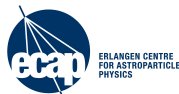


On consistent gauge-fixing conditions in polymerized gravitational systems

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Stefan Andreas Weigl (FAU)

joint work with Kristina Giesel (FAU), Bao-Fei Li (ZUT),
Parampreet Singh (LSU),



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Introduction

- Get to the physical sector by
 1. constructing observables: relational formalism^{1 2}
 2. or gauge-fixing $G_J = 0$
- Consider effective theories by³

$$\text{Polymerization: } A_a^j \mapsto h_a^j(A_a^j; \lambda_a^j) \text{ e.g. } \frac{\sin(\lambda A)}{\lambda}$$

Question: How to polymerize lapse and shift in a given model?

$$N_{\text{class}} = 1, N_{\text{class}}^a = \frac{b}{\gamma} \mapsto^4 N = 1, N^a = \frac{\sin(\lambda b)}{\lambda \gamma}$$

⇒ Check if stability equations are solved: $\dot{G}_J \stackrel{!}{\approx} 0$

¹Rovelli, *What is observable in classical and quantum gravity?*

²Dittrich, *Partial and complete observables for canonical general relativity*

³Ashtekar&Singh, *Loop Quantum Cosmology: A Status Report*

⁴Santacruz&Kelly&Wilson-Ewing, *Black hole collapse and bounce in effective loop quantum gravity*

Lemma 1: A simple model

Given a system where⁵

1. all G_J depend only on gravitational d.o.f.,
 2. only the connection is polymerized and h_a^j do not depend on partial derivatives,
 3. for at least one gauge-fixing condition we have $\frac{\partial G_I}{\partial t} \neq 0$
- \implies Gauge-fixing and Polymerization **do not** commute!

⁵Corichi&Singh, *Loop quantization of the Schwarzschild interior revisited*

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But: Replace third assumption by

3. for all G_J we have $\frac{\partial G_J}{\partial t} \approx 0$ and each of them only depend on one canonical pair

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or

3. temporal GF cond. only depend on matter d.o.f. and all other depend on only one canonical pair

\implies Gauge-fixing and Polymerization **do** commute!

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Stability Equations

Why is this the case?

$$\begin{aligned}\dot{G}_J &\approx \int N\{G_J, C\} + N^a\{G_J, C_a\} + \frac{\partial G_J}{\partial t} \\ &= \int N\left(\mathcal{F}_{J,0}^{A_1}\partial h_1 + \mathcal{F}_{J,0}^{A_2}\partial h_2 + \dots\right) + N^a\left(\mathcal{F}_{J,a}^{A_1}\partial h_1 + \dots\right) + \frac{\partial G_J}{\partial t}\end{aligned}$$

where

$$\mathcal{F}_{J,0}^{A_1} = \int d^3z \frac{\delta G_J}{\delta h_1(z)} \frac{\delta C(y)}{\delta E^1(z)} - \frac{\delta C(y)}{\delta h_1(z)} \frac{\delta G_J}{\delta E^1(z)}$$

- Red marked terms are equal to the polymerized classical terms
- non-temporal GF: need to factor out ∂h
- temporal GF: No ∂h allowed \Rightarrow non-polymerized variables

Lemma 2: Temporal Gauge-Fixing is key

Given a system:

1. we have at least one temporal GF condition $\frac{\partial G_I}{\partial t} \not\approx 0$,
2. which depends further on at most one canonical pair (Q^0, P_0) .
3. Polymerization is such that (no partial derivatives)

$$Q^0 \mapsto h_{Q^0}(Q^0; \lambda_{Q^0}), \quad P_0 \mapsto h_{P_0}(P_0; \lambda_{P_0})$$

$$\text{and that } \int d^3z \left(\frac{\delta h_{Q^0}(z')}{\delta Q^0(z)} \right) \left(\frac{\delta h_{P_0}(z'')}{\delta P_0(z)} \right) \not\approx \delta^{(3)}(z', z'')$$

\implies Gauge-fixing and Polymerization **do not** commute!

Note: This result holds for arbitrary polymerizations of the other canonical variables

Consequences

Given an effective system

- directly check if polymerized classical solutions of $N_{\text{class}}, N_{\text{class}}^a$ solve stability equations $\dot{G}_J \approx 0$
- if this is **not** the case,
 1. solutions N, N^a have non-standard polymerization
 2. reverse-engineering dynamical consistent GF conditions: non-stand. polymerization and/or wrong classical limit

⇒ Result: Physical Hamiltonian **changes** due to

1. polymerization ambiguities
2. different classical limit

Conclusion

Summary

- Analyzed whether GF and Polymerization commute
⇒ For most systems this is **not** the case
- **But** for some systems we can prove that this **is** the case
- Pay attention when polymerizing gauge fixed Hamiltonians (to not break dynamical consistency)

Outlook

- Operators of diffeomorphism constraints do not exist in full theory ⇒ work with $Q^{xx}C_xC_x$ and $\{G, \{G, Q^{xx}C_xC_x\}\}$
- Did not consider inverse triad corrections
- Extend analysis to quantized systems

Thank you for your attention!