On black holes and the cosmological constant $\Lambda > 0$

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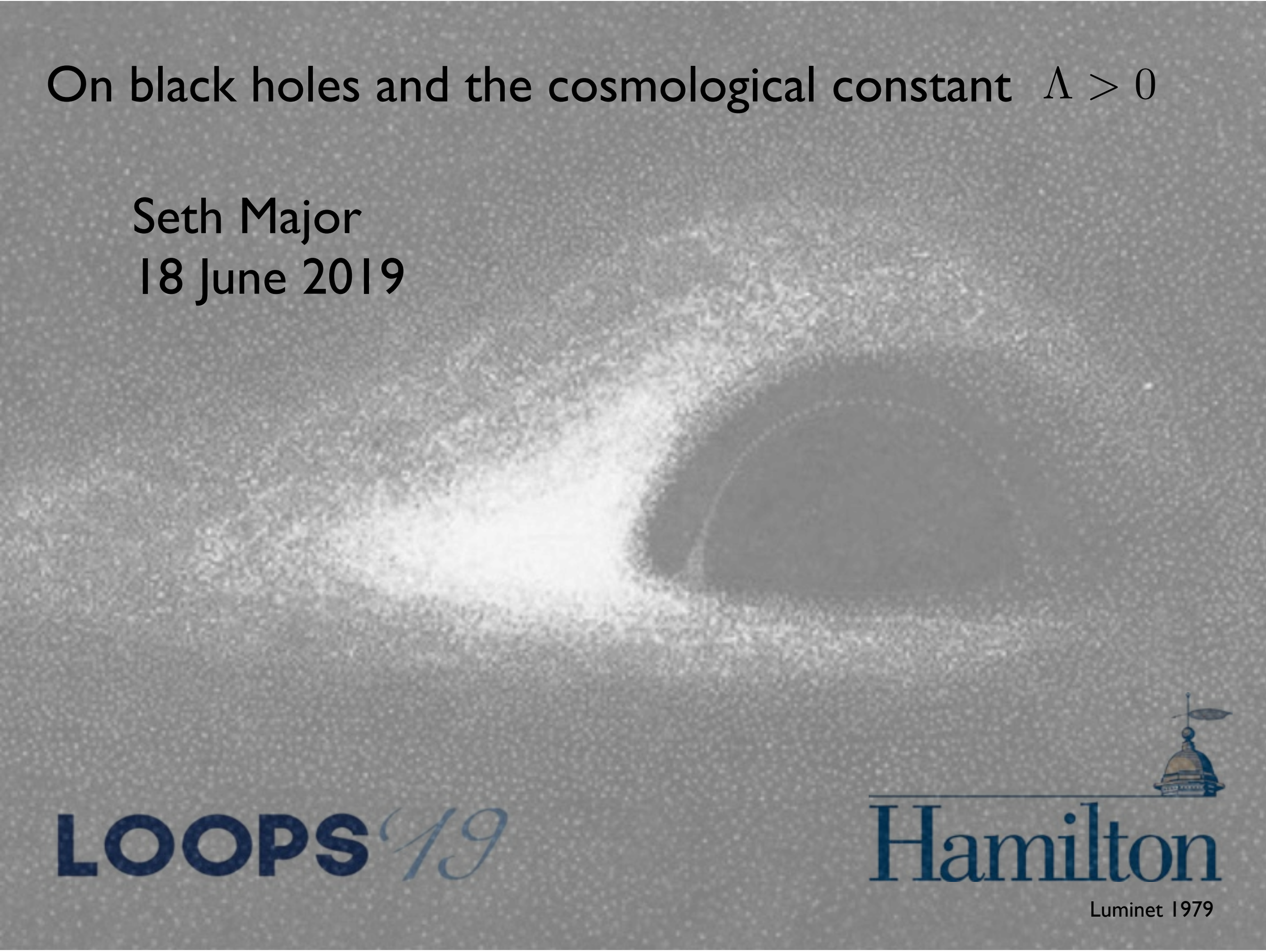


LOOPS'19

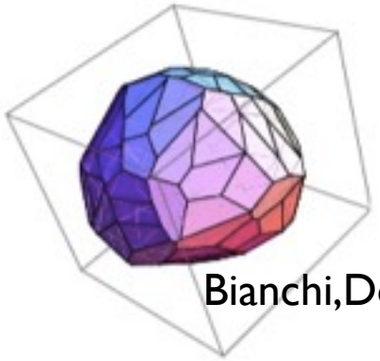
Hamilton

Luminet 1979

Motivation

Asymptotic safety and causal dynamical triangulations approaches rely on the existence of a cosmological constant.

One could build spacetime from homogeneously curved building blocks



Bianchi,Dona,Speziale

In 3+1 approach if blocks are solutions to Einstein's equations, scalar constraint has boundary terms - the Hamiltonian or quasi-local energy.

What can we learn about these ideas from black hole spacetimes?

- Could the existence of a cosmological constant affect the statistical mechanics of black holes?
- How is the quasi-local energy related to the local energy used in the statistical description?
- Report on work in progress generalizing earlier work (Frodden, Ghosh, Noui, Perez, Asin, Achor, Geiller, Major, Setter,...) to the Kottler (and Carter?) spacetime

Review of Kottler (1918) or *Schwarzschild-de Sitter* spacetime solution

$$ds^2 = -\left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)dt^2 + \left(1 - \frac{2M}{r} - \frac{\Lambda}{3}r^2\right)^{-1} dr^2 + r^2 d\Omega$$

- Exclusively in the regime where $0 < 9\Lambda M^2 < 1$
- Metric has two positive physical roots $r_H\,,\ r_\Lambda \quad 0 < 2M < r_H < 3M < r_\Lambda$

$$r_H = \frac{2}{\sqrt{\Lambda}}\cos(\alpha/3 + 4\pi/3) \quad \text{with} \quad \cos\alpha = -3M\sqrt{\Lambda}$$

Gibbons and Hawking 1977

$$r_\Lambda = \frac{2}{\sqrt{\Lambda}}\cos(\alpha/3)$$

with $\Lambda = 1.3 \times 10^{-52} m^{-2}$ and $\hbar\Lambda = 3.5 \times 10^{-122}$

- Cosmological horizon is on the same order as the size of the observable universe so there is *lots* of room for effective asymptotics.
- Two Killing vectors $\xi^a = (\partial_t)^a$ and $\psi^a = (\partial_\varphi)^a$ similar to Schwarschild and Kerr
- Kerr- de Sitter generalization is "Carter spacetime"

Family of static (or ZAMO) observers $\chi^a = \xi^a \left(+ \Omega \psi^a\right)$

• Using local first law framework (Frodden, Ghosh, Perez) - generalizing to Kottler (and Carter) spacetimes

$$U^a \to \frac{t^a}{\sqrt{1 - \frac{2M}{r} - \frac{\Lambda}{3} r^2}}$$

• Near horizon $r = r_H + \rho$

Proper distance from horizon

$$\ell = \int_{r_H}^{r_H + \rho} \sqrt{g_{rr}} dr \simeq 2\sqrt{2}\sqrt{M\rho}\left(1 + \frac{8\Lambda M^2}{3}\right)$$

Horizon area

$$A_H = 4\pi r_H^2 \simeq 16\pi M^2 \left(1 + \frac{8\Lambda M^2}{3}\right)$$

For static observers $\bar{\kappa} = \frac{\kappa}{\sqrt{-\chi_a \chi^a}} \simeq \frac{1}{\ell}$ includes Λ effects

Change in local energy $\delta E_{loc} \simeq \frac{\bar{\kappa}}{8\pi} \delta A_H \simeq \frac{1}{8\pi\ell} \delta A_H$

Quasi-local energies notoriously subtle!

• Brown-York quasi-local energy - the boundary term that arises in the variation of the action.
Depends on the trace of extrinsic curvature compared to reference spacetime

$$H_{\partial\Sigma}[N] = -\frac{1}{8\pi}\int_{\partial\Sigma} d^2x N\left(\sqrt{\sigma}K - \sqrt{\bar{\sigma}}\bar{K}\right) \qquad \sigma_{ij} \text{ metric on } \partial\Sigma$$

depends on how $\partial\Sigma$ is placed in foliation

At radius r=R in Kottler spacetime (assume vacuum is with Λ)

$$H_{\partial\Sigma}[N] = NR\left(\sqrt{1-\frac{\Lambda}{3}R^2} - \sqrt{1-\frac{2M}{r}-\frac{\Lambda}{3}R^2}\right) \simeq M\left(1+\frac{\Lambda}{6}R^2\right)$$

near horizon

$$H_{\partial\Sigma}[U,0] \simeq 2\sqrt{2}\frac{M^{3/2}}{\sqrt{\epsilon}} + \cdots + \frac{5}{4\sqrt{2}}(\epsilon M)^{3/2}\Lambda$$

$$\simeq \frac{A_H}{2\pi\ell}$$ includes leading order effects of Λ but there are factors of 2...

Better choices? Misner-Sharp, Komar, Kijowski, Liu-Yau...?

For now use Frodden, Ghosh, Perez energy

$$\delta E_{loc} \simeq \frac{\bar{\kappa}}{8\pi}\delta A_H \simeq \frac{1}{8\pi\ell}\delta A_H$$ with cosmological constant...

Quantum gravity with Λ

Turaev, Viro, Major, Smolin, Borrisov, Haggard, Han, Kaminski, Riello, Dupuis, Girelli, Dittrich, Bahr, Bianchi, Alexander, ...

- tantalizing connection between Chern-Simons theory and Turaev-Viro models
- based on these analogies we work with a deformed group with $k = \dfrac{6\pi}{\hbar\Lambda}$

Quantum group $SU(2)_k$ $\quad q = e^{2\pi i/(k+2)}$ a root of unity

"There are only three things you need to know..." (Mo Wilems)

Admissible spins $\quad j_k = \{0, \frac{1}{2}, 1, \ldots, \frac{k}{2}\}$

Quantum number $\quad [n] = \dfrac{q^{n/2} - q^{-n/2}}{q^{1/2} - q^{-1/2}} \qquad n = 2j$

Area for plaquette $\quad a = \dfrac{\gamma\hbar}{[2]}\sqrt{[n][n+2]}$ only an ansatz!

This is the source of the correction due to the cosmological constant

Given the speculative nature of the area operator, don't take numerical coefficients too seriously

Single plaquette partition function Z_1

- For single particle of geometry (plaquette in the boundary)

$$E_j = \epsilon\sqrt{j(j+1)} \qquad \text{with} \qquad \epsilon = \frac{\gamma\hbar}{8\pi\ell}$$

under q deformation

$$E_j = \frac{\epsilon}{[2]}\sqrt{[n][n+2]} \simeq \left(j+\tfrac{1}{2}\right)\left[1-\frac{1}{6}\left(\frac{\hbar\Lambda}{3}\right)^2\left(j+\tfrac{1}{2}\right)\right]$$

two limits - large j and small Λ

- Degeneracy

$$d_j = [n+1]$$

- Partition function $\quad n = 2j$

$$Z_1 = \sum_{n_k}[n+1]e^{-\beta\epsilon\sqrt{[n][n+1]}/[2]}$$

two limits - large j and large k $\qquad m = j + 1/2$

$$Z_1 \simeq 2\sum_m m\left[1-\tfrac{1}{6}\left(\tfrac{\hbar\Lambda}{3}\right)^2\right]e^{-\beta\epsilon m(1-(1/6)(\hbar\Lambda/3)^2\,m^2)}$$

Single plaquette partition function Z_1

a quick integration gives

$$Z_1 \simeq \left(\frac{1}{\beta\epsilon}\right)^2 \left[1 + \tfrac{1}{3}\left(\tfrac{\hbar\Lambda}{\beta\epsilon}\right)^2\right]$$

- For indistinguishable plaquettes

$$Z = \tfrac{1}{N!} Z_1^N$$

- Thermodynamic quantities - energy

$$U = -\partial_\beta \ln Z = \left(\tfrac{2N}{\beta}\right)\left[1 + \tfrac{1}{3}\left(\tfrac{\hbar\Lambda}{3}\right)^2\right]$$

entropy

$$S = \beta U - \ln Z \quad = \beta\epsilon \frac{A}{\gamma\hbar} - N \ln Z_1$$

$$\rightarrow \frac{A}{4\hbar} - N \ln Z_1$$

for local acceleration and Unruh temperature

Single plaquette partition function Z_1

So the first term is the expected BH entropy - what of the rest?

- A variety of approaches - fix Immirzi parameter, add chemical potential, include holographic dof, ...

For first approach ...

$$\left(\frac{4}{\gamma}\right)^2 \left[1 + \frac{1}{3}\left(\frac{4\hbar\Lambda}{\gamma}\right)^2\right] = 1$$

Depends on Λ ! Interpretation unclear

Or maybe renormalization? Microscopic effective Newton's constant could be renormalized relative to G_N perhaps like this

$$\frac{\gamma G}{G_N} = 1 - \frac{\gamma}{2}\ln\frac{\gamma}{4}\left[1 + \frac{1}{3}\left(\frac{4\hbar G\Lambda}{\gamma}\right)^2\right]$$

or with scale set by $\langle j \rangle$

On black holes and the cosmological constant

- Cosmological constant affects both the quasi-local energy and the statistical description of black holes

- Kottler (and Carter) spacetimes could have lessons for the larger goal constructing spacetime from fundamental blocks