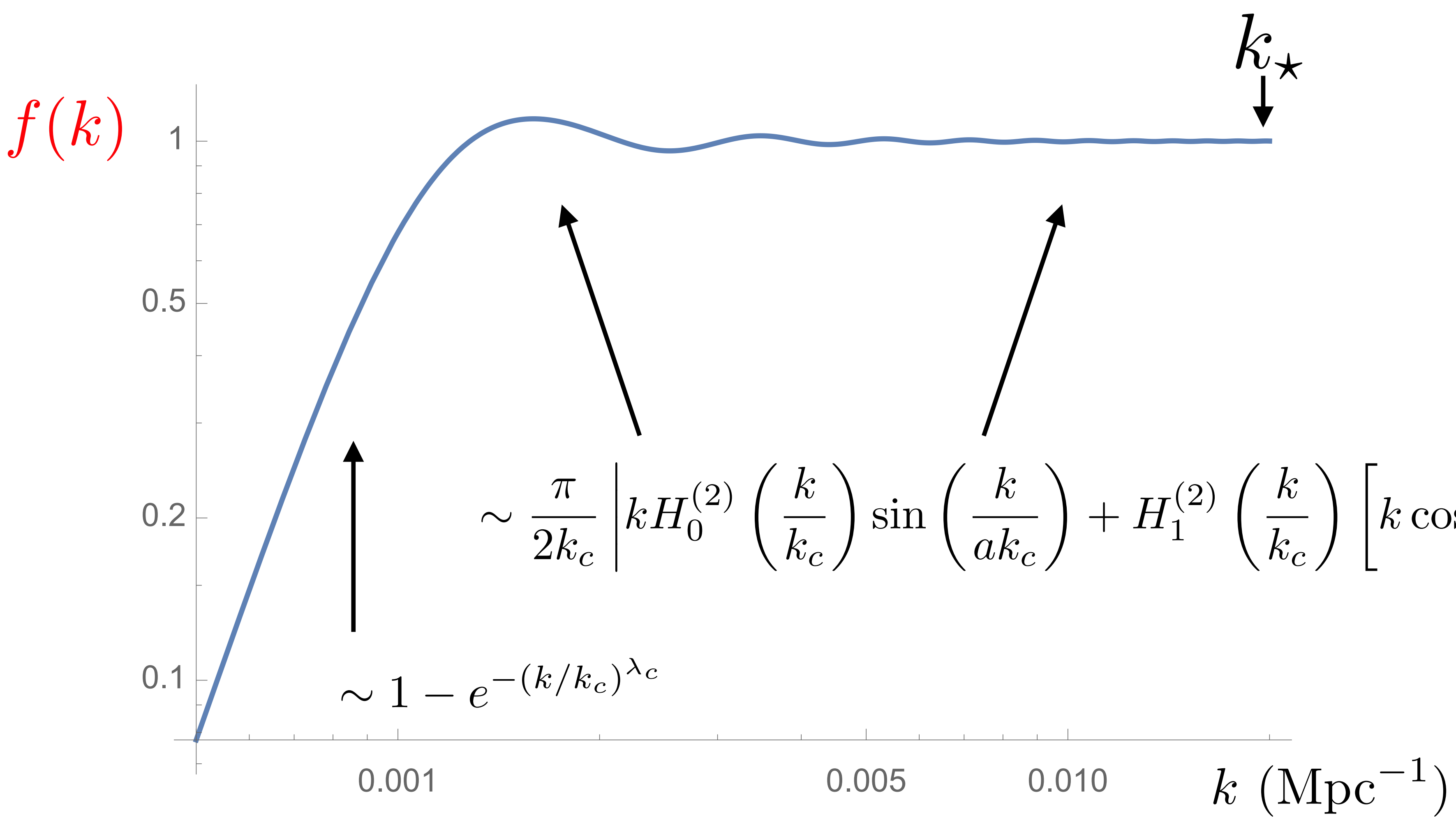


$$\sim \frac{\pi}{2k_c}\left| k H_0^{(2)}\left(\frac{k}{k_c}\right)\sin\left(\frac{k}{ak_c}\right) + H_1^{(2)}\left(\frac{k}{k_c}\right)\left[k\cos\left(\frac{k}{ak_c}\right) - \frac{k_c}{2}\sin\left(\frac{k}{ak_c}\right)\right]\right|^2$$

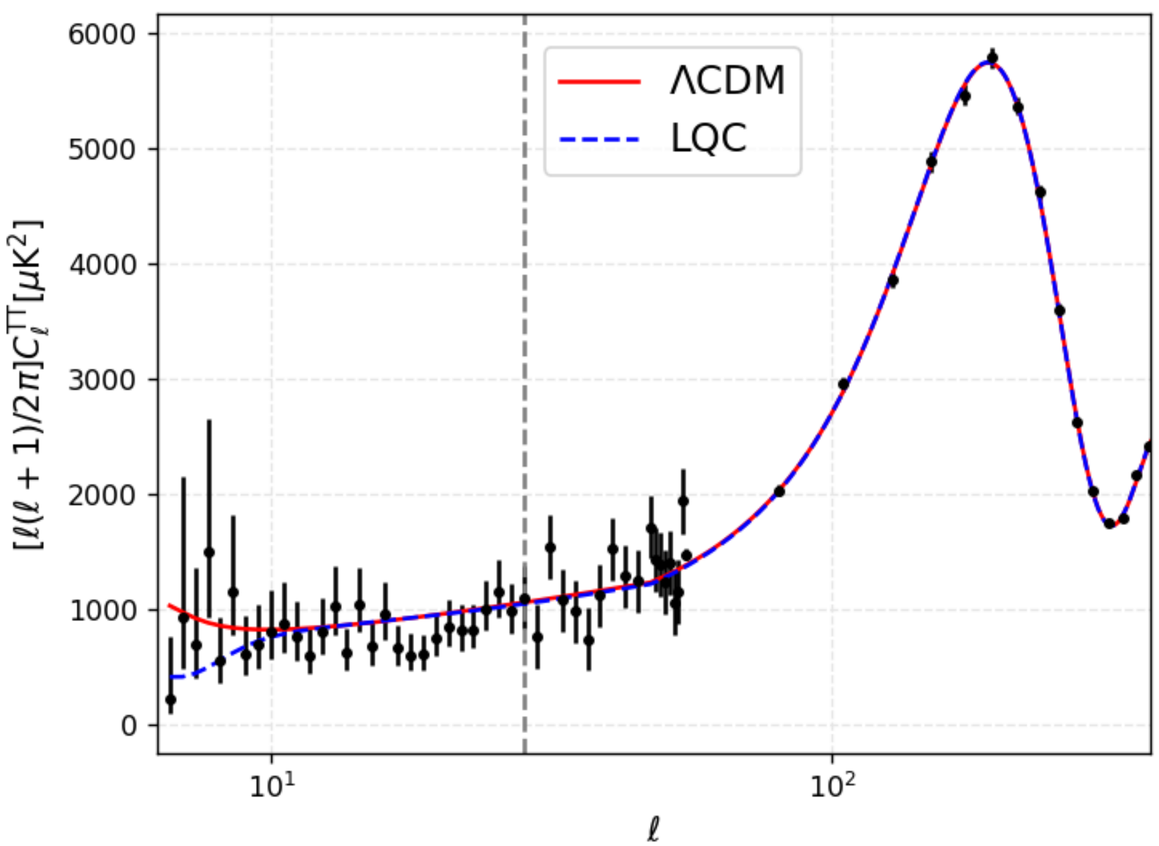
$\lambda_c \sim 3$

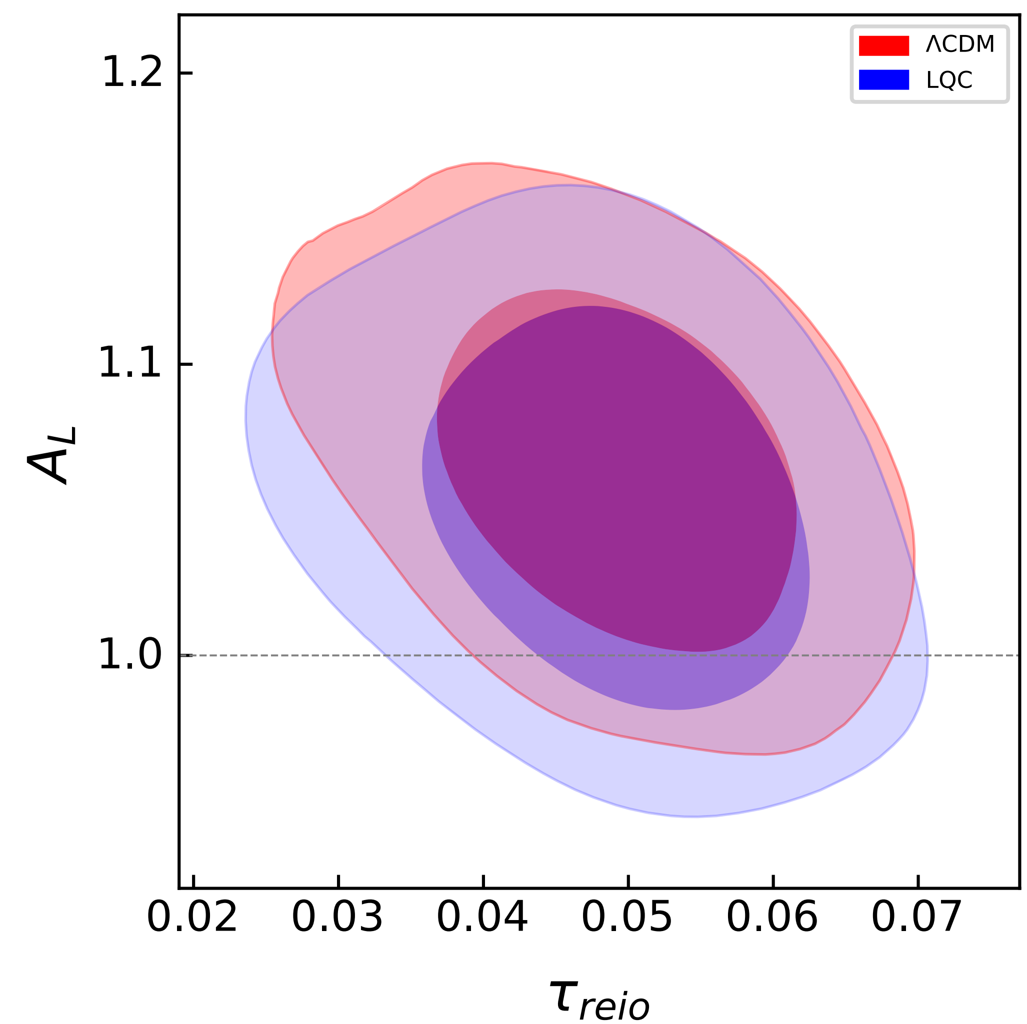
$k_c \sim 0.001$

$a \sim 1.3$

NO-VACUUM: PRELIMINAR PREDICTIONS

[Neves, Martín-Benito, Olmedo, w.i.p.]





- We confirm tension alleviation regarding power suppression at large angular scales, and $A_L = 1$ within 1σ (ΛCDM : $A_L > 1$ at 1.9σ) .

- Optical depth $\tau_{\text{reio}} \longrightarrow k_{LQC}$ [Gupt, 17]

OPEN QUESTIONS

- Universal behaviour independent on the choice of vacuum?

- How non-Gaussianities affect the predictions obtained with A-G and N-O vacua?

[more on non-Gaussianities: A. Wang's talk]

- Inflationary paradigm supplemented with a pre-inflationary bouncing dynamics, or other alternatives:

- Matter bounce / ekpyrosis [Wilson-Ewing, 13] [Li, Saini, Singh, 21]

but, is there a way to obtain a PPS compatible with observations just from first principles in LQC/LQG?

[Relation to Quantum Reduced Loop Gravity, Reduced Phase Space Quantization, Group Field Cosmology ...]

CONCLUSIONS

LQC aims to encode the main quantum gravity effects coming from LQG in cosmological settings.

LQC leads to a well-defined pre-inflationary dynamics, and therefore offers a framework to extract physical consequences of quantum gravity in cosmology.

We are making efforts trying to extract robust predictions from LQC, and determine the role played by our choices of initial conditions.

THANK YOU FOR YOUR ATTENTION

M.C. ESCHER