Ambiguities in polymer quantum black holes

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Aim of polymer models :

-Take into account the quantum corrections in an effective way.

Approach :

-In cosmology, we obtain a quantum theory by replacing c by $\frac{\sin(\bar{\mu}c)}{\bar{\mu}}$ and then quantize (LQC).

Issues :

-In LQC, the dynamic depends on the choice of the polymerisation function.

L.Amadei, A.Perez and S.Ribisi arXiv:2203.07044

A.Barrau, K.Martineau and C.Renevey arxiv:2109.14400

$\longrightarrow$ Is it the same for black holes ?

Hypothesis : Stationary, spherically symmetric and empty spacetime.

Line element : $ds^2 = -a(r)dt^2 + \frac{n(r)}{a(r)}dr^2 + b(r)^2 d\Omega^2$

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Change of variable : $v_1 = \frac{2}{3}b^3$, $v_2 = 2ab^2$, $P_1 = \frac{\dot{a}}{\sqrt{n}b}$, $P_2 = \frac{\dot{b}}{\sqrt{n}b}$

Hamiltonian constraint : $H = \sqrt{n}(3v_1P_1P_2 + v_2P_2^2 - 2) \approx 0$

This leads to the usual Schwarzschild solution.

What will happen when we polymerise ?

$$P_1 \rightarrow \frac{f_1(\lambda_1 P_1)}{\lambda_1} \text{ and } P_2 \rightarrow \frac{f_2(\lambda_2 P_2)}{\lambda_2}$$

Such that : $f_{1,2}(x) \overset{x \ll 1}{\sim} x$ and $f_{1,2}$ are 2π-periodic functions. The usual choice is the sin function.

N. Bodendorfer, F. M. Mele, and J. Munch arXiv:1902.04542

We want to investigate other choices :

-Is there still a bounce? Can there be more than one ?

-What can we say about the properties of this effective spacetime ?

Effective Hamiltonian : $\sqrt{n}\left(3v_1\frac{f_1}{\lambda_1}\frac{f_2}{\lambda_2}+v_2\frac{f_2^2}{\lambda_2^2}-2\right)\approx 0$

$$\Downarrow$$

$$\frac{dP_2}{dr}=-\sqrt{n}\frac{f_2^2}{\lambda_2^2}\ ,\quad \frac{dP_1}{dr}=-3\sqrt{n}\frac{f_1}{\lambda_1}\frac{f_2}{\lambda_2}\ ,\quad \frac{dv_1}{dr}=...\ ,\quad \frac{dv_2}{dr}=...$$

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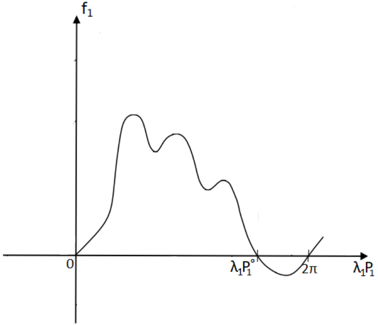
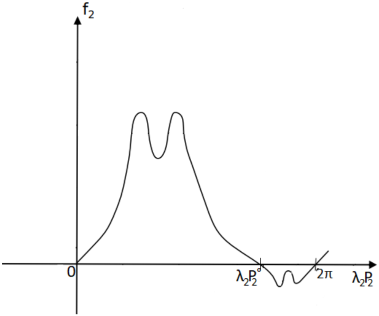
Dirac observable : $v_1\frac{f_1}{\lambda_1}=K_1\Rightarrow v_1=\frac{\lambda_1 K_1}{f_1}$

Solving the Hamiltonian constraint : $v_2\approx\frac{2\lambda_2^2}{f_2^2}-\frac{3K1\lambda_2}{f_2}$

General polymerisation functions :





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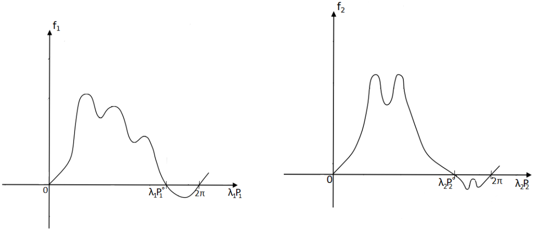
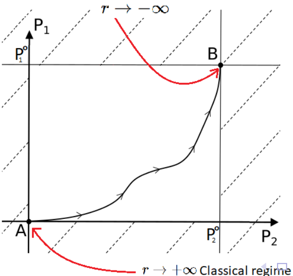
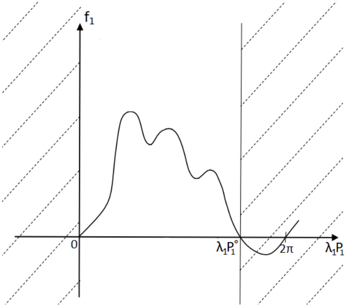
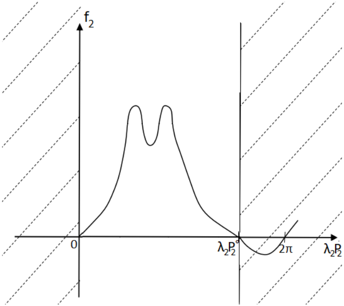
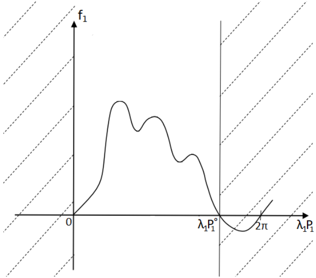


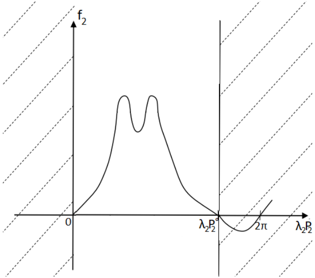
Dynamical evolution :




f_1
0
$\lambda_1 P_1^\circ$
2π
$\lambda_1 P_1$

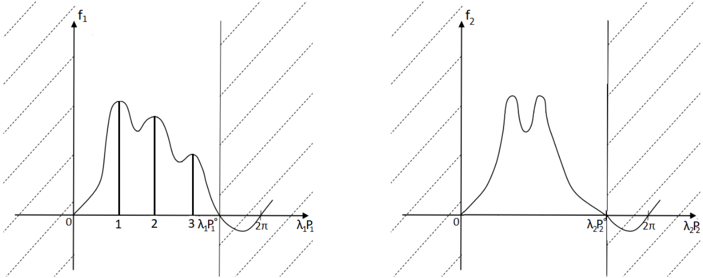

f_2
0
$\lambda_2 P_2^\circ$
2π
$\lambda_2 P_2$





Bounces : $v_1 = \frac{\lambda_1 K_1}{f_1} \Rightarrow b = \left(\frac{3\lambda_1 K_1}{2 f_1}\right)^{1/3}$

$\longrightarrow$ Local minimum for $b \Longleftrightarrow$ Local maximum for f_1.



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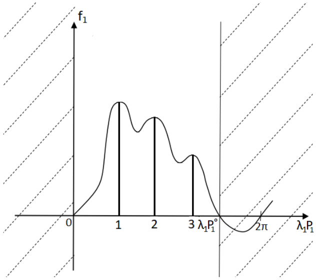

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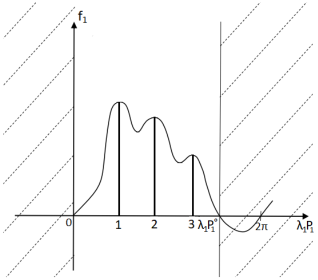

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We can constraint the choice for the polymerisation functions such that :

-The metric is Schwarzschild when $r \to \pm\infty$.

-The Kretschmann scalar does not diverges anywhere.

-The first correction to Schwarzschild is in agreement with the usual 1 loop correction.

Conclusion

The bad news :

-Many properties of the effective space time depend on the choice of the polymerisation functions.

The good news :

-The singularity is always replaced by a bounce.

-There is no divergence of the curvature in most of the cases.

-It is possible to constraint the choice of polymerisation functions.