Consistent and non-consistent deformations of gravitational theories

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Motivation

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- Understanding field theories with $U(1)^n$ symmetries and their applications (see the relevant papers by Madhavan on the quantization of the Hamiltonian constraint and also recent work by Bakhoda and Thiemann on $U(1)^3$ models).

- Revive and expand an old proposal by Smolin to develop a perturbative quantization of gravity based on the self-dual action.

An old proposal by Smolin

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- **Introduce Newton's gravitational constant** G in the curvature and covariant exterior derivative:

$$F^{KL} = \mathrm{d}\omega^{KL} + G\omega^{K}{}_{M} \wedge \omega^{ML}$$
$$De^{I} = \mathrm{d}e^{I} + G\omega^{I}{}_{J} \wedge e^{J}$$

  and rewrite the **self-dual action** in the form

$$S(e,\omega) = \int_{\mathcal{M}} \epsilon_{IJKL} e^{I} \wedge e^{J} \wedge \left(\mathrm{d}\omega^{KL} + G\omega^{K}{}_{M} \wedge \omega^{ML}\right)$$

  with **two terms**: a **lowest order** (cubic) term and an **interaction term** proportional to G.
- The hope: the field theory described by the lowest order term can be dealt with (classically and/or quantum mechanically) and **the interaction term added as a perturbation**. This could happen, for instance, if the cubic term gives rise to an **integrable** model.

Comparing approaches

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- **Standard perturbative setting:**
  - A specific **non-dynamical** (background) **metric** is introduced.
  - **Its particular properties play a central role**. For instance, if the Minkowski metric is used, its Poincaré symmetry allows us to interpret the **free Lagrangian** as describing **massless spin-2 particles** (gravitons).
  - The Fierz-Pauli term **is not diff-invariant** (but it has symmetries). It gives linear field equations and can be readily quantized.
  - **General relativity is a consistent deformation of the Fierz-Pauli Lagrangian,** but it is **not renormalizable**.
- **Smolin's approach:**
  - **No auxiliary objects** are introduced.
  - **Both terms** in the action are **diff-invariant**.
  - The cubic part is an *abelianized* version of the full action.

Is the full action a consistent deformation of the abelianized one?

Consistent deformations

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(Henneaux, Barnich, Brandt and collaborators)

- An action of the type $S = S_0 + gS_1 + g^2 S_2 + \cdots$ where g is a coupling constant is a consistent deformation of S_0 if:
  - The **number of d.o.f.** of S_0 and S coincide.
  - The **transformations** corresponding to the gauge symmetry of S are those of S_0 with (posibly) extra terms that vanish when $g \to 0$.
  - The **gauge algebra** of S resuces to that of S_0 when $g \to 0$.
- A familiar example: **Yang-Mills is a consistent deformation of the sum of several copies of the Maxwell Lagrangian.**

The problem

- Given a **gravitational action** written in terms of **tetrad/connection variables** (so that it has an "internal" symmetry group) we take its "internal abelianization".
- Check whether the original theory is **a consistent deformation** of the internally abelianized version.
- **First step:** Counting degrees of freedom (if the S_0 part of the action gives solvable field equations **covariant methods** are the most effective).
- **Second step:** Find the **gauge symmetries of the deformed action** or show that they do not exist.

Deformations of gauge symmetries back

- We define the **Euler forms** according to $\mathbf{d}S = \int_M E_{\varphi a} \wedge \mathbf{d}\varphi^a$ (φ_a are dynamical forms labeled by a and $\mathbf{d}S$ the variation of the action).
- **Gauge invariant directions** (vector fields) $\rightsquigarrow$ kernel of $\mathbf{d}S$.
- **The action:** $S = S^0 + gS^1$ (g coupling constant).
- **Deformed gauge symmetries** $\mathbb{Z}_\epsilon = \mathbb{Z}^0_\epsilon + g\mathbb{Z}^1_\epsilon$.
- Up to first order we have

$$0 = \mathbf{d}S(\mathbb{Z}_\epsilon) = \int_M \Big(\underbrace{E^0_{\varphi a} \wedge Z^{0a}_\epsilon}_{\mathbf{d}S^0(\mathbb{Z}^0_\epsilon)} + g\big(\underbrace{E^1_{\varphi a} \wedge Z^{0a}_\epsilon}_{\mathbf{d}S^1(\mathbb{Z}^0_\epsilon)} + \underbrace{E^0_{\varphi a} \wedge Z^{1a}_\epsilon}_{\mathbf{d}S^0(\mathbb{Z}^1_\epsilon)} \big) + O(g^2) \Big)$$

- To zero order in g this always holds. If we can find $\mathbb{Z}^1_\epsilon$ in such a way that it also holds to first order in g, then we have found **consistent deformations** of the gauge symmetry.
  - Notice that for φ_a such that $E^0_{\varphi a} = 0$ we must have $\mathbf{d}S^1(\mathbb{Z}^0_\epsilon) = 0$. As a consequence it is possible to write $\mathbf{d}S^1(\mathbb{Z}^0_\epsilon)$ as $\int_M E^0_{\varphi a} \wedge \mathrm{something}$.
  - Read off $\mathbb{Z}^1_\epsilon$ from $g \int_M \left(E^1_{\varphi a} \wedge Z^{0a}_\epsilon + E^0_{\varphi a} \wedge Z^{1a}_\epsilon \right) = 0$.

Deformations of gravitational theories back

- **We have considered several models in 2+1 and 3+1 dimensions:**
  - **2+1 gravity** and generalizations.
  - The **Husain-Kuchař** model.
  - The **Cartan-Palatini** action for GR.
  - The **Holst action**.
  - The **self-dual action**.
- In all the cases **the main goal is to introduce the abelianized model and check if the full actions are consistent deformations of it.**
- Another interesting (and hard) problem is to obtain **all the possible consistent deformations** of a given action. I will not discuss it here.

Results in 2+1 dimensions

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The Chern-Simons action generates many interesting models.

- **Theorem (Brandt, Barnich and Henneaux):** *The consistent deformations of abelian Chern-Simons actions are Chern-Simons actions based on arbitrary groups of the same dimension.*
- The 2+1 Palatini action and the 2+1 Husain-Kuchař model are consistent deformations of their Abelianized versions.

Results in 3+1 dimensions

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- The Husain-Kuchař model is a **consistent deformation** of its Abelianized version.
  - Integrability of the Abelian model?
  - Can this be used to understand the full model? (3-geometries...).
  - Can the Abelian model be used as a first step towards its perturbative quantization?
- The Cartan-Palatini and the Holst actions **are not** consistent deformations of their abelianized counterparts (which only describe **topological degrees of freedom**).
- On the other hand, the Euclidean or complex **self-dual** actions **are consistent deformations** of their abelianized versions.
- **In 3+1 dimensions some gravitational actions are consistent deformations of their abelianized versions but others are not.**

Some interesting questions

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1. Is there a **real** analogue of the selfdual action for **Lorentzian** signatures which is a **consistent deformation** of its abelianization?
2. If so, **is the abelianized model integrable**?
3. Is it possible to build a perturbative approach taking advantage of it, both at the classical and the quantum levels?

Thanks!

References

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