Toward black hole entropy in chiral loop quantum supergravity

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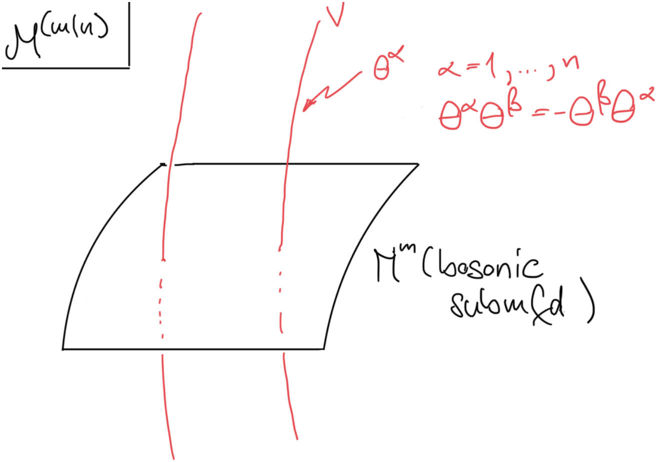
Section 1

Introduction

Supermanifold

Phase space of field theories containing both bosonic & fermionic d.o.f. has structure of **supermanifold**



Super Lie group & algebra

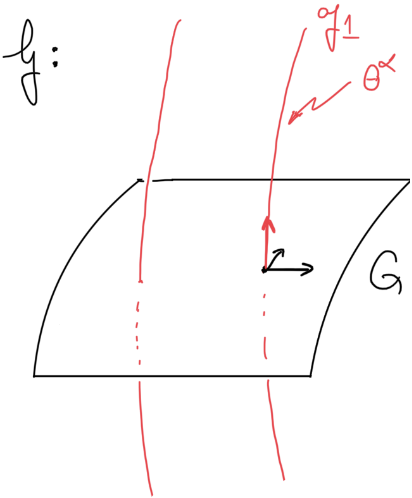
SUSY field theories: Field theories with underlying symmetry group given by a **supergroup**

- Lie superalgebra:

$$\mathfrak{g} := T_e\mathcal{G} = \mathfrak{g}_0 \oplus {\color{red}\mathfrak{g}_1}$$

- Lie bracket:

$$[X, Y] := XY - (-1)^{|X||Y|}YX$$



Supersymmetry

Super anti-de Sitter group $\mathrm{OSp}(\mathcal{N}|4)$

$$\mathfrak{osp}(1|4) = \underbrace{\mathbb{R}^{1,3} \ltimes \mathfrak{so}(1,3)}_{\mathfrak{g}_0} \oplus \underbrace{S_{\mathbb{R}}}_{\mathfrak{g}_1}$$

Generators: P_I, M_{IJ}, Q_α (Majorana spinor)

$$[P_I, Q_\alpha] = -\frac{1}{2L} Q_\beta (\gamma_I)^\beta{}_\alpha$$

$$[M_{IJ}, Q_\alpha] = \frac{1}{2} Q_\beta (\gamma_{IJ})^\beta{}_\alpha$$

$$\{Q_\alpha, Q_\beta\} = \frac{1}{2}(C\gamma_I)_{\alpha\beta} P^I + \frac{1}{4L}(C\gamma^{IJ})_{\alpha\beta} M_{IJ}$$

Further generators $Q^r_\alpha \to \mathcal{N}$-**extended SUSY**!

$AdS_4 := \{x \in \mathbb{R}^5 | -(x^0)^2 + (x^1)^2 + \ldots + (x^3)^2 - (x^4)^2 = -L^2\}$

LQG and supergravity

What has been done?

- Canonical SUGRA adapted to LQG [Jacobson '88]
- Hidden $\mathfrak{osp}(1|2)$-symmetry in constraint algebra, construction of $\mathfrak{osp}(1|2)$-valued connection [Fülöp '94, Gambini+Obregon+Pullin '96]
- Quantization [Gambini+Obregon+Pullin '96, Ling+Smolin '99]

But:
- Considerations remain formal
- Origin of hidden symmetry unclear
- Generalizations?

- Canonical theory for higher D and quantization [Bodendorfer +Thiemann+Thurn '11]

But:
- SUSY not manifest
- Formalism/constraints quite complicated

LQG and supergravity

My goals:

Follow [L+S]: Keep SUSY manifest as much as possible!

- Origin of $\mathfrak{osp}(1|2)$-symmetry? $\rightarrow$ Understand the geometric origins with a view towards generalizations:
  - Immirzi parameters
  - Higher $\mathcal{N} > 1$
  - Boundary theory $\rightarrow$ **black holes**
- Mathematically rigorous formulation, both classically and in quantum theory:
  - Construction of super spin-nets ($\rightarrow$ *super parallel transport*)
  - Structure of Hilbert spaces $\leftrightarrow$ relation to standard quantization of fermions in LQG

Section 2

Supergravity & boundary theory

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Supergravity

AdS (Holst-)Supergravity as 'constrained' gauge theory with gauge group $\mathrm{OSp}(\mathcal{N}|4)$ [D'Auria-Fré-Regge '80, Castellani-D'Auria-Fré '91, KE '21+'22]

Gauge field ($\mathcal{N} = 1$)

$$\mathcal{A} : \mathcal{M} \to \mathfrak{osp}(1|4), \qquad \iota : M \hookrightarrow \mathcal{M}$$

Decomposition

$$\mathcal{A} = \underbrace{\mathrm{pr}_{\mathfrak{g}_1} \circ \mathcal{A}}_{\psi} + \underbrace{\mathrm{pr}_{\mathbb{R}^{1,3}} \circ \mathcal{A}}_{e} + \underbrace{\mathrm{pr}_{\mathfrak{spin}(1,3)} \circ \mathcal{A}}_{\omega}$$

- e: co-frame ($\to$ metric)
- ω: spin connection
- ψ: (spin-3/2) Rarita-Schwinger field

Supergravity and LQG

super Holst-MacDowell-Mansouri action [KE+HS '21]

$$S_{\text{sH-MM}}[\mathcal{A}] = \int_M \iota^* \mathrm{STr}(F[\mathcal{A}] \wedge \mathbf{P}_\beta F[\mathcal{A}])$$

Super Holst-operator

$$\mathbf{P}_\beta := \mathbf{0} \oplus \mathcal{P}_\beta \oplus \mathcal{P}_\beta : \mathfrak{osp}(1|4) \to \mathfrak{osp}(1|4)$$

with

$$\mathcal{P}_\beta := \frac{\mathbb{1} + i\beta\gamma_5}{2\beta} \qquad \beta : \textit{Immirzi}$$

- $\rightarrow$ yields Holst action of $D = 4$, $\mathcal{N} = 1$ AdS-SUGRA + *bdy terms*
- **Can show:** Boundary theory **unique** by SUSY-invariance! [Andrianopoli+D'Auria '14, KE+HS '21]

Extended SUGRA and boundary theory

What about **extended** SUSY? → Consider $\mathcal{N} = 2$ [KE+HS '21]

Super Cartan connection

$$\mathcal{A} = e^I P_I + \frac{1}{2}\omega^{IJ} M_{IJ} + \Psi^\alpha_r Q^r_\alpha + \hat{A} T$$

$\hat{A}$: $\mathrm{U}(1)$ gauge field (*graviphoton*)

super Holst-MacDowell-Mansouri action ($\mathcal{N} = 2$)

$$S^{\mathcal{N}=2}_{\text{sH-MM}}[\mathcal{A}] = \int_M \iota^* \mathrm{STr}(F[\mathcal{A}] \wedge \mathbf{P}_\beta F[\mathcal{A}])$$

$$\mathbf{P}_\beta := \mathbf{0} \oplus \mathcal{P}_\beta \oplus \mathcal{P}_\beta \oplus \mathcal{P}_\beta \oplus \frac{1}{2\beta}(1 + \beta\star)$$

⇒ β literally has interpretation as θ-**ambiguity**!

Section 3

The chiral theory

Chiral Theory

In general: $\mathbf{P}_\beta$ destroys manifest SUSY-invariance!

$\rightarrow$ This changes in **chiral theory ($\beta = \mp i$)**

Holst projection [KE+HS '21]

$$\mathbf{P}_{-i}\,:\ \mathfrak{osp}(\mathcal{N}|4)_{\mathbb{C}} \rightarrow \mathfrak{osp}(\mathcal{N}|2)_{\mathbb{C}}$$

$$M_{IJ} \mapsto T_i^+ = \frac{1}{2}(J_i + iK_i)$$

$$Q_\alpha \mapsto Q_A$$

Q_A: left-handed Weyl-fermion

Chiral Theory

Super Ashtekar connection [KE+HS '21]

$$\mathcal{A}^+ := \mathbf{P}_{-i}\mathcal{A} = A^{+i}T_i^+ + \psi_r^A Q_A^r + \frac{1}{2}\hat{A}_{rs}T^{rs}$$

Chiral action

$$S_{\text{sH-MM}}^{\beta=-i} = \frac{i}{\kappa}\int_M \langle \mathcal{E} \wedge F(\mathcal{A}^+)\rangle + \frac{1}{4L^2}\langle \mathcal{E} \wedge \mathcal{E}\rangle + \underbrace{S_{\text{CS}}^{\mathrm{OSp}(\mathcal{N}|2)}(\mathcal{A}^+)}_{\text{boundary term}}$$

$\mathcal{E}$: *super electric field*

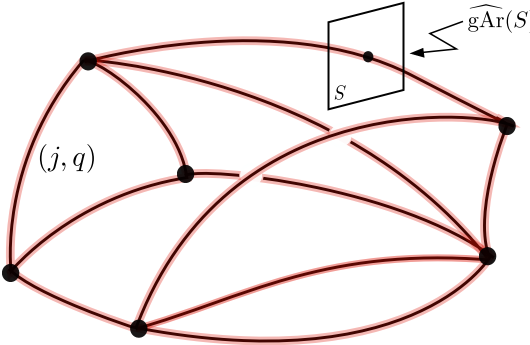
SUSY-invariance: Coupling bulk $\leftrightarrow$ boundary

$$\underset{\Longleftarrow}{F(\mathcal{A}^+)} = -\frac{1}{2L^2}\underset{\Longleftarrow}{\mathcal{E}}$$

Quantum theory: Super spin nets

What do we need for quantum theory?

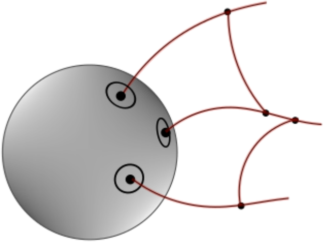
- Holonomies (parallel transport map) ✓
- Hilbert spaces (✓)
- Spin network states ✓



$$\widehat{\mathrm{gAr}}(S) \sim \begin{cases} i l_p^2 \sqrt{j\left(j+\frac{1}{2}\right)} & (\mathcal{N}=1) \\ i l_p^2 \sqrt{j^2-q^2} & (\mathcal{N}=2) \end{cases}$$

SUSY black holes in LQG

Holst-MM in the presence of inner boundary: [KE '22, KE+HS '22]



- Geometric theory induces super CS on inner boundary
- Gauge group: $G = \mathrm{OSp}(\mathcal{N}|2)$

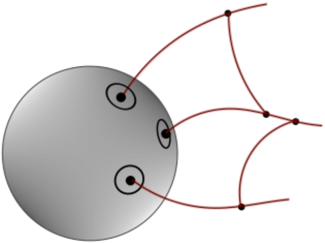
Hilbert space full theory

$$\mathcal{H}_{\gamma}^{\mathrm{full}} = \mathcal{H}_{\gamma}^{\mathrm{bulk}} \otimes \mathcal{H}_{\gamma}^{\mathrm{CS}}$$

Quantum boundary condition

$$\mathbb{1} \otimes \widehat{F}_{\underline{A}}(p) = -\frac{2\pi i}{\kappa k} \widehat{\mathcal{E}}_{\underline{A}}(p) \otimes \mathbb{1}$$

SUSY black holes in LQG



- For $\mathcal{N} = 1$: $G = \mathrm{OSp}(1|2)$
- **Issue:** G complex/non-compact $\rightarrow$ adapt strategy of [Perez et al '14, Noui et al '15]
- Consider compactification $\mathrm{UOSp}(1|2)$ and analytically continue $j \rightarrow -\frac{1}{4} + is$

$\mathrm{UOSp}(1|2)$ state counting

$$N = \frac{1}{2\pi} \int_0^{\pi} \mathrm{d}\theta \, \sin^2(2\theta) \left[4 - n + \sum_{i=1}^{p} n_i d_{j_i} \frac{\tan(d_{j_i}\theta)}{\tan\theta} \right] \prod_{l=1}^{p} \left(\frac{\cos(d_{j_l}\theta)}{\cos\theta} \right)^{n_l}$$

$\rightarrow$ *analytic continuation*

Entropy

$$S = \ln N = \frac{a}{4} + \ldots \qquad \rightarrow \text{[Bekenstein '73, Hawking '75]}$$

Section 4

Outlook

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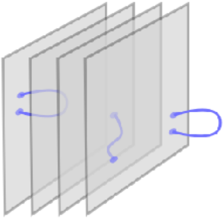
Outlook

- Generalization to $\mathcal{N} \geq 3$ (*in particular:* $\mathcal{N} = 4, 8$)
- Limit $L \to \infty$ (vanishing cosmological constant) [Concha+Ravera+Rodríguez '19]
- Simplification dynamics extending generalized GCD to super setting $\to$ Abhay's talk, Madhavan's talk
- (Charged) supersymmetric BHs $\leftrightarrow$ entropy calculations in string theory [Strominger+Vafa '96, Cardoso et al. '96, KE+HS '22]
- $\leftrightarrow$ Boundaries in string theory and **OSp**-super Chern-Simons theory [Mikhaylov+Witten '14]



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Quantum theory: Overview

- Quantization: study $\mathcal{A}^+$ and associated holonomies
- turns out $\rightarrow$ need additional **parametrizing supermanifold** $\mathcal{S}$ to incorporate **anticommutative** nature of fermionic fields

Super holonomies [KE '20+'21]

$$h_e[\mathcal{A}^+] = h_e[A^+] \cdot \mathcal{P}\exp\left(-\oint_e \mathrm{Ad}_{h_e[A^+]^{-1}}\psi^{(\tilde{s})}\right) : \mathcal{S} \rightarrow \mathcal{G}$$

- $\mathcal{A}^+ = A^+ + \psi$ and $e : [0,1] \rightarrow M \subset \mathcal{M}$
- $h_e[A^+]$: bosonic holonomy associated to A^+

Quantum theory: Overview

- $\mathcal{A}_{\mathcal{S},\gamma}$: set of generalized super connections

$$\begin{array}{ccc} \mathcal{A}_{\mathcal{S}',\gamma'} & \longrightarrow & \mathcal{A}_{\mathcal{S}',\gamma} \\ \downarrow & & \downarrow \\ \mathcal{A}_{\mathcal{S},\gamma'} & \longrightarrow & \mathcal{A}_{\mathcal{S},\gamma} \end{array}$$

- for fixed graph γ induces functor $\mathcal{A}_\gamma : \mathcal{S} \to \mathcal{A}_{\mathcal{S},\gamma}$
  $\to$ **Molotkov-Sachse-type smf.**
- $\to$ can study cylindrical functions and invariant measures on $\mathcal{A}_\gamma$
- covariance under reparametrization requires **Berezin-type integral** for fermionic d.o.f.
- $\to$ induces **Krein structure** on state space $\leftrightarrow$ strong similarities to quantization of fermions in standard LQG

Symmetry reduction

- supersymmetric minisuperspace models [D'Eath + Hughes '88+'92, D'Eath + Hawking + Obregon '93]
- hybrid homogeneous isotropic ansatz for bosonic and fermionic d.o.f.
- in general: fermions not compatible with isotropy
- **But**: in (chiral) LQC can exploit enlarged gauge symmetry!
- $\rightarrow$ natural interpretation in terms of homogeneous isotropic super connection [KE '20, KE+HS '20]

Symmetry reduced connection

$$\mathcal{A}^{+} = c\, \mathring{e}^{i}\, T_{i}^{+} + \psi_{A} \mathring{e}^{AA'}\, Q_{A'}$$

$\mathring{e}^{i}$: fiducial co-triad

- **Also**: can show that this is the most general ansatz consistent with reality conditions (contorsion tensor isotropic)

Loop quantum cosmology

- Construction of state space via super holonomies $h_e[\mathcal{A}^+]$ along straight edges of a fiducial cell
- $\Rightarrow$ motivates state space of quantum theory

Hilbert space

$$\mathcal{H} = \overline{H_{\mathrm{AP}}(\mathbb{C})}^{\|\cdot\|} \otimes \Lambda[\psi_{A'}]$$

- reality condition in quantum theory can be solved exactly![Wilson-Ewing '15, KE+HS '20]

Quantum right SUSY constraint

$$\widehat{S}^R_{A'} = \frac{3g^{\frac{1}{2}}}{2\lambda_m}|v|^{\frac{1}{4}}\left((\mathcal{N}_- - \mathcal{N}_+) - \frac{\kappa\lambda_m}{6g|v|}\widehat{\Theta}\right)|v|^{\frac{1}{4}}\widehat{\phi}_{A'}$$

- λ_m: quantum area gap (full theory)

Loop quantum cosmology

- Quantum algebra between left and right SUSY constraint closes and reproduces Poisson algebra exactly!

Quantum algebra

$$[\widehat{S}^{LA'}, \widehat{S}^R_{A'}] = 2i\hbar\kappa\widehat{H} - \frac{\hbar\kappa}{6g^{\frac{1}{2}}|v|^{\frac{1}{2}}}\widehat{\pi}_\psi^{A'}\widehat{S}^R_{A'}$$

- $\rightarrow$ fixes some of the quantization ambiguities
- semiclassical limit: $\lambda_m \rightarrow 0$ i.e. corrections from quantum geometry negligible
- $\rightarrow$ **Hartle-Hawking state** as solution of constraints $\Leftrightarrow$[D'Eath '98]

Hartle-Hawking state

$$\Psi(a) = \exp\left(\frac{3a^2}{\hbar}\right)$$