Symmetries of asymptotically flat spacetimes

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Based on work in collaboration with

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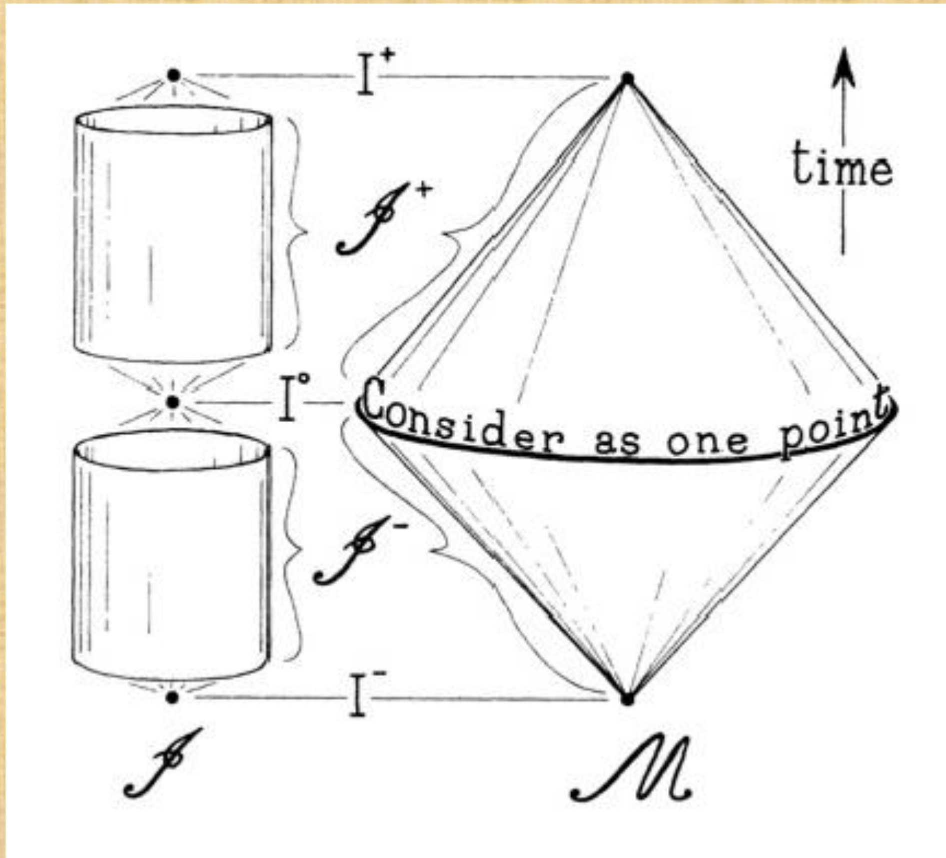
Asymptotically flat spacetimes

- Useful idealization to describe isolated systems in GR
- Underlies important ideas in classical and quantum gravity:
  - Gravitational Waves
  - BHs and their evaporation
- Asymptotic boundary allows definition of a rich class of observables:
- (classical/quantum) gravitational scattering



[Penrose '63]

Symmetries of asymptotically flat spacetimes (I)

- First systematic study of AF spacetimes [Bondi, van der Burg, Metzner '62; Sachs '62] revealed an ``enlarged Poincare group'' at null infinity

$$\mathrm{BMS} = \mathrm{Conf}(S^2) \ltimes C^\infty(S^2)$$

Lorentz group — SuperTranslations $\supset$ translations: $\ell = 0, 1$

- Physical consequences for classical and quantum gravity
  - Gravitational memory [Zeldovich Polnarev '74; Braginsky Thorne '87; Christodoulou '91; Strominger Zhiboedov '16]
  - Non-trivial vacua space of the gravitational field [Ashtekar '81]
  - + more…

Spacetime metric near null infinity

$$ds^2 \overset{r\to\infty}{=} -du^2 - 2du dr + r^2 \left(q_{ab} + \frac{1}{r} C_{ab} \right) dx^a dx^b + \cdots$$

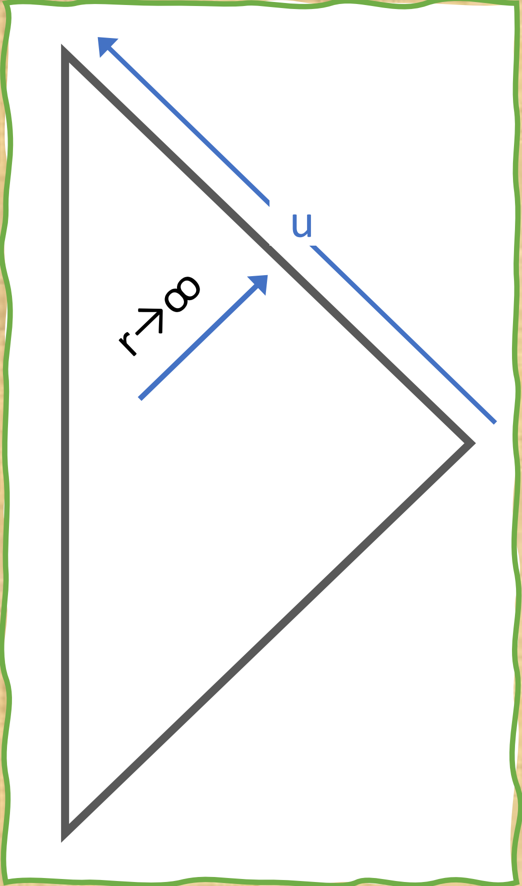
BMS generators:

- Lorentz: $\xi_V = V^a(x)\partial_a + \cdots, \quad V^a$: CKV of q_{ab}
- supertranslations: $\xi_f = f(x)\partial_u + \cdots$

$$\delta_V C_{ab} = \left(\mathcal{L}_V - \tfrac{1}{2} D_c V^c + \tfrac{u}{2} D_c V^c \partial_u \right) C_{ab}$$

$$\delta_f C_{ab} = f \partial_u C_{ab} - 2(D_a D_b)^{\mathrm{TF}} f$$

Inhomogeneous shift



- r: radial coordinate
- $u \sim t - r$: retarded time
- $x = x^a$: coordinates on S^2
- q_{ab}: unit sphere metric
- $C_{ab}(u, x)$: STF 2d tensor "shear"

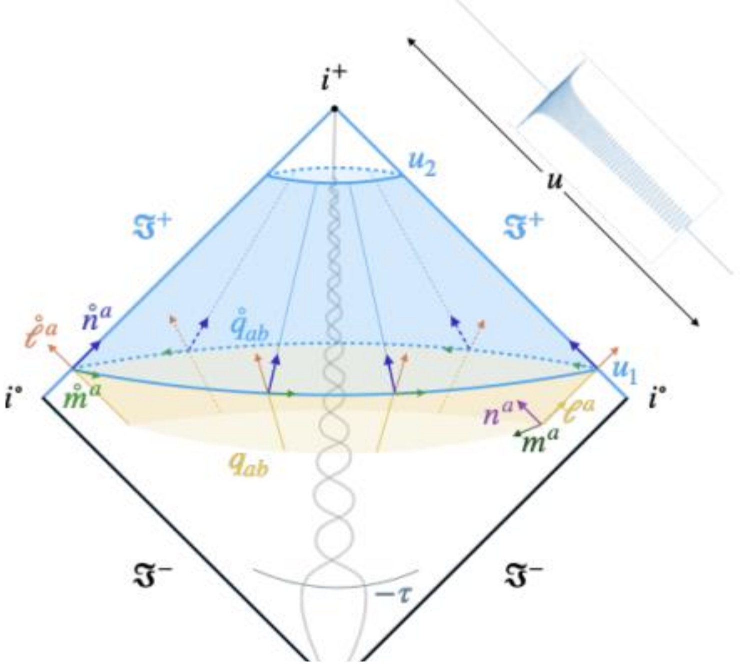
Gravitational memory

- In general $\lim_{u\to+\infty} C_{ab}(u,x) \neq \lim_{u\to-\infty} C_{ab}(u,x)$
- Formula for this difference has applications for GW observations/simulations [Favata '10; Ashtekar, De Lorenzo, Khera '19; Mitman et al '20; …]

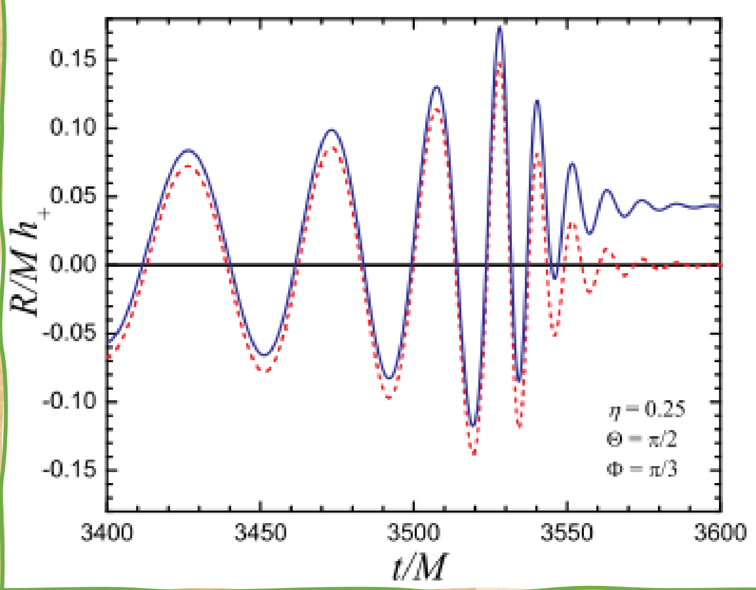
$$\Delta C_{ab}(x) = \partial_a n_\mu \partial_b n_\nu \left(4G \sum_{\text{"in particles"}} \frac{p_i^\mu p_i^\nu}{n \cdot p_i} - (\text{"out particles"}) - \frac{1}{2} \int du d^2x' N^{ab}(u,x') N_{ab}(u,x') \frac{n'^\mu n'^\nu}{n \cdot n'} \right)$$

$$n^\mu = (1, \hat{n})$$

- In frequency space, memory manifests as a 1/ω pole $\tilde{C}_{ab}(\omega) \overset{\omega\to 0}{=} \frac{i}{\omega} \Delta C_{ab} + \cdots$
- Classical counterpart of a ``soft theorem'' in scattering amplitudes [Weinberg '65]



[Ashtekar, De Lorenzo, Khera '19]



[Favata '10]

BMS charges and their conservation

- The space of shears at null infinity comes equipped with a natural symplectic structure that is invariant under BMS

[Ashtekar Streubel ’81]

$\Longrightarrow$ BMS generators or charges

supermomenta P_f and angular momenta $\mathring{J}_V$

- BMS Poisson Bracket algebra

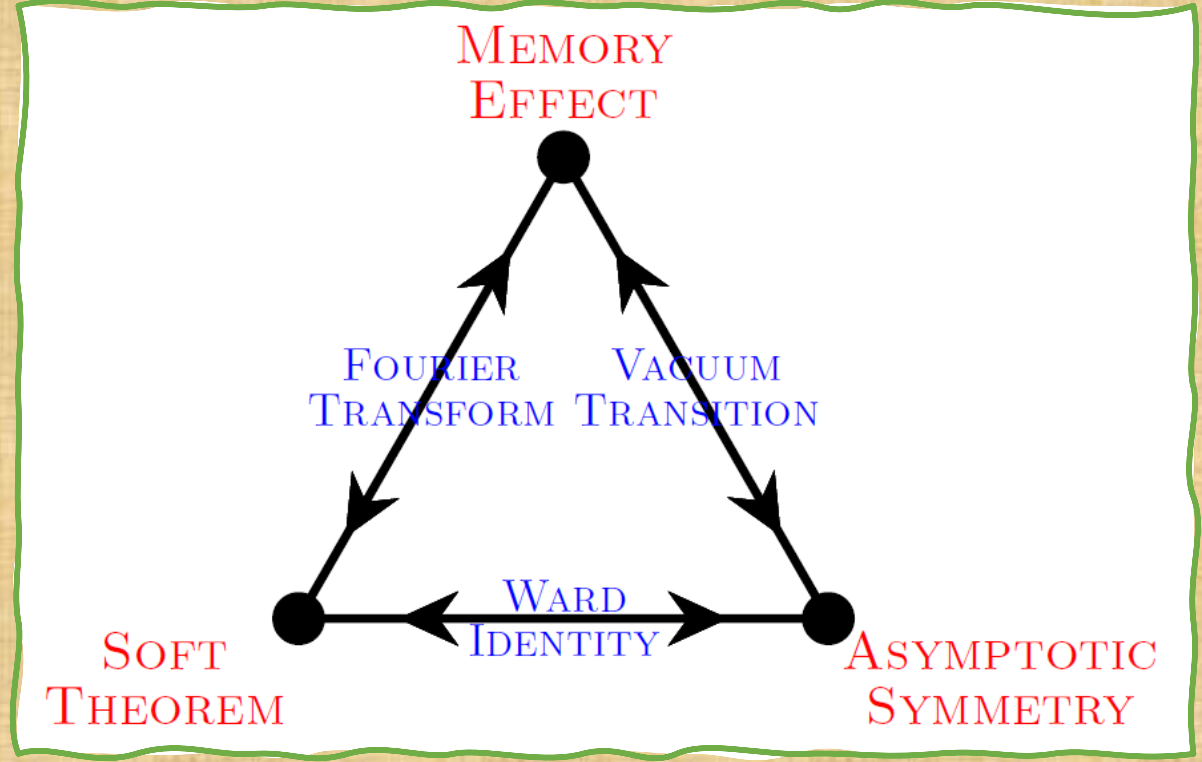
$$\{P_f, P_{f'}\} = 0, \quad \{\mathring{J}_V, P_f\} = P_{V(f)}, \quad \{\mathring{J}_V, \mathring{J}_{V'}\} = \mathring{J}_{[V,V']}$$

- There exists 2 copies of BMS algebras: at future and past null infinity
- Memory formula can be interpreted as a conservation of BMS charges → single BMS acting simultaneously at both infinities

[Strominger ’14] [He, Lysov, Mitra, Strominger ’14]

Strominger's infrared triangle(s)

[Strominger '17]



Symmetries of asymptotically flat spacetimes (II)

- Another well known asymptotic symmetry enhancement occurs in asymptotically AdS_3 spacetimes [Brown Henneaux ’86]

$$\text{Conf}(\partial AdS_3) \rightarrow \text{Conf}^*(\partial AdS_3)$$

- Analogous enhancement can be obtained in asym flat 4d [Barnich Troessaert ’10]

$$\text{Lorentz} = \text{Conf}(S^2) \rightarrow \text{Conf}^*(S^2) \qquad (\text{BMS} \rightarrow \text{Extended BMS})$$

SuperRotations

- Extended symmetry predicts a new soft thm [Kapey, Lysov, Pasterski, Strominger ‘14]

$$\tilde{C}_{ab}(\omega) \overset{\omega\to 0}{=} \frac{1}{\omega} C^{(0)}_{ab} + C^{(1)}_{ab} + \cdots$$

$C^{(0)}_{ab}$ → STs

$C^{(1)}_{ab}$ → SRs

- EBMS symmetry brings a CFT_2 perspective on scattering amplitudes
- Actively studied in the past few years (keyword: “celestial holography”)
- Properties of amplitudes reinterpreted in CFT language, leading to new insights. Example:

[Guevara, Himwich, Pate, Strominger ‘21]

Collinear limit in scattering amplitudes ⟷ OPE in putative CFT ⟶ Higher spin symmetries

[Freidel, Pranzetti, Raclariu ‘21]

- Initial developments restricted to tree-level amplitudes. Many current efforts to incorporate loop corrections (later in the talk)

Symmetries of asymptotically flat spacetimes (II+I)

- From the perspective of "local holography" (Freidel's talk) it is natural to consider a different extension: Generalized BMS [MC, Laddha '14]

$$\text{GBMS} = \text{Diff}(S^2) \ltimes C^\infty(S^2)$$

- Subleading soft thm suggests existence of non-trivial Diff(S^2) charges
- This requires treating the 2d metric at null infinity as dynamical (an example of "dynamical frame covariance"? Höhn's talk)
- Similar extension is implicit in the EBMS case

$$ds^2 \overset{r\to\infty}{=} -\frac{\mathcal{R}}{2}du^2 - 2dudr + r^2\left(q_{ab} + \frac{1}{r}(C_{ab} + uT_{ab})\right)dx^a dx^b + \cdots$$

[Barnich Troessaert '10; MC Laddha '16; Compere Long '16, Compere Fiorucci Ruzziconi '18]

- $\mathcal{R}$: scalar curvature of q_{ab}
- T_{ab} STF defined by $D^b T_{ab} = -\frac{1}{2}D_a\mathcal{R}$

Rewriting of tensor introduced in [Geroch '76]
Related to ``boundary stress tensor'' in AdS_3

- GBMS action on elementary fields:

$$\begin{aligned}\delta_V C_{ab} &= \left(\mathcal{L}_V - \tfrac{1}{2}D_cV^c + \tfrac{u}{2}D_cV^c\partial_u\right)C_{ab}\\ \delta_f C_{ab} &= f\partial_u C_{ab} - 2(D_aD_b)^{\mathrm{TF}}f + T_{ab}f\end{aligned}$$

$$\begin{aligned}\delta_V q_{ab} &= (\mathcal{L}_V - D_cV^c)q_{ab}\\ \delta_V T_{ab} &= \mathcal{L}_V T_{ab} - (D_aD_b)^{\mathrm{TF}}D_cV^c\end{aligned}$$

$$\begin{aligned}\delta_f q_{ab} &= 0\\ \delta_f T_{ab} &= 0\end{aligned}$$

- EBMS can be obtained as a singular limit of GBMS
- GBMS can be obtained by certain smearing of EBMS

[Donnay Pasterski Puhm '20]

Enlarged phase space and GBMS charges

- If GBMS is a symmetry: need to enlarge standard phase space at null infinity

$$C_{ab}(u,x) \to (C_{ab}(u,x), q_{ab}(x))$$

- GR symplectic structure diverges in this case
- Divergences may be absorbed in corner ``counterterms'' [Compere Fiorucci Ruzziconi '18; Donnay, Nguyen, Ruzziconi '22]
- Finite symplectic structure can be determined by consistency with GBMS algebra + soft thm [MC Peraza '20]

Enlarged phase space and GBMS charges

[MC Peraza'20]

$$\Omega = \Omega^{\mathcal{I}} + \Omega^{S^2}$$

$$\Omega^{\mathcal{I}} = \int_{\mathcal{I}} \sqrt{q}(\delta\partial_u C^{ab} \wedge \delta C_{ab})$$

$$\Omega^{S^2} = \int_{S^2} \sqrt{q}(\delta p^{ab} \wedge \delta q_{ab} + \delta\Pi^{ab} \wedge \delta T_{ab})$$

$$\overset{1}{N}_{ab}(x) := \int_{-\infty}^{\infty} u\partial_u C_{ab}(u,x)du$$

$$\begin{aligned} p^{ab} &= D^{(a}D_c\overset{1}{N}^{b)c} - \frac{\mathcal{R}}{2}\overset{1}{N}^{ab} + (\text{quadratic in } C)^{ab}|_{\partial\mathcal{I}} \\ \Pi^{ab} &= 2\overset{1}{N}^{ab} + \frac{1}{2}CC^{ab}|_{\partial\mathcal{I}}. \end{aligned}$$

$$J_V = J_V^{\mathcal{I}} + J_V^{S^2}$$

$$P_f = \int_{\mathcal{I}} dud^2x\sqrt{q}\partial_u C^{ab}\delta_f C_{ab}$$

$$J_V^{\mathcal{I}} = \int_{\mathcal{I}} dud^2x\sqrt{q}\partial_u C^{ab}\delta_V C_{ab}$$

$$J_V^{S^2} = \int_{S^2} d^2x\sqrt{q}\overset{1}{N}^{ab}\left[2\delta_V T_{ab} + D_{(a}D^c\delta_V q_{b)c} - \frac{\mathcal{R}}{2}\delta_V q_{ab}\right] + (\text{quadratic in } C_{ab})|_{\partial\mathcal{I}}$$

[MC Peraza'20; MC Laddha '21]

- To get simpler expressions, it is useful to consider the case

$$q_{ab} = 2dzd\bar{z}, \quad \delta q_{ab} = 0 \qquad T_{ab} = 0, \quad \delta T_{ab} \neq 0$$

Relevant sector for EBMS

$$J_V = J_V^{\mathcal{I}} + J_V^{S^2}$$

$$J_V^{\mathcal{I}} = \int_{\mathcal{I}} dud^2x\sqrt{q}\partial_u C^{ab}\delta_V C_{ab}$$

$$\overset{1}{N}_{zz} = \int_{-\infty}^{\infty} u\partial_u C_{zz} du$$

$$J_V^{S^2} = -2\int d^2z\,\partial_z^3 V^z(\overset{1}{N}^{zz} + \frac{1}{4}[C^{zz}C]_{\partial\mathcal{I}}) + c.c.$$

$$C_{zz}|_{u=\pm\infty} = -2\partial_z^2 C^{\pm}$$

- Global CKV $\leftrightarrow$ $\partial_z^3 V^z = 0$: J_V reduces to angular momentum
- New O(C^2) boundary term not present in the Barnich-Troessaert charge
  - Ensures covariance wrt supertanslations: $\delta_f J_V = P_{V(f)} = -\delta_V P_f$
  - Consistent with 1-loop correction to subleading soft thm

[Donnay, Nguyen, Ruzziconi '22; Pasterski '22]

- Soft theorems:
  - Classical and quantum
  - Tree vs loop level
  - Leading, subleading, sub-subleading, …
  - …..

Soft expansion of gravity amplitudes

Leading:

$$\mathcal{A}_{n+1} \overset{\omega\to 0}{=} \omega^{-1} S^{(0)} \mathcal{A}_n + \cdots$$

$$\lim_{\omega\to 0} \omega \mathcal{A}_{n+1} = S^{(0)} \mathcal{A}_n$$

[He, Lysov, Mitra, Strominger]

conservation law for P_f

$$S^{(0)} = \varepsilon_{\mu\nu} \sum_i \frac{p_i^\mu p_i^\nu}{n \cdot p_i}$$

[Weinberg]

Subleading (tree-level):

$$\mathcal{A}_{n+1} \overset{\omega\to 0}{=} \omega^{-1} S^{(0)} \mathcal{A}_n + \omega^0 \widehat{S}^{(1)} \mathcal{A}_n + \cdots$$

$$\lim_{\omega\to 0} \partial_\omega \left(\omega \mathcal{A}_{n+1}\right) = \widehat{S}^{(1)} \mathcal{A}_n$$

[Kapec, Lysov, Pasterski, Strominger; MC Laddha]

conservation law for J_V

[Cachazo-Strominger, White,...]

$$\widehat{S}^{(1)} = \varepsilon_{\mu\nu} \sum_i \frac{p_i^\mu n_\rho}{n \cdot p_i} \hat{J}_i^{\rho\nu}$$

Loop corrections to subleading soft thm

Two ways to think of them:

$$\frac{\mathcal{A}_{n+1}}{\mathcal{A}_n} \overset{\omega\to 0}{=} \omega^{-1} S^{(0)} - \log(\omega) S^{(\log)} + \cdots$$

[Sahoo, Sen]

$$\mathcal{A}_n^{(\epsilon)} = e^{\frac{1}{\epsilon} K_n} \mathcal{A}_n^{\text{IR-finite}}$$

[Weinberg]

$$\frac{1}{\epsilon} \sim -\log \Lambda_{\text{IR}}$$

$$\mathcal{A}_{n+1}^{(\epsilon)} \overset{\omega\to 0}{=} \left(\omega^{-1} S^{(0)} + \frac{1}{\epsilon} S^{(\log)} \right) \mathcal{A}_n^{(\epsilon)} + \cdots$$

[Bern, Davies, Nohle]

$S^{(\log)}$

$$K^{\rm reg}_{\rm gr} = \frac{i}{2}\sum_{\substack{a,b\\ b\neq a}}\frac{1}{4\pi}\ \ln\omega^{-1}\ \frac{\{(p_a.p_b)^2-\frac{1}{2}p_a^2p_b^2\}}{\sqrt{(p_a.p_b)^2-p_a^2p_b^2}}\left\{\delta_{\eta_a\eta_b,1}-\frac{i}{2\pi}\ln\left(\frac{p_a.p_b+\sqrt{(p_a.p_b)^2-p_a^2p_b^2}}{p_a.p_b-\sqrt{(p_a.p_b)^2-p_a^2p_b^2}}\right)\right\}. \qquad (6.26)$$

taken from [Sahoo Sen '18]

$$\log(\omega^{-1})S^{(\log)} = \widehat{S}^{(1)}_{gr}\ K^{\rm reg}_{\rm gr} + \frac{1}{4\pi}(\ln\omega^{-1}+\ln R^{-1})\left[i\sum_{\substack{b\\ \eta_b=-1}}k.p_b\sum_a\frac{\varepsilon_{\mu\nu}p_a^\mu p_a^\nu}{p_a.k}+\frac{1}{2\pi}\sum_a\frac{\varepsilon_{\mu\nu}p_a^\mu p_a^\nu}{p_a.k}\sum_b p_b.k\ \ln\frac{m_b^2}{(p_b.\hat{k})^2}\right]. \qquad (6.29)$$

$$\widehat{S}^{(1)} = \varepsilon_{\mu\nu}\sum_i\frac{p_i^\mu k_\rho}{k\cdot p_i}\hat{J}_i^{\rho\nu}$$

$$K = {\rm Re}K + i{\rm Im}K_n \implies S^{(\log)} = {\rm Re}S^{(\log)} + i{\rm Im}S^{(\log)}$$

${\rm Re}S^{(\log)}$: "quantum soft factor"

$i\,{\rm Im}S^{(\log)}$: "classical soft factor" → $1/u$ tail in the shear: $C_{ab} \overset{u\to\pm\infty}{=} C^{\pm}_{ab} + O(1/u)$

[Laddha, Sen'18]

Loop corrections and superrotations

- "Quantum" $S^{(\log)}$ term in pure gravity obtained from superrotation Ward Id applied to Loop-corrected amplitude [Donnay, Nguyen, Ruzziconi '22; Pasterski '22]

$$\mathcal{M} = \mathcal{M}_{\text{soft}}\,\mathcal{M}_{\text{finite}}$$

$$\mathcal{M}_{\text{soft}} = \langle \mathcal{W}_1 \dots \mathcal{W}_n \rangle = \exp\left[-\frac{1}{2}\sum_{i\neq j}^{n} \eta_i \eta_j \omega_i \omega_j \langle C^{(0)}(z_i,\bar{z}_i) C^{(0)}(z_j,\bar{z}_j)\rangle\right]$$

$$\langle C^{(0)}(z_i,\bar{z}_i) C^{(0)}(z_j,\bar{z}_j)\rangle = \frac{\hbar}{\epsilon}\frac{\kappa^2}{(4\pi)^2}\,|z_{ij}|^2 \ln|z_{ij}|^2\,.$$

[Himwich, Narayanan, Pate, Paul, Strominger '20]
[Nguyen Salzer '21]

- “Classical” $S^{(\log)}$ leads to a divergence in classical J_V !

$$\partial_u C_{ab}(u,x) \overset{|u|\to\infty}{=} O(1/u^2) \iff \widetilde{C}_{ab}(\omega,x) \overset{\omega\to 0}{=} O(1/\omega) + O(\ln\omega)$$

$$\overset{1}{N}_{zz} = \int_{-\infty}^{\infty} u\partial_u C_{zz} du = \infty$$

- Divergence is absent for global CKVs
- Related divergence in the angular momentum of asymptotic particles

$$r_a^\mu(\sigma) = \eta_a \frac{1}{m_a} p_a^\mu \sigma + c_a^\mu \ln|\sigma| + \cdots ;$$

$$\mathbf{J}_a^{\mu\nu} \simeq r_a^\mu(\sigma) p_a^\nu - r_a^\nu(\sigma) p_a^\mu + \text{spin} = (c_a^\mu p_a^\nu - c_a^\nu p_a^\mu) \ln|\sigma| + \cdots .$$

- Related Log(r) terms in the Bondi expansion of the metric [Damour ‘86; Christodoulou ’02; Kehrberger ‘21]
  - These lead to a divergent angular momentum aspect (but total angular momentum still finite) [Winicour ’85]

Superrotations after loop corrections: Summary

- Currently in the strange situation where quantum case is better understood than classical one
- An important ingredient so far missing in the analysis is the use of ``dressed'' asymptotic states for IR-finite S matrix [Ashtekar '86; Ware, Saotome, Akhoury '13;….]
- We have made some progress in understanding classical+quantum $S^{(\log)}_{QED}$ in an analogue problem in QED [MC, Laddha '19; Bhatkar '19 '20] but analogy has its limitations → need to go back to gravity! wip with Laddha and Bhatkar

Discussion

- The infrared sector of gravity exhibits a rich structure which has not yet been fully understood
- Interesting interplay of ideas from different areas: GW physics, amplitudes, quantum gravity,…
- ``Infrared triangles'' in other theories (Yang-Mills, QED,…) (Peraza's talk)
- Higher order soft theorems point towards an infinite tower of symmetries (Pranzetti's and Peraza's talks) (relation to "generalized GCL"? Ashtekar's talk)

Related topics I did not have time/knowledge to mention:

- Symm in asym flat gravity in d=3 [Barnich, Compere, Geiller, Gomberoff, Gonzalez, Oblak, Troessaert, …] and d>4 [Hollands Ishibashi '03 Tanabe, S. Kinoshita, and T. Shiromizu '11 Kapec Lysov Pasterski Strominger '17 Aggarwal '18 Colferai Lionetti '20 Campoleoni Francia Heissenberg '21 Capone '21]
- Group theoretical aspects of (extended) BMS [Barnich Ruzziconi '21 Prinz Schmeding '21…]
- Asym symm from the perspective of spatial infinity [Ashtekar Hansen '78 Virmani '11 Compere Dehouck '11 Troessaert '17 Prabhu '19, Henneaux,…]
- Approaches to Flat-space holography (….)

Thank you for listening!