Second law of the thermodynamics for field theories on null hypersurfaces

Antoine Rignon-Bret

Objectives

- Understand better why the Noether charge associated to the null Killing horizon is an entropy (Wald 93’, Iyer Wald 94’)

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- Can an analog of the second law of thermodynamics be written in general fields theories ?
- Deriving the Physical Process First Law (PPFL) up to second order terms from these general second laws

Noether theorem

Lagrangian : $L(\phi, \chi)$

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Dynamical fields

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Dynamical fields (ϕ)   Background fields (χ)

Noether theorem

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Dynamical fields — Background fields

Variation with respect to the diffeomorphism ξ →

$$\mathcal{L}_\xi A(\phi, \chi) = \frac{\delta A}{\delta \phi}\mathcal{L}_\xi \phi + \frac{\delta A}{\delta \chi}\mathcal{L}_\xi \chi$$

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$$\begin{cases} \delta L = \dfrac{\delta L}{\delta \phi}\delta\phi + d\Theta \\ \delta_\xi L = \dfrac{\delta L}{\delta \phi}\delta_\xi\phi + dI_\xi\Theta \end{cases}$$

with

$$\begin{cases} \Theta = p\delta q \\ I_\xi\Theta = p\delta_\xi q \end{cases}$$

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$$d(I_\xi\Theta - i_\xi L) = -\frac{\delta L}{\delta \phi}\mathcal{L}_\xi \phi - \frac{\delta L}{\delta \chi}\mathcal{L}_\xi \chi$$

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$\frac{\delta L}{\delta \chi}\mathcal{L}_\xi \chi$ = 0 if ξ is a symmetry of the environment

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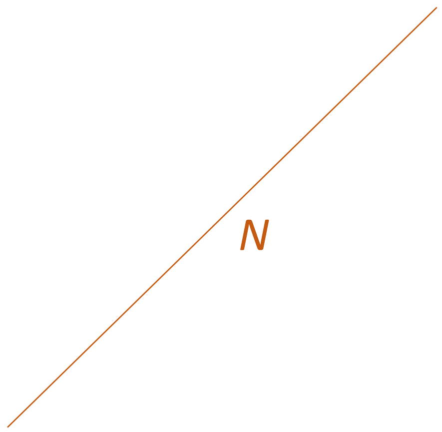
We can prove :

$$j_\xi = -\left(\frac{\delta L}{\delta \phi}\cdot A_\phi \cdot \xi\right)\cdot \varepsilon_M - \left(\frac{\delta L}{\delta \chi}\cdot A_\chi \cdot \xi\right)\cdot \varepsilon_M + dq_\xi$$

Current on a null hypersurface and entropy law



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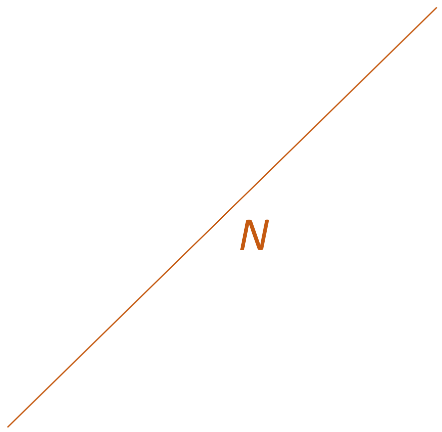

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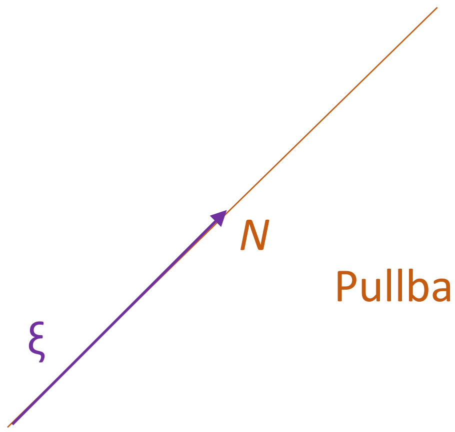

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Pullback on *N* :

$$dq_\xi = I_\xi \Theta + \left(\frac{\delta L}{\delta \chi} \cdot A_\chi \cdot \xi\right) \cdot \varepsilon_M$$

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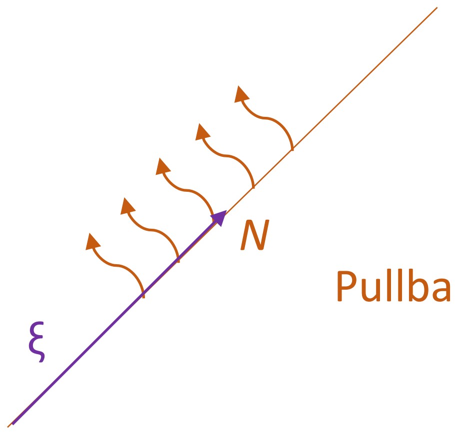

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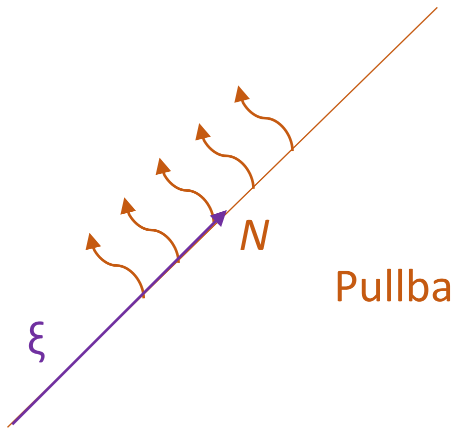

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$$\chi = g$$

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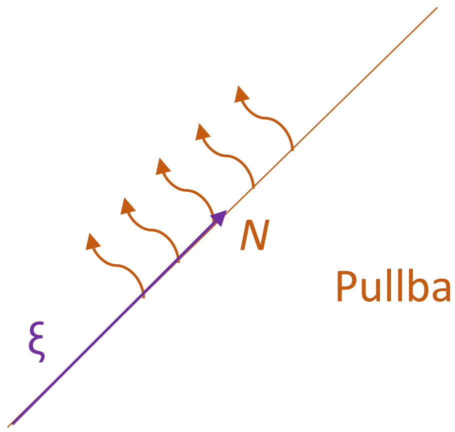

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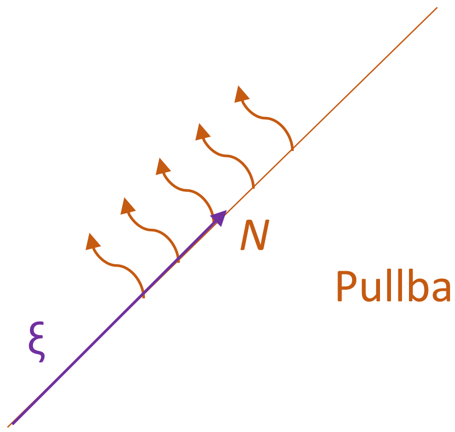
Current on a null hypersurface and entropy law



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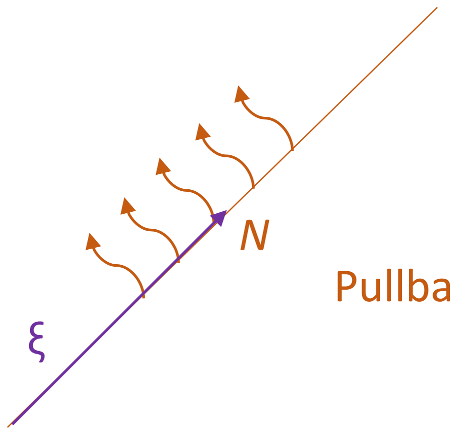
≥ 0 ← Null energy conditions

Current on a null hypersurface and entropy law



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Null energy conditions

$$dS = S_e + \underbrace{S_c}_{\geq 0}$$

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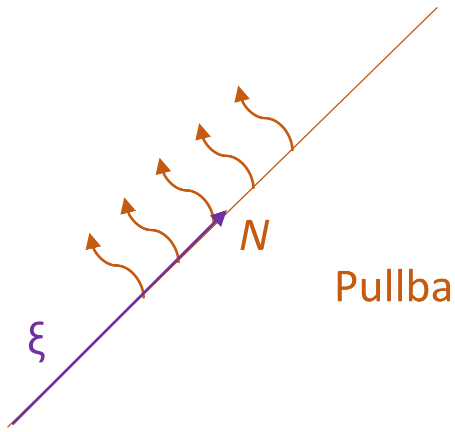

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Entropy creation terms come from the presence of background fields !

Gravity and PPFL

$$L = L^{EH}(g)$$

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From the previous analysis $\longrightarrow$

$$dq_\xi^{EH} = -I_\xi \Theta^{grav} - \left(\frac{\delta L^{EH}}{\delta g} \cdot A_g \cdot \xi\right) \cdot \varepsilon_M$$

Gravity and PPFL

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From the previous analysis →

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↓

$$dq_\xi = -I_\xi \Theta^{EH} + T_{\mu\nu}\xi^\mu n^\nu \varepsilon_N$$

With :

$$(q_\xi^{EH})_{\mu\nu} = -\frac{1}{16\pi}\varepsilon_{\mu\nu\rho\sigma}\nabla^\rho \xi^\sigma$$

$$q_\xi = q_\xi^{EH} - I_\xi \vartheta$$

$$I_\xi \Theta^{grav} = \int_N [(\sigma^{\mu\nu} - \frac{1}{2}\theta\gamma^{\mu\nu})\delta_\xi \gamma_{\mu\nu} + 2(\eta_\mu - \theta l_\mu)\delta_\xi n^\mu]\varepsilon_N - d\vartheta(\delta_\xi g_{\mu\nu})$$

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Linearized perturbation

$$
\begin{cases}
\phi = \varepsilon\tilde{\phi} \\
T = \varepsilon^2 \tilde{T} \\
g = \bar{g} + \varepsilon^2 \tilde{g}
\end{cases}
$$

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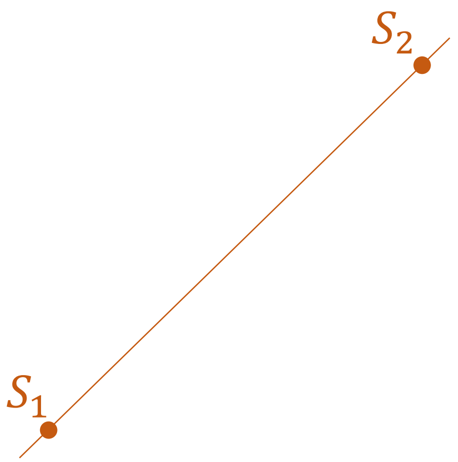

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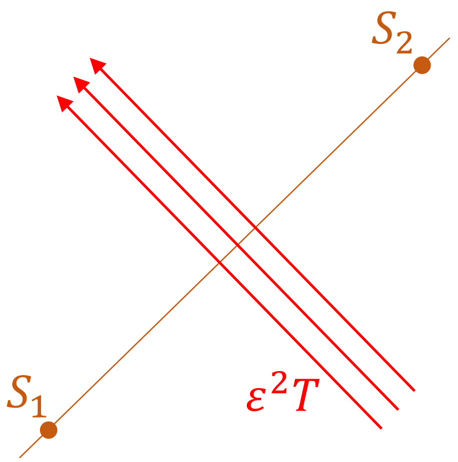

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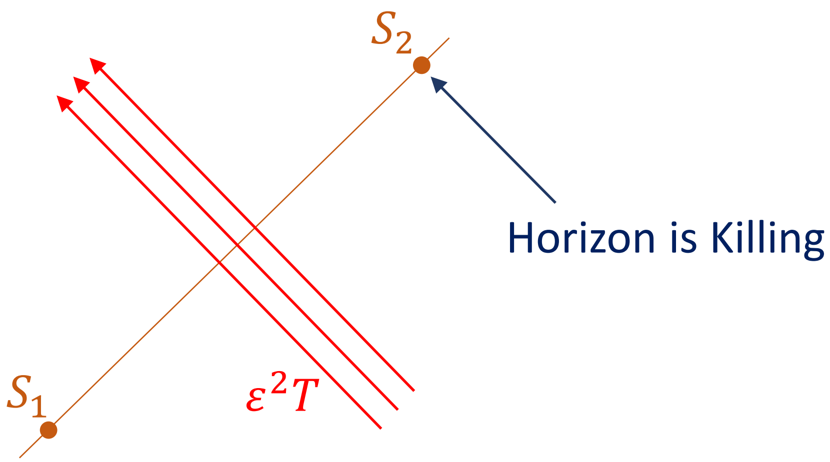
ξ is Killing



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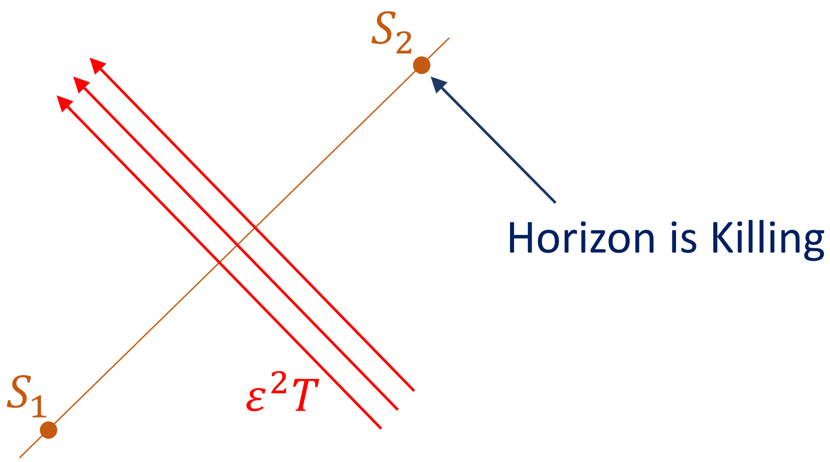
ξ is Killing (for $\bar{g}$)



$$dq_\xi = -I_\xi\Theta^{EH} + T_{\mu\nu}\xi^\mu n^\nu \varepsilon_N \quad \Longrightarrow \quad \frac{\bar{\kappa}}{2\pi}\Delta\left(\frac{A}{4}\right) = \varepsilon^2 T_{\mu\nu}\xi^\mu n^\nu + O(\varepsilon^4)$$