Phase Spaces for

Isolated Horizons

(and BH Entropy?)

Alejandro Corichi

Center for Mathematical Sciences, UNAM, Morelia, México

LOOPS'22, ENS-Lyon
July, 2022

Collaboration with: J.D. Reyes and T. Vukasinac.

PLAN

1. **Isolated Horizons**
2. **Self Dual Gravity**
3. **3+1 Decomposition**
4. **Different choices**

ISOLATED HORIZONS

Isolated Horizons is a formalism that describes black hole horizons in equilibrium. Its definition involves imposing geometric conditions on the spacetime, which imposes conditions on the variables describing the geometry.

In particular, a 3D hypersurface Δ should be null, non-expanding, and have some 'time translation symmetry' along its generator ℓ^μ.

The conditions imposed should be independent of which variables we use to describe gravity: $g_{\mu\nu}$ or e^I_μ.
But using co-tetrads e^I_μ introduces an extra internal, local, gauge symmetry $SO(3,1)$.

Question: Can we have consistent canonical description(s)?

Expectation: if Δ a boundary, there might be 'new' degrees of freedom thereon.

Important note: We are interested in quantization (a la LQG), so we need a canonical Hamiltonian formulation.

This means that we need to start with an action that is well defined and then go to a $3+1$ decomposition. Note: Doing a covariant analysis and claiming that to be the canonical result might not work!

The action depends on e_μ^I (and a gauge connection $w_{\mu I}^{J}$), and has possible boundary terms.
Ask again: Can we complete the program?

WELCOME BACK TO 1999!!!

Recall that within the very large literature on Isolated Horizons, only two papers, both from 1999, considered a canonical formalism (PSU group). After that, only the covariant Hamiltonian formalism has been used.

FIRST ORDER GRAVITY.

One can consider co-tetrads e^I_μ instead of $g_{\mu\nu} = e^I_\mu e^J_\nu \eta_{IJ}$. One can obtain a first order action by considering a $SO(3,1)$ connection ω, as independent variable. The action is (AC, Wilson-Ewing):

$$S_{\mathbf{H}}[e,\omega] = \frac{1}{2\kappa}\left[\int_{\mathcal{M}} \Sigma^{IJ} \wedge \left(F_{IJ} + \frac{1}{\gamma} \star F_{IJ}\right) - \int_{\partial\mathcal{M}} \Sigma^{IJ} \wedge \left(\omega_{IJ} + \frac{1}{\gamma} \star \omega_{IJ}\right)\right], \tag{1}$$

with

$$\Sigma^{IJ} := \star e^I \wedge e^J = \frac{1}{2}\epsilon^{IJ}{}_{KL}\, e^K \wedge e^L,$$ **and F_{IJ} curvature of ω_{IJ}**

For AF conditions, it is finite, differentiable, and admits a well defined covariant Hamiltonian formulation.
One can also define a consistent Hamiltonian formulation via a 3+1 splitting (AC, Reyes).

Consistency of the action on Δ. (AC, JD Reyes, Vukasinac)

- One has to be careful of the boundary conditions when asking for the action to be differentiable.

- Different possible actions: With or without boundary term, with or without Holst term $(1/\gamma = 0)$.

- Possible gauge reduction: Choice of one of the internal basis elements l^I to coincide with $\ell^I = e^I_\mu \ell^\mu$. Gauge is reduced to those transformations that leave direction of ℓ^I invariant.

3+1 Decomposition of Self-dual Action. (AC, Reyes, Vukasinac)

- Start with first order $SL(2,C)$ action. No boundary term on Δ.

- 3+1 Decomposition (for real), $SL(2,C)$ reduces to $SU(2)$.

- "Follow our noses" (Improved Regge-Teitelboim formalism) (AC, Vukasinac):

- Note: Contribution from the boundary to the symplectic structure depends on the details of the *canonical* action.

- If there is a contribution from the boundary, then we will have "Boundary DOF".

- Boundary term in the Canonical action takes the form:

$$S_{\mathbf{SD}}^{\Delta} = \int_{\Delta} \frac{1}{N} \left(t^{\mu} + \mathcal{A}_{\mu}^{\ AB}\right) \left(i\sqrt{2}\, \sigma^{a}{}_{AB}\, \tilde{e}_{a}^{\nu}\right) \ell_{\nu}{}^{2} \epsilon \, \mathrm{d}\lambda = - \int \mathrm{d}t \oint_{S_{\Delta}} t \cdot (\omega + V)^{\,2} \epsilon$$

- Here is the main point here: We can stare at this expression and conclude there are 4 cases:

- I) Leave term as it is: Regge Teitelboim tell us that the term is to be included into the Hamiltonian. Our formalism adds that there is no contribution to the symplectic structure from the horizon, or

- II) Define potentials for ω, V or both! 3 Cases!

$$\mathcal{L}_t \psi_R = t \cdot \mathrm{d}\psi_R \triangleq t \cdot \omega \,,$$

$$\mathcal{L}_t \psi_I = t \cdot \mathrm{d}\psi_I \triangleq t \cdot V \,.$$

Two descriptions

Regge Teitelboim case: Take

$$S^{\Delta}_{\mathbf{SD}} = -\int \mathrm{d}t \oint_{S_\Delta} t \cdot (\omega + V)\,{}^2\epsilon$$

as is, compute variations and so on. There are no boundary contributions to the symplectic structure $\Omega_1 = \int_\Sigma \mathrm{d}^3x\, \mathbb{d}\tilde{P}^a{}_{AB} \wedge \mathbb{d}A_a{}^{AB}$, **variation of** H **has a boundary term:**

$$\delta H_{\mathbf{SD,1}}|_{S_\Delta} = \oint_{S_\Delta} \left[\,(\delta\kappa_{(t)} + \delta(t \cdot V)\right]{}^2\epsilon\,. \tag{2}$$

First term is canceled by a BT arising from variations of bulk H**. In order to cancel the variation one needs to impose a condition on fields at the boundary. Boundary Hamiltonian gives Smarr mass for BH! No degrees of freedom at the boundary, no gauge, no diffeos!**

‘Define a potential’ case:

$$S^{\Delta}_{\mathbf{SD,2}} = -\int \mathrm{d}t \oint_{S_\Delta} \left(\kappa_{(t)} + \mathcal{L}_t \psi_I\right)^2 \epsilon \,. \tag{3}$$

Now (ψ_I, π) will appear in the boundary simplectic structure, and are elevated to BDOF: $\Omega^{\Delta}_{2} = \oint_{S_\Delta} \mathrm{d}^2\Theta \, \mathbb{d}\tilde{\pi} \wedge \mathbb{d}\psi_I$. First term goes to Hamiltonian. Variations of Hamiltonian behave the same as in previous case.

BUT, now Bdry contributions to variations do NOT all have to vanish. Now Gauss’ law does generate $U(1)$ gauge tranformations on V, which can be seen as the (gauge) boundary degree of freedom. No diffeos.

Since the first term is the same as in RT case, then the boundary contribution to the Hamiltonian yields the Smarr Mass.

What about Gauge? ($SU(2)$ vs $U(1)$ issue)

In the standard 3+1 decomposition of the self dual action, to obtain the canonical theory in terms of Ashtekar variables, one needs to introduce a 'time gauge' fixing. This means fixing an internal "timelike" direction. The gauge group is reduced from $SL(2,C)$ to $SU(2)$. Thus, the canonical theory on Σ involves a SU(2) gauge theory. If we see S_Δ, the 2D internal boundary (corner) of Σ ($S_\Delta = \Sigma \cap \Delta$), what are the gauge degrees of freedom there?
Do we recover $SU(2)$?

Is there a further gauge reduction?

If in the covariant theory the gauge group is the one induced by the AN-gauge, then compatibility with a canonical decomposition reduces the gauge group to $U(1)$.

What about the canonical theory? Just look at the Gauss' law and see what it generates at the horizon, to see what is gauge and what is not.

Result: Only in Case II do we recover some notion of gauge on horizon.

Thus,

In that case, the natural gauge group for WIH IS $U(1)$!

SUMMARY

- **The canonical analysis of SD gravity with a WIH can be performed, but**

- **There exist several consistent descriptions,**

- **They differ on the degrees of freedom on the boundary, and "gauge symmetries".**

- **Boundary contribution to the symplectic structure vary.**

- **We recover horizon Hamiltonian and energy.**

- $SU(2)$ **vs** $U(1)$ **issue under control: Need to do proper canonical analysis (not covariant and claim to be the canonical)**

- **Quantization?**

REFERENCES

Covariant Hamiltonian analysis:

AC, J.D. Reyes, T. Vukasinac arXiv:1612.01462 (CQG, 2017).

Extension of Regge-Teitelboim:

AC, T. Vukasinac arXiv:2001.06068 (CQG, 2020).

Canonical Hamiltonian analysis:

AC, J.D. Reyes, T. Vukasinac, stay tuned, tonight!).