On consistent gauge-fixing conditions in polymerized gravitational systems

Phys. Rev. D 105, 066023

Stefan Andreas Weigl(FAU)

joint work with Kristina Giesel(FAU), Bao-Fei Li(ZUT),
Parampreet Singh(LSU),

Funded by
Deutsche Forschungsgemeinschaft
German Research Foundation

21.07.2022, LOOPS 22 Lyon

Introduction

- Get to the physical sector by
  1. constructing observables: relational formalism[1] [2]
  2. or gauge-fixing $G_J = 0$
- Consider effective theories by[3]

$$\text{Polymerization:} \quad A_a^j \mapsto h_a^j\left(A_a^j; \lambda_a^j\right) \text{ e.g. } \frac{\sin(\lambda A)}{\lambda}$$

Question: How to polymerize lapse and shift in a given model?

$$N_{\text{class}} = 1, N_{\text{class}}^a = \frac{b}{\gamma} \quad \mapsto^{4} \quad N = 1, N^a = \frac{\sin(\lambda b)}{\lambda\gamma}$$

$\Rightarrow$ Check if stability equations are solved: $\dot{G}_J \overset{!}{\approx} 0$

---

[1] Rovelli, *What is observable in classical and quantum gravity?*

[2] Dittrich, *Partial and complete observables for canonical general relativity*

[3] Ashtekar&Singh, *Loop Quantum Cosmology: A Status Report*

[4] Santacruz&Kelly&Wilson-Ewing, *Black hole collapse and bounce in effective loop quantum gravity*

Lemma 1: A simple model

Given a system where[5]

1. all G_J depend only on gravitational d.o.f.,
2. only the connection is polymerized and h_a^j do not depend on partial derivatives,
3. for at least one gauge-fixing condition we have $\frac{\partial G_I}{\partial t} \not\approx 0$

$\Longrightarrow$ Gauge-fixing and Polymerization **do not** commute!

[5] Corichi&Singh, *Loop quantization of the Schwarzschild interior revisited*

Lemma 1: A simple model

Given a system where[5]

1. all G_J depend only on gravitational d.o.f.,
2. only the connection is polymerized and h_a^j do not depend on partial derivatives,
3. for at least one gauge-fixing condition we have $\frac{\partial G_I}{\partial t} \not\approx 0$

$\Longrightarrow$ Gauge-fixing and Polymerization **do not** commute!

But: Replace third assumption by

3. for all G_J we have $\frac{\partial G_J}{\partial t} \approx 0$ and each of them only depend on one canonical pair

[5] Corichi&Singh, *Loop quantization of the Schwarzschild interior revisited*

Lemma 1: A simple model

Given a system where[5]

1. all G_J depend only on gravitational d.o.f.,
2. only the connection is polymerized and h_a^j do not depend on partial derivatives,
3. for at least one gauge-fixing condition we have $\frac{\partial G_I}{\partial t} \not\approx 0$

$\Longrightarrow$ Gauge-fixing and Polymerization **do not** commute!

But: Replace third assumption by

3. for all G_J we have $\frac{\partial G_J}{\partial t} \approx 0$ and each of them only depend on one canonical pair

or

3. temporal GF cond. only depend on matter d.o.f. and all other depend on only one canonical pair

$\Longrightarrow$ Gauge-fixing and Polymerization **do** commute!

---

[5] Corichi&Singh, *Loop quantization of the Schwarzschild interior revisited*

Stability Equations

Why is this the case?

$$\overset{\bullet}{G}_J \approx \int N\{G_J, C\} + N^a\{G_J, C_a\} + \frac{\partial G_J}{\partial t}$$
$$= \int N\Big(\mathcal{F}^{A_1}_{J,0}\partial h_1 + \mathcal{F}^{A_2}_{J,0}\partial h_2 + ...\Big) + N^a\Big(\mathcal{F}^{A_1}_{J,a}\partial h_1 + ...\Big) + \frac{\partial G_J}{\partial t}$$

where

$$\mathcal{F}^{A_1}_{J,0} = \int \mathrm{d}^3 z\, \frac{\delta G_J}{\delta h_1(z)}\frac{\delta C(y)}{\delta E^1(z)} - \frac{\delta C(y)}{\delta h_1(z)}\frac{\delta G_J}{\delta E^1(z)}$$

- Red marked terms are equal to the polymerized classical terms
- non-temporal GF: need to factor out ∂h
- temporal GF: No ∂h allowed $\Rightarrow$ non-polymerized variables

Lemma 2: Temporal Gauge-Fixing is key

Given a system:

1. we have at least one temporal GF condition $\frac{\partial G_I}{\partial t} \not\approx 0$,
2. which depends further on at most one canonical pair (Q^0, P_0).
3. Polymerization is such that (no partial derivatives)

$$Q^0 \mapsto h_{Q^0}(Q^0; \lambda_{Q^0}), \qquad P_0 \mapsto h_{P_0}(P_0; \lambda_{P_0})$$

and that $\int d^3z \left(\frac{\delta h_{Q^0}(z')}{\delta Q^0(z)}\right)\left(\frac{\delta h_{P_0}(z'')}{\delta P_0(z)}\right) \not\approx \delta^{(3)}(z', z'')$

$\Longrightarrow$ Gauge-fixing and Polymerization **do not** commute!

Note: This result holds for arbitrary polymerizations of the other canonical variables

Consequences

Given an effective system

- directly check if polymerized classical solutions of $N_{\text{class}}, N^a_{\text{class}}$ solve stability equations $\dot{G}_J \approx 0$
- if this is **not** the case,
  1. solutions N, N^a have non-standard polymerization
  2. reverse-engineering dynamical consistent GF conditions: non-stand. polymerization and/or wrong classical limit

$\Longrightarrow$ Result: Physical Hamiltonian **changes** due to

1. polymerization ambiguities
2. different classical limit

Conclusion

Summary

- Analyzed whether GF and Polymerization commute
  $\Longrightarrow$ For most systems this is **not** the case
- **But** for some systems we can proof that this **is** the case
- Pay attention when polymerizing gauge fixed Hamiltonians (to not break dynamical consistency)

Outlook

- Operators of diffeomorphism constraints do not exist in full theory $\Longrightarrow$ work with $Q^{xx}C_xC_x$ and $\{G, \{G, Q^{xx}C_xC_x\}\}$
- Did not consider inverse triad corrections
- Extend analysis to quantized systems

Thank you for your attention!