LQG Dynamics: An Electric Shift in Perspective

Madhavan Varadarajan

(Raman Research Institute)

Introductory Remarks:

- Non-trivial gravitational dynamics is driven by the Hamiltonian constraint. Building on prior work, overcoming key obstacles, Thiemann constructed a Hamiltonian constraint operator in his QSD paper. Construction shows viability of novel techniques used to define QFT in absence of background classical sptime.
- Spectacular achievement but open problems remain:
  1. Many ambiguities in final operator action.
  2. Constraint commutator $[\hat{H}(M), \hat{H}(N)]$ does not reproduce correct lapse dependence of $\widehat{\{H(M), H(N)\}}$.(Desired dependence connected with implementation of spacetime covariance in quantum theory).
  3. Constraint acts ultralocally leading to doubts re:ability to propagate qg perturbations.
- Further progress was handicapped by absence of an instrinsically 3d picture of classical evolution generated by Ham constraint. (Recall:Existence of such a picture for diffeo constraints is what ensures their correct quantum implementation).
- Situation has changed recently (Abhay's talk)!!

Classical evolution time can now be thought of in terms of certain spatialdiffeomorphism like transformations generated by the Electric Shift$\sim \frac{NE_i^a}{q^\alpha}$.

A Ham constraint operator whose action on quantum states incorporates this classical feature would be physically compelling.

This talk is devoted to the discussion of a constraint action for Euclidean LQG which achieves this.

Hence, in the new constraint operator, quantum state transformations generated by the quantum Electric Shift operator $\widehat{Nq^{-\frac{1}{3}}E_i^a}$ will play a key role.

A few remarks with regard to the new constraint operator:

1. Ambiguities: Remarkably the new constraint oprtr action shares many features with Thiemann's QSD oprtr action. However, does not suffer from Perez's spin j ambiguity. Many of the remaining ambiguities arise from how we visualise Electric Diffeomorphisms in the quantum theory.
2. Spacetime Covariance: The new constraint action has been shown to be non-trivially anomaly free with the constraint commutator displaying the correct lapse dependent prefactor.
3. Propagation: As for QSD, the constraint oprtr action is ultralocal. Work over the last decade has shown that such ultralocality is perfectly consistent with propagation: **The Folklore is incorrect!**.

- With slight improvements, I anticipate that we may be able to construct Thiemann's Complexifier Operator which maps solns of Euclidean LQG to those of Lorentzian LQG and thereby may even be able to show existence of spacetime covariant, propagating physical states for Lorentzian LQG!

Plan:

1. **Review of Thiemann's procedure.**
2. **Derivation of the new constraint action: A flavor**
3. **Comments on Anomaly Free Commutators**
4. **Comments on Propagation**
5. **Concluding Remarks**

1.1 Thiemann's procedure in spirit:

- The Problem: Classical Ham constraint depends on local fields like curvature $F(x)$ of connection. Basic connection operators nonlocal holonomies. Classically: Extract F from δ size loop holonomy: $\lim_{\delta\to 0} \frac{h_{\text{small loop}}-1}{\delta^2}$.
  QMly: Limit does not exist on operators. Backgrd indep Hilbert space cant distinguish 'smaller', 'still smaller' loops.
- Solution: A 2 step procedure:
  Step 1: Approximate local connection dep fields in classical Ham constraint by small loop holonomies of coordinate size δ. Get $H_\delta(N)$ which agrees with $H(N)$ in $\delta \to 0$ limit.
  In $H_\delta(N)$ replace holonomies, triads by corresponding operators,get $\hat{H}_\delta(N)$.
  Step 2: Take $\delta \to 0$ limit in suitable operator topology/space of states. Idea is that even tho limits of individual bits and pieces dont exist, limit of conglomeration of approximants which make up $\hat{H}_\delta(N)$, exists.
  NOTE: Final operator action depends on choice of small loops and repn of its holonomy: Ambiguities!.

2. Derivation of the New Constraint Operator: A flavor

- Classical Constraint: $H(N) = \frac{1}{2}\int \epsilon^{ijk} \; Nq^{-\frac{1}{3}} E_i^a \; E_j^b F_{abk}$
  Usually: density wt 1 constraint with $q^{-\frac{1}{2}}$.
  Here higher density 4/3 constraint.
  In quantum theory for density 1 $\hat{H}_\epsilon(N)$, all factors of ϵ cancel.
  Higher density$\Rightarrow$ extra factors of $\hat{q} \sim \hat{V}/\epsilon^3$. Get overall factor of $1/\epsilon$ which is important for non-trivial constr commutators.

- Choose ordering: $\hat{H}(N) = \frac{1}{2}\int \epsilon^{ijk} \hat{F}_{abk} \; \hat{E}_j^b \; \widehat{NE_i^a q^{-\frac{1}{3}}}$.
  "Constraint oprt $\sim \int$ curvature$\times$ triad $\times$ elec shift.
  Evaluate on spin network state S.

- We will now implement Step 1 of Thiemann's procedure and construct the regulated constraint operator $\hat{H}_\epsilon(N)$ by suitably regulating operators above.

$$\hat{H}(N) = \frac{1}{2}\int \epsilon^{ijk}\hat{F}_{abk}\ \ \hat{E}^b_j\ \widehat{NE^a_i q^{-\frac{1}{3}}}.$$

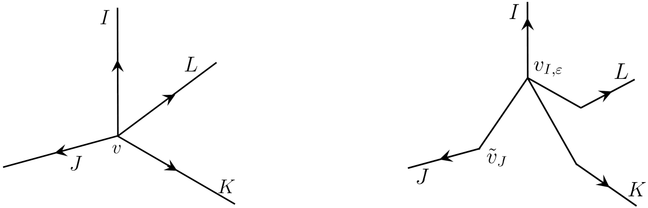
- $\widehat{NE^a_i q^{-\frac{1}{3}}}\ S$: $\hat{q}^{-1/3}$ defined thru Volume oprtr. Acts at vertices of S,changes intertwiner. $\hat{E}^a_i$ acts on each edge: insertion of τ_i on Ith edge + factor of edge tangent vector.
  Regulated Constraint Action at vertex v: $iN(v,\{x\})\sum_I \hat{e}^b_{I,\epsilon}\hat{X}_{i,I}S_\lambda$
  $\hat{e}^b_{I,\epsilon}$ is supported in ϵ nbhood of vertex, agrees with $\hat{e}^b_I$ at v as $\epsilon \to 0$.
- $\hat{E}^b_j$: Inserts τ_j in e_J in j_J repn + factor of Jth edge tangent vector.
- Integrating, get:
  $N(v)\sum_{I,J}\int_{e_J} dt_J \dot{e}^a_J(\hat{e}^b_{I,\epsilon}\hat{F}_{ab\ k})\epsilon^{ijk}(h_{e_J}(1,0)\tau_j)^{A_J}{}_{B_J}\frac{\partial \hat{X}_{i,I}S_\lambda(A)}{\partial h^{A_J}_{e_J\ B_J}}$
  Usual procedure would be to approximate $F_{ab\ k}$ for each k by trace of small loop holonomy with τ_k (in some arbitrarily chosen spin repn). This would not allow us to make further progress.
  Instead , replace $\epsilon^{ijk}\tau_j$ by $[\tau^k,\tau^i]$. This yields combination $\hat{e}^b_{I,\epsilon}\hat{F}_{ab\ k}\tau^k$ with k summed over. So we get the entire curvature and do not have to deal with each internal component separately!
  Note that τ_k above is in the same(spin j_J)repn as τ_j.

The $\widehat{curvature}$ contribution $\hat{e}^b_{I,\epsilon}\hat{F}_{ab\,k}\tau^k$

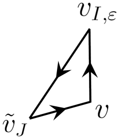
- $\hat{e}^b_{I,\epsilon}F^k_{ab}$ turns out to be exactly the Gauge Covariant Lie derivative of A with respect to $\hat{e}^a_{I,\epsilon}$. This allows us to approximate the curvature term $\hat{e}^b_{I,\epsilon}\hat{F}_{ab\,k}\tau^k$ in terms of an ϵ size finite transformation of A determined by $\hat{e}^b_{I,\epsilon}$.
- The finite transf involves the ϵ size diffeomorphism $\phi_{I,\epsilon}$ generated by $\hat{e}^b_{I,\epsilon}$. Since $\hat{e}^b_{I,\epsilon}$ comes from the quantum electric shift, we refer to $\phi_{I,\epsilon}$ as an Electric Diffeomorphism.
- Replacing the curvature term $\hat{e}^b_{I,\epsilon}\hat{F}_{ab\,k}\tau^k$ by its finite transformation approximant reduces the constraint action, astonishingly, to a simple expression involving a small loop holonomy approximant to the curvature: $h_{l_{IJ,\epsilon}} - 1$.

- The holonomy is the j_J repn. This fixes Perez's spin j ambiguity: The small loop spin label is tailored to that of the spin net edge!
- The small loop $l_{IJ,\epsilon}$ is constructed from the image of e_J by the Electric Diffeomorphism $\phi_{I,\epsilon}$.

$\phi_{I,\epsilon}$ is visualised as an abrupt pull along the Ith edge, consistent with fact that as $\epsilon \to 0$, $\hat{e}^b_{I,\epsilon}$ goes over to the Ith edge tangent which vanishes everywhere except at v.



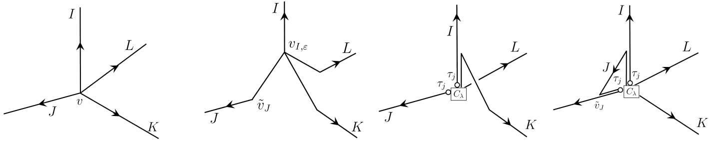
Action of $\phi_{I,\epsilon}$ in a nbrhood of vertex v in S



The loop $l_{IJ,\epsilon}$

Finally...

After a little more work the final picture is as follows:
The constraint action at a vertex of S deforms the vertex structure through the introduction of the small loops $l_{IJ,\epsilon}$. The vertex contributions are then summed over independently reflecting the ultralocality of the action. Pictorially, at a vertex v, the constraint generates:



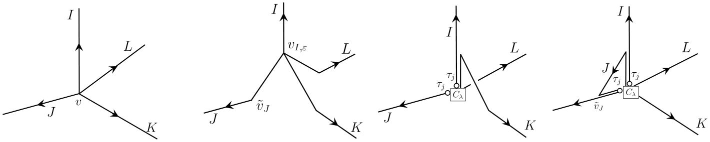
$S_{undeformed}$ $\quad$ $S^{(\epsilon)}_{electric\ diffeo}$ $\quad$ $S^{(\epsilon)}_{propagation}$ $\quad$ $S^{(\epsilon)}_{propagation}$

The first set of 'electric diffeomorphism' type spin nets is responsible for non-trivial anomaly free constraint algebra. Next two do not contribute to the constraint commutator but contribute to propagation.

4. The Issue of Anomaly Free Constraint Commutators

- Classical 3+1 Hamiltonian formulation of a theory of spacetime butchers spacetime into space and time.
- Question: Is there a structure in the Hamiltonian formulation which encodes the 4d spacetime covariance of the system?
  Answer (H-K-T): YES! The constraint algebra of such a system has a certain characteristic structure. Thus 4d covariance of 3+1 Hamiltonian GR is encoded in the structure of its constraint algebra.
- The most non-trivial part of this in Euclidean case is:
  $\{H(M), H(N)\} = D(\vec{A})$ where $D(\vec{A})$ is the diffeo constraint smeared with shift $A^a = q^{ab}(N\nabla_b M - M\nabla_b N)$. In quantum theory, if the constraint commutators reflect this structure so that
  $[\hat{H}(M), \hat{H}(N)] = i\hbar\widehat{D(\vec{A})}$, we expect the emergent classical theory to be spacetime covariant.
- To get $NdM - MdN$ prefactor require 2nd constraint to act on vertex created by first so that 2nd lapse evaluated at $v + \epsilon$ leading to desired $M(v+\epsilon)N(v) - N(v+\epsilon)M(v)$ (times $1/\epsilon$ from higher density!). This does not happen in QSD (Problem of Sptime Cov). But happens with new constraint...

- $\hat{H}_\epsilon(N)$ generates $S^{(\epsilon)}_{elec\ diffeo}$, $S_{undeformed}$, $S^{(\epsilon)}_{prop}$.



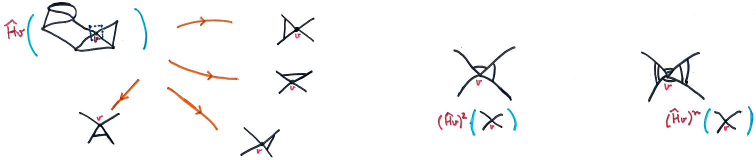
$S_{undeformed}$ $\quad$ $S^{(\epsilon)}_{electric\ diffeo}$ $\quad$ $S^{(\epsilon)}_{propagation}$ $\quad$ $S^{(\epsilon)}_{propagation}$

- Define $\hat{H}_\epsilon(N)$ on a suitable ‘habitat’ of states which live in the algebraic dual i.e $\Psi_f = \sum_{\bar{S}} c_{\bar{S},f} < \bar{S}|$.
  Ψ_f labelled by function f on Σ, spin net coefficients $c_{\bar{S},f}$ depend on evaluations of f on vertices of $\bar{S}$.
- Evaluate $\lim_{\epsilon\to 0} \Psi_f(\hat{H}_\epsilon(N)|S>)$. Schematically:

- $\Psi_f(|S^{(\epsilon)}_{prop}\rangle)$ vanishes as $\epsilon \to 0$.
- Evaluation of Ψ_f on $S^{(\epsilon)}_{electric\ diffeo}$, $S_{undeformed}$ gives:
  $\sim N(v)(f(v+\epsilon) - f(v)/\epsilon) \xrightarrow[\epsilon\to 0]{} N\partial f$.
- $\Psi_f(\hat{H}_{\epsilon'}(M)\hat{H}_\epsilon(N)|S>)|_{\epsilon',\epsilon\to 0} \sim M\partial(N\partial f)$.
- $\Psi_f([\hat{H}(M), \hat{H}(N)]|S\rangle) \sim (M\partial N - N\partial M)f$: get correct lapse dependence.

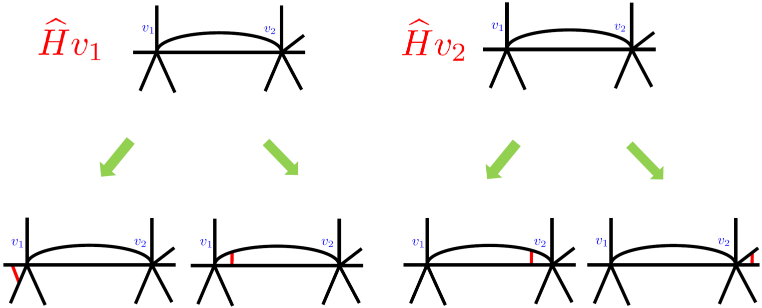
4. Propagation

The Hamiltonian constraint acts only at vertices of a spin net S.
At each vertex only a small neighborhood of v in S is affected.
Constraint action at one vertex indep of action at other.
Action is said to be ultralocal.



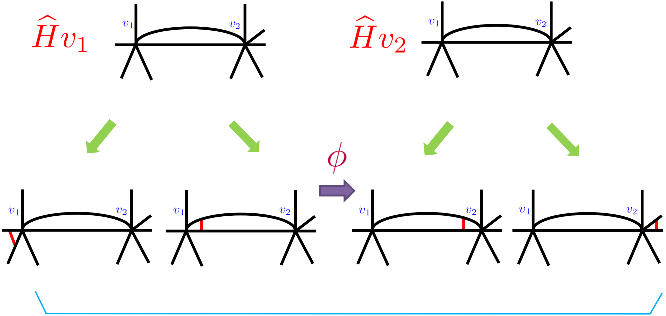
Clearly, repeated action of constraint only lead more ‘vertex embroidery’ but cannot propagate embroidery from one vertex of S to another. This lead to a folklore that ultra local action is incompatible with propagation and hence could not lead to the correct classical limit.

- Of course we do not have a true Hamiltonian, so propagation should be articulated in terms of properties of physical states.
  A physical state Ψ is a sum of bras:
  $\Psi = \sum_{\bar{S}} c_{\bar{S}} < \bar{S}|$ s.t. $\lim_{\epsilon \to 0} \Psi(\hat{H}_\epsilon(N)|S\rangle) = 0$ for all S
  Call states generated by action of $\hat{H}_\epsilon(N)$ on $|S\rangle$ as
  "Children of Parent $|S\rangle$".
  For $|S\rangle$ such that action of $\hat{H}_\epsilon(N)$ on $|S\rangle$ generates ket correspondents of these bras, nontrivial eqns result. Ψ will be linear combination of children of a set of parents.
- By considering lapses of support only around individual vertices of the parental graph, we get eqns at each vertex for coefficients $c_{\bar{S}}$.
- If these equations are such that the presence in Ψ of children with distortions at v_1 necessarily implies presence in Ψ of other children with distortions at v_2, then we say that distortions propagate from v_1 to v_2. Here by presence in Ψ I mean that coefficients of these children are non-vanishing in the sum representing Ψ.
- As we now illustrate, ultralocality implies no propagation.



Ultralocality implies no vertex coupled equations and no propagation.

This picture changes drastically with diffeo invariance!



VERTEX COUPLING OF EQUATIONS

Ultralocality is not a sharply defined notion in a diffeo inv setting!
States are diffeo inv and no-prop intuition **no longer holds**.

Visually, can put these states in sequence:

5. Concluding Remarks

- We constructed a constraint action for Euclidean LQG which carries an imprint of the key feature of classical dynamics, namely the role of the Electric Shift. Action has been shown to support anomaly free commutators, is expected to be consistent with propagation and there is enhanced control/understanding of ambiguities.
- Work in progress suggests that with minor improvements to constraint action and the habitat states, the density wt 1 constraint can also be shown to be non-trivially anomaly free. Improvements may also facilitate the implemnttn of Thiemann's Complexifier mapping of Euclidean to Lorentzian physical states.
- Many interesting issues to discuss: Should we demand state inv under electric diffeos? Does that remove moduli a la Rovelli-Fairbairn? Consequences of remaining ambiguities? Is the 'amount' of propagation too much/too little or just right? Repercussions for LQC? Complexifier based quantization of LQC?
- Suppose we show existence of propagating physical states of Lorentzian LQG for a complexifier rotated Euclidean constraint, what next?

A few useful references:

1. Classical Dynamics: Universe 7 (2021) 1, 13, e-Print: 2012.12094 [gr-qc] (Ashtekar, MV)
2. New action: Class.Quant.Grav. 38 (2021) 13, 135020, e-Print: 2101.03115 [gr-qc] (MV)
3. Non-trivial anomaly free commutators: e-Print: 2205.10779 [gr-qc] (MV: Comments Welcome!)
4. Propagation in LQG e-Print: 2112.03992 [gr-qc] (Thiemann, MV).
5. Propagation in PFT Class.Quant.Grav. 34 (2017) 1, 015012 e-Print: 1609.06034 [gr-qc] (MV)
6. Complexifier: Class.Quant.Grav. 13 (1996) 1383-1404 e-Print: gr-qc/9511057 [gr-qc] (T. Thiemann)
   Phys.Rev.D 53 (1996) 2865-2869, e-Print: gr-qc/9511083 [gr-qc] (A. Ashtekar)
   Class.Quant.Grav. 36 (2019) 1, 015016, e-Print: 1808.00673 [gr-qc] (MV).